R V COLLEGE OF ENGINEERING

(An autonomous institution affiliated to VTU, Belgaum)

DEPARTMENT OF MATHEMATICS

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS (MA211TB)

UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-1

I. Objective type questions:

1. If A is a 3×4 matrix then rank of A cannot exceed ____________.
2. Rank of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is __________.
3. Rank of identity matrix of order 4 is ___________ .
4. If the rank of the transpose matrix A is 3 then the rank of matrix A is _________.
5. Rank of singular matrix of order 5 is_________________.

II. Find the rank of the following matrices

1) $A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ **Answer: rank of A=2**

2) $A=\begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 3 \end{bmatrix}$ **Answer: rank of A=2**

3) $A=\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 2 & 0 & 4 & -2 \\ 12 & -3 & 9 & -3 \end{bmatrix}$ **Answer: rank of A=4**

4) Find the values of k such that the rank of the matrix A is 3, where

$$A=\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & k \end{bmatrix}$$

Answer: k=1

5) For which value of b the rank of the matrix

$$A=\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix} \text{ is } 2$$

b=2.

6) Find the rank of A, B, A+B, BA and AB if

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

Answer: rank of A=2, rank of B=1, rank of (A+B)=2, rank of (AB)=0, rank of (BA)=1.

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TUTORIAL SHEET-2

1. Test the consistency of the following system of equations
   2x+6y=-11
   6x+20y-6z=-3
   6y-18z=-1
   Answer: Inconsistent

2. Test the consistency of the following system and solve if the system is consistent

$x_1 + 2x_2 + x_3 = 2$
$3x_1 + x_2 - 2x_3 = 1$
$4x_1 - 3x_2 - x_3 = 3$
$2x_1 + 4x_2 + 2x_3 = 4$

   Answer: Consistent. $x_1 = 1,\ x_2 = 0, x_3 = 1.$

3. Find the value of k such that the following system of equations posses a non-trivial solution. Also find the solution of the system

$4x_1 + 9x_2 + x_3 = 0$
$kx_1 + 3x_2 + kx_3 = 0$
$x_1 + 4x_2 + 2x_3 = 0$

   Answer: $k = 1,\ x_1 = 2k, y =- k, z = k.$

4. Investigate the values of λ and μ so that the equations
   2x+3y+5z=9
   7x+3y-2z=8
   2x+3y+ λ z= μ
   have (a) Unique solution (b) Infinite number of Solutions (c) No Solution
   Answer: (a) $\lambda \neq 5$ (b) $\lambda = 5,\ \mu = 9$ (c) $\lambda = 5,\ \mu \neq 9$

5. Solve the system of equations by Gauss elimination method
   x-2y+3z=2
   3x-y+4z=4
   2x+y-2z=5
   Answer: $x = \frac{11}{5}, y =- \frac{7}{5}, z =- 1$

6. Solve the system of equations by Gauss elimination method

$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$

$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$

$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$

$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$

Answer: $x_1 = 3,\ x_2 = 1, x_3 = -2, x_4 = 1.$

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UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-3

1. Solve the following system of equations by Gauss –Jordon method
   2x+y+z=10
   3x+2y+3z=18
   x+4y+9z=16
   Answer: x=7, y=-9, z=5.
2. Find the inverse of a matrix A=[2 3 4 4 3 1 1 2 4] using Gauss-Jordan method.

   Answer: $A^{-1} = \left[- \ 2 \ \frac{4}{5} \ \frac{9}{5} \ 3 \ - \frac{4}{5} \ - \frac{14}{5} \ - 1 \ \frac{1}{5} \ \frac{6}{5} \right]$

3. Solve the system of equations by Gauss elimination method
   9x+2y+4z=20
   x+10y+4z=6
   2x-4y+10z=-15
   Answer: x=2.7372, y=0.9872, z=-1.6525

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS (MA211TB)

UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-4

1. Find the eigenvalues and eigenvectors of the matrix
   $A = [11 \ -4 \ -7 \, 7 \ -2 \ -5 \, 10 \ -4 \ -6 \,]$.

Answer: $\lambda = 0, 1, 2$ and $X_1 = [1\ 1\ 1\,]$, $X_2 = [1 \ -1\ 2\,]$, $X_3 = [2\ 1\ 2\,]$

2. Find the eigenvalues and eigenvectors of the matrix
   $A = [5 \ -2\ 0 \ -2\ 6\ 2\ 0\ 2\ 7\,]$.

Answer: $\lambda = 3, 6, 9$ and $X_1 = [2\ 2 \ -1\,]$, $X_2 = [2 \ -1\ 2\,]$, $X_3 = [1 \ -2 \ -2\,]$

3. The sum and product of the eigenvalues of the matrix $A = [2 \ -3\ 4 \ -2\,]$ are

Answer: 0 and 8.

4. If two eigenvalues of $[8 \ -6\ 2 \ -6\ 7 \ -4\ 2 \ -4\ 3\,]$ are 3 and 15, then the third eigenvalue is __________.

Answer: 0

5. If $A = [2\ 1\ 1\ 2\,]$, then the eigenvalues of A^{-1} are

Answer: 1 and $\frac{1}{3}$.

1. Find the largest eigenvalue and the corresponding eigenvector of the matrix
   $A = [1 \ -3\ 2\ 4\ 4 \ -1\ 6\ 3\ 5\,]$ by Rayleigh power method. (Perform 5 iterations)

   Answer:$AX^{(4)} = 6.941[0.341\ 0.039\ 1\,]$

2. Find the largest eigenvalue and the corresponding eigenvector of the matrix

   $A = [6 \ -2\ 2 \ -2\ 3 \ -1\ 2 \ -1\ 3\,]$ by Rayleigh power method taking initial eigenvector as

   $[1\ 1\ 1\,]^T$. (Perform 5 iterations)

   Answer: $AX^{(4)} = 6.941[0.341\ 0.039\ 1\,]$