Practice Problems

2. Summary Statistics (continued)

2.1 You have just completed a survey in which you asked 20 inmates at the Kingston Penitentiary how many years remain in their sentences. You obtain the following data: 0 1 1 1 5 3 0 3 4 3 5 1 1 1 3 3 6 2 5 7.

a. Construct a frequency distribution for the sample.

b. Construct the relative frequency Distribution for the sample.

c. What is the mean number of years remaining?

d. What is the median number of years remaining?

e. What is the mode

f. Calculate: (i) the sample variance; (ii) The sample standard deviation, and; (iii) the range.

g. What is the range of years remaining within 2 standard deviations of the mean?

2.2 Chebyshev's Theorem states that the percentage of measurements in a data set that fall within three standard deviations of their mean is:

a. 75% b. at least 75% c. 89% d. at least 89%

2.3 On a multiple choice test given to 200 psychology students the TA notes a class average of 75 percent and a standard deviation of 2.0 percent. Based on Chebyshev's Rule the minimum number of students who could have received marks between 70 and 80 is approximately equal to:

a. 32 b. 50 c. 132 d. 150 e. 168

3. Probability

3.1 A worker admits that on 10% of his shifts, he forgets to shut off the machine on his line. This causes the machine to overheat, increasing the probability that a defective product will be produced during the early morning run from 5% to 20%. The plant manager randomly selects a product from the early morning run and discovers it is defective. The probability that the worker forgot to shut off the machine the previous night is closest to

a. 0.10 b. 0.17 c. 0.22 d. 0.26 e. 0.31

3.2 Forty percent of cows are brown. Twenty percent of brown animals are cows. Ninety percent of animals are not brown. Farmer Joe has 300 animals. The number of his animals that are not brown but are cows is closest to:

a. 0 b. 9 c. 22 d. 35 e. None of these

3.3 A company has submitted bids on two separate contracts, A and B. The company feels that it has a 50% chance of winning contract A and a 40% chance of winning contract B. Furthermore, the company believes that it has a 60% chance of winning contract B given that it wins contract A. If the company wins contract B, the probability that it will not win contract A is:

a. 0 b. 0.125 c. 0.25 d. 0.375 e. None of these

Answers to Practice Problems

2. Summary Statistics (continued)

2.1 Note: $\sum_{i=1}^{20} x_i = 55, \quad \sum_{i=1}^{20} x_i^2 = 231, \quad n = 20$

a-b. Answers to be taken up during the introduction to Excel and STATA.

c. What is the mean number of years remaining?

$\bar{x} = 55/20 = 2.75$

d. What is the median number of years remaining?

The location of the 50th percentile is 10.5.
The median is half way between the 10th and 11th data point.
$median = 3 + 0.5(3 - 3)$
$= 3$

e. What is the mode

The mode is 1.

f. Calculate: (i) the sample variance; (ii) the sample standard deviation, and; (iii) the range.

$s^2 = 4.197$

$s = 2.049$

$range = 7$

g. What is the range of years remaining within 2 standard deviations of the mean?

$2s = 4.098$

$\bar{x} - 2s = -1.348$

$\bar{x} + 2s = 6.848$

All but one data point is between the range of (-1.348, 6.848). The proportion of observations between 2 standard deviations is 19/20=95%.

2.2 Solution: d

Chebyshev's Theorem: $\Pr\left[|\mu - x| \le k\sigma\right] \ge 1 - \frac{1}{k^2}$

So, $\Pr[|x - \mu| \le 3\sigma] \ge 1 - \frac{1}{9} = 0.89$

2.3 Solution: e

Chebyshev's Theorem:

$\Pr[|X - \mu| \le k\sigma] \ge 1 - \frac{1}{k^2} \Rightarrow \Pr[|X - 75| \le 2.5\sigma] \ge 1 - \frac{1}{2.5^2} = 0.84$

So, $200 \times 0.84 = 168$

3. Probability

3.1 Answer: e

D=defective product
S=shut-off machine

We know: P(D/S) = 0.05 $\Rightarrow P(D/S) = \frac{P(D \cap S)}{P(S)} = 0.05$

P(D/$\overline{S}$) = 0.2 $\Rightarrow P(D/\overline{S}) = \frac{P(D \cap \overline{S})}{P(\overline{S})} = 0.2$

P($\overline{S}$) =0.1 => P(S) = 0.9

We want to know P($\overline{S}$/D)

$$P(\overline{S}/D) = \frac{P(\overline{S} \cap D)}{P(\overline{S} \cap D) + P(S \cap D)}$$

$P(\overline{S} \cap D) = 0.20P(\overline{S}) = 0.02$
$P(S \cap D) = 0.05P(S) = 0.045$
$P(\overline{S} \cap D) = 0.02/[0.02 + 0.045] = 0.31$

3.2 Solution: b

n=300 P($\overline{B}$)=90% P($B|C$) = 40% P($C|B$) = 20%

$P(C \cap \overline{B}) = P(\overline{B}|C) \cdot P(C)$

$$P(C) = \frac{P(B \cap C)}{P(B|C)} = \frac{P(C|B) \cdot P(B)}{P(B|C)} = 0.05$$

$P(\overline{B}|C) = 1 - P(B|C) = 60\%$

So, $P(C \cap \overline{B}) = 60\% \cdot 05\% = 0.03 \Rightarrow n \cdot 0.03 = 9$

3.3 Solution: c

P(A)=0.5, P(B)=0.4, P(B|A)=0.6

P(A ∩ B)=P(A)P(B|A)=(0.5)(0.6)=0.3

P(A|B)=P(A ∩ B)/P(B)=0.3/0.4=0.75

$P(\overline{A}|B)$=1-P(A|B)=1-0.75=0.25