

# **Estimation of Effects of Endogenous Time-Varying Covariates: A Comparison Of Multilevel Modeling and Generalized Estimating Equations**

PROPOSAL

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## Introduction

Across a wide range of disciplines, researchers analyze clustered longitudinal, observational data to investigate prospective causal relationships between variables. To address such data, the psychological sciences most commonly resort to the multilevel linear model (MLM) (McNeish et al., 2017),<sup>[1]</sup> which is characterized by its separation of stable between-person differences from possible trends in the dynamics of within-person fluctuations (Hamaker & Muthén, 2020). Conversely, other fields, such as biostatistics and econometrics often favour generalized estimating equations (GEE, McNeish et al., 2017). While some interdisciplinary research has introduced and compared these methods (McNeish et al., 2017; Muth et al., 2016; Yan et al., 2013),<sup>[2]</sup> much of it remains superficial, lacking a focus on the specific conditions under which these models can produce meaningful inferences.

Recent evidence has shed light on an issue present in both methods,<sup>[3]</sup> where controlling for moderators of the treatment effect—that depend on prior treatment—may yield biased causal estimands (Pepe & Anderson, 1994; Qian et al., 2020). Such moderators fall under the umbrella of *time-varying endogenous covariates*. As a result of including these covariates in the model, ordinary interpretations of the coefficients are no longer valid (Qian et al., 2020, p. 3). According to Diggle (2002), this issue not only pertains GEE and MLM, but *all* longitudinal data analysis methods. However, due to a schism of the disciplines that employ these methods, such critiques of the MLM not reach the applied researcher. And when it does, the technical nature and jargon of the literature in other fields may prevent these researchers from figuring out how and when these problems occur.

<sup>[4]</sup> This indicates a need to understand (1) the consequences of including these covariates in analyses on multilevel longitudinal data, such as GEE, MLM and dynamic structural

# Summary of Comments on Estimation of Effects of Endogenous Time-Varying Covariates: A Comparison Of Multilevel Modeling and Generalized Estimating Equations

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"(...) which, in the context of longitudinal data analysis, decomposes observed variance into person-specific specific variance (i.e., the between-person component) and person- and time-specific variance (i.e., the within-person component)."

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Ik dacht zelf dat dit duidelijker is voor de toegepaste onderzoeker. Wanneer we het over decompositie hebben raken we misschien mensen kwijt.

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Dit klinkt wel erg algemeen aanvullend. Kan je hier specifieker welke aanvulling wij hebben t.o.v. Muth et al. (2016) en Yan et al. (2013)? Ik weet bijvoorbeeld niet of in die andere papers ze uitgebreid ingaan op het probleem met endogenous covariates? Ik zou de zin dan ook op die manier insteken: "While there exists some interdisciplinary research comparing MLM and GEE, this comparins does not involve how both frameworks take endogenous covariates into account."

Nu heb je het probleem met endogenous covariates nog niet geïntroduceerd, dus deze zin moet nog worden aangepast, maar iets in deze vorm denk ik.

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Eens, ik zal dit specifieker verwoorden

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Je hebt het hier specifiek over moderator. Maar ik zou het eigenlijk omdraaien en dus algemener beginnen. Het algemenere probleem waar Qian et al. (2020) het over hebben is de inclusie van endogenous covariates in een MLM. In het empirische voorbeeld van Qian et al. (2020) hebben ze ook een time-varying moderator die endogenous is, maar dit is een speciaal geval van "endogenous covariates".

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volgens Qian et al (2020) zijn moderators juist het probleem, zie abstract: "However, directly applying linear mixed models is problematic because potential moderators of the treatment effect are potentially endogenous---that is. may depend on prior treatment."

Later word een time-varying endogenous covariate gedefinieerd als een covariate die afhankelijk is van voorgaande treatment of outcome.

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equation modeling (DSEM; a modeling framework based on MLM) and (2) how this issue may be addressed by researchers that perform longitudinal data analysis. Accordingly, this paper explains how the inclusion of time-varying covariates can yield faulty inferences and intends to establish guidelines on how this may be prevented. More specifically, the current project addresses the following research question: *to what extent does the inclusion of endogenous variables in multilevel linear models and generalized estimating equations result in biased estimands?* In line with the literature (Diggle, 2002; Pepe & Anderson, 1994; Qian et al., 2020), we expect that the inclusion of endogenous time-varying covariates in longitudinal data analyses that involve <sup>2</sup>a marginal interpretation may result in bias that—depending on the circumstances—can promote the potential for faulty inferences.

## Analytic Strategy

To uncover the undesirable effects of endogenous covariates and investigate robustness against these effects, we will carry out simulations in which data will be generated according to several increasingly complex causal questions. These scenarios will be visually represented using directed acyclic graphs and analyzed using GEE, MLM and DSEM. We will start out with a scenario of the basic MLM—where a time-varying outcome is regressed on one time-varying predictor and stable between person differences are modeled with a random intercept—and increase the complexity until we reach the scenario discussed by Qian et al. (2020) (i.e., MLM that includes a time-varying endogenous covariate). The primary interest of this simulation study is the comparative performance in terms of bias and the secondary interest is the efficiency in mean squared error (MSE).


<sup>3</sup>We rely on large sample sizes to avoid potential artifacts and estimation issues that may

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Deze zin lijkt een beetje dubbelop, waarbij zin 1 zegt dat iets nodig is, en zin 2 zegt dat we datgeen wat nodig is, ook gaan doen. Een andere structuur helpt wellicht:

"The goal of this project is twofold. First, we present the issue of including endogenous covariates in analyses involving GEE, MLM and DSEM (a popular modeling framework in the social sciences based on MLM) in a didactical manner and in a psychological context. Second, we establish guidelines on how researchers can prevent this issue in their longitudinal data analysis."


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Is dit nodig?

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Dit is iets waar ook niet vaak over wordt gepraat in de MLM literatuur, het verschil tussen marginal en conditional models. Ik denk dat je hier ook wat over kunt zeggen. Dat er in de MLM literatuur duidelijk onderscheid wordt gemaakt tussen verschillende manieren van centereren, en het effect van cross-level interacties op parameter interpretaties, terwijl in de GEE literatuur de focus meer schijnt te liggen op de marginal interpretation en conditional interpretation van model parameter. Om GEE en MLM beter te kunnen vergelijken moeten deze debates ook duidelijker aan elkaar gelinkt worden.


Dit is een punt wat denk ik duidelijker naar voren mag komen in je research report, maar voor nu even op de plank kan.


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klopt, goed idee

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Oh? Waarom? Welke sample sizes had je in gedachten?

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werd ook vermeld in Hamaker en Muthen (2020) in fixed/random effects paper. Maar we kunnen de sample size ook variëren.

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occur when using small samples ([Bou & Satorra, 2018](#)).

Statistical analyses pertaining to the GEE and basic MLM will be performed in R, version 4.2.0 ([R Core Team, 2022](#)). To fit the GEE, the R-packages `geepack` ([Halekoh et al., 2006](#)) and `gee` ([Carey et al., 2024](#)) will evaluate several different working correlation structures, including independent, exchangeable, AR(1) and unstructured. To fit the basic MLM, the R-package `lme4` ([Bates et al., 2015](#)) will be employed, where we will both evaluate restricted maximum likelihood estimation and ordinary maximum likelihood estimation. Extensions of the MLM from the DSEM framework will be fitted using `Mplus`, version 8.10 ([Muthén & Muthén, 1998](#)).

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