

Search for Particles with Anomalous Charge in the IceCube Detector

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- To bla bla bla

Abstract

Bla bla bla Ik zou graag ook mezelf willen bedanken Of dat je in het middelbaar leert dat je eigenlijk een bundel bent van elektronen en protonen en bla bla bla, maar van zodra je fysica studeert besef je dat je eigenlijk een bundel trillingen in velden bent.

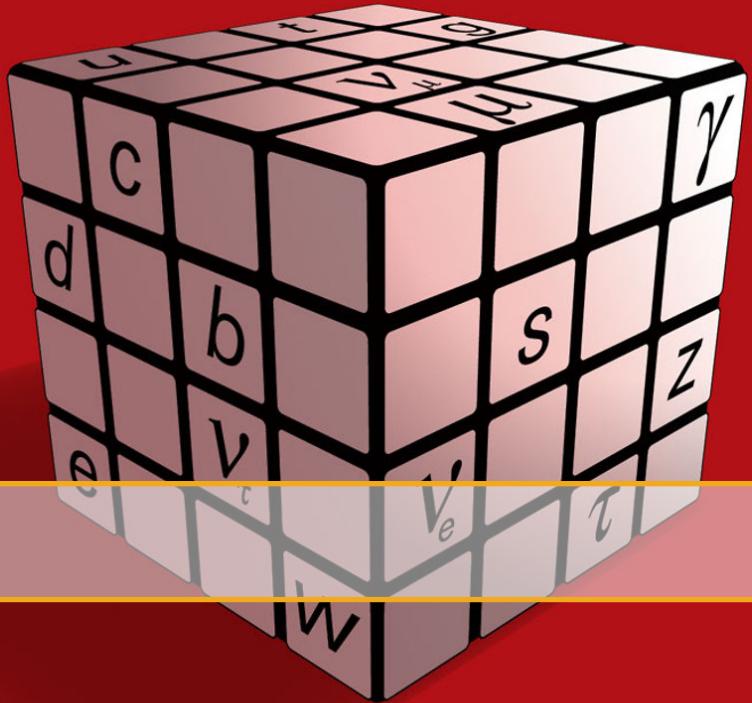


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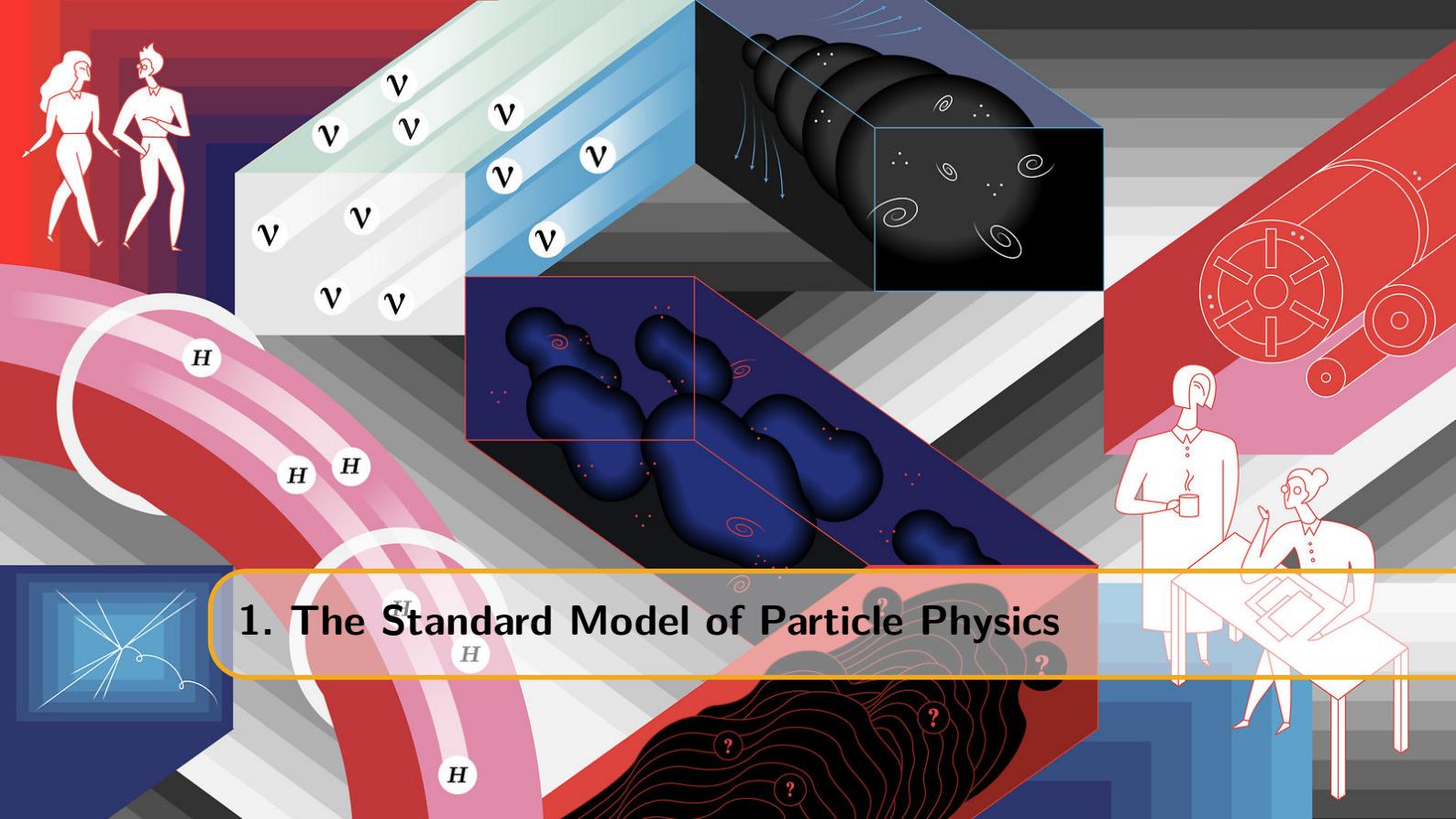
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Theory and Experiment

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1. The Standard Model of Particle Physics

The aim of this chapter is to give a summary of the theoretical framework that is used in particle physics. This framework was developed in stages throughout the latter half of the 20th century and is known as the Standard Model of particle physics. The Standard Model is a quantum field theory which is able to describe most of what is seen in particle physics experiments and proved to be successful in predicting later experimental discoveries. In this chapter a brief historical overview of the development of this theory will be given together with a limited description in order to familiarize the reader with concepts that will be used throughout this work. For a more in-depth and exhaustive discussion I refer to Refs. [1, 2, 3, 4]. [Sections bla bla bla](#)

1.1 What we call matter: fermions

Physics (from Ancient Greek: φυσική - *physikē*, “knowledge of nature”) is the natural science that studies matter. Matter is made up by *atoms* (from Greek: *atomos*, “indivisible”)* which can bind together into molecules which results into what is around us in our everyday life. Atoms are made up of a positively charged *nucleus* and surrounded by one or more *electrons* which are bound to the nucleus. The nucleus is made up of one or more *protons* and typically an approximately equal amount of *neutrons*. Because of their similar characteristics protons and neutrons are often referred to as *nucleons* and together they make up of more than 99.94% of an atom’s mass. Nucleons are made up of smaller particles called *quarks*[†] which are, as far as we know, *fundamental particles*. This means that we believe there are no smaller substructures making up these objects and are in essence mathematically best described as infinitely small. Because of this they are often referred to as pointlike. In the Standard Model (SM) these particles are characterized in our three-dimensional space as *fermions* which have odd half-integer spins obeying the laws of quantum mechanics. The spin of a particle is often illustrated with its classical counterpart in which an object is spinning and thus carries an intrinsic angular momentum. This analogy cannot be extrapolated to pointlike particles but the property happens to hold the same units as the classical orbital momentum. The spin of a particle seems to be

*Coined by ancient Greek philosophers Leucippus and his pupil Democritus who believed matter was made up of discrete units.

[†]The word “quark” originally appeared in the novel *Finnegans Wake* written by the Irish author James Joyce (1882–1941). The protagonist of the book dreams that he is serving beer to a drunken seagull. Instead of asking for “three quarts for Mister Mark” the inebriated bird says “three quarks for Muster Mark”. Murray Gell-Man had the habit of using names like “squeak” and “squark” for peculiar objects and after encountering the sentence in the book the name struck him as appropriate since the hypothetical came in threes.

just another property particles have, like charge or mass. Fermions follow Fermi-Dirac statistics and therefore obey the Pauli exclusion principle. As a consequence fermions cannot occupy the same place at the same time. (More formally, no two fermions may be described by the same quantum numbers.) This agrees with our macroscopic observations of matter in everyday life: people cannot walk through walls!

In total the SM distinguished 24 different fermions which can be subdivided into two distinct classes: *quarks* and *leptons*. There are six quarks (up, down, charm, strange, top and bottom), and six leptons (electron, electron neutrino, muon, muon neutrino, tau and tau neutrino), along with the corresponding antiparticle of each of these. A summary of the particles in the Standard Model is given in Figure 1.1.

1.1.1 Leptons

Leptons* can be subdivided into two classes: electromagnetically charged particles (e , μ and τ) and the neutral neutrinos (ν_e , ν_μ and ν_τ). Because of their charge, electrons are the well known particles combining into atoms together with nucleons. Being the lightest of the three, the electron is said to be part of the first *generation* together with the electron neutrino. Muons differ only from electrons in mass[†] and make up the second generation together with muon neutrinos. Similarly tau particles and tau neutrinos define the third generation. All leptons have a corresponding antiparticle indicated by a positive charge (i.e. e^+) or a bar (i.e. $\bar{\nu}_e$). Neutrinos[‡] are proven to have a very small mass **REFERENCE** and interact only using the weak force (Section **HE?**) making them inherently very hard to detect.

1.1.2 Quarks

The six quarks are called up, down, charm, strange, top and bottom quarks ((u, d), (c, s), (t, b)). One generation is made up of a particle with charge $1/3$ and one with $2/3$. This is again visualized in Figure 1.1. The difference between generations is essentially again the bare mass of the particles. Because quarks also interact through the strong force (see Section **????**) they combine into *hadrons*[§] of which the nucleons are best known. Because of their color charge and the intrinsic behaviour of the strong force quarks cannot be observed freely: they always combine into color neutral particles, a property called *confinement*. When a hadron, with it's constituent quarks, is pulled apart the attractive force between the quarks does not fall down rapidly because gluons carry color charge. When these particles are pulled apart far enough it becomes energetically more favorable to produce new quark-antiquark pairs which again combine into color neutral particles[¶]. The energy requirement for the production of new particles is far below that to separate the quarks far enough from each other to observe them separately.

Antiparticles are again denominated with a bar (i.e. \bar{u}). Because of their ability to interact with the strong force; particle accelerators in the 19???'s led to the discovery of a plethora of possible combinations. Something which is often referred to as the "particle zoo".

1.2 How particles communicate: interactions

There are four fundamental interactions known to exist: gravity and electromagnetism, which produce significant long-range forces, and the strong and weak force which only express themselves

*λεπτός (leptos) meaning thin, delicate, lightweight, or small. These particles don't need to bind to each other, which keeps them "thin" in a certain sense. Originally leptons were considered the "light" particles and hadrons the "heavy" particles, but the discovery of the tau lepton in 1975 broke that rule

[†]This characteristic of often referred to as *lepton universality* [??]

[‡]The name is a wordplay. The Italian word for neutron (neutrono) sounds like the word neutral (neutro) with an augmentative suffix (-one) tacked on the end. That is, it sounds something like "big neutral" to Italian ears. Replace the augmentative suffix -one with the diminutive suffix -ino and you have a "little neutral", which is a good description of what a neutrino is — a diminutive neutral particle.

[§]αδρός (adros) meaning thick, robust, massive, or large. This name alludes to the ability of the point-like quarks to bind together and form particles that are "thick" in a certain sense.

[¶]This process is called *hadronization* and results into the production of "jets" in particle accelerators [5]

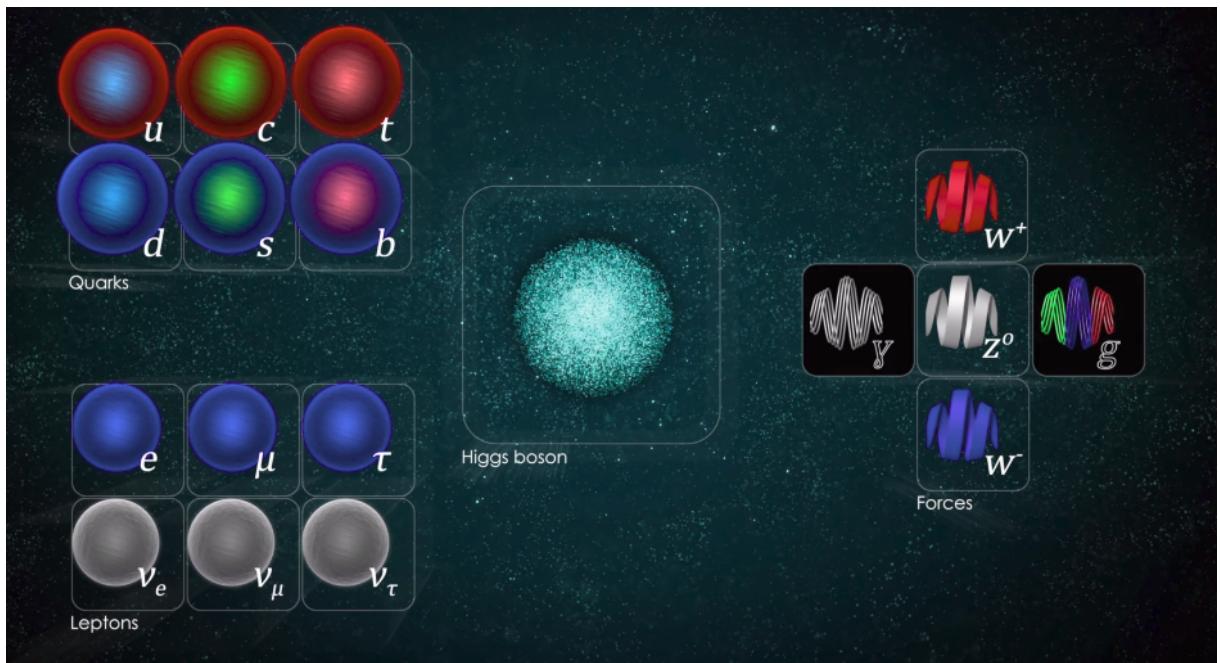


Figure 1.1: info from: <https://home.cern/about/physics/standard-model>

Interaction	Gravitation	Weak	Electromagnetic	Strong	
		Electroweak		Fundamental	Residual
Acts on:	Mass/Energy	Flavor	Electric charge	Color charge	Atomic nuclei
Experiencing	All particles	All fermions	Electrically charged	Quarks, Gluons	Hadrons
Mediation:	Not yet observed	W^+, W^-, Z	γ	Gluons	π, ρ, ω mesons
Strength	10^{-35}	10^{-11}	1	10^2	/
Long-distance	$\frac{1}{r^2}$	$\frac{1}{r} e^{-m_{W,Z} \cdot r}$	$\frac{1}{r^2}$		r^*

Table 1.1: Bla bla bla. Ref: <https://web.archive.org/web/20160304133522/> <https://www.pha.jhu.edu/> dfehling A nice webpage explaining... :<https://profmattstrassler.com/articles-and-posts/particle-physics-basics/the-known-forces-of-nature/the-strength-of-the-known-forces/> <http://web.mit.edu/sahughes/www/8.022/lec01.pdf>

at (sub)atomic distances and govern nuclear interactions. These are explained in more detail below and an overview is given in Table 1.2.

Particles interact with each other through the exchange of *gauge bosons* or *force carriers*^{||}. Gauge bosons are bundles of energy (*quanta*) and can be seen as excitations of one of the force fields. Fields are a mathematical approach used by physicists to describe what we observe in experiments.

Although the use of fields is very natural, the concept might feel a bit unfamiliar. In the following the known forces are described in more detail. Although it plays less of a role in subatomic physics, the theory of gravity is added for completeness but is mainly used to explain the concept of fields more in depth.

Gravity

Gravity (from Latin/old French: *gravitas*/grave, “weighty, heavy”) is the phenomenon in which objects with mass are attracted to each other. Gravitation is famously described by the general theory[†] of general relativity proposed by Albert Einstein in 1915. Compared to the other forces

^{||}A classic way of looking at force carriers is imagining two people standing in a boat. The force carrier is a heavy ball which can be thrown from one person to the other. Doing so both persons will move in opposite direction.

[†]A scientific theory is an explanation of an aspect of the natural world that can be repeatedly tested, in accordance with the scientific method, using a predefined protocol of observation and experiment. Established scientific theories have withstood rigorous scrutiny and embody scientific knowledge. [REF WIKI?](#)

gravity is intrinsically very weak* and is not described in the SM (see Section???). Because of this, gravity is often left out in discussions of particle physics experiments. It can however be used to explain the concept of a field in a very natural way.

First published on July 5th 1686 in Newton's work *Philosophiae Naturalis Principia Mathematica* ("the Principia"), a very good description of gravitation was already well known centuries ago. The equation of the force exerted by two massive bodies takes the following form:

$$F = G \frac{m_1 m_2}{r^2} \quad (1.1)$$

where F is the gravitational force acting between two objects, m_1 and m_2 are the masses of the objects, r is the distance between the centers of their masses, and G is the gravitational constant.

Newton's law states that two massive bodies will exert a force onto one another proportional to their masses but inversely proportional to the square of the distance between them. Newton realized this would mean that at any given instant in time this would mean that all massive objects in the universe would know of every other object in the universe where it is located†. Because of this Newton himself believed his explanation could not be the final answer. The answer is fully described in Einstein's work **RELATIVITY** but was already explained by Pierre-Simon Laplace in 1783. Gravitation is the slope of a field that pervades space and because of this one only needs to know the value of the field in a local region to calculate the attractional force. Massive objects do not "feel" each other but distort space and time in such a way that objects are attracted "fall" towards each other. Field theory makes it able to treat the laws of physics as local instead of action at a distance.

To date it is not possible for gravity to be described in the framework of quantum field theory like the other fundamental forces in a compatible way with the theory of general relativity. The gauge bosons from such a quantum field theory for gravity are referred to as *gravitons*.

Electromagnetism

The electromagnetic field (from Ancient Greek: ἥλεκτρον *ēlektron*, "amber", and μαγνῆτις λίθος, *magnetislithos*, which means "Magnesian stone")‡ presents itself in the electrical and magnetical forces. In the late 1870's the publication of James Clerk Maxwell's *A Treatise on Electricity and Magnetism* showed that the interactions of negative and positive charges are mediated by one force. Particles carrying a quantity (charge) of one of these forces can attract each other or repel.

Similar to the theory of gravity, the electromagnetic field pervades all around us and the interaction of nuclei, which have a positive electric charge, and electrons makes up most of what is described in chemistry. The force carrier of electromagnetism is called a *photon*, or in other words: light.

Weak force

The weak force is one aspect of the overarching electroweak theory which combines electromagnetism and the weak force. As opposed to gravity and electromagnetism it only takes place at very small subatomic distances§. One well known phenomenon that is described by the weak force is *beta decay* in which free neutrons decay into protons and produce an extra electron and anti-electron neutrino. Another beautiful example of the weak force is the driving mechanism in the Sun's thermonuclear process which makes it shine. This process cannot be explained by chemical processes but with the fusion of hydrogen into deuterium. Two protons are squeezed together into a He atom which consists of a proton and a neutron. The conversion of the proton into a neutron can be explained by the weak force.

*Two magnets that fit in the palm of your hand can deliver a force which is of similar strength than what the whole of Earth exerts on a human body.

†Imagine the attraction of the Moon towards the Earth: how are both "communicating" to each other?

‡In 1641 Athanasius Kircher titled one of the chapters in his book Magnus [BLABLABLA]: "Elektromagnetismos i.e. On the Magnetism of amber, or electrical attractions and their causes" in which the ???

§The reason being that the force carriers are massive, see more info in section???

The force carriers of the weak force are the W^+ , W^- and Z bosons. Because of the mass of these particles the coupling of particles with the weak field is inversely proportional to the square of their mass the force seems to be very weak, hence it's name*.

The weak force also carries some peculiar properties which are unique in a number of respects:

- It is the only theory that violates parity symmetry and even does so maximally.
- It's force carriers are massive as opposed to all other force carriers.
- It is the only force capable of changing quarks from one family into a quark of another family.

Strong force

As indicated in Section 1.1 nuclei are made up of protons and neutrons. However, the forces described in this section up to now cannot explain how they can make up a stable combination. The positive/neutral electromagnetic charge of the protons/neutrons would even suggest the opposite. Protons and neutrons are made up by quarks which carry a quantity which is called *color charge*. Particles carrying a color charge participate in interactions of the strong force. Due to the principle of *self interaction* the strong force only manifests itself on very small scales[†]. When nucleons are squeezed together (either due to high temperatures or pressure) and come close enough the quarks that make up the nucleons interact and make up the binding energy between nucleons. The force carriers of the strong force are called *gluons* which carry a color charge themselves and are massless.

Aside from holding nucleons together the strong force is also responsible for around 99% of the mass of the nucleon mass. The binding energy (which includes the kinetic energy of the quarks and the energy of the gluon fields that bind the quarks together).

A note about bosons

As opposed to fermions, which obey Fermi-Dirac REF statistics and cannot occupy the same quantum state, bosons follow Bose-Einstein statistics[‡]. Bosons carry integer spins ($s = 0, 1, 2$, etc.), fermions carry half-integer spins ($s = 1/2, 3/2$, etc.). As a result, bosons have no problem occupying the same place at the same time. (More formally, two or more bosons may be described by the same quantum numbers.) As the particles that make up light and other forms of electromagnetic radiation, photons are the bosons we have the most direct experience with. In our everyday experience, we never see beams of light crash into one another. Photons can go through each other with no effect.

1.3 The Standard Model in theory

A lot from Mandel and Saw: QFT

The Standard Model is a *quantum field theory*, meaning its fundamental objects are fields of a quantum nature which are defined at all points in spacetime. These fields are

- fermion fields, ψ , which account for “matter particles”;
- electroweak boson fields, W^1, W^2, W^3 and B ;
- gluon field, G^a ; and
- Higgs field, ϕ

Quantum field theory treats particles as excited states of one of these underlying fields, so called *field quanta*. The difference between classical and quantum fields is that they are operator-valued. Classical fields can in principle take on distinct values at each point in space whereas a quantum field accommodates observations of quantum mechanics as:

- energies are quantized, meaning that only discrete energy values are possible,
- these discrete energy levels are equally spaced,

*Iets van Fermi? Dat hij dacht dacht het een eenpuntsinteractie was?

[†]As opposed to the weak force where the short distance is explained due to the mass of the force carriers.

[‡]The name boson originates from Paul Dirac REFCOMMENTED OUT who wanted to commemorate the contributions of Indian physicist Satyendra Nath Bose who, together with Albert Einstein, theorized the characteristics of elementary particles that follow the Bose-Einstein statistics.

- the lowest achievable energy is not equal to absolute zero, but has a zero-point energy* (REFERENCE)?.

REFERENCE: <https://arxiv.org/pdf/hep-ph/0609174.pdf>, chapter 2 The dynamics of the quantum state and the fundamental fields are determined by the Lagrangian density \mathcal{L} . Writing the time and space coordinates in the form $(t, \mathbf{x}) = (x^0, x^1, x^2, x^3) = x^\mu$ the equations of motion of these fields can be written as:

$$\frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathcal{L}}{\partial (\partial \phi / \partial x^\mu)} \right] - \frac{\partial \mathcal{L}}{\partial \phi} = 0, \quad (1.2)$$

which follow from the principle of least action REFERENCE. The lagrangian function depends on the fields and how these fields change in spacetime: $\mathcal{L}(\phi, \nabla\phi)$. Quantization of these fields can be obtained by interpreting the coordinates and momenta as Heisenberg operators, and subjecting these to canonical commutation relations REFRENENCE MANDEL AND SHAW.

Furthermore the Standard Model is a gauge theory in which the Lagrangian is invariant under certain Lie groups (referred to as the symmetry group or the gauge group of the theory) of local transformations. For quantized gauge groups the quanta of the gauge fields are referred to as *gauge bosons*. A gauge theory is a mathematical model that has a gauge freedom of some of the mathematical degrees of freedom are redundant. In other words: different mathematical expressions describe the exact same physical system and in that sense unphysical. An experiment could never uniquely determine their values, even in principle. If the phase of the wavefunction is changed by a different amount at each point in spacetime and the physics remains unchanged, the Lagrangian is said to follow a *local phase symmetry*.[†].

The Standard Model is defined by the local $SU(3) \times SU(2) \times U(1)$ gauge symmetry. Each factor gives rise to three fundamental forces:

1.3.1 **SU(3): quantum chromodynamics**

The quantum chromodynamics (QCD) sector defines the interactions between quarks and gluons. Since leptons do not carry colour charge they do not participate in this interaction. The Dirac Lagrangian of the quarks coupled to the gluon fields is given by

$$\mathcal{L}_{QCD} = \sum_{\psi} \bar{\psi}_i \left(i\gamma^\mu \left(\partial_\mu \delta_{ij} - ig_s G_\mu^a T_{ij}^a \right) - m_\psi \delta_{ij} \right) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}, \quad (1.3)$$

where

ψ_i is the Dirac spinor of the quark field, where $i = r, g, b$ represents the color charges,

γ^μ are the Dirac matrices REFERENCE,

G_μ^a is the 8-component ($a = 1, 2, \dots, 8$) $SU(3)$ gauge field,

T_{ij}^a are the 3×3 Gell-Mann matricesREF, generators of the $SU(3)$ color group,

$G_{\mu\nu}^a$ are the field strength tensors for the gluons,

g_s is the strong coupling constant.

1.3.2 **SU(2) × U(1): electroweak**

Definitie van electroweak charge, Y?

The electroweak sector is a Yang-Mills gauge theoryREF with the symmetry group $SU(2)_L \times U(1)$. The Lagrangian is given by

*This is in accordance with the well known Heisenberg uncertainty principle which states that because of the zero-point energy, the position and momentum of a particle are not fixed but have a small range of variance: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$.

[†]See appendix A

$$\begin{aligned}
\mathcal{L}_{EW} &= \sum_{\psi} \bar{\psi} \gamma^{\mu} \left(i\partial_{\mu} - g' \frac{1}{2} Y_W B_{\mu} - g \frac{1}{2} \vec{\tau}_L \vec{W}_{\mu} \right) \psi - \frac{1}{4} W_a^{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
&= \bar{Q}_i i D_{\mu} \gamma^{\mu} Q_i + \bar{u}_i i D_{\mu} \gamma^{\mu} u_i + \bar{d}_i i D_{\mu} \gamma^{\mu} d_i + \bar{L}_i (i D_{\mu} \gamma^{\mu}) L_i + \bar{e}_{R,i} (i D_{\mu} \gamma^{\mu}) e_{R,i} \\
&\quad - \frac{1}{4} W_a^{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu},
\end{aligned} \tag{1.4}$$

where

B_{μ} is the U(1) gauge field,

Y_W is the weak hypercharge - the generator of the U(1) group,

\vec{W}_{μ} is the 3-component SU(2) gauge field,

$\vec{\tau}_L$ are the Pauli matrices - infinitesimal generators of the SU(2) group - with subscript L to indicate that they only act on left-chiral fermions,

g' (weak isospin) and g (weak hypercharge) are the U(1) and SU(2) coupling constants respectively,

$W^{a\mu\nu}$ ($a = 1, 2, 3$) and $B^{\mu\nu}$ are the field strength tensors for the weak isospin and weak hypercharge fields

Q, u and d are the left-handed doublet, right-handed singlet up right-handed singlet down quark fields,

L and e are the left-handed doublet and right-handed singlet electron fields.

The field strengths are given by

$$\begin{aligned}
W_{\mu\nu}^a &= \partial_{\mu} W_{\nu}^a - \partial_{\nu} W_{\mu}^a + g \epsilon^{abc} W_{\mu}^b W_{\nu}^c, \\
B_{\mu\nu} &= \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu},
\end{aligned}$$

and the covariant derivative for the left- and right-handed leptons by,

$$\begin{aligned}
D_{\mu} L_L &= \left(\partial_{\mu} - i \frac{g}{2} \tau_a W_{\mu}^a - ig' Y B_{\mu} \right) L_L, \\
D_{\mu} e_R &= \left(\partial_{\mu} - i \frac{g'}{2} Y B_{\mu} \right) e_R,
\end{aligned}$$

where T_a

It is worth noting no terms are included for fermion masses. These would have the form of $m \bar{\psi} \psi$ but are forbidden as they would break the $SU(2)_L \times U(1)$ gauge invariance. Neither is it possible to add explicit mass terms for the U(1) and SU(2) gauge fields.

1.3.3 Brout-Englert-Higgs mechanism

To come to a viable description of the elementary particles one is required to introduce masses into a chiral theory. The masses of the W and Z bosons are explained by the use of the Brout-Englert-Higgs mechanism formulation. Introducing one or more scalar fields, the Higgs fields, which can acquire a vacuum expectation value, it is possible to spontaneously break a symmetry in the Lagrangian. We say that electroweak symmetry is broken down to electromagnetism. According to the Goldstone theorem REF, every spontaneously broken continuous symmetry results in a massless scalar particle, the Goldstone boson. Hence, the number of Goldstone bosons in a theory is equal to the number of broken generators of the symmetry group.

Since the electroweak theory after symmetry breaking should contain three massive gauge bosons (W^+, W^- and Z) the scalar fields of the Higgs fields should contain at least three degrees of freedom. The simplest approach to do this is by introducing a complex, scalar $SU(2)$ doublet Φ with positive hypercharge ($Y = \frac{1}{2}$),

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \tag{1.5}$$

Similar to the $SU(2)$ symmetry of the EW theory four new gauge particles are introduced: H^+, H^-, H^0 and H .

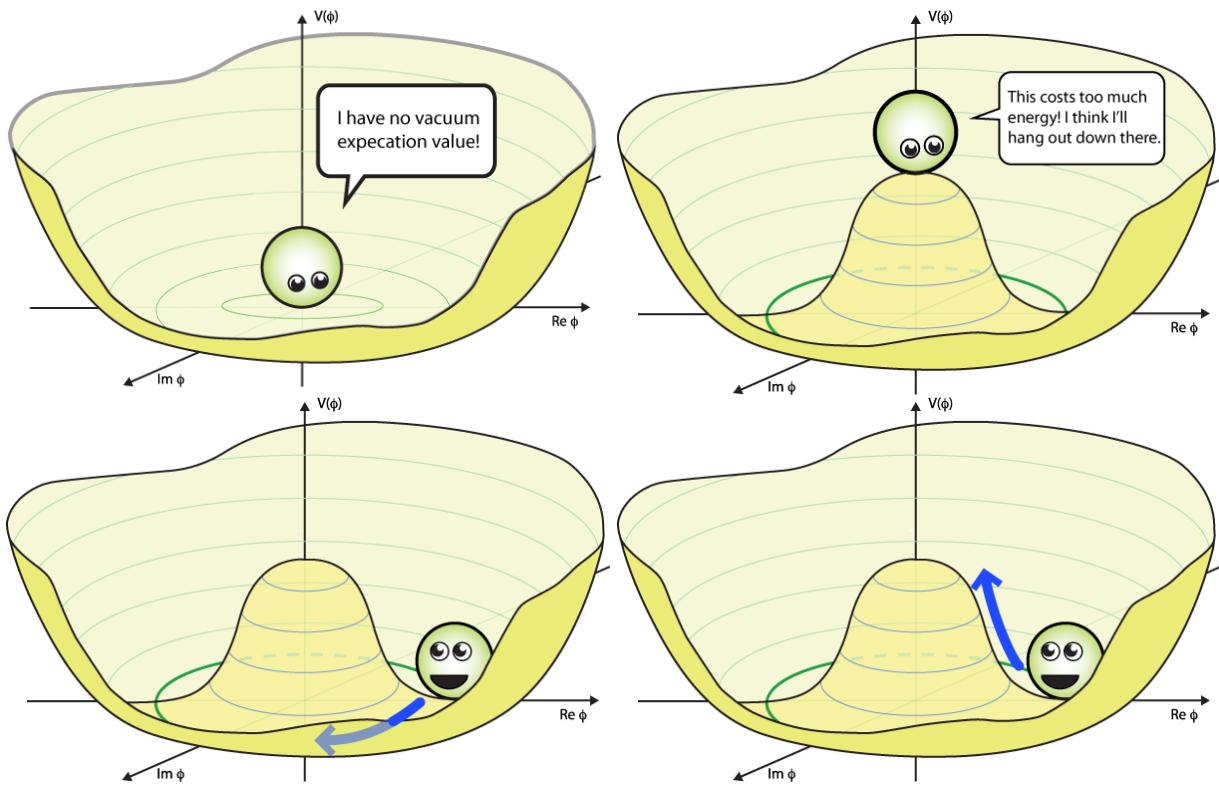


Figure 1.2: In this example the Higgs potential is illustrated in function of a complex scalar field (2D). The principal is the same for a complex scalar doublet but a lot harder to visualize. Top left: the Higgs potential with $\mu^2 > 0$, there is no vacuum expectation value. Top right: the scalar field will move to the lowest possible energy state. Bottom left: a flat direction in the potential corresponds to a massless Goldstone mode (remember there are two extra scalar fields meaning there are a total of 3). Bottom right: the concave shape of the potential near the minimum defines the Higgs boson mass.

Massive bosons

Having introduced this scalar doublet one needs to add the corresponding Lagrangian term to the electroweak Lagrangian from eq. 1.4,

$$\mathcal{L}_S = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi), \quad \text{with } V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \quad (1.6)$$

The first term is the kinetic term and the second corresponds to the potential of the scalar field*. The quartic term in the potential needs to be positive to ensure an absolute minimum in the Lagrangian. The quadratic term can either be positive or negative, depending on if $\mu^2 > 0$ or $\mu^2 < 0$. This is illustrated in Fig. 1.2. In the former case the scalar potential has an absolute minimum at the origin:

$$\langle 0 | \Phi | 0 \rangle \equiv \Phi_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (1.7)$$

From eq. 1.6 and 1.7 one can see that it the kinetic term does not give rise to massive particles†. In the case of $\mu^2 < 0$, the minimum is no longer located at the origin of the fields: $\partial_{|\Phi|} V(|\Phi|) = 0$

*The form of the potential is not known from first principles but is the simplest form that can explain the spontaneous symmetry breaking mechanism.

†Substitute $v = 0$ in eq. 1.13.

for $|\Phi| = \sqrt{-\frac{\mu^2}{2\lambda}}$, hence one possible solution is*

$$\langle 0 | \Phi | 0 \rangle \equiv \Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, v = \sqrt{-\frac{\mu^2}{\lambda}}. \quad (1.8)$$

v is referred to as the *vacuum expectation value* to reflect that the Higgs field is always “on”. To investigate the terms we can expand the field around the minimum:

$$\Phi(x) = \begin{pmatrix} \theta_2(x) + i\theta_1(x) \\ \frac{v+H(x)}{\sqrt{2}} - i\theta_3(x) \end{pmatrix} = e^{i\theta_a \tau_a} \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}. \quad (1.9)$$

Implementing this into eq. 1.6 would yield the existence of unphysical fields $\phi_{1,2,3}$ that give rise to three extra degrees of freedom that were not present in the original Lagrangian[†]. Since a change of variables cannot alter the number of d.o.f. of a system one can conclude that three fields do not represent physical fields. They can be removed by fixing a gauge, the unitary gauge, which breaks the original symmetry of the system!

$$\Phi(x) \rightarrow e^{-i\theta_a \tau_a} \Phi(x) = \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}, \quad (1.10)$$

where we have introduced a new scalar field $H(x)$. After inserting this in the kinetic part of the scalar Lagrangian (eq. 1.6), and redifining the gauge fields as

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W^1 \mp iW_\mu^2), \\ Z_\mu &= \frac{1}{g^2 + g'^2} (gW_\mu^3 - g'B_\mu), \\ A_\mu &= \frac{1}{g^2 + g'^2} (gW_\mu^3 + g'B_\mu), \end{aligned} \quad (1.11)$$

we find for the kinetic part of the scalar Lagrangian:

$$|D_\mu \Phi|^2 = \frac{1}{2} (\partial_\mu H)^2 + \frac{1}{2} g^2 (v + H)^2 W_\mu^+ W^{-\mu} + \frac{1}{8} (v + H)^2 (g^2 + g'^2) Z_\mu Z^\mu. \quad (1.12)$$

Since mass terms enter this equation in the general form of $M_W^2 W_\mu W^\mu$ for the W -bosons and $\frac{1}{2} M_Z^2 Z_\mu Z^\mu$ for the Z boson, the mass terms of the gauge bosons after spontaneous symmetry breaking can be written down as:

$$\begin{aligned} M_W &= \frac{1}{2} vg, \\ M_Z &= \frac{1}{2} v \sqrt{g^2 + g'^2}, \\ M_A &= 0, \end{aligned} \quad (1.13)$$

where it is clear that the photon remains massless[‡].

*It is not possible for the charged part of the fields to have a vev as this would not be in agreement with electromagnetism.

[†]In eq. 1.6 the vector fields are massless and each give rise to 2 d.o.f. The vev would make the three vector fields massive, thus adding 3 d.o.f. and introduce three unphysical fields.

[‡]Because the W and Z bosons are massive it costs energy to produce them and so the weak force is only really effective over a short distance. This is in contrast to the massless photons that result into a long range electromagnetic force. Thus the Higgs is responsible for the “weakness” of the weak force.

Corollary 1.3.1 — Eating the gauge bosons. A vev can give rise to massless Goldstone bosons. These particles correspond to the infinite possibilities of it's phase in the potential. However, this would also lead to extra degrees of freedom in the Lagrangian. Therefore one has to fix a gauge resulting in the disappearance of the bosons and making the vector bosons of the original fields massive.

Specifically, the two massless W^1, W^2 bosons in the electroweak theory (2×2^a polarizations = 4 d.o.f.) and two charged Higgses (2 d.o.f.) sum to a total of six degrees of freedom. In the broken theory, we have two massive W^+, W^- bosons (2×3 polarizations) which again total to six degrees of freedom.

Similarly, the W^3, B (2×2 d.o.f.) and H^0 (2 d.o.f.) combine into the neutral Z (massive, 3 d.o.f.), the γ (2 d.o.f.) and the scalar H (1 d.o.f.).

Similar to the nucleons having a mass which is much greater than the summed mass of it's constituents the Higgs fields gives rise to the mass of these gauge bosons. In the case of nucleons it is primarily the potential energy of the strong force that is responsible for the total mass following $E = mc^2$. The coupling of the gauge bosons to the Higgs field gives a sense of inertia to the particle. The particle does not float freely in vacuum but interacts with the ever present Higgs field (with a vev $\neq 0$) making it massive.

^aA massless particle cannot have a third polarization, similar to a photon: it is traveling at the speed of light so it cannot have a polarization in the direction of propagation, only longitudinal.

Using the potential term in eq. 1.6 together with the vev in eq. 1.10 we find for the Lagrangian of the Higgs boson:

$$\mathcal{L}_H = \frac{1}{2} (\partial_\mu H) (\partial^\mu H) - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4. \quad (1.14)$$

The first term again corresponds to the kinetic term whereas the third and forth refer to the three- and four-point self-interactions of the Higgs respectively. Scalar masses have the general form $\frac{1}{2}m\phi^2$; the Higgs boson mass is thus equal to

$$m_H = 2\lambda v^2 = -2\mu^2, \quad (1.15)$$

and needs to be determined experimentally.

Working through the interaction terms of the Lagrangian, one can show that the electric charge e is related to the couplings of the weak isospin g and hypercharge g' .

$$e = g \sin \theta_W = g' \cos \theta_W, \quad (1.16)$$

where the Weinberg angle is denoted as θ_W and indicates the magnitude of rotation of the boson planes after spontaneous symmetry breaking:

$$\begin{pmatrix} \gamma \\ Z \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}, \quad (1.17)$$

and is related to the weak isospin and hypercharge:

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \text{ and } \sin \theta_W = \frac{g'}{\sqrt{g'^2 + g^2}}. \quad (1.18)$$

references: <https://www.quantumdiaries.org/2011/11/21/why-do-we-expect-a-higgs-boson-part-i-electroweak-symmetry-breaking/> and <https://www.nikhef.nl/~ivov/HiggsLectureNote.pdf>

Massive fermions

A term like $-m\bar{\psi}\psi = -m [\bar{\phi}_L\psi_R + \bar{\psi}_R\phi_L]$, where we have decomposed the equation into the left- and right-handed chiral states[§] is not gauge in the Lagrangian. The left-handed fermions

[§] $\psi_L = P_L\psi = \frac{1-\gamma^5}{2}\psi$ and $\psi_R = P_R\psi = \frac{1+\gamma^5}{2}\psi$

from an isospin doublet and the right-handed fermions form isospin singlets. They transform differently under $SU(2)_L \times U(1)_Y$:

$$\begin{aligned}\chi_L &\rightarrow \chi'_L = \chi_L e^{i\tau_L^z \vec{W} + i\alpha Y} \\ \psi_R &\rightarrow \psi'_R = \psi_R e^{i\alpha Y}\end{aligned}. \quad (1.19)$$

It is possible for the fields to couple to the complex Higgs doublet defined in 1.5 by adding Yukawa couplings. This results into terms which are *singlets* under $SU(2)_L$ and $U(1)_Y$:

$$\mathcal{L}_{Yuk} = \lambda_e \overline{L}_L \Phi e_R - \lambda_d \overline{Q}_L \Phi d_R - \lambda_u \overline{Q}_L \tilde{\Phi} u_R + h.c. \quad (1.20)$$

where we have introduced the conjugate of Φ , $\tilde{\Phi} = i\tau_2 \Phi^*$ which has a negative hypercharge. After spontaneous symmetry breaking (eq. 1.10), we find:

$$\begin{aligned}L_{Yuk} &= -\frac{1}{\sqrt{2}} \lambda_e (\bar{\nu}_e, \bar{e}_L) \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} e_R + \dots \\ &= -\frac{1}{\sqrt{2}} \lambda_e (v + H(x)) \bar{e}_L e_R + \dots,\end{aligned} \quad (1.21)$$

where we highlighted only the electron part. Fermion mass terms have the general form $m_f \bar{f}_L f_R + h.c.$. Therefore one finds:

$$m_e = \frac{\lambda_e v}{\sqrt{2}}, \quad m_u = \frac{\lambda_u v}{\sqrt{2}}, \quad m_d = \frac{\lambda_d v}{\sqrt{2}}. \quad (1.22)$$

The mass of the fermions is again not predicted as the Yukawa coupling parameters are free parameters.

1.3.4 Particle mixing

In equation 1.20 we introduced Yukawa coupling constants to explain the mass of fermions. In its most general realizations these couplings are not constants but matrices. This will introduce possible mixing of *flavor? eigenstates* into different *mass eigenstates*. Let us write out the second term in equation 1.20

$$\begin{aligned}\lambda_d \overline{Q}_L \Phi d_R &= Y_{ij}^d Q_{Li}^I \bar{\Phi} d_{Rj}^I = Y_{ij}^d \overline{(\text{up-type down-type})}_{Li}^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} (\text{down-type})_{Rj}^I \\ &= \begin{pmatrix} Y_{11} \overline{(u \ d)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & Y_{12} \overline{(u \ d)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & Y_{13} \overline{(u \ d)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\ Y_{21} \overline{(c \ s)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & Y_{22} \overline{(c \ s)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & Y_{23} \overline{(c \ s)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\ Y_{31} \overline{(t \ b)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & Y_{32} \overline{(t \ b)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & Y_{33} \overline{(t \ b)}_L^I \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \end{pmatrix} \cdot \begin{pmatrix} d_R^I \\ s_R^I \\ b_R^I \end{pmatrix}\end{aligned} \quad (1.23)$$

where the superscript I implies that the fermion fields are expressed in the *interaction (flavor)* basis. The subscript i stands for the three generations.

This means that after symmetry breaking the quark mass terms break down into

$$\begin{aligned}-\mathcal{L}_{Yuk}^{\text{quarks}} &= Y_{ij}^d \overline{d}_{Li}^I \frac{v}{\sqrt{2}} d_{Rj}^I + Y_{ij}^u \overline{u}_{Li}^I \frac{v}{\sqrt{2}} u_{Rj}^I + \dots \\ &= M_{ij}^d \overline{d}_L^I d_{Rj}^I + M_{ij}^u \overline{u}_L^I u_{Rj}^I + \dots\end{aligned} \quad (1.24)$$

where we have omitted the hermitian conjugate terms and the Higgs field interaction terms. Note that the u - and d -terms in the equation still each represent the three up- and down-type quarks

repectively. There is mixing between the flavor fields as there is no reason why the matrix M should be diagonal*.

To obtain proper mass terms one has to diagonalize the mass matrices M^u and M^d and find proper eigenstates. We introduce unitary matrices V as follows

$$\begin{aligned} M_{diag}^d &= V_L^d M^d V_R^{d\dagger} \\ M_{diag}^u &= V_L^u M^u V_R^{u\dagger} \end{aligned} \quad (1.25)$$

which can be done when the matrices V are unitary ($V_L^{d,u\dagger} V_L^{d,u} = \mathbb{1}$). Equation 1.24 can now be expressed as follows:

$$\begin{aligned} -\mathcal{L}_{Y_{uk}}^{\text{quarks}} &= \overline{d_{Li}^I} M_{ij}^d d_{Rj}^I + \overline{u_{Li}^I} M_{ij}^u u_{Rj}^I + \dots \\ &= \overline{d_{Li}^I} V_L^{d\dagger} V_L^d M_{ij}^d V_R^{d\dagger} V_R^d d_{Rj}^I + \overline{u_{Li}^I} V_L^{u\dagger} V_L^u M_{ij}^u V_R^{u\dagger} V_R^u u_{Rj}^I + \dots \\ &= \overline{d_{Li}^I} \left(M_{ij}^d \right)_{diag} d_{Rj}^I + \overline{u_{Li}^I} \left(M_{ij}^u \right)_{diag} u_{Rj}^I + \dots \end{aligned} \quad (1.26)$$

where the V matrices have been absorbed in the quark flavor eigenstates and have formed mass eigenstates. These mass eigenstates, which are the eigenstates one sees in experiments, couple differently to the gauge fields of the weak interaction. Working out one term from equation 1.4 the mixing of the flavor eigenstates is clearly visible

$$\begin{aligned} \mathcal{L}_{kinetic}(Q_L) &= i \overline{Q_{Li}^I} \gamma_\mu D^\mu Q_{Li}^I \\ &= \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma_\mu W^{-\mu} d_{Li}^I + \frac{g}{\sqrt{2}} \overline{d_{Li}^I} \gamma_\mu W^+ \mu u_{Li}^I + \dots \\ &= \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \left(V_L^u V_L^{d\dagger} \right)_{ij} \gamma_\mu W^{-\mu} d_{Li}^I + \frac{g}{\sqrt{2}} \overline{d_{Li}^I} \left(V^d V^{u\dagger} \right)_{ij} \gamma_\mu W^+ \mu u_{Li}^I + \dots \end{aligned} \quad (1.27)$$

The combination of matrices $\left(V_L^d V_L^{u\dagger} \right)_{ij}$, a unitary 3×3 matrix is known as the *Cabibbo-Kobayashi-Maskawa (CKM)* mixing matrix. By convention, the interaction and flavor eigenstates of the up-type quarks are chosen to be equal. The down-type quarks are therefore chosen to be rotated:

$$\begin{aligned} u_i^I &= u_i \\ d_i^I &= V_{CKM} d_i \end{aligned} \quad (1.28)$$

or explicitly:

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (1.29)$$

Because the choice of the global phases of the quark fields is arbitrary and the matrix is unitary, the nine unknown complex elements can be reduced to three real numbers and one phase[†]. The matrix is most often written as:

$$V = \begin{pmatrix} c_{12}c_{13} & & s_{12}c_{13}s_{13}e^{-i\delta} & \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} - c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & & c_{23}c_{13}, & \end{pmatrix} \quad (1.30)$$

*The question and answer of flavor/mass mixing can be put as: “Why is there mixing? Because it can.”

[†]This phase is responsible for *CP-violation*.

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$ for $i < j = 1, 2, 3$ and δ is the CP -violating phase.

The mixing of the flavor quantum states is necessary to explain charged current interactions changing the strangeness with one [6] and CP -violating processes [7].

Without going into detail it is worth noting that a similar matrix exists that connects the lepton flavor and mass eigenstates. In contrast to the quarks, the down-type interaction doublet states are chosen to be the same as the mass eigenstates. The mixing of the mass and interaction eigenstates is in the neutrino sector. This matrix is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [8].

1.4 A success story

Over the course of multiple decades the Standard Model was built up into an extremely comprehensive theory. The first building blocks necessary for its construction came from experiments in the early 19th century when its quantum characteristics became more apparent. The theory was formulated as a gauge theory in the 1960s and 1970s and for decades it has been rigorously tested and checked, leading to extremely accurate experimental precision measurements which agree with the theory. Apart from precision measurements it has lead to predictions of particles and their interactions which could only be tested years or decades after they were first proposed. In the following I give a brief overview of some astonishing experimental results.

1.4.1 Wave-particle duality

One of the most striking features of the SM is that particles may be partly described in terms of not only particles, but also of waves. The classical concepts of “particles” or “waves” is insufficient in describing the behaviour of quantum-scale objects. In the late 18th and early 19th century physicist were puzzled by the successful approaches to well known problems of electromagnetic phenomena, which were widely accepted as fields, with quantizations (particles). Black body radiation and the photoelectric effect are described below. In QFT particles are defined as excited states of a field. The wave nature is evident in the calculated *probability distributions* for a given reaction.

1.4.1.1 Photoelectric effect

At the close of the 19th century J.J. Thomson found that electricity is caused by *particles* that can fly through vacuum. But, since electromagnetism was known to be a wave that is generated by a changing electric or magnetic *field*, a particle description of electricity and charge was, in a way, flawed by construction. A first step towards quantization came when the ultraviolet catastrophe seemed to be solved if black-body radiation is not explained by the classical equipartition theorem (Appendix B) but a quantization of the electromagnetic field: Planck’s law. Planck denounced these particles of light as a limitation of his approximation, not a property of reality. It was only later, when Einstein combined this theory with the photoelectric effect (which was not understood), that light quanta/photons became more accepted.

The photoelectric effect describes that when a material is shined upon by light, electrons or other free carriers can be emitted. In a classical theory perspective, an alteration in the intensity of light would induce changes in the kinetic energy of the particles emitted from the material. Instead, these particles are dislodged only by the impingement of photons when those photons reach or exceed a threshold frequency/energy. Changing the intensity of the light bundle only changes the amount of particles released, but does not alter its kinetic energy. This is easy to explain in the quantum view and resulted in the Einstein’s Nobel Prize in 1921.

1.4.1.2 Double-slit experiment

The double-slit experiment was first performed by Thomas Young in the early 19th century. If a beam of light is shined through two slits of a screen onto a second screen behind the first an interference pattern can be seen as illustrated in Fig. 1.3. This proved the wavelike nature of light.

In later years similar experiments for electrons were done to prove the wavelike properties of particles [11, 12]. The same double-slit experiment was finally performed in the 60s [13] and for

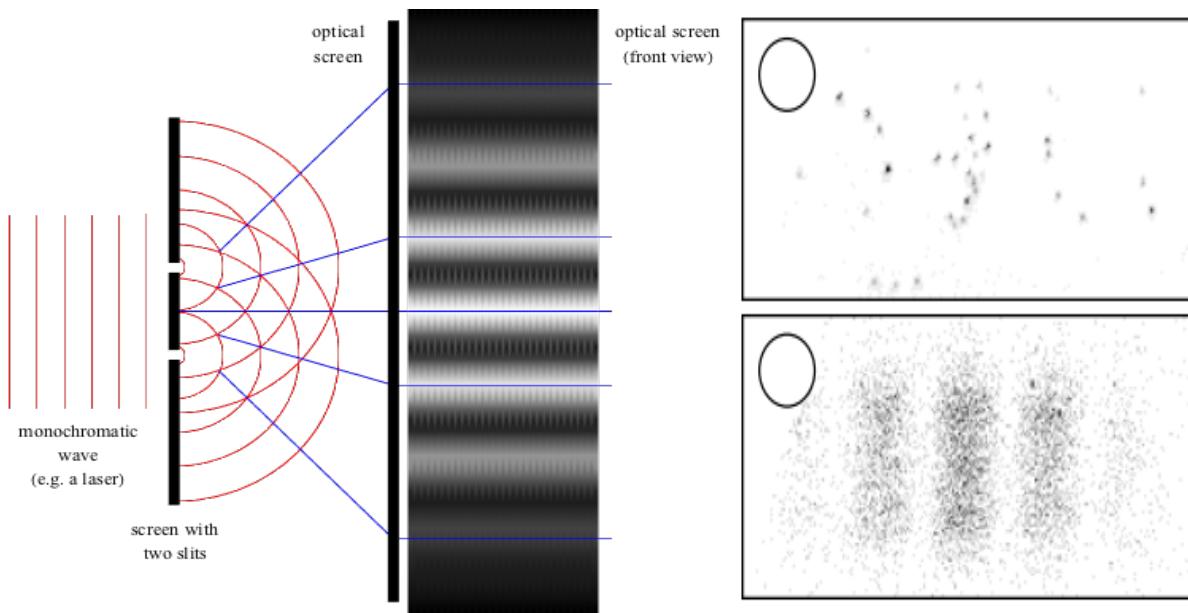


Figure 1.3: Left: Schematic view of interference pattern as in Young's experiment[9]. Right: (top) a single 20 ms time frame showing several individual photon arrival points. Right: (bottom) image obtained after integrating for a few seconds: the diffraction pattern emerges[10].

single electrons in 1974 [14]. Single photon [10] (shown in Fig. 1.3) and single electron double-slit experiments show the same results, illustrating the wave probability interpretation of particles and fields.

1.4.2 Predictions

By 1932, scientists knew that atoms were made up by protons, neutrons and electrons. Together with the photon a total number of four particles were known. Four grew to five when Anderson REF discovered the existence of positrons (which were predicted by Dirac REF). Then came the pion REF and muon. By the 1960s there were hundreds of “fundamental particles” with no good guiding principles to link them together. They were often referred to as the “particle zoo”.

By a series of insights by several individuals, the Standard Model as a quantum field theory became more widely accepted. Since then, the SM has predicted the results of experiment after experiment. Some of them are:

- Neutral weak currents. Postulated by A. Salam, S. Weinberg and S. Glashow, the theory of electroweak interactions predicted the existence of a new type of weak interaction, in which the reacting particles do not change their charges. The first observation was made in 1973 at the European nuclear research laboratory, CERN.
- Weak gauge bosons. Again postulated by the abovementioned people. These particles were also discovered in CERN, 1984.
- Heavy quarks. To explain the CP violations in kaon decays, M. Kobayashi and T. Maskawa predicted the existence of a third generation of quarks: the *top* and *bottom* quarks. The bottom quark was discovered in 1977 at Fermilab. It took another 18 years for the top quark to be found in the same institute.
- Gluons. The gauge bosons of quantum chromodynamics were discovered in 1978 and 1979.
- Higgs boson. On July 4, 2012, physicists at CERN announced the discovery of the only fundamental particle predicted by the Standard Model which was not yet discovered.

1.4.3 Precision tests

Inconsistencies between experiment and theory can be signs of wrong or incomplete theories. One example is the *Lamb shift*, a difference in energy between two energy levels of the hydrogen atom which was not predicted by the Dirac equation. This phenomenon is explained with the

Parameter	Experimental value	Theoretical value	Standard deviation
m_t [GeV]	172.74 ± 0.46	172.96 ± 0.45	-0.5
m_W [GeV]	80.387 ± 0.016	80.358 ± 0.004	1.8
	80.376 ± 0.033		0.6
	80.370 ± 0.019		0.6
Γ_W [GeV]	2.046 ± 0.049	2.089 ± 0.001	-0.9
	2.195 ± 0.083		1.3
m_H [GeV]	125.14 ± 0.15	125.14 ± 0.15	0.0
$g_V^{\nu e}$	-0.040 ± 0.015	-0.0398 ± 0.0001	0.0
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5063	0.0
τ_τ [fs]	290.75 ± 0.36	290.39 ± 2.17	0.1
$\frac{1}{2}(g_\mu - 2 - \frac{\alpha}{\pi})$	$(4511.18 \pm 0.77) \times 10^{-9}$	$(4508.63 \pm 0.03) \times 10^{-9}$	3.3
M_Z [GeV]	91.1876 ± 0.0021	91.1884 ± 0.0020	-0.4
Γ_Z [GeV]	2.4952 ± 0.0023	2.4942 ± 0.0008	0.4
$\Gamma_Z(had)$ [GeV]	1.7444 ± 0.0020	1.7411 ± 0.0008	-
$\Gamma_Z(inv)$ [MeV]	499.0 ± 1.5	501.44 ± 0.04	-
$\Gamma_Z(l^+l^-)$ [MeV]	83.984 ± 0.086	83.959 ± 0.008	-

Table 1.2: Observables compared with the SM best fit predictions. Errors are the total (experimental plus theoretical) uncertainties. Results are taken from Tables 10.4 and 10.5 in [27]

theory of quantum electrodynamics. REF. Another example.....

Because of this, experimentalists are continuously testing theoretical parameters of the Standard Model. These precision tests are most often done for the theory of QED. With the use of *renormalization theory* many parameters of the theory can be calculated in great detail. High-precision measurements of various observables have been performed at LEP 1 and SLC [15, 16, 17, 18, 19, 20] for physics at the Z-boson mass ($\sqrt{s} \approx M_Z$) and other observables at Tevatron [21, 22], LEP 2[22], ATLAS[23, 24] and CMS[25, 26]. Some of them are listed in Table 1.2. From this it is clear that the SM is very consistent with what we see in experiments.

ook hydrogen atom: anders verdwijnt electron binnen $10^{**}-12$ sec! precision measurements in icecube?

The most spectacular successes of renormalization theory were the calculations of the anomalous magnetic moment of the electron and the Lamb shift in the spectrum of hydrogen.

Neutrino cross-section: IceCube/Nature <https://www.nature.com/articles/nature24459>

Beautiful interplay of forces: i.e. strong force necessary to KEEP the neutrons as they are. Without it all neutrons would become protons! On the other hand; protons and neutrons would weigh the same if?

Fun fact: entire lagrangian somewhere in a footnote?

Gargamelle experiment: weak neutral currents: hint of Z boson discovery? Another decade for direct detection

Weak bosons: proof that there are 3 and only 3 families

Leuk: bestaat ook niet zoiets als negatieve zwaartekracht. Nochtans wel alle andere flavors negatief en positief

1.5 The need for physics beyond the Standard Model

Despite its incredible success many physicists believe the Standard Model is certainly not the full story. There are a number of features that seem arbitrary, but also some that cannot be explained by the theory alone. Below, I give a list of open questions:

- Why are there **three families** for both leptons and quarks?
- What is the **cause of the symmetries** we see in the Standard Model. Why is, for example, QCD not an SU(4) gauge theory?

- There are a number of **parameters** in the SM which cannot be explained by first principles. We have no good explanation why the top quark is 75 000 times heavier than the up quark. Why is the vev of the Higgs potential 246 GeV? Why is the Higgs mass 125 GeV? There are in total 19 parameters in the SM which are tuned by experiments and can be found in Table 1.3.
- Why does the Higgs potential have this **Mexican hat shape**? In other words, why is $\mu^2 < 0$ in $\lambda (\Phi^\dagger \Phi)^2 + \mu^2 (\Phi^\dagger \Phi)$ negative? Also, the vev, the mass of the Higgs boson and the mass of the fermions due to the Yukawa couplings all appear in Table 1.3. This makes us believe there is something we do not fully understand about the BEH mechanism.
- Right-handed neutrinos can be introduced into the SM. They are singlets with respect to the strong and weak interaction and would therefore not carry an electric charge, weak hypercharge or weak isospin. Due to this lack of charge, right-handed neutrinos would be extremely difficult to detect. They have Yukawa interactions with other leptons and the Higgs boson but its coupling would be extremely small. Neutrinos can become massive with Dirac mass terms in the same way charged leptons become massive in the BEH mechanism. Their **extremely small masses** suggest another mechanism in which the very light left-handed neutrinos are accompanied with extremely heavy right-handed neutrinos. This mechanism is called the Seesaw mechanism and requires the addition of Majorana mass terms*.

Aside from these, there are a number of unexplained phenomena that probably cannot be explained in a simple extension of the Standard Model but need a non-trivial approach.

For example:

- It is a natural assumption that the universe is neutral with all conserved charges. Both the SM and general relativity give no explanation on the **matter-antimatter imbalance** we see in the universe. The Big Bang is expected to produce equal amounts of matter and anti-matter, yet we see that the observable universe consists almost exclusively out of baryonic matter[†]. The most likely explanation is that in the early universe physical laws we know today were absent or have acted differently. The observed CP-violation is insufficient to account for the observed baryon asymmetry of the universe given the limits on baryon number violation.
- The stars, planets, interstellar clouds, etc. we see in space consist of baryonic matter. Assuming general relativity is the correct theory to describe gravity on cosmological scales, the Lambda-CDM model REF predicts that the matter we see is only around 15% of the total matter present in our visible universe REF PLANCK, zie Nadja. To explain the galaxy rotation curves [28], galaxy velocity dispersions [29], galaxy cluster masses [30], gravitational lensing [31] and many more, we predict that around 85% of the mass is not yet observed. This matter is referred to as **dark matter** as it cannot interact electromagnetically because it would have already been observed otherwise. No known particles in the SM can explain this phenomenon.
- Similar to dark matter the Lambda-CDM model predicts that the total energy in the visible universe should consist mostly out of a constant energy density for the vacuum called **dark energy**. 5% of the total energy consists of baryonic matter, 26% should be dark matter and the remaining 69% of dark energy is necessary to explain the expansion of the universe[‡].
- General relativity is generally accepted to describe gravity on cosmological scales. Thusfar is has not been possible to describe **gravity** on a quantum scale as is the case for the Standard Model and still be valid on very large scales. The inclusion of the graviton would

*Oscillation of the massive neutrinos is described in the PMNS matrix and adds 7 new parameters to the SM: the three mass terms $m_{\nu 1}, m_{\nu 2}, m_{\nu 3}$, the mixing angles θ_{12}, θ_{13} and θ_{23} and the CP-violating phase δ_{CP}

[†]Why are there protons, neutrons and electrons everywhere while it is perfectly possible for antiprotons and antineutrons to form atomic nuclei with positrons?

[‡]If there is only matter and the Big Bang acceleration only happened in the beginning of the creation of the universe, then one would expect the expansion to diminish due to the gravitational pull of matter. Measurements REF say the opposite is true: the universe is expanding and in an accelerating rate.

for example not recreate what is observed experimentally without other modifications to the SM which have not been observed REF. Contradictory to popular belief, it is not true that general relativity and quantum mechanics are incompatible. There is a need for a more complete theory beyond the range of their combined applicability[32].

- Why is the CP-violation in the strong interaction extremely small or even zero?
- Often referred to as a muon g-2 anomaly there are possible hints of new physics as the theoretical prediction of magnetic moment of the muon and experimental values have a small but significant offset [33].
- Why is there much more mixing in the lepton sector (PMNS) compared to the quark sector (CKM)?
- To explain the apparent quantum fluctuations on cosmological scales together with the horizon REF, flatnessREF and magnetic monopoleREF problems we have a theory of exponential expansion of space in the early universe: **cosmic inflation**. The theory states that there was between 10^{-36} and 10^{-32} seconds after the Big Bang a rapid exponential expansion happened. This could explain the apparent thermal equilibrium between parts of the visible universe which are not in causal contact with each other and the even distribution of the cosmic microwave background. The hypothetical field that is thought to be responsible for inflation, inflaton, is not yet observed and would be an extension of the Standard Model.
- With the use of renormalization theory REF it is possible to show that bare parameters should not be the same as parameters measured in experiments. These parameters, as the mass of particles, depends on the energy scale at which they are probed and physics far beyond the scope of the probed energy scale can influence these parameters. An example is the screening effect which is described in more detail in section 1.5.1. Similarly one-loop corrections to the Higgs boson mass* will have radiative corrections with a quadratic dependence on the cutoff scale. Virtual particles in one-loop corrections can have infinite momenta which should contribute to the total mass of the Higgs boson. Since we expect new physics to be present at energies close to the Planck mass ($\approx 10^{18}$ GeV) these loop corrections should push the Higgs mass to similar energy ranges. But, we see that the Higgs mass is around 125 GeV, which would mean that there are other parameters which should almost *exactly* cancel these absurdly large numbers. This is called *fine-tuning*, and it's the intuition of most physicists that this incredible fine-tuning has a deeper, yet unknown, meaning. This problem is often referred to as *the hierarchy problem*.

Many of these problems can be seen as “environmental”. Why there are three families and why so many parameters in the Standard Model have no fundamental explanation could just be because it's just the way it is. Maybe there is a multiverse, a plethora of universes with similar Standard Models, which have slightly or vastly different parameters. Some questions might even be impossible to answer because of a lack of statistics: we only have one universe and mankind has not been around very long in the timescale of the universe. This consideration might be valid but again not answers all our questions, it does not solve the question around dark matter for example.

This argument should not prevent us in trying to find a more general theory for the Standard Model and general relativity. A better explanation could be fairly simple, but infinitely hard as well. There is only one way in trying to find a better understanding: experiments.

Unification is the most popular approach in describing physics beyond the Standard Model. Unification would mean that well established theories are low-energy approximations of a more grand unified theory. Historically this has worked very well: the unification of celestial gravitation of Kepler with terrestrial gravitation of Galileo into universal gravitation, unification of electricity, magnetism and later optics into electromagnetism. Gravity was overhauled by the much broader theory of general relativity. Lastly, the birth of gauge theories have combined QED and the weak interaction into the combined electroweak theory. The similarities in QCD and the electroweak,

*Fermions and bosons are not effected by higher energy physics in the same way as a scalar particle is. There is a logarithmic dependence.

Parameter	Description	Value
m_e	Electron mass	511 keV
m_μ	Muon mass	105.7 MeV
m_τ	Tau mass	1.78 GeV
m_u	Up quark mass	1.9 MeV
m_d	Down quark mass	4.4 MeV
m_c	Charm quark mass	1.32 GeV
m_s	Strange quark mass	87 MeV
m_t	Top quark mass	173.5 GeV
m_b	Bottom quark mass	4.24 GeV
θ_{12}	CKM 12-mixing angle	13.1°
θ_{23}	CKM 23-mixing angle	2.4°
θ_{13}	CKM 13-mixing angle	0.2°
δ_{CP}	CKM CP violation phase	0.995
g_1 or g'	U(1) gauge coupling	0.357
g_2 or g	SU(2) gauge coupling	0.652
g_3 or g_s	SU(3) gauge coupling	1.221
θ_{QCD}	QCD vacuum angle	≈ 0
v	Higgs vacuum expectation value	246 GeV
m_H	Higgs mass	125 GeV

Table 1.3: Bla

both being gauge theories, has led people to believe a unification is possible. This would unify the forces and particles known from the Standard Model into a *Grand Unified Theory* or GUT. A theory which would add gravity is called a *Theory Of Everything* or TOE.

1.5.1 Running of the coupling constants

The coupling constants in the Standard Model are actually not constants. They depend on the energy of the system. Quantum fluctuations in vacuum have a non negligible contribution in the apparent charge of particles. The effects of screening and anti-screening are visualized in Fig. 1.4 and can be mathematically formulated as a beta function. The function encodes the dependence of a coupling parameter, g , on the energy scale μ :

$$\beta(g) = \frac{\partial g}{\partial \log(\mu)}. \quad (1.31)$$

More coupling in shorter distances (i.e. higher energies) would give rise to a positive beta function and is the case in QED. In QCD gluons carry a color charge and enter the beta function with a negative sign:

$$\beta(g) = - \left(11 - \frac{2n_f}{3} \right) \frac{g^3}{16\pi^2}, \quad (1.32)$$

with n_f the number of fermions that participate in the strong interaction. This decrease in coupling strength in function of energy scale is called *asymptotic freedom*. Alternatively we can write down the coupling constants in function of the energy

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(m_Z) + \frac{b_i}{2\pi} \log \frac{Q}{m_Z}, \quad (1.33)$$

with $b_1 = -\frac{4}{3}n_g - \frac{1}{2}\log m_Z$, $b_2 = \frac{22}{3} - \frac{4}{3}n_g - \frac{1}{6}$ and $b_3 = \frac{11}{-4}$ the U(1), SU(2) and SU(3) constants in which n_g denotes the number of quark and lepton generations and n_h is the number of Higgs doublet fields. These three couplings seem to converge leading people to believe a more universal theory at higher energies would unify these parameters into one and is shown in Fig. ??.

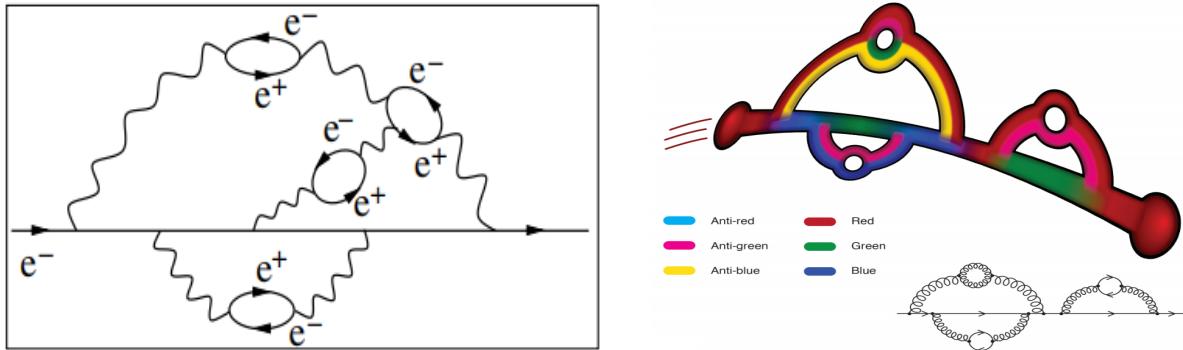


Figure 1.4: Left: virtual pairs of positrons and electrons can be produced in vacuum. Positrons are attracted to the bare electron resulting into an apparent lower charge of the bare particles from larger distances. This effect is referred to as *screening*. Right: quark and antiquark pairs have a similar screening effect in color charge as the case for electrical charges. There is an anti-screening effect due to gluons which “extract” color from the bare quark and is stronger than the screening effect of quark-antiquark pairs. At small distances the dilution of the initial color charge is larger than at large distance. This drawing should not be taken literally and the concept of self-interaction is very hard to illustrate. One could interpret the initial red color with a gluon emission as a conservation of the red color, thus the red quark can only become blue if a gluon with a red and anti-blue color is emitted (gluons need to be color+anti-color pairs). In the loop two more gluons are present but cannot have a combined color: anti-green with red and green with anti-blue is a possible where green and anti-green have a net zero color charge. Figures from [34].

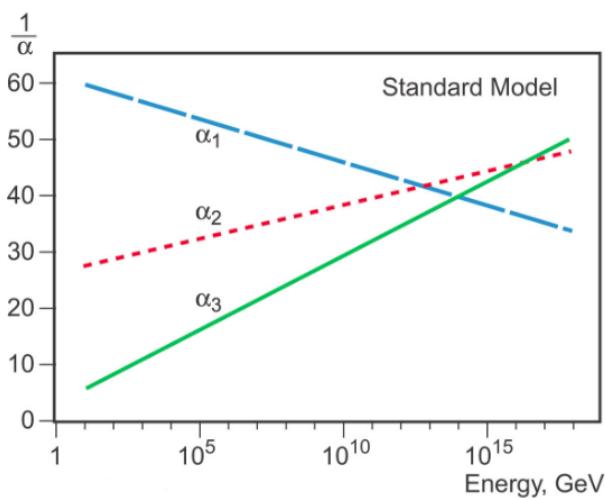


Figure 1.5: Running of the coupling constants in the Standard Model. Figure from [35].

1.5.2 Unifying theories

Linking the seemingly arbitrary parameters in the Standard Model has been ongoing for the last couple of decades. Linking these theories is not easy since they exhibit very different behaviors. Electromagnetism is long-ranged, the weak force is short-ranged and the strong force is weak in high-energy environments such as the early universe and strong where the probing energy is low. Many GUTs predict that quarks and leptons are part of a single representation of a gauge group with one single hypercharge and would explain why the electric charge of electrons and protons seem to be exactly the same REF.

The simplest GUT is SU(5), which would break down into the Standard Model at lower energies due to spontaneous symmetry breaking. Other possible extensions are SO(10)REF , SU(8)REF and O(16)REF Lie groups.

Without experimental results there is still much ongoing debate into which theory is the correct one. Multiple viable theories still remain. A subset of a handful are given in the following sections.

Supersymmetry

Supersymmetric models impose a new symmetry, supersymmetry (SUSY), that relates fermions and bosons. Every fermion would have a, yet unseen, bosonic supersymmetric partner and vice versa. This theory is highly motivated due to it's ability in cancelling the quadratically divergent terms in the mass correction of the Higgs boson. Unbroken SUSY would lead to partners with the same mass as the particles we know from the SM and would have been discovered long ago. Because of this one assumes the symmetry to be broken.

The last couple of years SUSY was regarded as the most promising extension of the SM with tremendous efforts from big collaborations such as the experiments at the Large Hadron Collidor (LHC) in search for proof. To this date no evidence for SUSY has been found.

Little Higgs

String Theory

1.5.3 How to look for new physics

In general one could say there are two possible ways to look for new physics. Almost all of the physics in our Solar system can be explained with what we know from the Standard Model. Interactions in controlled laboratory environments are currently on the order of 10 TeV in the experiments at the LHC. This could still be well below the energy levels to produce new exotic particles. In the *energy frontier* it is the goal to reach the highest energies possible in order to get as close to the energy requirements where new physics become more prominent. As a consequence more and more cosmic ray experiments have found an interest in searches for physics beyond the Standard Model. This is sometimes called the *cosmic frontier*. This analysis tries to explore this possibility in more detail for the IceCube experiment. Cosmic ray experiments have the disadvantage that they are not fully contained experiments and information is lost as the primary interaction is unknown and important parameters as energy, direction, type,... of the particle have to be reconstructed.

The other approach tries to extract information from precision experiments and are therefor reliant on limiting statistical and systematical uncertainties. In these experiments the intensity of the beam of particle accelerators is pushed to their highest values and is therefor referred to as the *intensity frontier*. This strategy tries to generate huge numbers of particles needed to study rare or exotic subatomic processes. Rare processes could gain us a lot of information on unknown physics. Some parameters which can be calculated in the SM have a small offset in what is measured in experiment. It is possible that new physics enter in Feynman diagrams and have a non-negligible contribution, however rare they are.

Waarom fysica? De vraag van Higgs veld: als er een globaal minimum is kunnen we allemaal dood gaan. Ook zelfde filmpje

I'd like to finish this chapter with quotes from Steven Weinberg in an interview with NovaREF about his vision on string theory. It shows the apparent stalemate phycisist seems to find themselves into: there are no theoretical breakthroughs regarding long standing problems.

"I believe that there is a simple theory that governs everything—the four forces we know about, perhaps other forces as well. I'm not sure that's true. It may be that nature is irreducibly messy. I'm sure that we should assume it's not, because otherwise we're never going to find a fundamental theory. But even so, we're not guaranteed that we'll find it. We may not be smart enough. Dogs are not smart enough to understand quantum mechanics. I'm not sure that people are smart enough to understand the whatever-it-is that unifies everything. I think we probably are, because of our ability to link our minds through language, but I'm not certain.

There was a marvelous period from, I'd say, the mid-'60s until the late '70s when theoretical physicists actually had something to say that experimentalists were interested in. Experimentalists made discoveries that theoretical physicists were interested in. Everything was converging toward a simple picture of the known particles and forces, a picture that eventually became known as the Standard Model. I think I gave it that name. And it was a time when graduate students would run through the halls of a physics building saying they had discovered another particle and it fit the theories, and it was all so exciting.

Since the late '70s, I'd say, particle physics has been in somewhat of a doldrums. Partly it's just the price we're paying for the great success we had in that wonderful time then. I think cosmology now, for example, is much more exciting than particle physics. The string theorists are trying to push ahead without much support from relevant experiments, because there aren't any relevant experiments that can be done at the kind of scales that the string theorists are interested in. "

- Lorentz violation? Nature of Teppei: <https://www.nature.com/articles/d41586-018-05931-2>



2. Theoretical Motivation of the Analysis

As seen in Chapter 1 there is much ongoing debate which beyond the Standard-Model physics models could help explain questions we do not have answers for. Over the last decades this quest has proven to be non-trivial since many anticipated accelerator experiments have not given any clear hints towards physics which cannot be explained by the Standard Model. A big part of the physics community is trying it's best to help answer these riddles and dedicated experiments have been constructed in their search for new physics. Other collaborations try to make use of their detector in the most efficient way possible. These experiments most often try to look for beyond the Standard-Model physics by searching for signals in their detector which could not be explained by the particles we know today. One example, and also being the subject of this work, is to try to look for particles which have a lower electromagnetic non-zero charge than the charged particles of the Standard Model.

2.1 Introduction

As seen in Chapter 1 all free particles have an electromagnetic charge which is a multitude of the absolute electron charge, e , equal to $1.602 \times 10^{-19} C$. Elementary particles such as (anti)quarks have fractional charges equal to $\pm \frac{1}{3}e$ and $\pm \frac{2}{3}e$ but have never been seen as isolated particles due to *confinement* as explained in Section [sub:quarks]. No other particles are expected to have a charge $< e$ and non-zero and are perfect candidates for searches for beyond the Standard Model. Different experiments have sought for these anomalously charged particles and are referred to as *Lightly Ionizing Particles (LIPs)* or *Stable Massive Particles (SMPs)*. Throughout this work the latter denomination is used, indicating they do not rapidly decay and have masses significantly higher than the lightest leptons.

2.2 Theory

In Section 1.5.2 I have introduced possible extensions of the Standard Model. One of the simplest possible extensions of the $SU(3) \times SU(2) \times U(1)$ group is the $SU(5)$ gauge group. It is the smallest Lie group that can contain the group of the Standard Model without introducing any new fermions. It could explain charge quantization REF, has complex representations and can accommodate fractional charges. In this scheme new vector bosons, usually called X and Y bosons, occur with charges $\frac{4}{3}$ and $\frac{1}{3}$. Extensions of the $SU(5)$ models allow for color singlet particles with charges $\frac{1}{3}$ and $\frac{2}{3}$ [36]. Other possible extentions are the $SU(7)$ [37], $SU(8)$ [38],

$\text{SO}(14)$ [39], $\text{SO}(18)$ [40], $\text{SO}(10) \times \text{SO}(8)$ [41].

It should be noted that the simplest form of an $\text{SU}(5)$ gauge group is already highly constrained as proton decay is allowed but experimental results have shown the lifetime to be $> 1.67 \times 10^{34}$ years ($\tau(p \rightarrow \pi^0 e^+)$) and $> 6.6 \times 10^{33}$ years.

There are also some string theories massive particles with a fractional charge are also predicted [42, 43], which was later confirmed to occur very often in certain compactifications [44].

More recently there has been an increasing interest in searches for millicharged particles. New particles could couple to the Standard Model via a “kinetic mixing” or “hypercharge portal” [45, 46]. And in recent years, they were studied as possible candidates for dark matter [47, 48, 49, 50]. The charges of these particles are however often $< 10^{-3}e$ and no ideal candidates in neutrino Cherenkov experiments. It is possible to look for them in neutrino experiments [51] but are more targeted toward future experiments such as DUNE [52] and SHiP [53]. A more detailed explanation of these particles can be found in [54]

There are possibly many other possible extensions but go beyond the scope of this work. One could just keep in mind that no free particles with an anomalous charge less than e are expected, and if seen would give clear hints of beyond the Standard-Model physics and would help in finding a more clear picture of what is possibly lurking beyond the realms of our understanding.

https://ac.els-cdn.com/S0370269314001257/1-s2.0-S0370269314001257-main.pdf?_tid=7f8b3af4-8845-417fa219-703f382cb092&acdnat=1535556263_162a6333d0a615412be21aa5fc3e5720

2.3 Properties of the signal

Because there are many possible scenarios in what these particles are, originate from or are produced one has to make certain assumptions about the properties of the signal. A particle traveling at the speed of light with a lifetime < 0.1 seconds traversing a detector will not give the same signal properties as one that has a very long lifetime. Therefore, I have chosen that the particles I am looking for

- behave leptonically, similar to muons,
- have a long lifetime and will not decay within the detector, or have a very low probability,
- follow an energy spectrum with a spectrum of -2^* ,
- are assumed to produce a uniform flux in angle space close to the detector.

These assumptions are consistent with previous searches which are mentioned in Section 2.4. The behaviour of these particles in the detector will depend on the charge (see section???) and, to a lesser extent, the mass. In this work I have chosen to look for particles with a

- charge of $1/3$, $1/2$ and $2/3$,
- mass of 10 GeV, 100 GeV, 1 TeV, 10 TeV and 100 TeV,

where I have referred to the charge of the particles as relative to the absolute electron charge, e^\dagger . The possible combinations result into a total of 15 unique signal samples which will be searched for.

2.4 Previous searches

There are several ways on can assume to produce fractional charge particles. Different assumptions lead to different possible searches with previous and current detectors. In the following I will give the results of several experiments. More information and a very good overview can be found in

2.4.1 Searches with accelerators and fixed targets

The total energy of the interaction should be large enough to produce particles of a certain mass. The square of the centre of mass energy is given by:

*More information about spectra can be found in section???????

[†]This will be done throughout this work from this point on.

Energy (GeV)	Charges sought	Collider	Reference
1-1.4	$\frac{2}{3}$	VEPP-2M	[57]
29	$\frac{1}{3}, \frac{2}{3}$	PEP	[58]
130-209	$\frac{2}{3}, \frac{4}{3}, \frac{5}{3}$	LEP	[59]
130-136, 161 and 172	$\frac{2}{3}$	LEP	[60]
91.2 (m_Z)	$\frac{2}{3}, \frac{4}{3}$	LEP	[61]
91.2 (m_Z)	$\frac{4}{3}$	LEP	[62]

Table 2.1: Highest-energy fractional charge particle searches in electron-positron colliders. No evidence for fractionally charged particles was found. From ??.

$$\begin{aligned} s &= (p_1 c + p_2 c)^2 \\ &= m_1^2 c^4 + m_2^2 c^4 + 2E_1 E_2 - 2\vec{p}_1 \cdot \vec{p}_2 c^2, \end{aligned} \quad (2.1)$$

where $p_{1,2}$ are the four-momenta of the two particles and c is the speed of light. Assuming E is the energy of the incoming particle and m the mass of a target particle in rest, the maximum mass reach of a search is given by:

$$m_{max} \approx \sqrt{2mE}. \quad (2.2)$$

If I is the incoming particle from the input beam and N a nucleus then the production of exotic particles can be depicted as

$$I + N \rightarrow F + X, \quad (2.3)$$

where F stands for the fractional charged particle and X for the other particles which are produced in the interaction. No experiments that used accelerators and fixed targets found evidence for the existence of fractional charge particles [55]. The highest-energy search used muons with a muon beam of 200 GeV, resulting in an m_{max} of 19 GeV/c² [56].

2.4.2 Colliders

Particle colliders can reach much higher energies than most fixed-target experiments. The maximal mass of new particles in a storage ring which is colliding particles of energy E , Eq. 2.1 gives

$$s = 4E^2. \quad (2.4)$$

There is a big difference in lepton and hadron accelerator experiments as much less particles are being produced in the former due to the absence of strong interactions. The production is “cleaner” and the sought particles are easier to distinguish from other productions. But, it is more difficult to reach higher energies* for lepton accelerators. An overview of electron-positron colliders is given in Tab. 2.4.2. No evidence for fractionally charged particles was found.

Experiments that use proton-antiproton colliders have reached large masses but have also found no evidence of fractionally charged particles. An overview is given in Tab. 2.4.2.

A more recent search was performed at the LHC, a proton-proton collider, when operating at an energy of 7 TeV. No evidence of particles with fractional charge was found. An upper limit of 95% confidence level was set for particles with electric charge $\frac{2}{3}$ up to a mass of 310 GeV and 140 GeV for those with charge $\frac{1}{3}$ [66].

*The radiative power of synchrotron radiation scales with a factor of m^{-4} : particles with low mass lose much more energy in circular accelerators with a fixed radius.

Energy (TeV)	Charges sought	Collider	Reference
0.54	$\frac{1}{2}, \frac{2}{3}$	SPS	[63]
1.8	$\frac{3}{2}, \frac{4}{3}$	Tevatron	[64]
1.8	$\frac{1}{3}, \frac{2}{3}$	Tevatron	[65]

Table 2.2: Highest-energy fractional charge particle searches in proton-antiproton colliders. No evidence for fractionally charged particles was found. From ??

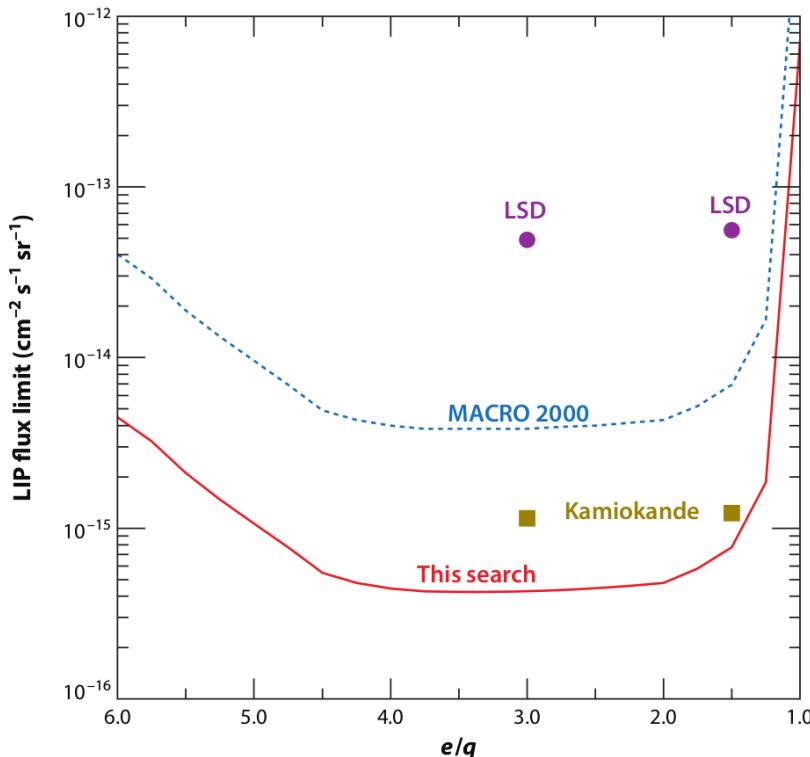


Figure 2.1: Upper limits on fluxed of particles close to the repective detectors. LIP stands for *Lightly Ionizing Particles*. From [68]

2.4.3 Searches for particles with telescopes

There are several ways particles with a fractional charge could be produced in cosmological events;

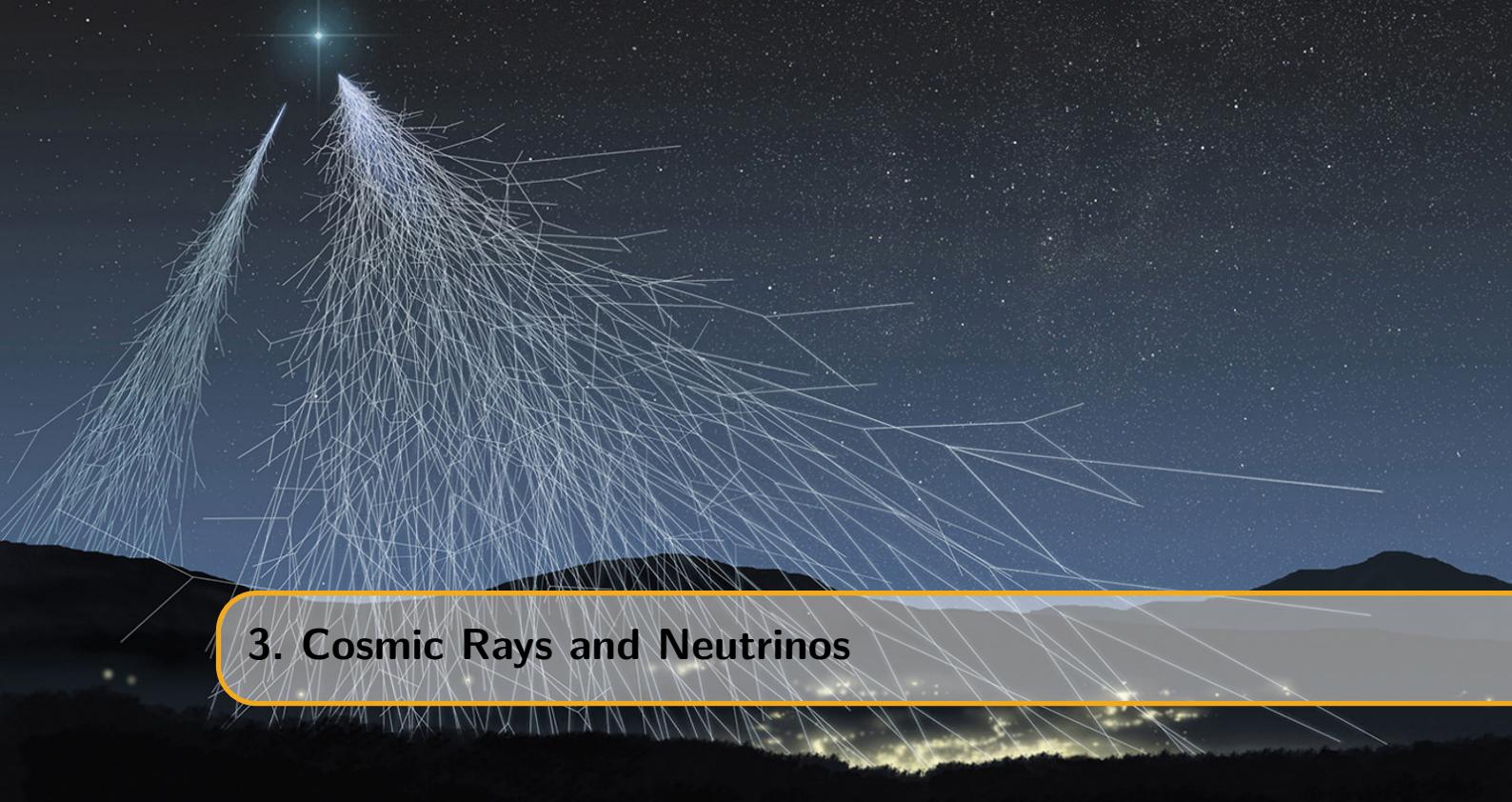
- The particles were produced early on in the Universe and are a stable component of the present material;
- The particles are rare but can be continuously produced in high-energetic astrophysical event; or
- The particles are produced in cosmic ray processes on Earth.

Because there is no clear preference in one of these possibilities most telescope experiments therefore express their search sensitivity in function of an incoming flux close to the detector in units of $\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. This analysis has adopted the same search strategy and aims to improve upon previous results. The most stringent upper limit was realized by the MACRO experiment found on the arXive which compares results from older searches and can be found in Fig. 2.4.3. The best published result is set by Kamiokande II [w]ith upper limits of $2.1 \times 10^{-15} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ and $2.3 \times 10^{-15} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ for particles with charges $\frac{1}{3}$ and $\frac{2}{3}$ respectively [67].

Hmmm... Deze heb je nog nooit bekeken: <https://arxiv.org/pdf/1601.04004.pdf> +references!

Particle physicists don't look for a needle in a haystack as what is sometimes used for an

analogy to make clear in what we are doing. We are in fact looking at hay in a haystack. The hay we are looking for has slightly different properties. It can be a bit drier or a bit longer, but it takes an enormous amount of time and clever thinking to be able to distinguish the hay from normal hay. But sometimes errors can slip in, or you can have a very long normal piece of hay while you could have smaller pieces of new hay so finding a new one doesn't give you a hundred percent assurance that you've indeed found something new. You need to find enough of the new ones to be able to say that there are too many found that could be obtained by pure chance from the normal set.



3. Cosmic Rays and Neutrinos

Cosmic rays almost exclusively refer to particles with a finite rest mass. The term *rays* was historically wrongly attributed to these particles as they were thought to be mostly electromagnetic radiation. The interest of cosmic rays within the field of particle physics and modern particle physics is clear: multiple new particles were discovered from the interactions at energies which were higher than most experiments could reach. Positrons, muons, pions and kaons were first discovered in cosmic ray experiments in the 1930s and 40s. Today, high-energy cosmic ray interactions are still of interest as the highest energies of these particles go beyond what is feasible at the most powerful accelerators such as the LHC. Neutrinos are expected to be produced together with cosmic rays near the source or close to Earth making neutrino astronomy a powerful and important part of modern day astronomy. In this chapter I will give BLA BLA BLA. For a more exhaustive description of cosmic rays I refer the reader to [69].

3.1 Cosmic rays

3.1.1 Discovery of cosmic rays

With the use of electrometers, Victor Hess performed multiple ground-breaking balloon flight experiments in 1912 to prove that the amount of radiation increases with altitude [70]. This was in strong contradiction with the widespread belief that radiation on Earth's surface mostly originates from radioactive substances in its crust. Hess concluded that an extremely penetrating radiation existed. He described this radiation to be coming from space which then enters Earth's atmosphere which proved to be correct but it was wrongfully attributed to electromagnetic radiation by Robert Millikan in the 1920s [71].

Hess later ruled out the possibility that cosmic rays originate from the Sun as his observations showed no particular differences in night and day and during solar eclipses. In the late 1920s, first evidence was found that cosmic rays were charged due to a variation of their intensity with latitude [72]. This indicated that they were deflected by the geomagnetic field.

3.1.2 What are cosmic rays?

Cosmic rays are, almost exclusively, the collection of nuclei which are stripped of their electrons, making them electrically charged, heavy particles. Around 90% of the particles are ionized hydrogen atoms, or protons. 9% are alpha particles and 1% are nuclei of heavier elements. There is a striking resemblance between the relative abundance of cosmic rays and elements in the

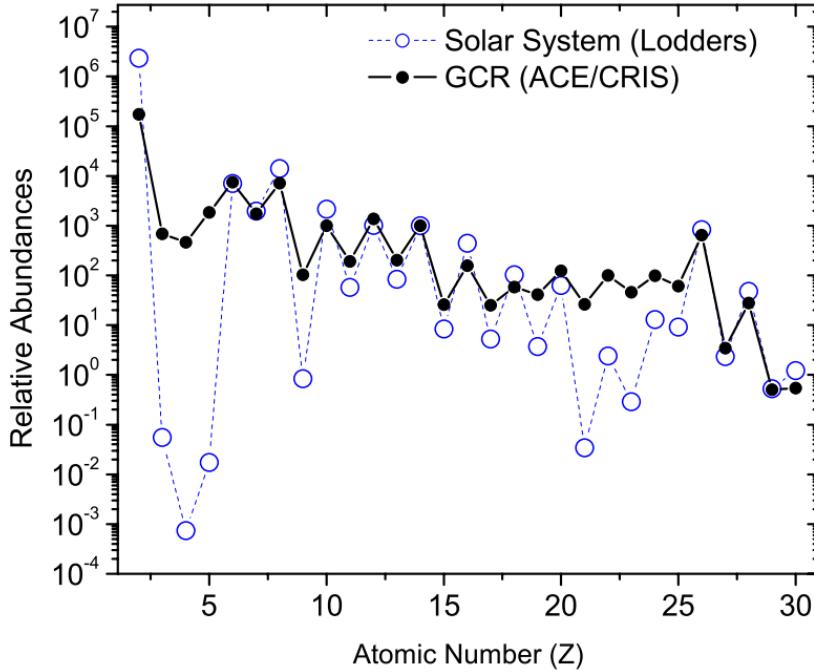


Figure 3.1: The cosmic ray elemental abundances measured on Earth compared to the solar system abundances, all relative to carbon = 100. Figure from ACE news archive [73].

Solar System as seen in Fig. 3.1.2. A much smaller fraction of incoming particles are electrons, positron and antiprotons.

There are however two important differences between cosmic rays and elements from our Solar System. Firstly, the two groups of elements Li, Be, B and Sc, Ti, V, Cr, Mn are many orders of magnitude more abundant in cosmic rays than in the solar system. This is due to their absence in stellar nucleosynthesis and are therefore not expected to be produced in large numbers. More massive cosmic rays (mainly C, O and Fe) can produce these nuclei in the process of *spallation*. They are produced by collisions of cosmic rays with the interstellar medium. Therefore, these nuclei are sometimes referred to as *secondary nuclei*. The second difference is that nuclei with $Z > 1$ are much more abundant with respect to hydrogen for cosmic rays. This phenomenon is not yet well understood but could be attributed to the difficulty to ionize hydrogen, necessary for acceleration processes.

The amount of cosmic rays seen on Earth is expressed in units of $[m^{-2}s^{-1}sr^{-1}]$. We can see in Fig. 3.1.2 that the cosmic ray flux follows a energy power law spectrum:

$$dN/dE \propto E^{-\gamma} dE, \quad (3.1)$$

where γ is called the *spectrum index*. Because of the steepness of the spectrum it is often multiplied by a higher power of energy*.

We can divide the global spectrum in four regions. Between 10 GeV and 1 PeV the differential spectrum index is around -2.7. From 10 PeV to 1 EeV it is around -3.1. Above 10 EeV the spectrum again flattens to an index around -2.6 and an apparent cutoff region is present at around 10^{20} eV. The transition of this first to second region is referred to as the *knee* at around 3 PeV. The second to third region transition is referred to as the *ankle*. It is possible to describe the full cosmic-ray spectrum with sources within our galaxy. A more generally accepted theory is that the knee in the spectrum originates from the end of a population of particles which are accelerated within our Milky Way [75]. A sometimes referred to at around 100 PeV is the *second*

*The broad range in both energy and flux should convince the reader that many types of detectors are necessary to study the behaviour of cosmic rays. Low-energy particles are abundant and high-energy particles are much more rare. Both the energy and the incoming flux will determine the type and size of the detector.

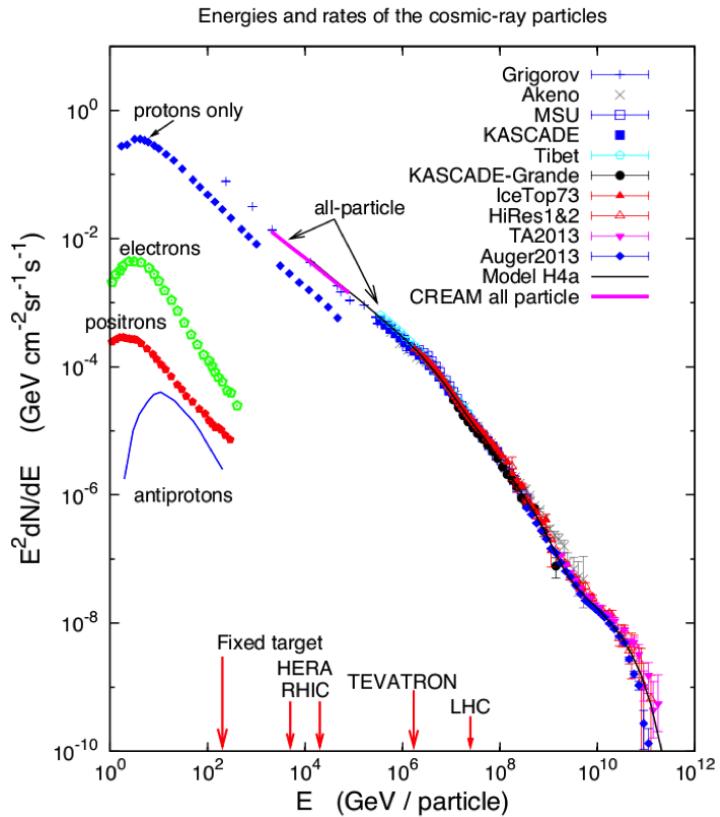


Figure 3.2: Spectrum of cosmic rays at Earth. The all-particle spectrum measured by different experiments is plotted together with the proton-only spectrum. Subdominant contributions to the total flux from electrons, positrons and antiprotons as measured by the PAMELA experiment are also shown. Figure from [74].

knee, believed to be a feature of the iron drop-off. The origin of cosmic rays has been a topic of discussion for many years. We know now that most particles originate from sources in the local galaxy, having spent on average 10^7 years in diffusive motion in the interstellar medium [75]. This is consistent with the resemblance of the relative abundances of cosmic rays and elements from our Solar System. However, there is no general consensus about the origin of the cosmic rays with energies above 3×10^{18} eV. In the following, the abovementioned energy regions are discussed in more detail.

3.1.2.1 Solar modulation

In the Solar System a stream of charged particles is released from the Sun. This stream is mostly made up of electrons, protons and alpha particles with kinetic energies ranging between 0.5 and 10 keV. Within this solar wind plasma there is a magnetic field. Cosmic rays coming in to the Solar System interact with these particles and magnetic field. The influence is greatest on particles with the lowest charges. This effect is called *solar modulation*. In effect, we see a strong suppression of cosmic rays at energies of 10 GeV and below.

3.1.2.2 Galactic component

The most probable acceleration mechanism for cosmic rays originating from our Galaxy is by shocks driven by expanding supernova remnants [76]. From the ratio of primary to secondary nuclei it can be inferred that cosmic rays travel distances thousands of times greater than the thickness of the disk of the Galaxy. There is also an apparent decrease in the amount of matter that is traversed by cosmic rays with higher energies than with lower. Higher-energy cosmic rays seem to spend less time in the Galaxy than lower-energy ones and suggests that cosmic rays are accelerated before most propagation occurs [69].

The way the spectrum is fit is not set in stone. Here I will use the convention used by Gaisser, Stanev and Tilav described used in reference [75]. The spectrum is subdivided in three populations. The first population corresponds to the particles accelerated by supernova remnants, with the knee signaling the cutoff of this population. The second population is a higher-energy galactic component of unknown origin. The third generation will be described in more detail in ???. Assuming that the primary spectrum depends on the *magnetic rigidity**,

$$R = \frac{pc}{Ze}, \quad (3.2)$$

where Ze is the charge of a nucleus of total energy $E_{tot} = pc$ and relates to the gyroradius of a particle in a given magnetic field B as

$$r_L = \frac{R}{B}. \quad (3.3)$$

If there is a characteristic rigidity, R_e above which a particular acceleration process reaches a limit, then the feature will show up in total energy first for protons, then for helium and so forth for heavier nuclei according to

$$E_{tot} = Ze \times R_e. \quad (3.4)$$

This effect is visualised in Fig. 3.1.2.2 and indicates that as one population reaches its maximum the composition becomes heavier. The second knee, reported by KASCADE-Grande [77] and GAMMA [78] could be explained with an “iron knee” bump.

3.1.2.3 Extragalactic component

The flux at the highest energies is exceedingly small. The number of events per year at energies above 5×10^{19} eV is around one per square kilometer per century. There are only two experiments in the world capable of detecting the highest-energy cosmic rays in a statistical meaningful way: Telescope Array, located in the Northern Hemisphere (area of ≈ 700 km 2) and the Pierre Auger Observatory in the Southern Hemisphere (area of ≈ 3000 km 2).

*An assumption which is experimentally favored over other assumptions. Rigidity is an appropriate variable for interpreting changes in the spectrum due to propagation and acceleration in magnetic fields.

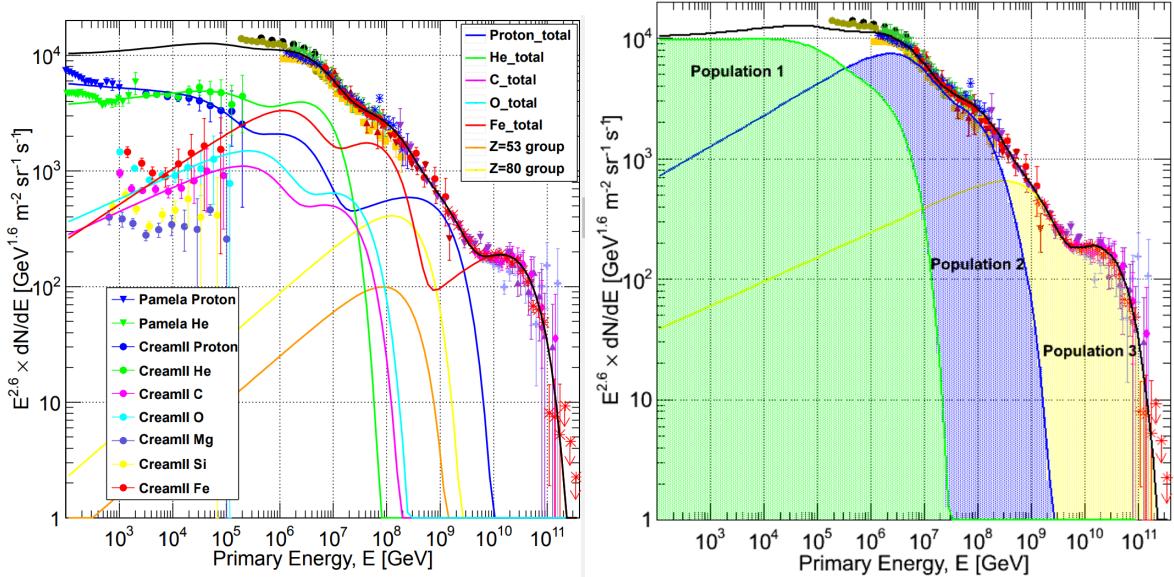


Figure 3.3: Blub

Both experiments see a suppression of the flux above 6×10^{19} eV. The exponential cutoff is consistent with the expected Greisen-Zatsepin-Kuzmin (GZK) effect [79, 80] where cosmic rays interact with the cosmic microwave background radiation (CMB)

$$\gamma_{CMB} + p \rightarrow \Delta^+ \rightarrow p + \pi^0 \quad (3.5)$$

or

$$\gamma_{CMB} + p \rightarrow \Delta^+ \rightarrow n + \pi^+. \quad (3.6)$$

Particles with energies above 5×10^{19} eV would interact with the CMB, leading to an exponential cutoff (but if the incoming particles above these energies would be relatively young it is still possible for them to reach the detector). The Pierre Auger experiment reported to see higher compositions at the highest energies [icrc2017:pa]. If the particle is a nucleus with A nucleons, then the GZK limit applies to its nucleons, which carry only a fraction $1/A$ of the total energy. For iron nuclei this would for example result in a limit of 2.8×10^{21} eV. In contrast, the TA experiment interpreted their data as implying a light primary composition (mainly p and He) at the highest energies. Both experiments use a different interpretations for crucial quantities of these measurements and a thorough joint analysis conducted by both experiments states that, at the current level of statistics and understanding of systematics, both data sets are compatible with being drawn from the same parent distribution [27].

The Pierre Auger Observatory also reported evidence in an anisotropic distribution of the arrival directions of the highest-energy cosmic rays [81] in a direction where the distribution of galaxies is relatively high and does not coincide with the galactic plane. These observations, together with our lack of known possible sources within our galaxy for these ultra-high energies shows compelling evidence that these particles have an origin from outside our galaxy. From pion decay there is also an expected flux from extragalactic neutrinos (more information in section 3.3.3 and ??). The flux, spectrum and angular distribution of the excess neutrino signal detected by IceCube between ≈ 50 TeV and ≈ 2 PeV are inconsistent with those expected for Galactic sources [82].

To put it simply, understanding cosmic rays and where they originate can help us answer fundamental questions about the origins of the universe, our galaxy and ourselves. To put it in the words of Carl Sagan:

“The nitrogen in our DNA, the calcium in our teeth, the iron in our blood, the carbon in our apple pies were made in the interiors of collapsing stars. We are made of starstuff.”

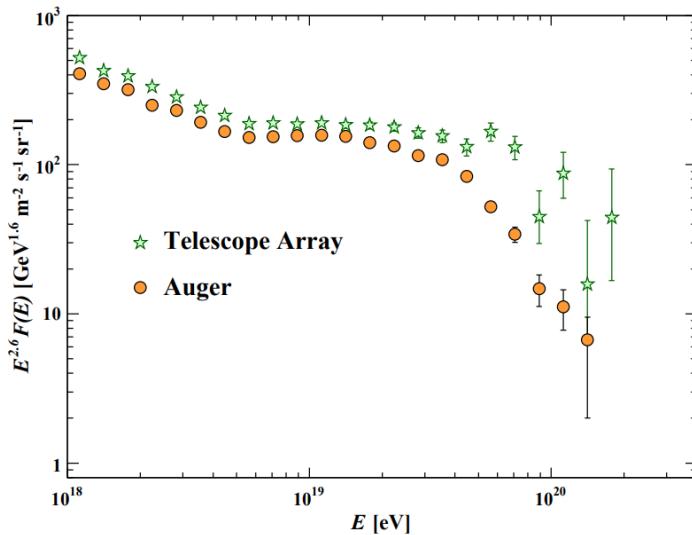


Figure 3.4: Expanded view of the highest energy portion of the cosmic-ray spectrum from data of the Telescope Array and the Pierre Auger Observatory [pdg2018].

3.1.3 Acceleration mechanism

How cosmic rays got their signature slope in the energy spectrum and its intricate details have been under discussion for multiple decades. To this date there is no clear picture how these particles are accelerated in full detail. It is beyond the scope of this work to give a comprehensive overview of all possible acceleration mechanisms or possible sources. Most calculations are left out and for a more detailed discussion the reader is referred to specialized books or the references in the text.

The acceleration of the particles can be subdivided into two questions. First, where are the particles accelerated? Does it happen on large scales, cosmological distances in galaxies or near specific sources? Secondly, how are these particles exactly accelerated? What is the driving mechanism? Since primary cosmic rays are all electromagnetically charged particles these mechanisms should clearly be sought for in places where electric and/or magnetic fields play a dominant role.

3.1.3.1 Galactic accelerators

With their approximate energy density around 0.5 eV/cm^3 in our local galaxy, the bulk of cosmic ray acceleration could very well be explained by **supernovae**. This density results into a total power of around

$$L_{CR} = 7 \times 10^{40} \text{ erg/s}, \quad (3.7)$$

where erg is a unit often used in astronomy*. If one assumes a supernova explosion of around one per every 30 years then the total power output of type II supernovae with a mass output of around ten times the mass of the Sun at a velocity close to $5 \times 10^8 \text{ cm/s}$ would result in a power of

$$L_{SN} \sim 3 \times 10^{42} \text{ erg/s}. \quad (3.8)$$

These numbers are not set in stone and hold large uncertainties, but it shows that with an acceleration efficiency on the order of a couple of percent supernova explosions are a prominent source of energetic cosmic rays, if not the dominant one.

3.1.3.2 Extragalactic accelerators

We will see in Section 3.1.4.1 that the maximum energy from shock acceleration by a supernova remnant is insufficient to explain UHECR. As explained in Section 3.1.4.1, particles can be

* $1 \text{ erg} = 10^{-7} \text{ J}$.

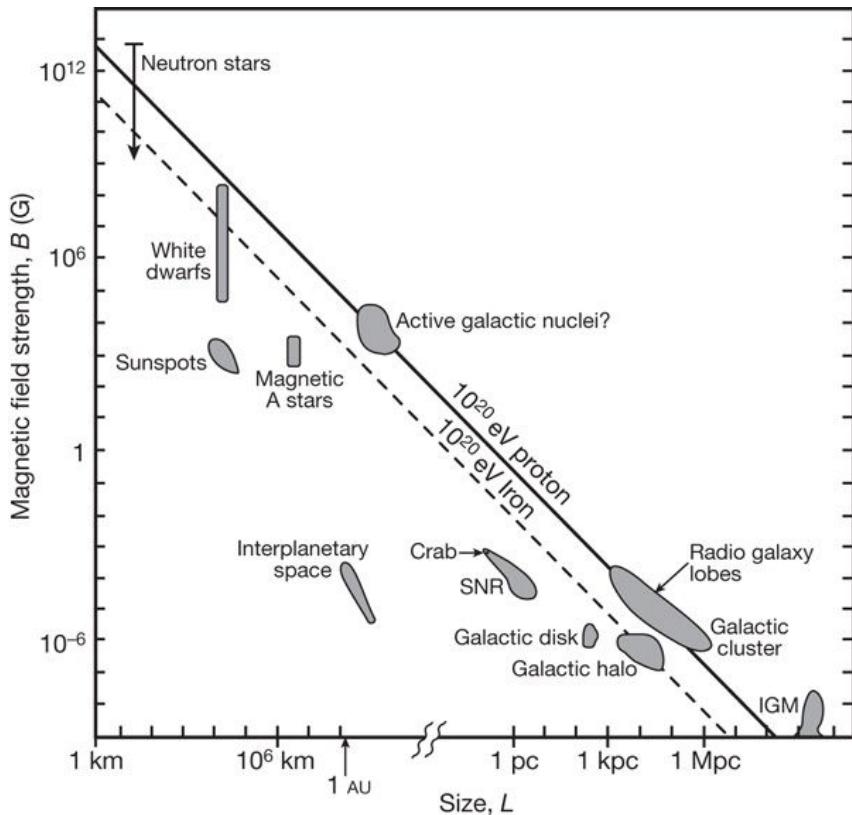


Figure 3.5: The Hillas plot of potential cosmic ray accelerators locates objects according to size and magnetic field. Objects to the left of the diagonal lines cannot accelerate particles to 10^{20} eV (proton: solid, iron: broken). Image obtained from HIER "REF" TOEVOEGEN, OVERAL EIGENLIJK [84]

accelerated if the trajectory of the particles can be changed and energy can be transferred multiple times. The magnetic fields responsible for the course change of these particles has to be sufficient in magnitude in order for these particles not to escape and go beyond the grasp of the source responsible for the acceleration. This limitation is expressed by the gyroradius in the accelerator, $r_L = E/ZeB$ similar to Eq. 3.3, requiring it to be smaller than the radius of the accelerator: $r_L < R$ or $E < ZeBR$.

Even if only qualitative, this relation provides an interesting criterion to identify possible sources of UHECRs by looking at the accelerator related term BR . This was done in a classic paper by Hillas [83], illustrated in the more recent Fig. 3.1.3.2. Accelerators necessary to explain the amount of UHECRs are not populated (enough) in our galaxy, making them more likely to be of extra-galactic origin. **Active galactic nuclei, blazars and gamma ray bursts** (en andere als je die hebt toegevoegd later) are therefore also briefly explained.

3.1.4 Sources

3.1.4.1 Supernova (remnants)

Supernovae can be broadly subdivided into two categories: type I and type II. Type I supernova explosions happen in binary star systems. In those systems one of the two stars is a carbon-oxygen white dwarf which accretes matter from the second star. When the total mass of the white dwarf reaches the Chandrasekhar REF limit of around 1.44 solar masses it cannot longer hold itself under the gravitational pressure and collapses in on itself. Within seconds, the carbon component in the white dwarf starts nuclear fusion and enough energy is released to produce an explosion brighter than the Sun with a factor of around 5 billion. A resulting shock wave can reach up to around 3% the speed of light.

Type II supernova explosions differ by being single star systems. When a star reaches the end of its lifecycle the subsequent fusion reactions reach to a halt. If the star has enough

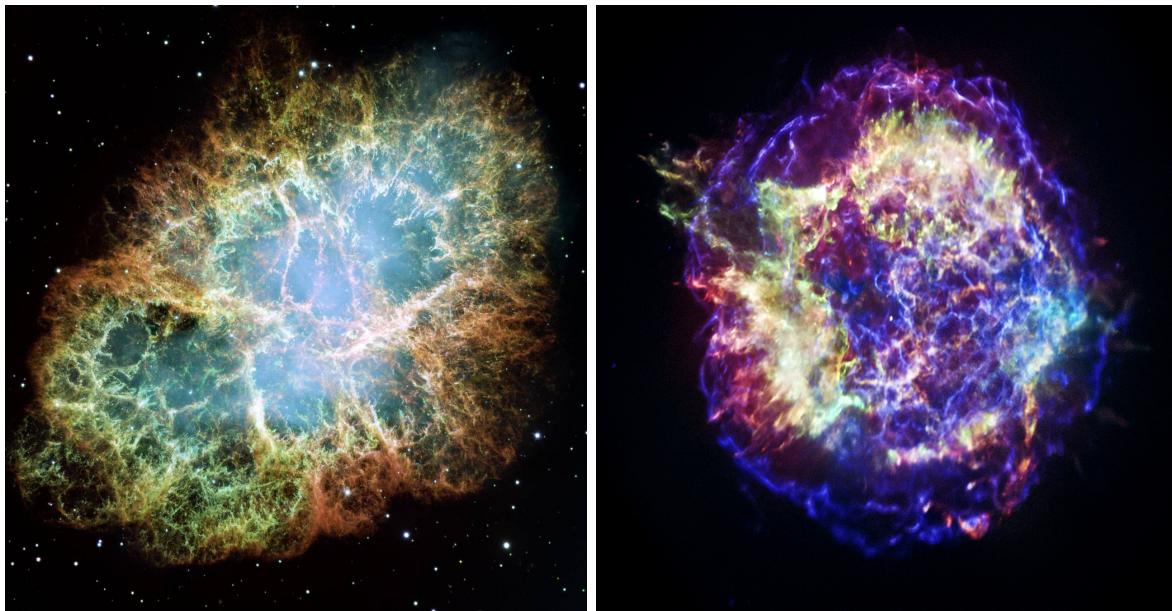


Figure 3.6: Left: the Crab Nebula is the supernova remnant approximately one thousand years old. The supernova was noted by Chinese astronomers in the year 1054 AD. Right: Chandra X-ray observatory picture of the Cassiopeia A supernova remnant (pictures from NASA).

mass (at least 8 times the mass of the Sun), it is possible for the inner core to again reach the Chandrasekhar limit and collapse in on itself due to the lack of *electron degeneracy*. Without the outward pressure of nuclear fusion reactions and the support of the core, the outer layers of the star collapse under the gravitational pressure. The compression of the electrons and protons into neutrons results into a very hot, dense, neutron core. The velocity of the inwards falling layers can reach to a staggering 23% of the speed of light and recoil when hitting the remaining core. Neutrinos are produced in this violent core collapse and the outward going shockwave hits the remaining outer layers forming the supernova explosion *.

Because of their brightness supernovae within our galaxy can be seen with the naked eye (provided they are not too far away). The last recorded supernova from our galaxy was by Johannes Kepler in 1604 but earliest recordings go back to 185 AD by Chinese astronomers[†]. The question remains how supernovae can serve as cosmic ray accelerators. In 1949, Enrico Fermi proposed a mechanism where particles can gain energy by collisions with moving interstellar ionized gas clouds. Only later, it was realized that a large, plane shock front moving with a certain velocity is able to accelerate charged particles much more efficiently. This first mechanism results into an energy transfer proportional to the squared velocity of the cloud and is thus called *second order Fermi acceleration*. Shock front acceleration energy transfer is proportional to the velocity and is called *first order Fermi acceleration*. Supernova remnants provide an explanation for the origin of these shock fronts.

First- and second-order Fermi acceleration

Suppose we have a magnetic cloud in the interstellar medium travelling with a certain velocity \vec{V} and a particle with velocity \vec{v} enters the cloud under an angle θ_1 (see Fig. 3.1.4.1). If we assume collisionless scattering can occur (no energy is dissipated from the particle to the cloud) due to the magnetic fields in the cloud, the magnitude of the momentum in the rest frame of the cloud will not change ($E'_1 = E'_2$, where we the apostrophe denotes the cloud rest frame). From special relativity we know that:

* Astronomer Carl Sagan once said, "The nitrogen in our DNA, the calcium in our teeth, the iron in our blood, the carbon in our apple pies were made in the interiors of collapsing stars. We are made of starstuff."

[†]From observations of other galaxies supernovae are expected to occur, on average, once every thirty years. Not all of these will be visible to the naked eye, but would almost certainly be observable with modern astronomical telescopes.

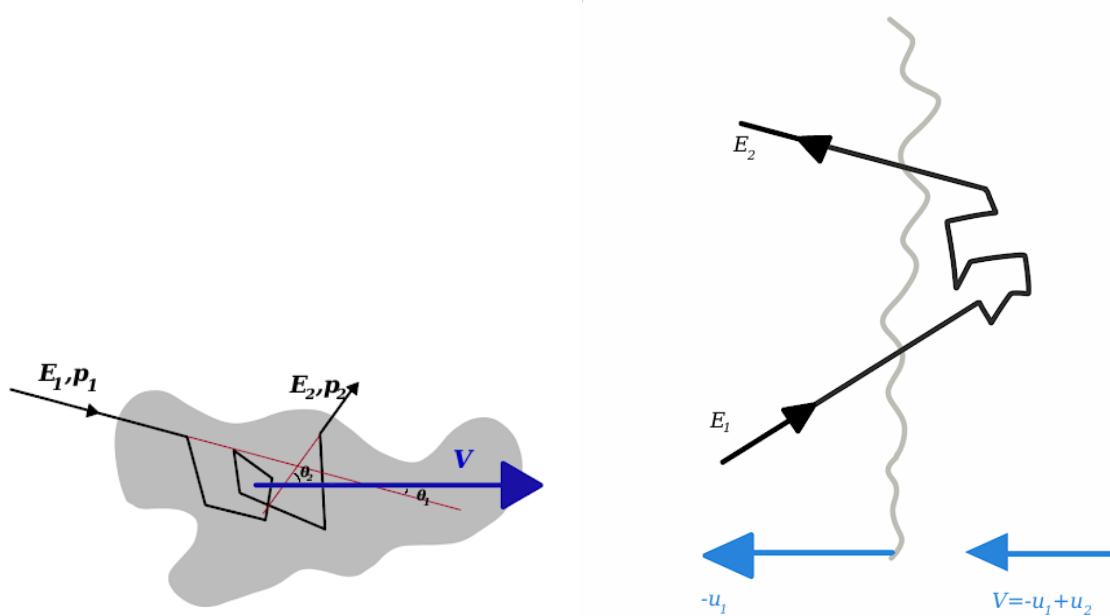


Figure 3.7: Left: magnetic cloud. Right: shock waves typically have magnetic inhomogeneities both preceding (downstream) and following them (upstream). If a charged particles travels through the shock wave it can gain velocity through first-order Fermi acceleration. In the illustration a particle travels from upstream to downstream and back upstream. At every back and forth movement the particle effectively gains in energy. For a particle with a velocity u_1 relative to the shock front the front seems to come at him with velocity $-u_1$. The downstream medium has a velocity relative to the shock front of $u_2 < u_1$ making it seem coming towards the particle with velocity $u_1 - u_2$.

$$\begin{aligned} E'_1 &= \gamma \left(E_1 - p_{1,\parallel} V \right) \\ &= \gamma E_1 (1 - \beta \cos \theta_1), \end{aligned} \quad (3.9)$$

with $\beta = V/c$ and γ the Lorentz factor. Similarly and using $E'_1 = E'_2$

$$\begin{aligned} E_2 &= \gamma E'_2 (1 + \beta \cos \theta'_2) \\ &= \gamma^2 E_1 (1 - \beta \cos \theta_1) (1 + \beta \cos \theta'_2) \end{aligned} \quad (3.10)$$

and

$$\frac{\Delta E}{E} = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta \cos \theta_1 + \beta \cos \theta'_2 - \beta^2 \cos \theta_1 \cos \theta'_2}{1 - \beta^2} - 1. \quad (3.11)$$

By hypothesis, the escaping particles are isotropic in the cloud frame: $\langle \cos \theta'_2 \rangle = 0$. One can show that $\langle \cos \theta_1 \rangle = -\frac{\beta}{3}$ [69], leading to

$$\frac{\Delta E}{E} = \frac{4}{3} \frac{\beta^2}{1 - \beta^2} \approx \frac{4}{3} \beta^2, \quad (3.12)$$

showing that for molecular clouds the energy gain is indeed proportional to the square of β for second-order Fermi acceleration.

If a particle is incoming to an expanding shock (see Fig. 3.1.4.1) $\langle \cos \theta'_2 \rangle$ is equal to $2/3$, leading to

$$\frac{\Delta E}{E} = \frac{\frac{4}{3}\beta + \frac{13}{9}\beta^2}{1 - \beta^2} \approx \frac{4}{3}\beta, \quad (3.13)$$

where β is now equal to $u_1 - u_2$ as explained in the caption of the figure. We have shown that for shock fronts the energy gain is indeed proportional to β for first-order Fermi acceleration. From both the outcome as the discussion it is clear that the energy gain enters through relativistic effects, making the intuitive approach not straightforward.

Power

The energy gain of a “single collision” results into the power law spectrum when considering a process in which a test particle increases its energy by an amount proportional to its energy with each encounter. Let us assume $\Delta E = \xi E$, after n encounters:

$$E_n = E_0 (1 + \xi)^n, \quad (3.14)$$

where E_0 is the energy when the particle first enters the accelerator medium. To reach a certain energy E' , the particles must encounter a number of collisions

$$n(E') = \frac{\ln \left(\frac{E'}{E_0} \right)}{\ln (1 + \xi)}. \quad (3.15)$$

To reach energies of E' or higher, the number of collisions will be proportional to

$$\begin{aligned} N(\geq E') &\propto \sum_{m=n}^{\infty} P_{present}(m) = \sum_{m=n}^{\infty} (1 - P_{esc})^m \\ &= (1 - P_{esc})^n \left((1 - P_{esc}) + (1 - P_{esc})^2 + \dots \right) \\ &= \frac{(1 - P_{esc})^n}{P_{esc}}, \end{aligned} \quad (3.16)$$

where $P_{present}$ is the probability of a particle still being present in the accelerator and P_{esc} the probability of the particle to escape after per collision. Making use of $a^{\ln b} = e^{\ln a \ln b} = b^{\ln a}$ and inserting Eq. B.4

$$N(\geq E') \propto \frac{1}{P_{esc}} \left(\frac{E'}{E_0} \right)^{-\gamma}, \quad (3.17)$$

with

$$\gamma = \frac{\ln \left(\frac{1}{1-P_{esc}} \right)}{\ln (1+\xi)} \approx \frac{P_{esc}}{\xi}. \quad (3.18)$$

The power law spectrum becomes visible in the derivative of the number of particles in energy

$$\frac{dN}{dE} \sim E^{-(\gamma+1)}, \quad (3.19)$$

in agreement with Eq. 3.1. Shock wave fronts have an expected $\gamma \approx 1$ giving rise to a different spectrum to what is seen on Earth but which can be explained by propagation from the source to Earth (this is beyond the scope of this work). The spectrum from Fermi shock acceleration is thus expected to follow an E^{-2} powerlaw behaviour.

Seems plausible all galactic CRs are accounted for my supernovae. This is supported with the realization that first-order Fermi acceleration naturally produces a spectrum of cosmic rays close to what is observed.

Maximum energy

The highest energies that particle can be accelerated to can be defined by:

- The differential energy gain per collision dE/dt
- The total time the particle can be accelerated
- . The energy gain is given by

$$\frac{dE}{dt} = \frac{\xi E}{T_{cycle}}, \quad (3.20)$$

where T_{cycle} is the charactersitic time for one acceleration cycle. T_{cycle} depends on the diffusion coefficients and velocities of the upstream and downstream regions and is set to $T_{cycle} \geq 20E/(3u_1ZeB)$ by Lagage and Cesarsky [85] for a strong shock and arguing that the diffusion lenth, λ_D cannot be smaller than the Larmor radius of the parcticle. Particles with a Larmor radius greater than the irregularities holding a magnetic field are not prone to be heavily influenced by them. Lagage and Cesarsky therefore concluded that

$$E_{max} \leq \frac{3}{20} \frac{u_1}{c} ZeB(u_1 T_{ST}), \quad (3.21)$$

where T_{ST} is the Sedov-Taylor time where particles are less prone to escape and is ~ 1000 years. For $u_1 \sim 10^9$ cm/s [86] and $B \sim 3\mu G$ the Lagage and Cesarsky limit reads

$$E_{max} \leq Z \times 2.4 \times 10^5 GeV. \quad (3.22)$$

3.1.4.2 Active Galactic Nuclei

Active Galactiv Neuclei (AGNs) are no stars at the end of their life cycle but active black holes located in the center of galaxies. In most older galaxies the stars at the center have reached the end of life and have gone supernova leaving behind white dwarfs or black holes. It is believed that most massive galaxies have supermassive black holes in their centers by the accretion of matter from surrounding large gas clouds [87, 88]. Their masses in current models range from 10^6 to 10^{10} Solar masses [89].

The efficient conversion from gravitational potential energy to kinetic energy and radiation make AGNs the most luminous persistent sources of electromagnetic radiation in the universe, and as such very good means in discovering distant objects. The accretion discs heat up due to



Figure 3.8: Left: artist impression of a blazar which is recently been linked to a possible origin of extragalactic neutrinos [TXS]. Illustration from DESY, Science Communication Lab. Right: Image from the Hubble telescope where we see a jet streaming out from the center of galaxy M87.

the inward falling and produce light peaking in the ultraviolet waveband. Certain emission lines are also expected due to the radiation from excited cold atomic material. Some accretion discs produce jets which point opposite each other and their direction is defined by either the spin of the black hole, the accretion disc or a combination of both. The most powerful AGNs are classified as *quasars* and AGNs with a jet pointing toward the Earth is called a *blazar*.

Charged particles have large cyclotron radii in AGNs and the relativistic jets could provide the necessary mechanisms to accelerate particles to ultra-high energies. Pierre Auger hinted to a correlation of the highest-energy cosmic rays with the positions of nearby active galactic nuclei [90]. Recently, a collaborative effort of IceCube, Fermi-LAT, MAGIC and others observed a coincidence of high-energy neutrinos and a blazar making them very good candidates of sources of extragalactic neutrinos [91].

Misschien nog net iets meer uitwerken hoe UHECRs hier uit kunnen komen?

3.1.4.3 Gamma Ray Bursts

The most catastrophic deaths of massive stars or mergers of neutron-neutron stars or a neutron star and a black hole result into Gamma Ray Bursts (GRBs). GRBs are named after the burst of gamma rays that is followed by a longer-lived afterglow of electromagnetic radiation at longer wavelengths. These bursts are the most energetic explosions in the electromagnetic spectrum and occur when a high-mass star collapses to form a neutron star or black hole. A typical burst releases as much energy in a few seconds than the Sun will do in its entire 10 billion-year lifetime and temporarily outshines the rest of the galaxy*. GRBs are isotropically distributed, making them extragalactic in origin [93].

An often used model to explain how charged particles could reach extremely high energies is called the *fireball model*. This internal-external shock model assumes that kinetic energy of an ultra-relativistic flow is dissipated in internal collisions. When the shock hits the surrounding matter it is slowed down and gives rise to the signature afterglow [94]. After initial the initial progenitor phase (see below) a plasma of photons, electrons, positrons and baryons develops into the formed jets. In this initial phase, the fireball is radiation-dominated and optically thick for photons, making it invisible in the electromagnetic spectrum. Due to radiative pressure the fireball expands at relativistic speeds (γ -factors > 100) to the point that it becomes more and more transparent. If the central engine produces multiple shocks with different velocities there will be internal shocks, which give rise to the observed burst emission. In this mechanism, the ultra-relativistic matter can transfer its kinetic energy to the acceleration of particles, explaining

*GRBs were first discovered in the late 1960s by accident. The Vela satellites had additional gamma ray detectors designed to detect very fast bursts of gamma rays which are expected to be produced by nuclear tests in space [92]

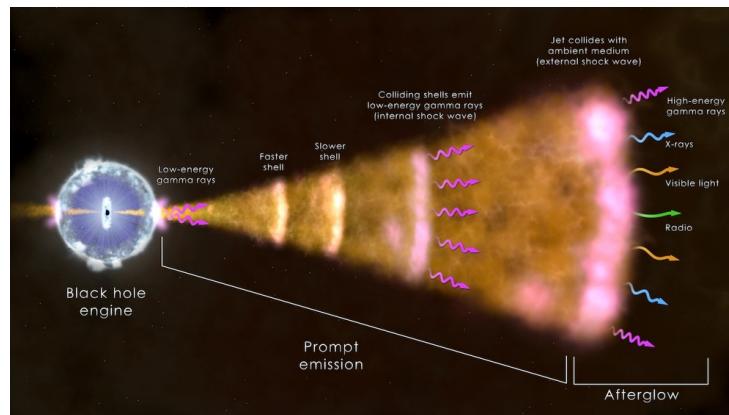


Figure 3.9: In the fireball model Image from NASA Goddard space flight center.

cosmic ray production. Later shocks of the jets with surrounding matter would explain the signature afterglow seen in GRBs.

Although there is still much ongoing discussion, GRBs are sometimes subdivided into two regions: *long gamma ray bursts* ($t_{burst} > 2\text{ s}$) and *short gamma ray bursts* ($t_{burst} < 2\text{ s}$). Long bursts originate from collapsars: a massive star core-collapse forms a black hole and surrounding matter is pulled into an accretion disk. Short bursts hint to progenitors that are extremely compact, where neutron-neutron star or neutron star and black hole mergers are the most probable explanation. The recent detection of the gravitational waves can provide a significant contribution to the understanding of these sources [95, 96, 97, 98, 99].

3.1.4.4 Starburst galaxies

Galaxies that undergo an episode of large-scale star formation, are called *starburst galaxies*. Most of these are in the midst of a merger close encounter with another galaxy. Several experiments have shown their gamma ray emission at several hundred GeV to be two to three orders of magnitude of that in our own Galaxy [100, 101]. Galactic scale winds from the central regions are possible sources for cosmic ray acceleration.

3.1.4.5 Galaxy clusters

When galaxies are bound together by gravity they are referred to as *galaxy clusters*. They can contain around 100 to 1000 galaxies and have typical mass ranges around $10^{14} - 10^{15}$ Solar masses. Through merging and accretion of dark matter and baryonic gas, galaxy clusters are expected to generate powerful shock waves on large scales. Shocks with significant velocities could provide the necessary conditions for cosmic ray acceleration [IeC \textbullet 1538-4357-689-2-L105].

3.1.5 Propagation

Doen of niet?

3.2 Air showers

When primary cosmic ray particles hit the Earth's atmosphere they give rise to a large shower of secondary particles. At low- to mid-energy ranges the abundance of cosmic rays is large enough for these showers to be analyzed with balloon or satellite experiments. As indicated in section 3.1.2, the flux of high-energy cosmic rays is so small, there is a need for very large-scale detectors, measuring kilometers in instrumented area.

The interaction length of nuclei with high energies is too small for them to be able to further than tens of kilometers in height. They will interact with an atmospheric nucleus and produce secondary particles. These particles, on their turn, decay or further interact with the atmosphere and give rise to an *extensive air shower* if the production of new particles is large enough. Some of these particles will be stopped, but others are capable of reaching the Earth

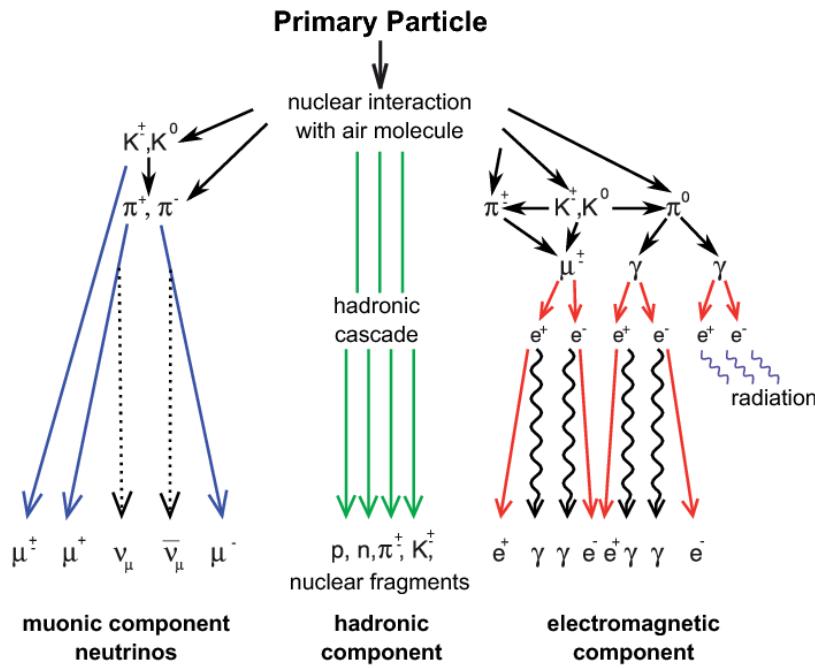


Figure 3.10: Schematic view of an extensive air shower with a clear distinction between the three components. Image from KASKADE collaboration.

or even penetrate deep inside it. Although air showers are of significant importance in cosmic ray studies, we will only give a brief summary of the most noteworthy features and its main importance for this analysis. An air shower has three components: the hadronic, muonic and electromagnetic. The hadronic component can be seen as the core of the shower consisting of high-energy hadrons. The interactions and subsequent decays of these hadrons fuel the electromagnetic and muonic parts. A schematic overview is given in Fig. 3.2. If the primary particle is a photon the shower is made up almost exclusively of an electromagnetic component. Because the lateral size of an electromagnetic cascade is caused by multiple scattering of electrons and positrons the lateral size of these showers is relatively small (radius around 1 km for a transversely downgoing 100 TeV photon). In hadronic cascades, on the other hand, the lateral size is caused by the transverse momenta of the secondary particles making these showers much larger (radius around 4km for a transversely downgoing 100 TeV proton) [102].

3.2.1 Hadronic component

When a proton interacts with a nucleus it interacts with a proton or neutron and will most often produce charged or neutral pions

$$\begin{aligned} p + N &\rightarrow p + N + k\pi^+ + k\pi^- + r\pi^0, \\ p + N &\rightarrow n + N + (k + 1)\pi^+ + k\pi^- + r\pi^0, \end{aligned} \quad (3.23)$$

where N stands for a nucleon of an atmospheric nucleus and k and r are the multiplicities of the produced pions. The example for heavier nuclei from this is straightforward. On average, one-third of the hadron production will be neutral pions which decay immediately into electromagnetic particles

$$\pi^0 \rightarrow \gamma + \gamma \quad (3.24)$$

$$(98.8\%) \text{ or} \quad (3.25)$$

$$\pi^0 \rightarrow e^+ + e^- + \gamma \quad (3.26)$$

$$(1.17\%). \quad (3.27)$$

The other two-thirds will be charged particles and have a lot longer lifetime, making them much more probable to interact with air nuclei. After having traveled a distance corresponding to their mean interaction length, charged particles interact again with air nuclei if their energy is great enough. 90% of these charged particles are new pions and 10% of the daughter particles are kaons. Pions almost exclusively decay into muons ($\pi^+ \rightarrow \mu^+ + \nu_\mu$) and the most dominant kaon decay modes are (similar for K^-) [pdg]

$$K^+ \rightarrow \pi^+ + \pi^0 \quad (20.7\%), \quad (3.28)$$

$$K^+ \rightarrow \mu^+ + \nu_\mu \quad (63.6\%), \quad (3.29)$$

$$K^+ \rightarrow \pi^0 + e^+ + \nu_e \quad (5\%), \quad (3.30)$$

$$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu \quad (3.4\%), \quad (3.31)$$

$$(3.32)$$

where the first decay mode fuels the hadronic component further. The remaining decay modes enter in the EM and muonic components. The total number of hadrons reaching sea level is very small and when they do, they are immediately stopped.

3.2.2 Muonic component

Muons are the dominant component of particles reaching sea level (around 80%). Most muons which are produced in an EAS are able to reach so far due to their relativistic velocities and lifetime of $2.2 \mu\text{s}^*$. They have relatively low ionization losses compared to electrons, making them very penetrating and are therefore referred to as the *hard component*. Muons can also decay and contribute to the electromagnetic component via

$$\begin{aligned} \mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu, \\ \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu. \end{aligned} \quad (3.33)$$

3.2.3 Electromagnetic component

At each hadronic interaction, slightly more than a third of the energy goes into the electromagnetic component. Since most hadrons re-interact, eventually most of the primary energy finds its way into the electromagnetic component. Muons can produce *delta electrons* or electron-positron pairs from pair production (see section??).

At energies above a few MeV photons interact with matter via pair production and convert into an electron-positron pair. High-energy electrons and positrons primarily emit photons via bremsstrahlung. These two processes are repeated until the photons fall below the pair production threshold and bremsstrahlung energy loss starts to dominate. Because electrons lose their energy fast they are almost immediately stopped when they reach dense matter (Earth's surface) and hence referred to as the *soft component*.

3.2.4 Neutrino component

Leeg laten en pas beschrijven hieronder?

3.3 Neutrinos

As by-products of cosmic ray collisions with matter, neutrinos provide incontrovertible evidence for hadronic acceleration. Since these particles are weakly-interacting they can escape much denser environments and hold crucial information about the origins of their production environments. Because these particles barely interact, their detection is difficult. Similarly to cosmic rays, neutrinos cover a broad range in energy (see Fig. ??), calling for different types of detector to cover this large spectrum.

*The half-survival length of 5 GeV muons is $L = \ln(2) \times \gamma \times 2.2\mu\text{s} \times 0.9998 \times c = \gamma \times 456 \text{ m} \approx 23 \text{ km}$. The relativistic time dilation is of crucial importance here!

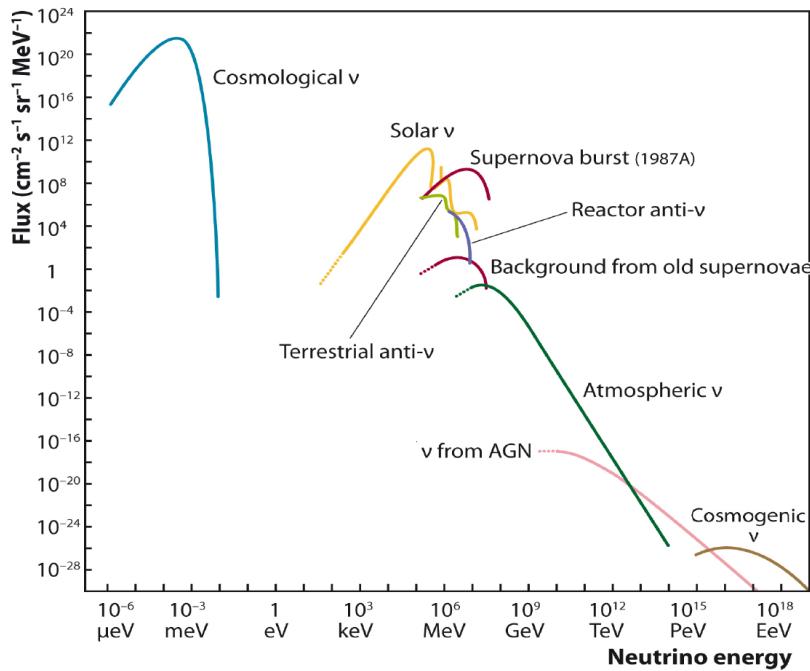


Figure 3.11: Plot illustrating several neutrino fluxes which cover a huge range of energy. Illustration from [Katz:2011ke].

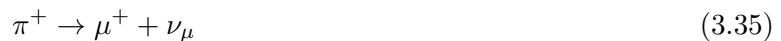
Cosmic rays are deflected in inter- and extragalactic magnetic fields and therefore their arrival direction at Earth does not hold much pointing information (Fig. 3.3, left). Light ranging from radio to gamma rays on the electromagnetic spectrum is of a crucial importance in astrophysics but has its limitations. Photons can be absorbed by interstellar medium, or are trapped in opaque sources. At higher energies ($\approx 10^{14}$ eV) photons interact and produce electron-positron pair ($\gamma + \gamma \rightarrow e^+ + e^-$). Unless the sources are closeby no photons are capable of reaching Earth (see Fig. 3.3, right). Neutrinos escape from the sources more easily and are not deflected by magnetic fields making them key messengers in identifying cosmic ray accelerators. In the following we will go over the different types of neutrinos which are detectable on Earth.

3.3.1 Conventional

Neutrinos are produced in large abundances in air showers as explained in 3.2. The neutrinos that are produced with low to high energies (\approx MeV to PeV range) are called *atmospheric* or *conventional* neutrinos. They are primarily produced in pion on kaon decay. Due to helicity effects, pion and kaon decay to electron(s) (neutrinos) is strongly suppressed compared to decays into muon(s) (neutrinos). As a result, the ratio of electron neutrinos to muon neutrinos is a factor of around two

$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \approx 2, \quad (3.34)$$

which should be clear when we look at the example of pion decay where the muon decays as well



The most referred to calculations for the atmospheric neutrino flux was done by Honda et al. [Honda:2006qj].

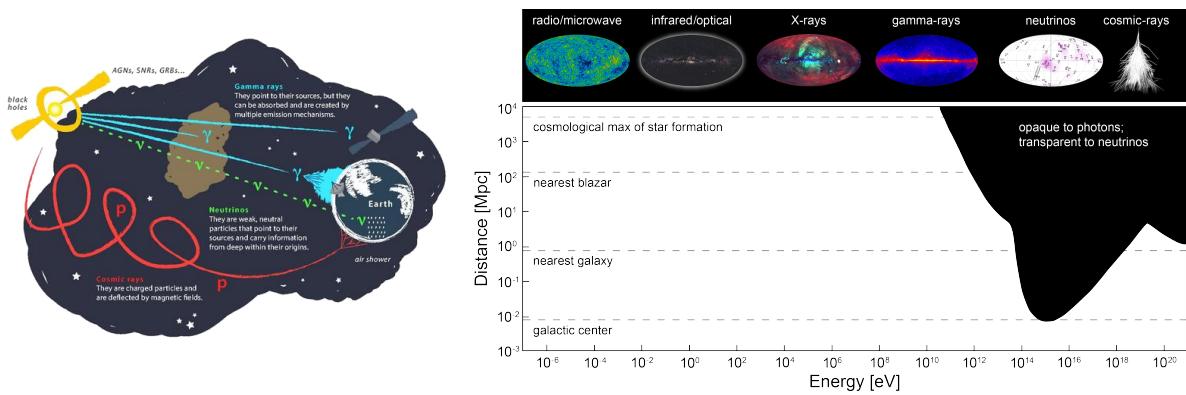


Figure 3.12: Left: Artist impression of the path that several types of particles travel before reaching Earth. Right: Illustration of the visibility of sources in function of their distance and the photon energy. The signature dip in the photon visibility comes from the pair production peak when photons interact with the CMB. Both illustrations from the IceCube collaboration.

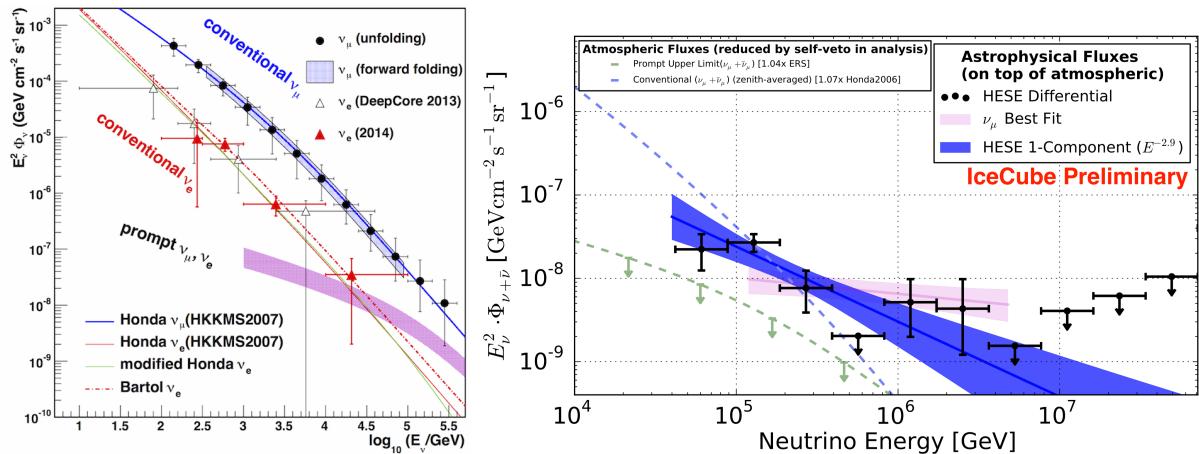


Figure 3.13: Left: Measurement from the IceCube collaboration showing the difference in ν_e and ν_μ flux. Right: Measured differential astrophysical flux using contained events (points) and a fit do that data (blue line and band), compared with the best fit obtained from through-going ν_μ (pink line and band). From Ref. [103]

3.3.2 Prompt

Charmed mesons, also called D mesons, are the lightest particles that contain charm quarks*. Hints of charm particles were first seen in cosmic rays in 1971 by Niu et al. [104]. The production of these particles is strongly suppressed, but are expected to exhibit a harder spectrum than conventional neutrinos do. These mesons have short lifetimes (hence the name: prompt) and decay into neutrinos independent of their energy and arrival direction. Therefore, their energy spectrum is expected to follow that of primary cosmic rays. Their contribution at higher energies can be non-negligible or even become dominant. To this date is has not been possible to observe this prompt component, but remains an interesting signal in diffuse neutrino searches and could contribute significantly to background expectations. The most referred to calculations for the prompt neutrino flux was done by Endberg et al. [105].

3.3.3 Astrophysical

Astrophysical neutrinos are expected to be created when cosmic rays interact close to their interaction sites. Because they are neutral and are unlikely to be absorbed, astrophysical neutrinos are expected to reveal more information about these sources. To first order, these neutrinos

* $D^0 : c\bar{u}, \bar{D}^0 : u\bar{c}, D^+ : c\bar{d}, D^- : d\bar{c}$

would follow the spectrum of cosmic rays at their production. As indicated in section 3.1.4.1, this is equal to an E^{-2} powerlaw spectrum from Fermi shock acceleration. The majority of these neutrinos are expected to arise from decays from pions which were created in these cosmic-ray interactions ($\pi \rightarrow \mu + \nu_\mu$) followed by the muon decay ($\mu \rightarrow e + \nu_e + \bar{\nu}_\mu$). The resulting flavor ratio fraction is $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ at the source. Neutrino oscillations across cosmological distances give an expected $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$ expectation at Earth. As can be seen in Fig. [fig:neutrinospectrum2].

The spectrum is expected to follow the hardest spectrum of the three and therefore dominates at the highest energies.

3.3.4 Other neutrino sources

3.3.4.1 Cosmological

Similar to the photons from the CMB, neutrinos would have been able to decouple from matter only seconds after the Big Bang. Due to the expansion of the universe, the temperature of the neutrinos has dropped to $\approx 1.95K$ due to redshift. At these energies direct detection is near to impossible. They have no measureable effect in large-scale neutrino detectors.

Mooie uitleg van redshift: <https://www.forbes.com/sites/startswithabang/2016/09/09/cosmic-neutrinos-detected-confirming-the-big-bangs-last-great-prediction/>

3.3.4.2 Solar

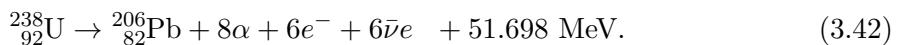
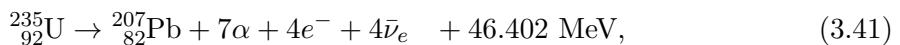
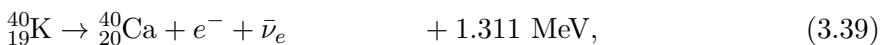
Nuclear fusion in the Sun is responsible for the production of electron neutrinos and is the largest contribution of neutrinos which can be detected on Earth. 86% of neutrinos are produced by the proton-proton reaction

$$p + p \rightarrow d + e^+ + \nu_e. \quad (3.37)$$

The remaining part is produced by reactions which involve more heavy particles. Having energies < 20 MeV these neutrinos have no measurable effect in neutrino telescopes.

3.3.4.3 Terrestrial neutrinos

Radioactive decays from ^{40}K , ^{232}Th and ^{238}U account for almost all geoneutrinos. The reactions give rise to neutrinos from beta decay [106]



The maximal energies of these neutrinos are again too low to give a contribution to neutrino telescopes.

3.3.4.4 Reactor neutrinos

Nuclear reactors harness energy from the splitting (fission) of heavy nuclei into lighter fission products. These neutron-rich daughter particles undergo beta decays ($n \rightarrow p + e^- + \bar{\nu}_e$). Reactor neutrinos are therefore always antineutrinos. The energy spectrum reaches a maximum around 10 MeV, making them again invisible for neutrino telescopes.

3.3.4.5 Supernova neutrinos

The core collapse of stars where electrons and protons are compressed into neutrons as described in ??? dissipates most of it's energy in the production of neutrinos ($e^- + p^+ \rightarrow n + \nu_e$) [107]. Depending on the distance of the source to the detector supernovae are only visible in a collective raise of the dark noise of the apparture [Kopke:2011xb].

3.4 Cosmic ray and neutrino detectors

Heel kort: pg 5 Gaisser boek. Zo IC introduceren.

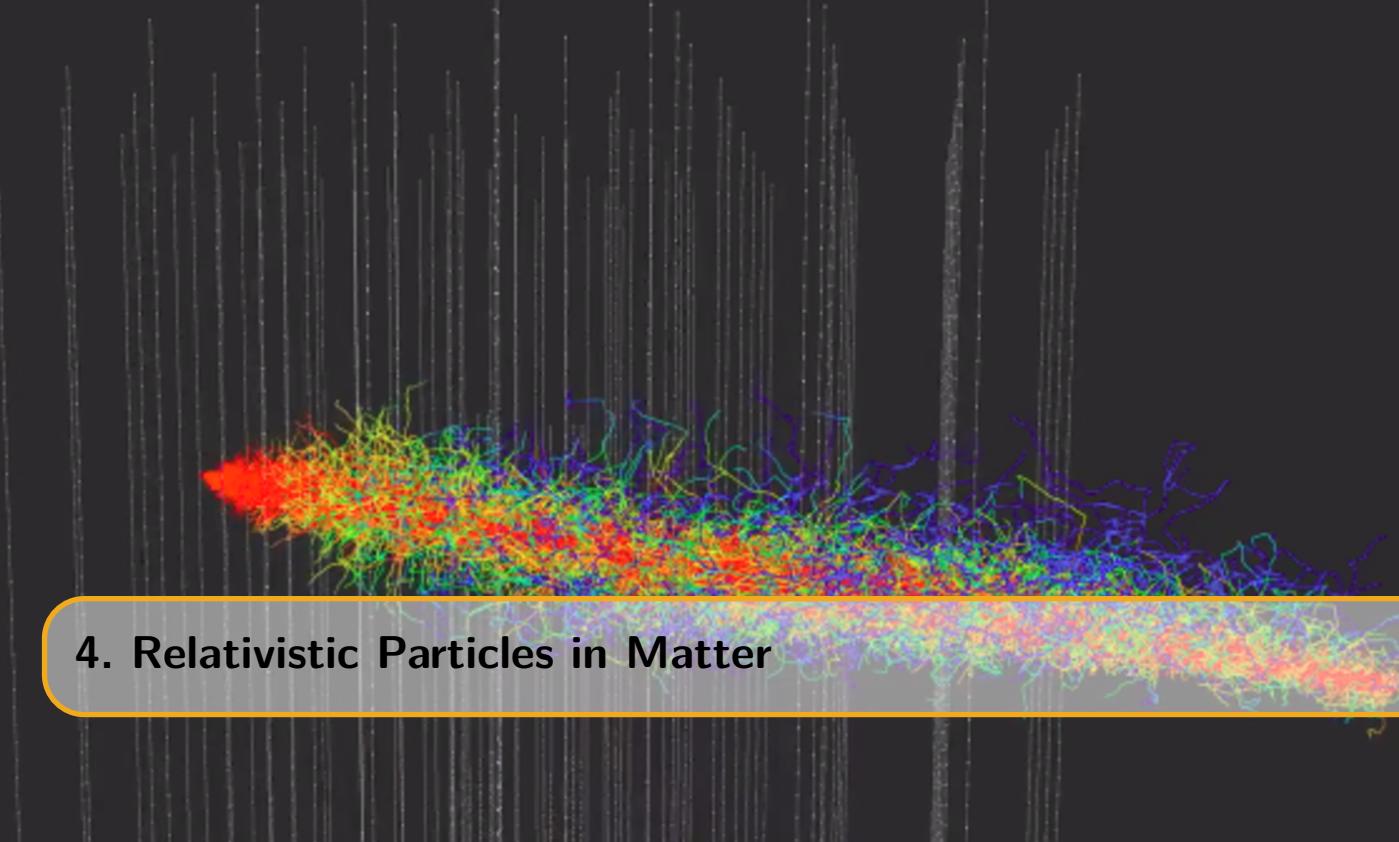
VERITAS, HESS, MAGIC, HAWC, PA, TA, Super-K, Antares, IceCube

Ook dat plotje waarbij je toont dat neutrinos interessanter zijn in hoge E omdat fotonen onzichtbaar worden!

The IceCube Collaboration instead tested the principle using neutrinos. Neutrinos interact with matter through the weak force — one of the four fundamental forces of nature. The influence of the weak force is limited to minute distances. As a result, interactions between neutrinos and matter are extremely improbable, and a neutrino can easily traverse the entire Earth unimpeded. This poses a challenge for physicists trying to study these elusive particles, because almost every neutrino will simply pass through any detector completely unnoticed.

In neutral-current reactions neutrinos lose energy, but are not absorbed

hier tot pg 50?



4. Relativistic Particles in Matter

Neutrinos are notoriously difficult to detect. When high-energy neutrinos interact with matter, they produce secondary particles that travel fast enough to produce Cherenkov radiation. As indicated in section 3.4 cubic-sized experiments try to exploit this properties by using natural ice or water as the instrumented volume. In this chapter an overview of the behaviour of relativistic matter is given. The experiment that is used here is the IceCube detector, therefore there will be a larger focus on ice.

4.1 Neutrino interactions

Neutrino interactions with matter in both the charged current (CC) and neutral current (NC) interactions. In the former, the mediator particle is a charged W-boson resulting in a charged lepton in the final state. In the latter, the mediator particle is the neutral Z-boson. Both interaction types have a resulting hadronic component as daughter particles. The interactions can be written as

$$\nu_l (\bar{\nu}_l) + N \xrightarrow{W^+} l^- (l^+) + X^{+(-)} \quad (CC) \quad (4.1)$$

$$\nu_l (\bar{\nu}_l) + N \xrightarrow{Z} \nu'_l (\bar{\nu}'_l) + X \quad (NC), \quad (4.2)$$

where l is the lepton flavor (e, μ, τ), N denotes the initial hadronic state of the nucleus and X the final hadronic state. These interactions are illustrated in Fig. 4.1.

The charged leptons and hadrons lead to light production via gamma ray production and Cherenkov radiation. With the right material it is possible to detect this light production and reconstruct some of the neutrino's characteristics.

4.2 Cherenkov effect

From Einstein's works on special and general relativity it follows that the speed of light in vacuum, c , is a universal constant. The speed of light in matter can be significantly lower than that. If a particle travels through a dielectric medium at a speed which is greater than the phase velocity of light in that medium, electromagnetic radiation is emitted. This radiation is called *Cherenkov radiation* and is named after the first person who was able to detect it experimentally,

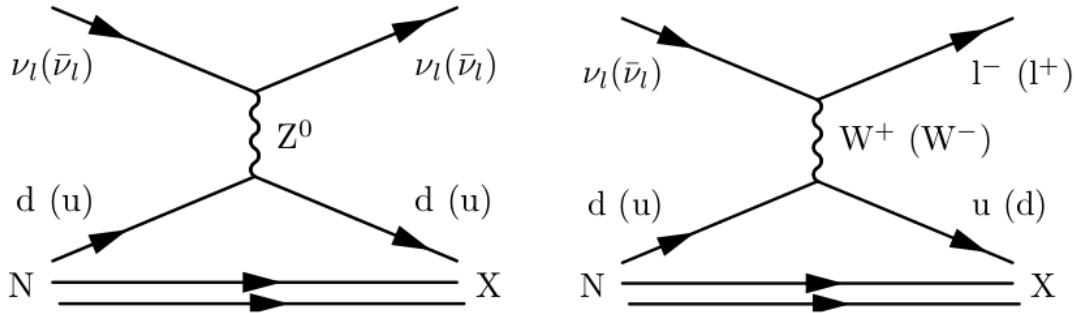


Figure 4.1: Feynman diagrams of the NC (*left*) and CC (*right*) OVERAL SCHUINE LETTERS LINKSRECHTS? neutrino interactions. l is the lepton flavor (e, μ, τ), N denotes the initial hadronic state of the nucleus and X the final hadronic state. The antineutrino interactions are given in between brackets.

Pavel Cherenkov. He was awarded the Nobel Prize in 1958 for his findings together with Frank and Tamm on their theoretical work on the subject [108].

As can be seen in Appendix [ch:planck], equation [eq:wave], the velocity of a propagating wave is given by

$$\nabla^2\psi = \frac{1}{v^2} \frac{\partial\psi}{\partial t}, \quad (4.3)$$

where ψ is the wave and v its group velocity. From Maxwell's equations and some vector calculus it is straightforward to find that the wave equation for electromagnetic radiation becomes

$$\nabla^2 E = \mu\epsilon \frac{\partial E}{\partial t}, \quad (4.4)$$

where E is the electric field and μ and ϵ the permeability and permittivity of the medium respectively. From these equations it is clear that for light in a dielectric medium

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r\epsilon_r}} \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_r\epsilon_r}} \times c \leq c, \quad (4.5)$$

where $1/\sqrt{\mu_0\epsilon_0} = c$ and μ_r and ϵ_r are the relative (to vacuum) permeability and permittivity respectively and are ≥ 1 . These terms are also written as the refractive index $n = \sqrt{\mu_r\epsilon_r}$: $v = c/n$.

When a charged particle moves inside a dielectric medium, it excites the molecules of the medium to the higher levels and excited states. The molecules emit photons in the form of electromagnetic radiation upon returning back to their ground state. According to the *Huygens principle*, the emitted waves move out spherically at the phase velocity of the medium (which can be less than the speed of light in vacuum). If the motion of the particle is slow, the radiated waves bunch up slightly in the direction of motion, but they do not cross. However if the particle moves faster than the light speed, the emitted waves add up constructively leading to a coherent radiation at angle θ_c with respect to the particle direction; Cherenkov radiation. The coherent interference is enough to be visible to the naked eye*. The signature of the effect is a cone of emission in the direction of particle motion. Fig. 4.2 shows a schematic of the Cherenkov radiation showing the typical spherical wavefront and the resultant radiation.

From the figure we can derive that

$$\cos \theta_c = \frac{\frac{c}{n}\Delta t}{v\Delta t} = \frac{c}{vn} = \frac{1}{\beta n}. \quad (4.6)$$

Because $-1 \leq \cos \theta_c \leq 1$, the velocity of the charged particle must be $v \geq c/n$. Typical values of n are on the order of 1-2, requiring the particles to be relativistic in order to emit Cherenkov

*The typical blue light in the cooling water at nuclear reactor sites is also due to this Cherenkov radiation.

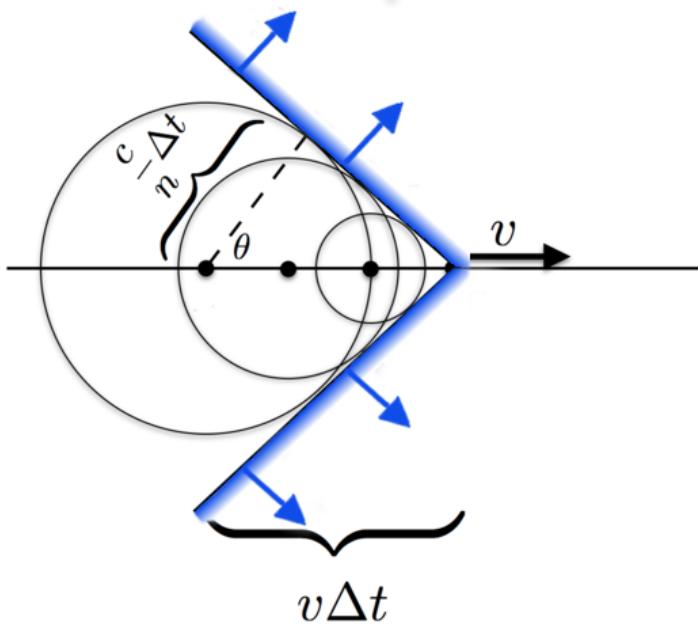


Figure 4.2: Schematic overview of Cherenkov radiation from a particle traveling at a velocity v .

radiation. The number of photons produced per unit path length of a particle with charge ze and per unit energy interval of the photon was calculated by Frank and Tamm, and is often referred to as the Frank-Tamm equation

$$\begin{aligned} \frac{d^2N}{dEdx} &= \frac{\alpha z^2}{\hbar c} \sin^2 \theta_c = \frac{\alpha^2 z^2}{r_e m_e c^2} \left(1 - \frac{1}{\beta^2 n^2(E)}\right) \\ &\approx 370 \sin^2 \theta_c(E) \text{ eV}^{-1} \text{ cm}^{-1} \quad (z=1), \end{aligned} \quad (4.7)$$

where r_e is the classical electron radius, m_e the electron mass and α the fine-structure constant. Equivalently, this equation can be written in function of the wavelength of the photon

$$\frac{d^2N}{dxd\lambda} = \frac{2\pi\alpha z^2}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right). \quad (4.8)$$

As an example, air Cherenkov telescopes such as MAGIC, H.E.S.S and VERITAS look for the direct and indirect Cherenkov light from gamma rays and cosmic rays. Because the refractive index of air is close to 1 (1.000293 at sea level and smaller with increasing height) the opening angle of the Cherenkov cone is small ($\approx 1^\circ$). The particles need to be pretty relativistic in order for Cherenkov radiation to occur*.

In water and ice, the refractive index is ≈ 1.33 , making $\beta_{min} = 0.75$ and $E_{min} = 1.51 \cdot m_0$. Experiments using water or ice as the interaction medium are Super-Kamiokande, ANTARES and the IceCube experiment.

4.3 Propagation

As described in section 4.1, neutrinos will give rise to several types of interactions in the surrounding medium. There are three characteristic signatures which are the main interest in the IceCube detector.

*Let us assume the refractive index of air at sea level, than from $E^2/m^2 = \gamma^2 = 1/(1 - \beta^2)$ it follows the minimal energy is ≈ 41 times its rest mass. Since the refractive index decreases in function of height, the energies of particles interacting with the atmosphere must be even higher.

4.3.1 Cascades

A charged current neutrino interaction the energetic electron will give rise to a shower of gamma rays (bremsstrahlung), positrons and electrons (pair production). Positrons and electrons will in their turn emit new gamma rays and this process continues until the photon energies fall below the pair production threshold. Because electrons/positrons lose their energy fast, they are almost immediately stopped, giving an electromagnetic cascade a spherical shape. In the case of neutral current events, the breakup of the struck nucleus leads to charged byproducts. These byproducts can reinteract in the medium and produce neutral pions that decay into gamma rays. These particles again die out quickly, resulting in a spherical emission of light.

4.3.1.1 Energy loss

Er stond echt wel ergens iets, ook thesis nalezen bij muon stuk. Mss dingen verkeerd of incompleet HIERE

4.3.2 Muon track

Muons are produced in charged current muon neutrino interactions and travel much further than electrons and positrons. The relativistic muon will produce light according to the Frank-Tamm equation 4.8, resulting in *direct Cherenkov radiation*. Ionization, bremsstrahlung, pair production and photonuclear interactions (see section 4.4) are also capable of producing relativistic secondary particles that will produce *indirect Cherenkov radiation*. Both effects result in a Cherenkov cone with a diffuse light emitting out from the track in all directions behind it.

4.3.2.1 Energy loss

Below 1 TeV, muons will lose most of its energy to ionization losses. A charged particle traversing matter will ionize the material around it. When the energy transfer is high enough, electrons can be stripped away from their atoms, resulting in *delta electrons*. As can be seen in Fig. 4.3.2.1, ionization losses have only a very weak energy dependence. It is therefore very difficult to distinguish for example a 50 GeV from a 500 GeV muon as the direct Cherenkov light production will be similar (Eq. 4.8) and the energy loss will be from the energy-independent ionization.

Above 1 TeV, however, the muon will on average lose more energy to stochastic effects*. Here, effects such as bremsstrahlung, pair production and the photonuclear effect dominate over ionization. Indirect Cherenkov production starts to dominate and make the energy estimation much easier.

The average energy loss along the muon trajectory can be parametrized by

$$-\frac{dE}{dx} = a + b \cdot E_\mu, \quad (4.9)$$

where a and b are obtained by fitting and can be found in Table 4.3.2.1 † ???. The muon range can be found by integrating Eq. 4.9

$$R_\mu \approx \frac{1}{b} \ln \left(\frac{E_\mu}{E_{th}} + 1 \right), \quad (4.10)$$

with $E_{th} = a/b = 720$ GeV, the energy threshold above which stochastic effects are dominant.

4.4 Energy loss formulae

Zie thesis Dima en PPC paper.

4.4.1 Ionization

Bethe Bloch

*In this context we mean the energy losses are not deterministic: it is impossible to know when an interaction of this kind will occur. One can only make estimations on their *expected* effects.

†Mwe stands for “meter water equivalent”, a unit often used in cosmic ray physics. A detector shielded by matter equal to 100 mwe would be equally shielded from cosmic rays if it were 100 meters below water.

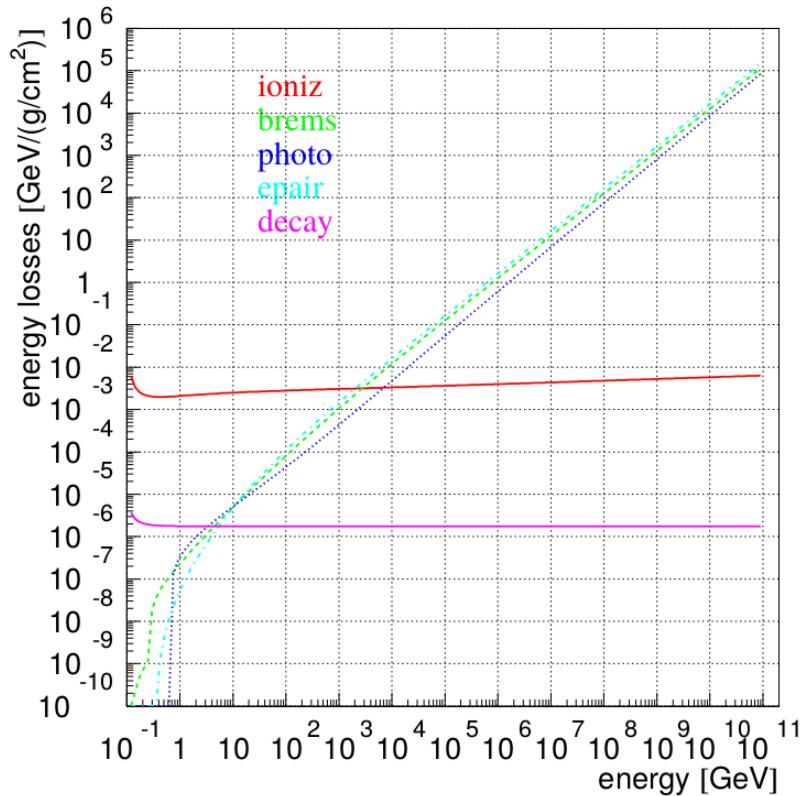


Figure 4.3: Muon energy loss from ionization (upper solid curve), bremsstrahlung (dashed), photonuclear (dotted), pair production (dashed-dotted) and decay (lower solid curve).

Medium	$a \left(\frac{\text{GeV}}{\text{mwe}} \right)$	$b \left(\frac{10^{-3}}{\text{mwe}} \right)$
Air	0.281	0.347
Ice	0.259	0.363
Fr. Rock	0.231	0.436
St. Rock	0.223	0.463

Table 4.1: Best fits for muon energy loss parameters a and b from Eq. 4.9. Fits from [Chirkin:2004hz].

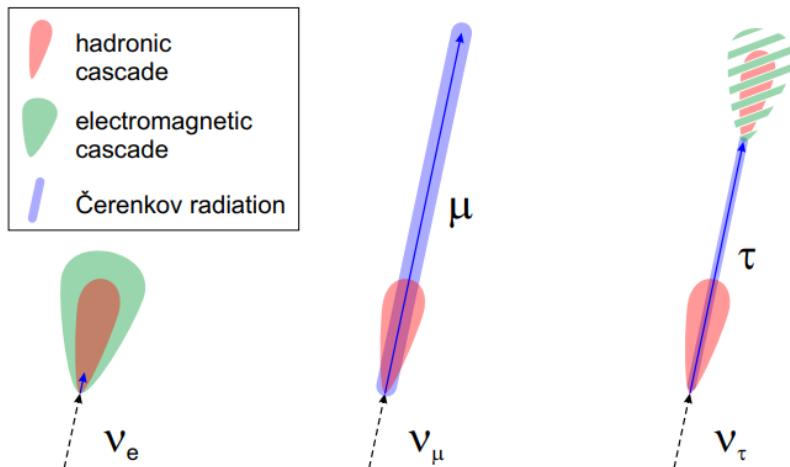


Figure 4.4: Schematic view of the neutrino signatures in matter. At each interaction point there is a hadronic cascade (red). Every hadronic cascade has electromagnetic sub-showers which are not illustrated here. Muons and energetic taus can give rise to tracks. Illustrations from [Wallraff].

5. The IceCube Experiment

hier tot pg 75?

Ook de voornaamste zoektochten van IceCube vermelden en duiden. Bv de point sources, astrophysical (Sam zijn thesis bv) etc.

Iets van wavelength dependence van Frank-Tamm:

$$\frac{dN}{dx} = \int_{\lambda_1}^{\lambda_2} \frac{2\pi\alpha}{\lambda^2} \sin^2(\theta_c) d\lambda = 2\pi\alpha \sin^2(\theta_c) \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right). \quad (5.1)$$

Copy paste van Wikipedia: Data from the Fermi Space Telescope (2013)[3] have been interpreted as evidence that a significant fraction of primary cosmic rays originate from the supernova explosions of stars.[4] Active galactic nuclei also appear to produce cosmic rays, based on observations of a neutrino and gamma rays from blazar TXS 0506+056 in 2018.[5][6]

5.0.1 Muon tracks

Ook muonen van atmosfeer, in hfdstk 4 enkel over muonen van neutrinos gebabbeld. intro The track-like structure of the signal allows to find a good “lever arm” as the deposited charge on the earliest and latest DOMs provide strong constraints on the event’s position, time and direction. As a result, directional reconstruction..... Muons can transit the entire detector, making it difficult to reconstruct the energy of the muon and the parent neutrino. Starting muon tracks are challenging, but more useful to infer the initial energy of the neutrino. Highly relativistic muons are stochastic in nature and the energy loss depends on the energy of the particle. Using equation ??? it is possible to have a better energy reconstruction.

5.0.2 Cascades

The light output has a slight asymmetry in the direction of motion, making directional reconstructions challenging. However, these interactions are often calorimetric and therefore allow for nearly complete measurements of the energy and result in a good energy resolution.

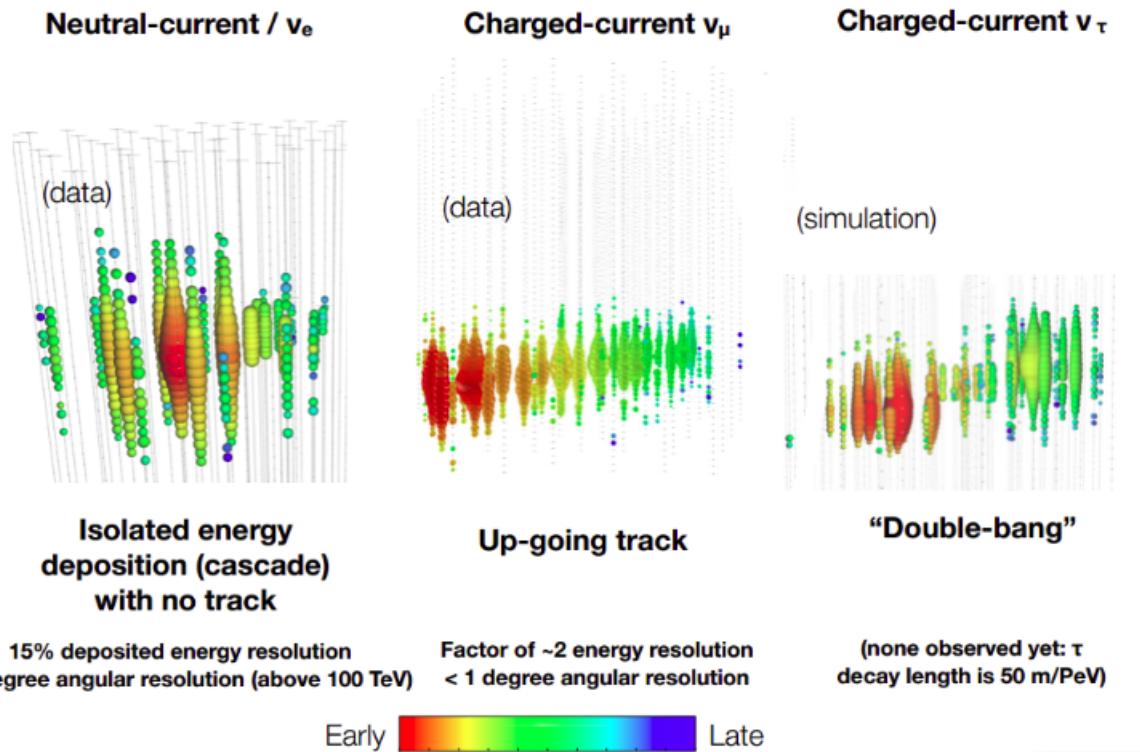


Figure 5.1: Neutrino interactions in the IceCube detector have two distinct types of interactions: cascades and tracks. Energetic taus are theorized to have double-bang signatures but have not undeniably been seen. Colors determine the timing of hits in the optical modules in the ice. Illustrations from the IceCube collaboration.

Simulation, Data Processing and Analysis

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6. Event Generation, Simulation and Reconstruction

6.1 PPC/PROPOSAL

Hier ook indirect cherenkov light component

6.2 IceHive

6.3 Line-Fit

6.4 SPE and MPE

6.5 Millipede

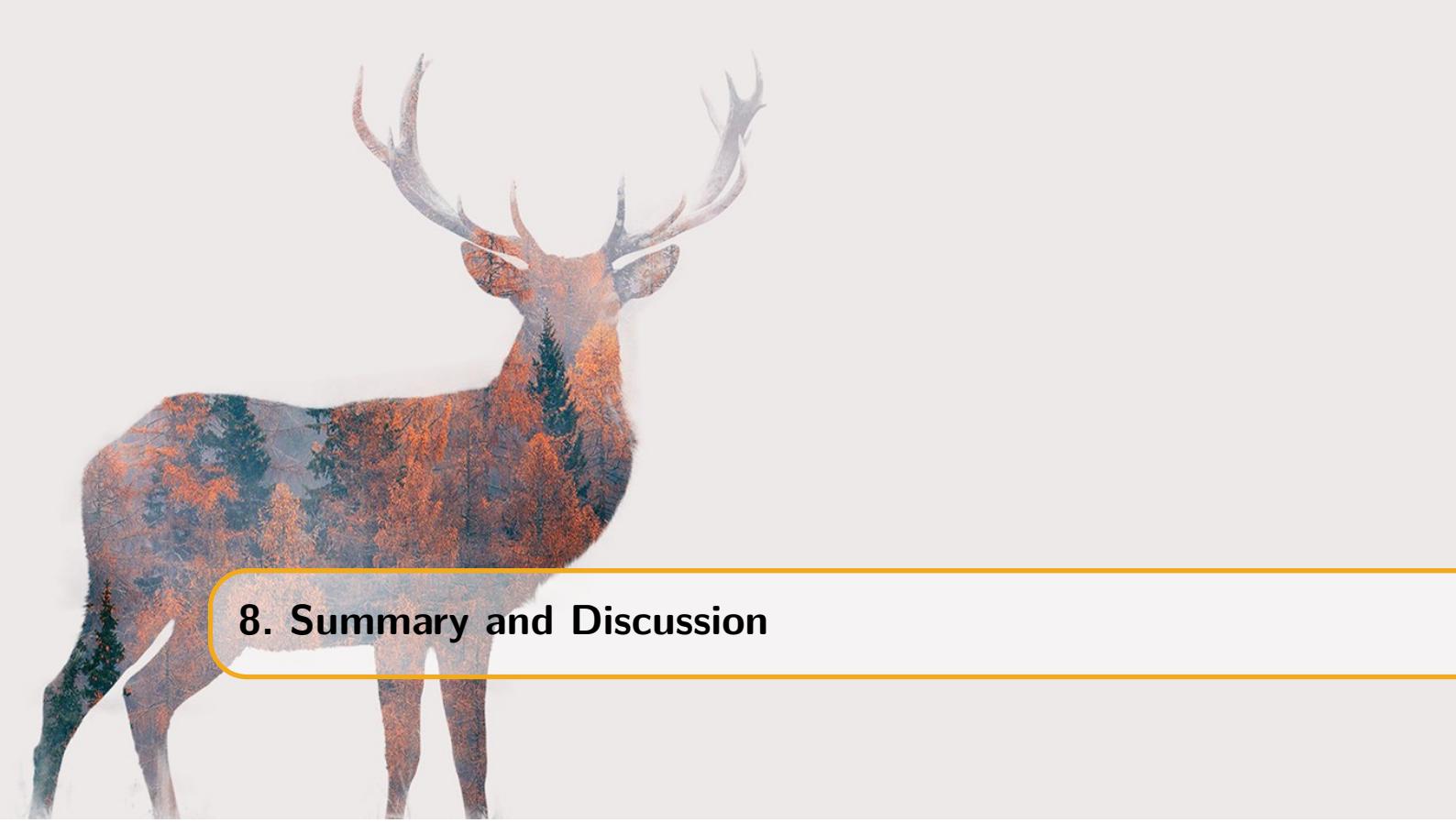
6.6 Boosted Decision Tree Classifiers



7. The SPACE Analysis

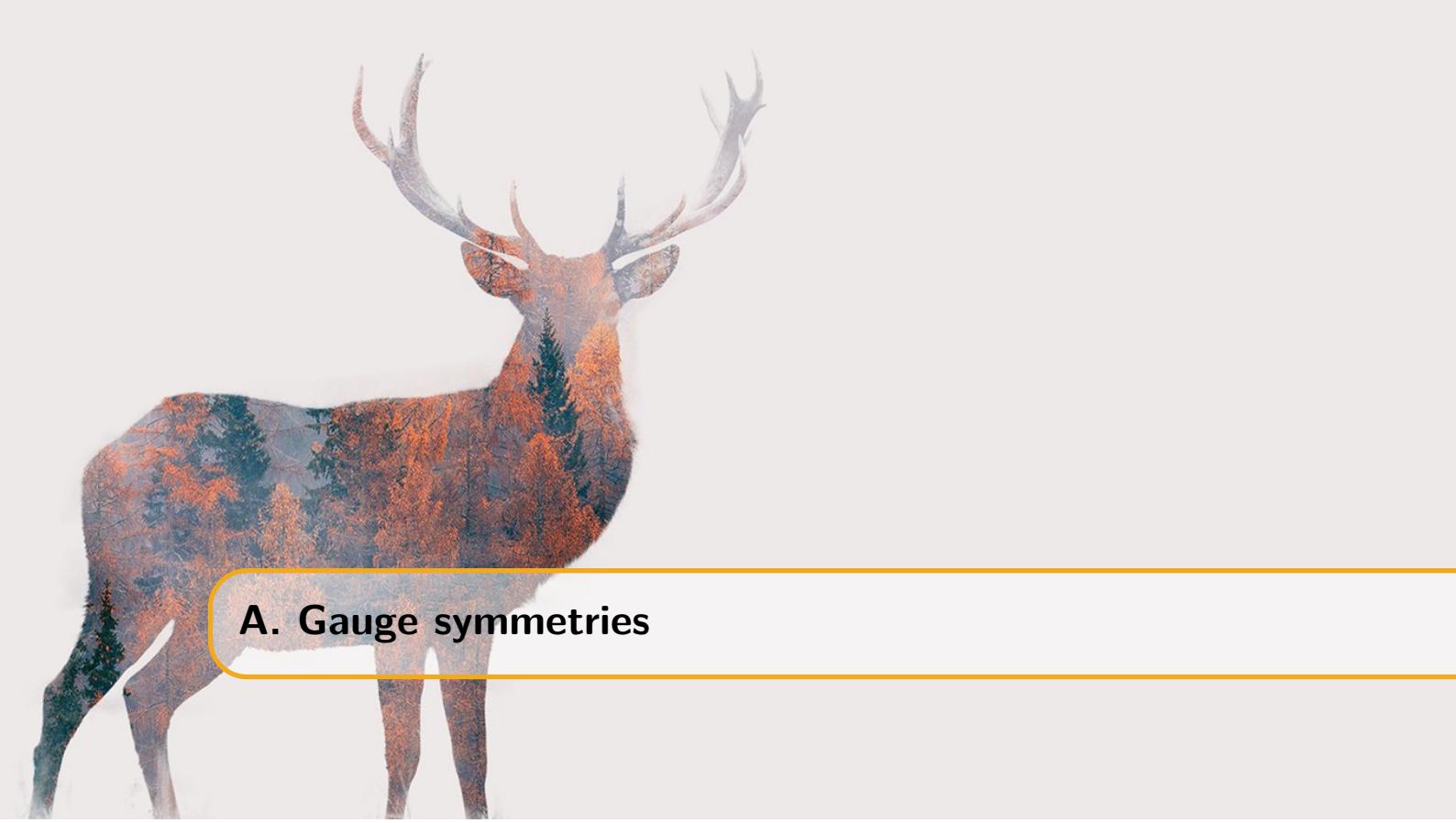
De uitleg van temperature modulations is goed in Sam zijn thesis <https://arxiv.org/pdf/physics/0312102v1.pdf>
<https://arxiv.org/pdf/1310.1284.pdf>

- 7.1 Motivation**
- 7.2 General strategy**
- 7.3 Event cleaning**
- 7.4 Variables**
- 7.5 BDT results**
- 7.6 Pull validation**
- 7.7 Systematic Uncertainties**
- 7.8 Results**



8. Summary and Discussion

Appendices

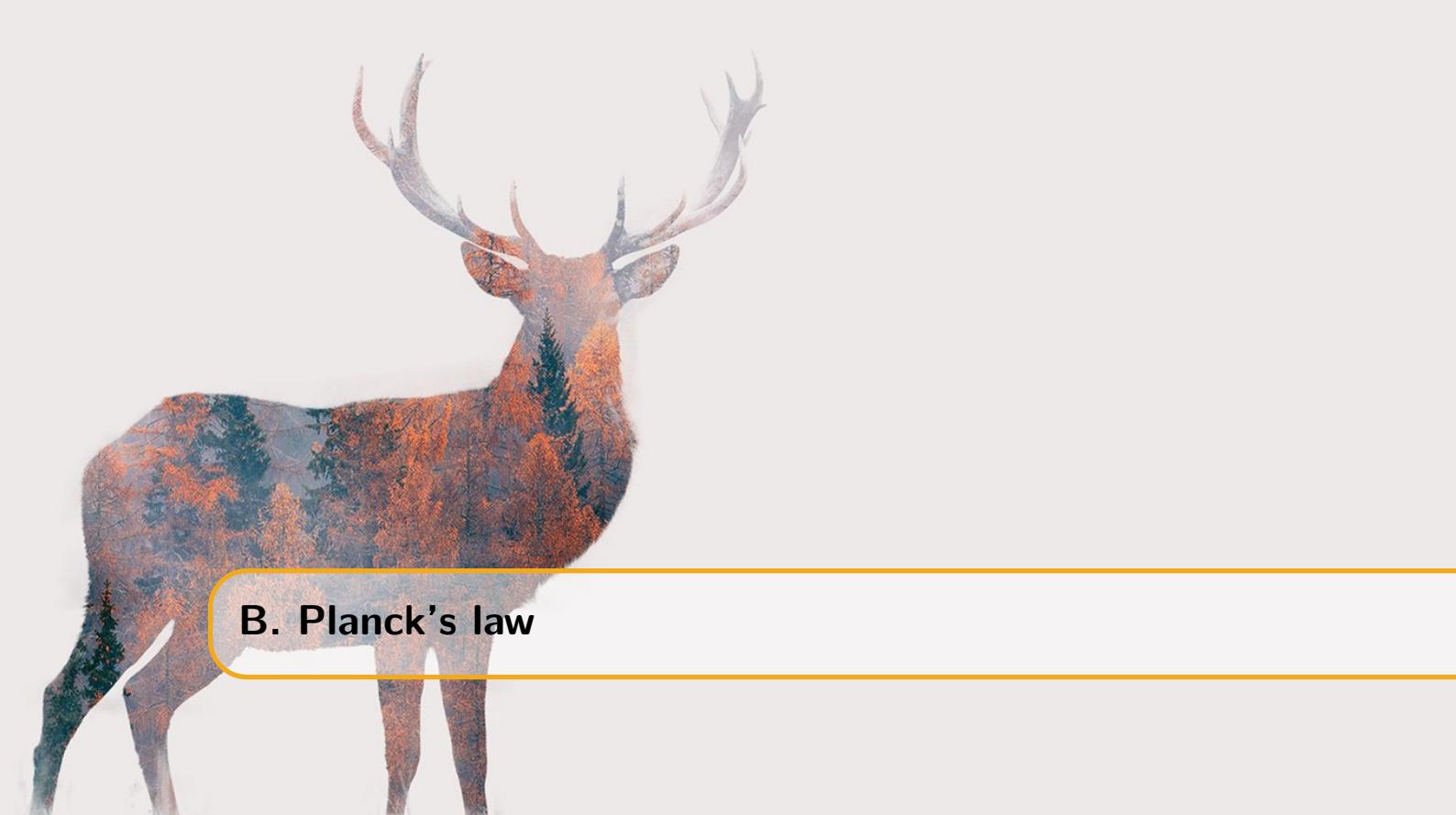


A. Gauge symmetries

The difference between global and local symmetries are not straightforward for everybody. In this appendix I try to give a better view of the matter.

Imagine that at each point in space and time there is a circle attached to it. If one shifts all circles of all points with a fixed angle the underlying physics hasn't changed. If we look at the whole in a different angle, nothing seems to be changed as everything holds the same relative orientation. This is a global symmetry. For local symmetries we instead shift each circle through a different angle, but an angle that changes smoothly from point to point and in a way that we can say how that angle is varying between different nearby regions. Then it will turn out that we can describe that rotation angle by means of a so-called gauge field, which just lets us transport the charged scalar field from one point in space time to another, taking account of how the rotation angle of the circle is changing. A gauge is a kind of coordinate system that is varying depending on the location with respect to some underlying space. In physics we are almost always concerned with space-time as the underlying space, and we are typically interested in theories that are invariant with respect to the choice of gauge or coordinate system.

Dan wat uitleg vanuit je QFT boek en de dingen hieronder: Je wilt je derivative anders doen werken in je theory onder een transformatie, maar daarvoor heb je een veld nodig. M.a.w.: dankzij een veld heb je lokale ijktransformatie mogelijk!



B. Planck's law

bron: <http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/rayj.html>

B.1 Electromagnetic waves in a cubical cavity

Suppose we have EM waves in a cavity at equilibrium with its surroundings. These waves must satisfy the wave equation in three dimensions:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}. \quad (\text{B.1})$$

The solution must give zero amplitude at the walls. A non-zero value would mean energy is dissipated through the walls which is in contradiction to our equilibrium assumption. A general solution takes the form of

$$\Psi(x, y, z, t) = \Psi_0 \sin k_1 x \sin k_2 y \sin k_3 z \sin k_4 t, \quad (\text{B.2})$$

which, after requiring $k_n L = n\pi$ with $n = 0, 1, 2, \dots$ and $k_4 \frac{\lambda}{2c} = \pi$, leads to

$$\Psi(x, y, z, t) = \Psi_0 \sin \left(\frac{n_1 \pi x}{L} \right) \sin \left(\frac{n_2 \pi y}{L} \right) \sin \left(\frac{n_3 \pi z}{L} \right) \sin \left(\frac{2\pi c t}{\lambda} \right). \quad (\text{B.3})$$

From the wave equation it is easy to find that

$$n^2 = n_1^2 + n_2^2 + n_3^2 = \frac{4L^2}{\lambda^2}, \quad (\text{B.4})$$

which span up a sphere in “n-space” with a volume of $\frac{1}{8} \frac{4}{3} \pi n^{3/2}$, where the first term originates from the positive nature of $n_{1,2,3}$. Because there are two possible polarizations of the waves one has to multiply with an additional factor 2. The number of modes per unit wavelength is equal to

$$\frac{dN}{d\lambda} \times \frac{1}{L^3} = \frac{d}{d\lambda} \left[\frac{8\pi L^3}{3\lambda^3} \right] \times \frac{1}{L^3} = - \left[\frac{8\pi}{\lambda^4} \right]. \quad (\text{B.5})$$

B.1.1 Classical approach

Following the principle of equipartition of energy, each standing wave mode will have an average energy kT with k the Boltzmann constant and T the temperature in Kelvin. The energy density is then:

$$\frac{du}{d\lambda} = -kT \frac{8\pi}{\lambda^4}. \quad (\text{B.6})$$

In function of frequency $\nu = \frac{c}{\lambda}$:

$$\frac{du}{d\nu} = -\frac{c}{\lambda^2} \frac{du}{d\lambda} = \frac{8\pi k T \nu^2}{c^3}, \quad (\text{B.7})$$

also known as the Rayleigh-Jeans law*. Problem: divergence

B.1.2 Quantum approach

The energy levels from a quantized harmonic oscillator are equal to

$$E_r = h\nu \left(r + \frac{1}{2} \right) = \frac{hc}{\lambda} \left(r + \frac{1}{2} \right) \quad \text{with } r = 0, 1, 2, \dots \quad (\text{B.8})$$

Implementing eq. B.4

$$E = \left(r + \frac{1}{2} \right) \frac{hc}{2L} \sqrt{n_1^2 + n_2^2 + n_3^2} \quad (\text{B.9})$$

According to statistical physics the average energy is now not equal to kT but follows a probability distribution

$$p(\nu, r) = \frac{e^{-r h \nu}}{\sum_{r=0}^{\infty} e^{-r h \nu}}, \quad (\text{B.10})$$

where we reference to the ground state of the oscillator: $E'_r = E_r - E_0$.

The average energy is now:

$$\begin{aligned} \langle E(\nu) \rangle &= \sum_{r=0}^{\infty} E(\nu, r) \cdot p(\nu, r) = \frac{\sum_{r=0}^{\infty} r h \nu e^{-r h \nu}}{\sum_{r=0}^{\infty} e^{-r h \nu}} \\ &= \frac{h \nu}{e^{h \nu / k T} - 1} \end{aligned} \quad (\text{B.11})$$

Substituting this for kT in eq. B.7 we find Planck's equation:

$$\frac{du}{d\nu} = \frac{8\pi h \nu^3}{c^3} \frac{h \nu}{e^{h \nu / k T} - 1} \quad (\text{B.12})$$

*This is often quoted per unit of steradian, which results in $\frac{2kT\nu^2}{c^3}$

9. Some useful things for LaTeX

9.1 Definitions

This is an example of a definition. A definition could be mathematical or it could define a concept.

Definition 9.1.1 — Definition name. Given a vector space E , a norm on E is an application, denoted $\|\cdot\|$, E in $\mathbb{R}^+ = [0, +\infty[$ such that:

$$\|\mathbf{x}\| = 0 \Rightarrow \mathbf{x} = \mathbf{0} \quad (9.1)$$

$$\|\lambda\mathbf{x}\| = |\lambda| \cdot \|\mathbf{x}\| \quad (9.2)$$

$$\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\| \quad (9.3)$$

9.2 Remarks

This is an example of a remark.

R The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

9.3 Corollaries

This is an example of a corollary.

Corollary 9.3.1 — Corollary name. The concepts presented here are now in conventional employment in mathematics. Vector spaces are taken over the field $\mathbb{K} = \mathbb{R}$, however, established properties are easily extended to $\mathbb{K} = \mathbb{C}$.

9.4 Propositions

This is an example of propositions.

9.4.1 Several equations

Proposition 9.4.1 — Proposition name. It has the properties:

$$|||\mathbf{x}|| - ||\mathbf{y}||| \leq ||\mathbf{x} - \mathbf{y}|| \quad (9.4)$$

$$|| \sum_{i=1}^n \mathbf{x}_i || \leq \sum_{i=1}^n ||\mathbf{x}_i|| \quad \text{where } n \text{ is a finite integer} \quad (9.5)$$

9.4.2 Single Line

Proposition 9.4.2 Let $f, g \in L^2(G)$; if $\forall \varphi \in \mathcal{D}(G)$, $(f, \varphi)_0 = (g, \varphi)_0$ then $f = g$.

9.5 Examples

This is an example of examples.

9.5.1 Equation and Text

■ **Example 9.1** Let $G = \{x \in \mathbb{R}^2 : |x| < 3\}$ and denoted by: $x^0 = (1, 1)$; consider the function:

$$f(x) = \begin{cases} e^{|x|} & \text{si } |x - x^0| \leq 1/2 \\ 0 & \text{si } |x - x^0| > 1/2 \end{cases} \quad (9.6)$$

The function f has bounded support, we can take $A = \{x \in \mathbb{R}^2 : |x - x^0| \leq 1/2 + \epsilon\}$ for all $\epsilon \in]0; 5/2 - \sqrt{2}[$. ■

9.5.2 Paragraph of Text

■ **Example 9.2 — Example name.** Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris. ■

9.6 Exercises

This is an example of an exercise.

Exercise 9.1 This is a good place to ask a question to test learning progress or further cement ideas into students' minds. ■

9.7 Problems

Problem 9.1 What is the average airspeed velocity of an unladen swallow?

9.8 Vocabulary

Define a word to improve a students' vocabulary.

Vocabulary 9.1 — Word. Definition of word.

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