FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND STATISTICS (MAT211CT)

Multivariable Functions and Partial Differentiation

TUTORIAL SHEET-1

1. 1. If $x = r\cos\theta, y = r\sin\theta$, then $\left(\frac{\partial y}{\partial \theta}\right)^2 + \left(\frac{\partial x}{\partial \theta}\right)^2 =$ ________ **Ans**: r
2. If $z = x\sin y + y\sin x$, then $\frac{\partial^2 z}{\partial x \partial y} =$ ______________________ **Ans**: $\cos y + \cos x$
3. If $z = e^{2x^2+xy}$, then $\frac{\partial z}{\partial y} =$ ______________________ **Ans**: xe^{2x^2+xy}
4. The steady state temperature of a metal sheet is $T(x, y) = x^2 - a^2y^2$. The values of 'a' for which $T(x, y)$ satisfies the Laplace equation $T_{xx} + T_{yy} = 0$ are ___________. **Ans**: $\pm a$
5. If $u = y\cos(xy)$ then $\frac{\partial u}{\partial y}$ at the point $(1, \pi)$ is _______________. **Ans**: -1
6. If $V = f(x - ct) + g(x + ct)$ where f and g are arbitrary functions of $x - ct$ and $x + ct$ respectively and c is a constant, then show that $\frac{\partial^2 V}{\partial t^2} = c^2 \frac{\partial^2 V}{\partial x^2}$
7. If $u = \frac{x+y}{x-y}$,verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
8. If $u = ae^{-gx}\sin(nt - gx)$, where a, g and n are positive constants and $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$, show that $g = \sqrt{\frac{n}{2\mu}}$.
9. If V is the volume and S is the total surface area of rectangular box of length x, breadth y and height z, find
   (i) the rate of change of V with respect to x if y =4 and z= 12,
   (ii) the rate of change of S with respect to z if x = 3 and y = 4.
   Ans: (i) 48 (ii)14
10. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.

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TUTORIAL SHEET-2

1. Given $z = xy^2 + x^3y$ where x and y are functions of t with $x(1) = 1$, $y(1) = 2$, $x'(1) = 3$ and $y'(1) = 4$. The value of $\frac{dz}{dt}$ at t = 1 is ________________. **Ans**: 16
2. For the implicit function $(\cos x)^y = (\sin y)^x$, $\frac{dy}{dx} =$ _______.

   Ans:$-\left[\frac{y(\cos x)^{y-1}(-\sin x)-(\sin y)^x \log \sin y}{(\cos x)^y \log(\cos x)-x(\sin y)^{x-1}\cos y}\right]$
3. Given 't' represents time and $u = x^2 - y^2, x = \frac{1}{t}, y = e^t$ then the rate of change of u with respect to 't' is ____________________ . **Ans**: $\frac{-2}{t^3} - 2e^{2t}$
4. For the implicit function $e^x - e^y = 2xy$, $\frac{dy}{dx} =$ ________________. **Ans**: $\left[\frac{e^x-2y}{e^y+2x}\right]$
5. Given, $x^2 + y^2 + 3xz = 1$ and $x + y = 1$, then $\frac{dz}{dx} =$ _______ **Ans**: $\frac{-(2x+3)}{3} + \frac{2y}{3x}$
6. If $z = z(x,y), x = e^u \sin v, y = e^v \cos v$, then $\frac{\partial z}{\partial u}$ __________. **Ans**: $\frac{\partial z}{\partial x} e^u \sin v$
7. If $u = xyz$ where $x = e^{-t}, y = e^{-t} \sin^2 t, z = \sin t$, then find $\frac{du}{dt}$.

   Ans: $e^{-t}\sin^2 t(3\cos t - 2\sin t)$
8. If $z = x^2 + 2xy + 4y^2$ and $y = e^{3x}$, find $\frac{dz}{dx}$. **Ans**: $2(x + e^{3x}) + 2(x + 4e^{3x})3e^{3x}$
9. If z is a function of x and y and if $x = e^u \sin v, y = e^u \cos v$, prove that

   (i) $\frac{\partial z}{\partial u} = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

   (ii) $\frac{\partial z}{\partial x} = e^{-u}\left(\sin v \frac{\partial z}{\partial u} + \cos v \frac{\partial z}{\partial v}\right)$
10. If $= f(x-y, y-z, z-x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
11. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, determine $\frac{\partial^2 u}{\partial x \partial y}$.

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TUTORIAL SHEET-3

1. Match the following:

i)	If $x = e^{u}\sin v, y = e^{v}\cos v$ then $J\left(\frac{x,y}{u,v}\right) =$ _______________ .	a)	$\frac{1}{4v\sin 2u}$
ii)	If (a,b) is a stationary point of f(x,y) and $f_{xx} = 3$, $f_{xy} = 2$ and $f_{yy} = 2$ at this point then the nature of (a,b) is____.	b)	minimum
iii)	The nature of the point (1, -1) to the function $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$ is __________________________ .	c)	$e^{u+v}(\sin v\cos v - \sin^2 v)$
		d)	$\frac{v\sin 2u}{4}$
iv)	If $\frac{\partial(x,y)}{\partial(u,v)} = v\sin 2u$ and $\frac{\partial(x,y)}{\partial(r,\theta)} = \frac{1}{4}$ then $\frac{\partial(u,v)}{\partial(r,\theta)} =$_______________.	e)	Neither maximum nor minimum
		f)	Saddle point
		g)	maximum
		h)	$e^{u+v}(\sin v\cos v - \sin^2 v) - e^{u}\cos v$

Ans: (i) - (c) (ii) –(g) (iii) – (e) (iv) – (a)

2. Find the extreme values of $\sin x + \sin y + \sin(x+y)$. **Ans:** Maximum value at $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$, maximum value is $\frac{3\sqrt{3}}{2}$
3. Find the volume of largest rectangular parallelepiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.Ans: Maximum volume $=\frac{8abc}{3\sqrt{3}}$
4. Find the maximum and minimum distances of the point (1, 2, 3) from the sphere $x^2 + y^2 + z^2 = 56$ using Lagrange's Method of undetermined multipliers. **Ans:** Minimum distance at (2, 4 , 6) = $\sqrt{14}$, Maximum distance at (-2,-4,-6), maximum distance = $\sqrt{126}$
5. Show that $u = \frac{x^2-y^2}{x^2+y^2}, v = \frac{2xy}{x^2+y^2}$ are functionally dependent and find the relation between them. Ans: $u^2 + v^2 = 1$.
6. For $u = xyz, v = yz + zx + xy, w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, Ans: $(y-z)(z-x)(x-y)$.
7. If $x = e^{v}\sec u, y = e^{v}\tan u$, then verify that $J' = 1$.