



DEPARTMENT OF MATHEMATICS
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS
MA211TB

UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-1

I. Objective type questions:

- If A is a 3×4 matrix then rank of A cannot exceed _____.
- Rank of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is _____.
 $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
- Rank of identity matrix of order 4 is _____.
- If the rank of the transpose matrix A is 3 then the rank of matrix A is _____.
- Rank of singular matrix of order 5 is _____.

II. Find the rank of the following matrices

- $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 1 & 2 \\ 3 & 4 & 5 & 8 \\ 1 & 3 & 10 & 14 \\ 1 & 1 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 7 & 3 \end{bmatrix}$
Answer: rank of A=2
- $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 4 & 5 & 8 \\ 1 & 3 & 10 & 14 \\ 1 & 1 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 7 & 3 \end{bmatrix}$
Answer: rank of A=2
- $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 4 & 5 & 8 \\ 1 & 3 & 10 & 14 \\ 1 & 1 & 1 & 3 \end{bmatrix}$
 $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 7 & 3 \end{bmatrix}$
Answer: rank of A=4

4) Find the values of k such that the rank of the matrix A is 3, where

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{Answer: } k=1$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & k \end{bmatrix}$$

5) For which value of b the rank of the matrix

$$\begin{bmatrix} 1 & 5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} \text{ is } 2 \times 2$$

b

$$\begin{bmatrix} 1 & 3 \\ 0 & 10 \end{bmatrix}$$

$$b=2.$$

6) Find the rank of A, B, A+B, BA and AB if

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 1 \\ 5 & 10 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Answer: rank of A=2, rank of B=1, rank of (A+B)=2, rank of (AB)=0, rank of (BA)=1.



UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-2

1. Test the consistency of the following system of equations

$$2x+6y=-11$$

$$6x+20y-6z=-3$$

$$6y-18z=-1$$

Answer: Inconsistent

2. Test the consistency of the following system and solve if the system is consistent

$$x_1 + 2x_2 + x_3 = 2$$

$$3x_1 + x_2 - 2x_3 = 1$$

$$4x_1 - 3x_2 - x_3 = 3$$

$$2x_1 + 4x_2 + 2x_3 = 4$$

Answer: Consistent. $x_1 = 1, x_2 = 0, x_3 = 1$.

3. Find the value of k such that the following system of equations possesses a non-trivial solution.

Also find the solution of the system

$$4x_1 + 9x_2 + x_3 = 0$$

$$x_1 + 3x_2 + x_3 = 0$$

$$x_1 + 4x_2 + 2x_3 = 0$$

Answer: $x = 1, y = 2, z = -2, w = 5$.

4. Investigate the values of x and y so that the equations

$$2x+3y+5z=9$$

$$7x+3y-2z=8$$

$$2x+3y+yz=5$$

have (a) Unique solution (b) Infinite number of Solutions (c) No Solution

Answer: (a) $x \neq 5$ (b) $x = 5, y = 9$ (c) $x = 5, y \neq 9$

5. Solve the system of equations by Gauss elimination method

$$x-2y+3z=2$$

$$3x-y+4z=4$$

$$2x+y-2z=5$$

Answer: $x = \frac{11}{5}, y = -\frac{7}{5}, z = -1$

6. Solve the system of equations by Gauss elimination method

$$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$$

$$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$$

$$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$$

$$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$$

Answer: $x_1 = 3, x_2 = 1, x_3 = -2, x_4 = 1$.



1. Solve the following system of equations by Gauss –Jordan method

$$2x+y+z=10$$

$$3x+2y+3z=18$$

$$x+4y+9z=16$$

Answer: $x=7, y=-9, z=5$.

2. Find the inverse of a matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ -2 & 4 & 9 \\ 5 & 5 & 5 \end{bmatrix}$ using Gauss-Jordan method.

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$$\begin{bmatrix} 1 & 6 & 5 \\ -1 & 5 & 5 \end{bmatrix} \quad 3$$

Answer: A^{-1}

$$-^4_5 - ^{14}_5$$

3. Solve the system of equations by Gauss elimination method

$$9x+2y+4z=20$$

$$x+10y+4z=6$$

$$2x-4y+10z=-15$$

Answer: $x=2.7372, y=0.9872, z=-1.6525$

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DEPARTMENT OF MATHEMATICS

**FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS
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UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-4

1. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \end{bmatrix}$.

Answer: $\lambda_1 = 1$, $\lambda_2 = -1$, $\lambda_3 = 2$
 $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$, $\begin{bmatrix} 5 & -2 & 0 \\ 10 & -4 & -6 \end{bmatrix}$

2. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} -2 & 6 & 2 \end{bmatrix}$.

Answer: $\lambda_1 = 3$, $\lambda_2 = 6$, $\lambda_3 = 9$
 $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $\begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$

1

$$\begin{bmatrix} -2 & -2 & 0 & 2 & 7 \end{bmatrix}$$

3. The sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$

Answer: 0 and 8.

5. If $A = \begin{bmatrix} 2 & 1 \\ 4 & -2 \end{bmatrix}$ are

4. If two eigenvalues of A

$\lambda_1 = 3$ and $\lambda_2 = 15$, then the third eigenvalue is

Answer: 0

$\lambda_1 = 2$, then the eigenvalues of A^{-1} are

Answer: 1 and $\frac{1}{3}$.

1. Find the largest eigenvalue and the corresponding eigenvector of the matrix

$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by Rayleigh power method. (Perform 5 iterations)

Answer: $\lambda_{\max}^{(4)} = 6.941$

$$\begin{bmatrix} 0 \\ 0.039 \\ 1 \end{bmatrix}$$

2. Find the largest eigenvalue and the corresponding eigenvector of the matrix

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh power method taking initial eigenvector as

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$. (Perform 5 iterations)

Answer: $\lambda_{\max}^{(4)} = 6.941$

$$\begin{bmatrix} 0 \\ 0.039 \\ 1 \end{bmatrix}$$

$$\frac{1}{0.0391}$$