

# **Estimation of Effects of Endogenous Time-Varying Covariates: A Comparison Of Multilevel Linear Modeling and Generalized Estimating Equations**

Research Report

**Ward B. Eiling (9294163)**

Supervisors: Ellen Hamaker and Jeroen Mulder

*Master's degree in Methodology and Statistics for the Behavioural,  
Biomedical and Social Sciences*

*Utrecht University*

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# 1 Introduction

Across a wide range of disciplines, researchers analyze clustered longitudinal, observational data to investigate prospective causal relationships between variables. When analyzing such data, the psychological sciences most commonly resort to the multilevel linear model (MLM, [McNeish et al., 2017](#)), which—in the context of longitudinal data analysis—separates observed variance into stable between-person differences and within-person fluctuations ([Hamaker & Muthén, 2020](#)). Conversely, other fields, such as biostatistics and econometrics often favour generalized estimating equations (GEE) for the analysis of longitudinal data ([McNeish et al., 2017](#)). Despite some cross-disciplinary efforts to compare these methods ([McNeish et al., 2017](#); [Muth et al., 2016](#); [Yan et al., 2013](#)), their scarcity may leave researchers with limited guidance in choosing the most suitable approach for their application.

A recent study by Qian et al. ([2020](#)) highlighted an issue present in both methods—except for GEE with working independence—where controlling for *time-varying endogenous covariates* may lead to biased causal estimates. A time-varying covariate is *endogenous* if it is directly or indirectly influenced by prior treatment or outcome, meaning its value may be determined by earlier stages of the process ([Qian et al., 2020](#)). As a result of including these covariates in these models, ordinary interpretations of the coefficients are no longer valid ([Qian et al., 2020, p. 3](#)). According to Diggle ([2002](#)), this issue not only pertains GEE and MLM, but *all* longitudinal data analysis methods.

However, due to a divide between the disciplines that employ these methods, such critiques of the MLM appear to have largely failed to reach the applied researcher in psychology. One specific reason might be that the technical jargon in other disciplines makes

it difficult for researchers to recognize when and how these issues emerge<sup>1</sup>. Therefore, this report aims to understand and explain the issue of including endogenous covariates in analyses involving GEE and MLM in a psychological context. To achieve this aim, the current investigation will employ (a) graphical tools such as the directed acyclic graph (DAG) and path diagram to assess causal identification assumptions, as well as (b) simulations with additional scenarios to pinpoint the issue.

Accordingly, the following research questions will be addressed with sub-questions being specified to isolate the issue described in the Qian et al. (2020):

- (1) When does the inclusion of endogenous variables in multilevel linear models result in biased estimates of the treatment effect?
  - (a) How is the bias in the treatment effect affected by the removal of the interaction  $\beta_1$  from generative model 3?
  - (b) How is the bias in the treatment effect affected by the removal of the random slope  $b_{i2}$  from generative model 3?
- (2) When does the inclusion of endogenous covariates in multilevel linear models result in a discrepancy between conditional and marginal interpretations of the treatment effect?

Research questions 1 and 2 will be investigated by employing MLM and GEE estimation respectively.

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<sup>1</sup>For instance, the term ‘endogeneity’ in econometrics, while related, has a distinct meaning from that of an endogenous variable, which can lead to confusion.

## 2 Methods

To obtain a better understanding of the issue exposed by Qian et al. (2020), two methods were employed. First, graphical methods were used provide insight into the presence and extent of bias with potential violation of assumptions: (a) path diagrams were used to evaluate the conditional independence assumption and (b) directed acyclic graphs (DAGs) were used to evaluate the backdoor criterion (Pearl, 1988, 2009). Second, a simulation study was performed to reproduce the results for the generative models (GMs) from Qian et al. (2020) and to further isolate the issue using additional GMs.

### 2.1 Data Generation

In the simulation Qian et al. (2020) considered three generative models (GMs), all of which have an endogenous time-varying covariate. In GM1 and GM2, the endogenous covariate  $X_{it}$  equals the previous outcome  $Y_{it}$  plus some random noise, so the *conditional independence* assumption is valid. In GM3, the endogenous covariate depends directly on  $b_{i0}$ , violating the assumption. To isolate the issue in GM3, we consider two variations on this model: GM3A, where the random slope  $b_{i2}$  for the treatment  $A_{it}$  is removed; GM3B, where the interaction term  $\beta_1 A_{it} X_{it}$  is removed. Note that the conditional independence assumption is violated in either of these variations. The details of the generative models are described below. We follow the notation of Qian et al. (2020) to allow for direct comparison, but rewrite the equations into within- and between-person models (see Raudenbush & Bryk, 2002). We accompany the equations of the GMs with graphical representations, where random effects are represented by grey circles, observed variables by squares and relationships across variables by arrows. The path diagrams of the three data generating models shows the discrepancies between the different generative models—

especially concerning the interaction effects—more clearly than DAGs.

### 2.1.1 Generative Model 1

In GM1, we considered a simple case with only a random intercept and a random slope for  $X_{it}$ . The outcome is generated according to the following repeated-observations or within-person model (level 1):

$$Y_{it+1} = \pi_{0i} + \pi_{1i}X_{it} + \pi_{2i}A_{it} + \pi_{3i}A_{it}X_{it} + \epsilon_{it+1}$$

with the person-level or between-person model (level 2):

$$\pi_{0i} = \alpha_0 + b_{i0}, \quad b_{i0} \sim \mathcal{N}(0, \sigma_{b0}^2),$$

$$\pi_{1i} = \alpha_1,$$

$$\pi_{2i} = \beta_0 + b_{i2}, \quad b_{i2} \sim \mathcal{N}(0, \sigma_{b2}^2),$$

$$\pi_{3i} = \beta_1.$$

By substitution, we get the single equation model:

$$\begin{aligned} Y_{it+1} &= \pi_{0i} + \pi_{1i}X_{it} + \pi_{2i}A_{it} + \pi_{3i}A_{it}X_{it} + \epsilon_{it+1} \\ &= (\alpha_0 + b_{i0}) + \alpha_1X_{it} + (\beta_0 + b_{i2})A_{it} + \beta_1A_{it}X_{it} + \epsilon_{it+1} \\ &= \alpha_0 + \alpha_1X_{it} + b_{i0} + A_{it}(\beta_0 + \beta_1X_{it} + b_{i2}) + \epsilon_{it+1}. \end{aligned}$$

The random effects  $b_{i0} \sim \mathcal{N}(0, \sigma_{b0}^2)$  and  $b_{i2} \sim \mathcal{N}(0, \sigma_{b2}^2)$  are independent of each other.

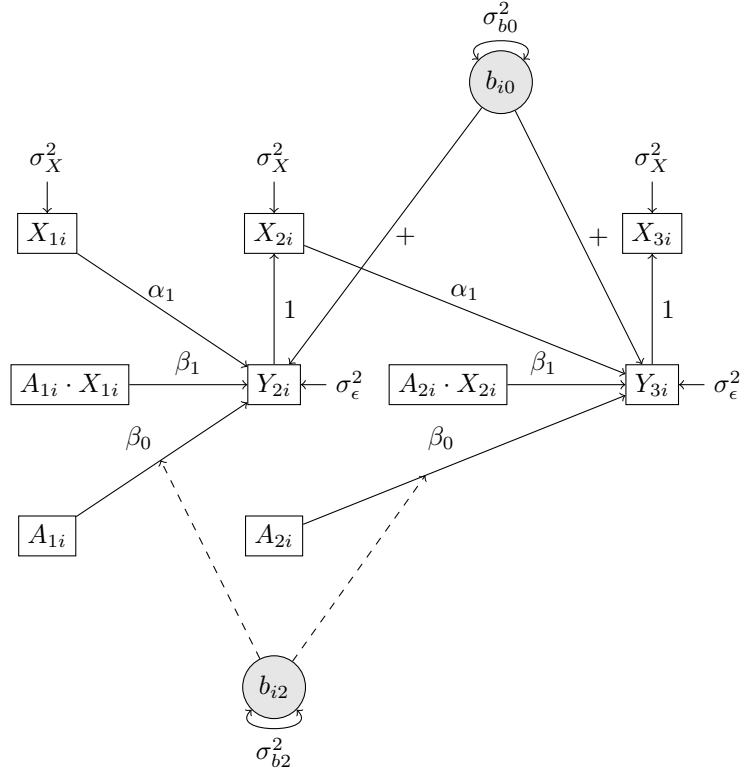
The covariate is generated as  $X_{i1} \sim \mathcal{N}(0, 1)$ , and for  $t \geq 2$ ,

$$X_{it} = Y_{it} + \mathcal{N}(0, 1).$$

The randomization probability  $p_t = P(A_{it} = 1 \mid H_{it})$  is constant at  $1/2$ . Thus,  $A_{it} \sim \text{Bernoulli}(0.5)$  for  $i = 1, \dots, N$  and  $t = 1, \dots, T$ . The exogenous noise is  $\epsilon_{it+1} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ .

Figure 1 shows the path diagram for GM1.

Figure 1: Path diagram for Generative Model 1 ( $t = 1, 2, 3$ )



### 2.1.2 Generative Model 2

In GM2, we considered the case with a random intercept and random slopes for (1) covariate  $X_{it}$ , (2) treatment  $A_{it}$ , and (3) the interaction between  $A_{it}$  and  $X_{it}$ ; and with a time-varying randomization probability for treatment. The outcome is generated according to the same repeated-observations model presented in GM1. However, the person-level

model is different:

$$\pi_{0i} = \alpha_0 + b_{i0}, \quad b_{i0} \sim \mathcal{N}(0, \sigma_{b0}^2),$$

$$\pi_{1i} = \alpha_1 + b_{i1}, \quad b_{i1} \sim \mathcal{N}(0, \sigma_{b1}^2),$$

$$\pi_{2i} = \beta_0 + b_{i2}, \quad b_{i2} \sim \mathcal{N}(0, \sigma_{b2}^2),$$

$$\pi_{3i} = \beta_1 + b_{i3}, \quad b_{i3} \sim \mathcal{N}(0, \sigma_{b3}^2).$$

By substitution, we get the single equation model:

$$\begin{aligned} Y_{it+1} &= \pi_{0i} + \pi_{1i}X_{it} + \pi_{2i}A_{it} + \pi_{3i}A_{it}X_{it} + \epsilon_{it+1} \\ &= (\alpha_0 + b_{i0}) + (\alpha_1 + b_{i1})X_{it} + (\beta_0 + b_{i2})A_{it} + (\beta_1 + b_{i3})A_{it}X_{it} + \epsilon_{it+1} \\ &= \alpha_0 + \alpha_1X_{it} + b_{i0} + b_{i1}X_{it} + A_{it}(\beta_0 + \beta_1X_{it} + b_{i2} + b_{i3}X_{it}) + \epsilon_{it+1}. \end{aligned}$$

The random effects  $b_{ij} \sim \mathcal{N}(0, \sigma_{bj}^2)$ , for  $j = 0, 1, 2, 3$ , are independent of each other.

The covariate is generated as  $X_{i1} \sim \mathcal{N}(0, 1)$ , and for  $t \geq 2$ ,

$$X_{it} = Y_{it} + \mathcal{N}(0, 1).$$

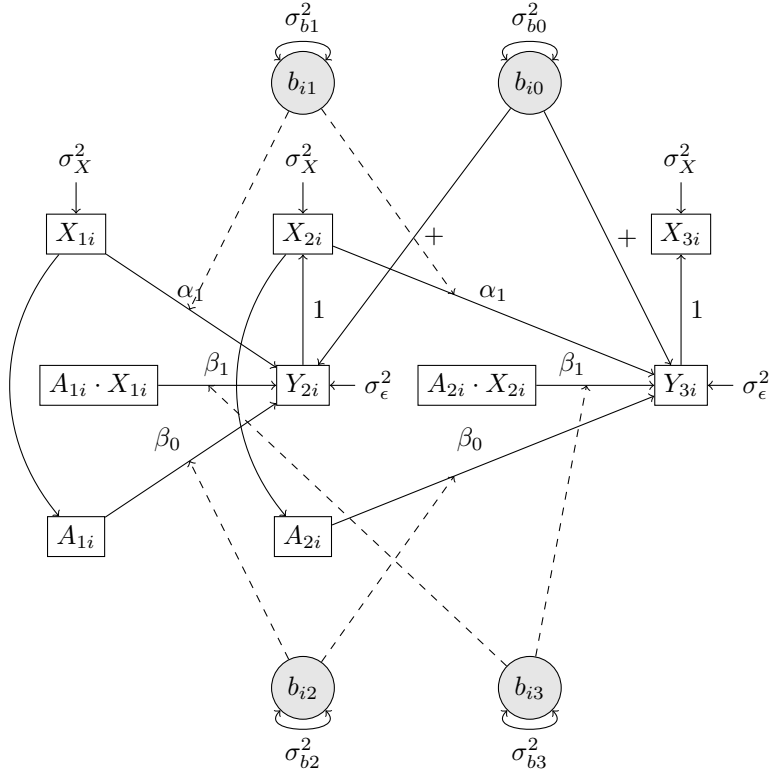
The randomization probability depends on  $X_{it}$ :

$$p_t = P(A_{it} = 1 \mid H_{it}) = \begin{cases} 0.7 & \text{if } X_{it} > -1.27, \\ 0.3 & \text{if } X_{it} \leq -1.27, \end{cases}$$

where the cutoff  $-1.27$  was chosen so that  $p_t$  equals 0.7 or 0.3 for about half of the time. In other words, if the value of the covariate for any given person and time point is above the cutoff, the probability of receiving the treatment  $p_t$  is 0.7; otherwise, it is 0.3. Accordingly,  $A_{it} \sim \text{Bernoulli}(p_t)$  for  $i = 1, \dots, N$  and  $t = 1, \dots, T$ . The exogenous noise is  $\epsilon_{it+1} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ .

Figure 2 shows the path diagram for GM2.

Figure 2: Path diagram for Generative Model 2 ( $t = 1, 2, 3$ )



### 2.1.3 Generative Model 3

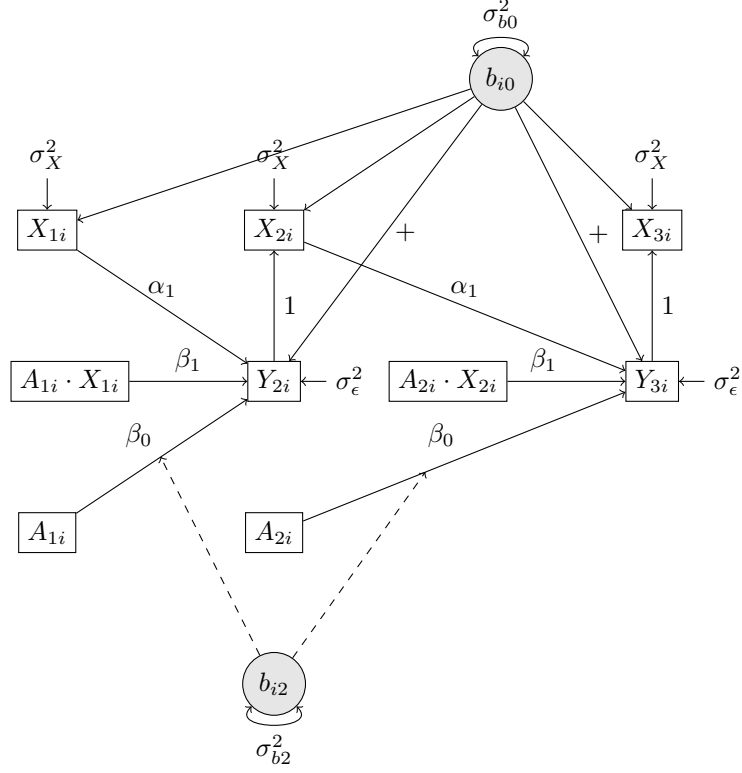
GM3 is the same as GM1, except that the covariate  $X_{it}$  depends directly on  $b_{i0}$ :



$$X_{i1} \sim \mathcal{N}(b_{i0}, 1), \quad X_{it} = Y_{it} + \mathcal{N}(b_{i0}, 1) \text{ for } t \geq 2.$$

Figure 3 shows the path diagram for GM3.

Figure 3: Path diagram for Generative Model 3 ( $t = 1, 2, 3$ )

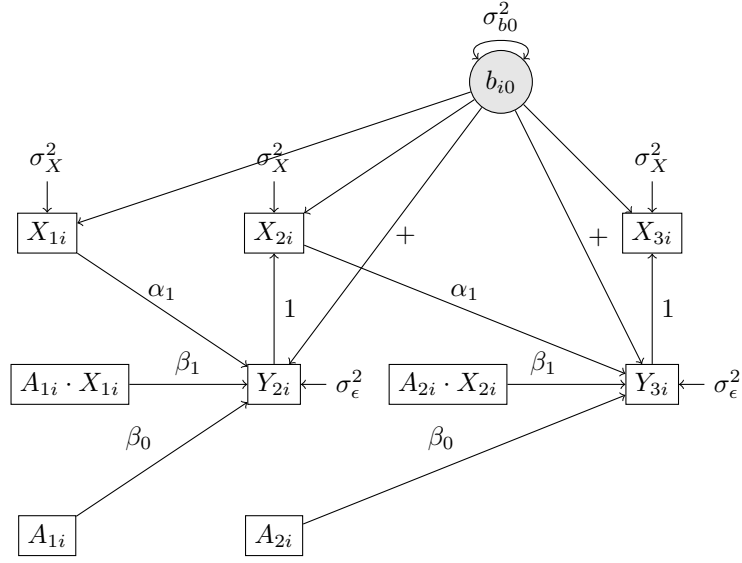


#### 2.1.4 Generative Model 3A

GM3A is the same as GM3, except that the random slope  $b_{i2}$  for the treatment  $A_{it}$  is removed. The single equation model then becomes:

$$Y_{it+1} = \alpha_0 + \alpha_1 X_{it} + b_{i0} + A_{it}(\beta_0 + \beta_1 X_{it}) + \epsilon_{it+1}.$$

Figure 4: Path diagram for Generative Model 3A ( $t = 1, 2, 3$ )

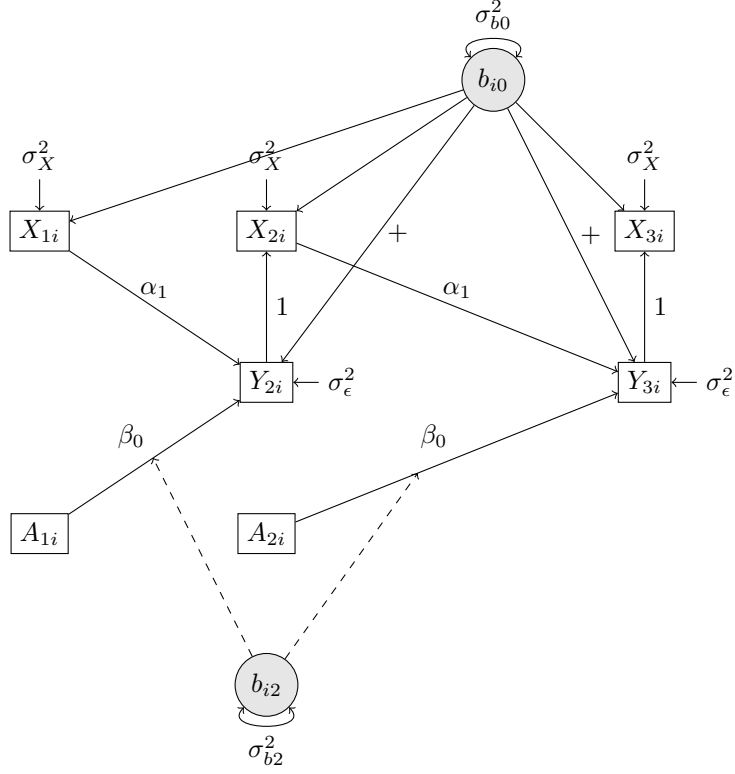


### 2.1.5 Generative Model 3B

GM3B is the same as GM3, except that the interaction term  $\beta_1 A_{it} X_{it}$  is removed. The single equation model then becomes:

$$Y_{it+1} = \alpha_0 + \alpha_1 X_{it} + b_{i0} + A_{it}(\beta_0 + b_{i2}) + \epsilon_{it+1}.$$

Figure 5: Path diagram for Generative Model 3B ( $t = 1, 2, 3$ )



### 2.1.6 Parameter Values

The following parameter values were adapted from Qian et al. (2020):

$$\alpha_0 = -2, \quad \alpha_1 = -0.3, \quad \beta_0 = 1, \quad \beta_1 = 0.3,$$

$$\sigma_{b0}^2 = 4, \quad \sigma_{b1}^2 = \frac{1}{4}, \quad \sigma_{b2}^2 = 1, \quad \sigma_{b3}^2 = \frac{1}{4}, \quad \sigma_\epsilon^2 = 1.$$

## 2.2 Path Diagrams and Conditional Independence

Qian et al. (2020) proposes the use of the conditional independence assumption to identify whether bias may occur, which is given by:

$$X_{it} \perp (b_{i0}, b_{i1}) \mid H_{it-1}, A_{it-1}, Y_{it}.$$

where  $H_{it}$  refers to the history of the set of covariates, which in this case are all observations of covariate  $X_{it}$  prior to the current timepoint  $t$ . This allows  $X_{it}$  to be endogenous, but the endogenous covariate  $X_{it}$  can only depend on the random effects through variables observed prior to  $X_{it}$ . If the only endogenous covariates are functions of prior treatments and prior outcomes, then the assumption automatically holds.

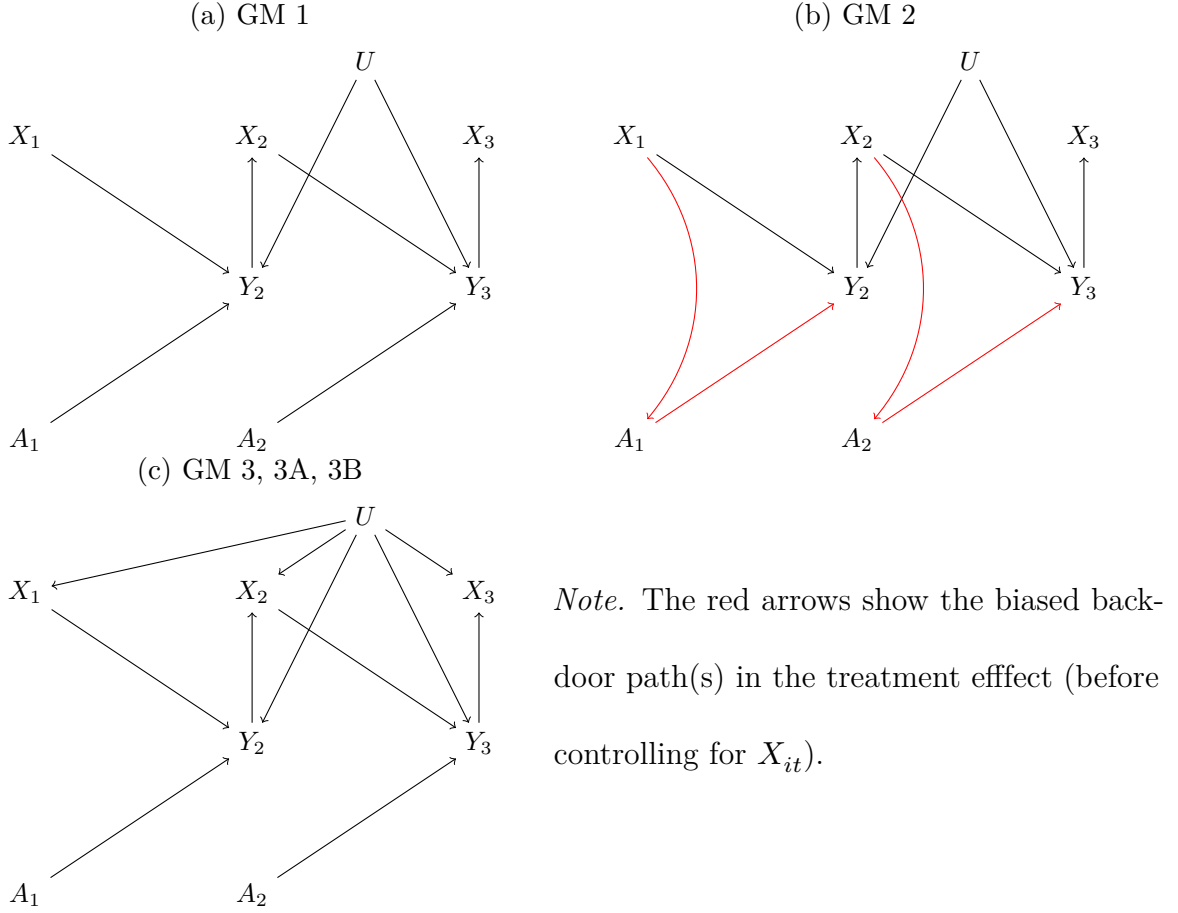
When inspecting Figure 1 and Figure 2, we may notice that  $X_{it}$  becomes independent of the random effects after conditioning on  $Y_{it}$ . On the other hand, we can see that this assumption is violated in GM3/3A/3B, as  $X_{it}$  depends directly on  $b_{i0}$  and can thus not be made independent of the random effects by conditioning on prior variables such as  $Y_{it}$  (see Figure 3, Figure 4 and Figure 5). Thus, we would expect biased estimates of the treatment effect for GM3/3A/3B.

## 2.3 Backdoor Criterion and DAGs

DAGs are a useful tool for representing causal relationships between variables and to evaluate the assumptions needed for causal identification. According to the backdoor criterion (Pearl, 1988, 2009), a requirement for causal identification, causal effects can be identified by blocking non-causal paths through conditioning on intermediate variables (e.g., controlling or matching). If any non-causal paths cannot be blocked due to omitted variables or measurement error, treatment and outcome remain linked via backdoor paths, leading to biased estimates of the treatment effect (Kim & Steiner, 2021).

We formulated the DAGs in `dagitty`, where the random disturbance  $b_{0i}$  was represented by the node U (e.g., Kim & Steiner, 2021). The DAGs for the first three observations of the three data generating models are presented in Figure 6.

Figure 6: DAGs for Generative Models 1, 2, 3/3A/3B ( $t = 1, 2, 3$ )



When applying Pearl's backdoor criterion to GM1/3/3A/3B, it may be observed that there exists no backdoor path in the treatment effect  $A_{it} \rightarrow Y_{it+1}$ , as  $A_{it}$  does not have any parents. While we need not control for covariate  $X_{it}$  to obtain an unbiased total effect, doing so should not introduce bias.

On the other hand, in GM2, there is a backdoor path in the treatment effect:  $A_{it} \leftarrow X_{it} \rightarrow Y_{it+1}$  (see Figure 6b). More specifically,  $X_{it}$  is a confounder in the relationship between  $A_{it}$  and  $Y_{it+1}$ . However, controlling for  $X_{it}$  blocks this backdoor path, making the treatment effect unbiased. In other words, the history of covariate  $X_{it}$  is a sufficient adjustment set for the treatment effect.

The results of Qian et al. (2020) show that GM3 is the only model with bias in the treatment effect. However, the backdoor criterion failed to identify this bias, as there is no

backdoor path in the treatment effect. This may be explained by the fact that the DAG does not impose restrictions based on (a) the random slopes and (b) interaction effects. Concerns regarding the use of Pearl’s backdoor criterion in situations with interaction effects have been voiced by several people (see Weinberg (2007); Attia et al. (2022)).

## 2.4 Data Analysis

We evaluated the performance of the models across a total of 30 different settings, each replicated 1,000 times, by systematically varying the following factors:

- **Generative Models (GM):** 1, 2, 3, 3A, 3B
- **Number of timepoints (T):** 10, 30
- **Sample size (N):** 30, 100, 200

All data generation and estimation was performed in R, version 4.4.2 (Team, 2024). After the generation of data generation for any given setting, several models were fit. To fit the standard MLM, the `lmer` function from the R-package `lme4` (Bates et al., 2015) was employed with restricted maximum likelihood estimation. For the MLM, the analytical models were equivalent to each of the respective data-generating models. To fit the GEE with the “exchangeable”, “independent” and “AR(1)” working correlation structures, the `geeglm` function from the R-package `geepack` (Halekoh et al., 2006) was employed with the identity link function. Since the random effects are not explicitly modelled in GEE, the analytical GEE models simply contain only the fixed effects of the generative model at hand.

### 3 Results

`?@tbl-simulation_results` presents the simulation results for each of the generative and analytical models. The estimates for the analytical MLM may be interpreted in terms of bias. Here we find that there is little to no bias for GM1/3a/3b, small bias for GM2 and substantial bias for GM3. Thus, once we remove either the dependency of the random intercept with the covariate (GM1), the random slope  $b_{i2}$  (GM3A) or the interaction  $\beta_1$  (GM3B) from GM3, the bias disappears or becomes extremely small. This bias in GM3 decreases as the number of timepoints  $T$  increases from 10 to 30. Note that the MLM model fitting success rates are poor for GM2, ranging from 0.92 to 0.09, implying that for the latter case, only 90 of the 1000 models were fitted.

For the GEE with independence, the values refer to the difference between the estimated unbiased marginal effect and the specified conditional effect. Here we find that there is a difference between these effects of around .09 for GM1, 0.03 for GM3 and close to zero for GM3A/GM3B.

The GEE models fitted successfully for all settings.

Table 1: Simulation results for  $N = 200$  and  $\beta_{0,MLM} = 1$  over 1000 replications, with  $T = 10$  and  $T = 30$ .

GM	Characteristics	$\hat{\beta}_{0,MLM} - \beta_{0,MLM}$		$\hat{\beta}_{0,GEE-ind} - \beta_{0,MLM}$	
		$T = 10$	$T = 30$	$T = 10$	$T = 30$
1	Includes random intercept and random slope for treatment	0.003	0.001	0.086	0.090
3	Model 1 with dependency random intercept and covariate	-0.051	-0.023	0.033	0.032
3a	Model 3 without random slope $b_{2i}$	0.002	-0.000	-0.001	-0.000
3b	Model 3 without interaction effect $\beta_1$ (between treatment and covariate)	0.005	0.001	0.003	0.001



GM	T	N	MLM_bias	MLM_sd	GEE-Ex_bias	GEE-Ex_sd	GEE-AR1_bias	GEE-AR1
1	10	30	0.000	0.238	0.080	0.235	-0.042	0
1	10	100	-0.012	0.129	0.071	0.129	-0.037	0
1	10	200	0.003	0.093	0.087	0.090	-0.015	0
1	30	30	-0.001	0.203	0.086	0.205	-0.091	0
1	30	100	-0.007	0.107	0.084	0.110	-0.106	0
1	30	200	0.001	0.079	0.095	0.079	-0.103	0
2	10	30	0.011	0.282	-0.116	1.017	-0.354	0
2	10	100	0.005	0.147	-0.095	1.183	-0.367	0
2	10	200	0.008	0.103	0.066	1.219	-0.248	0
2	30	30	0.000	0.220	-373.623	15220.410	-7.737	6866
2	30	100	-0.014	0.114	-891.751	35797.928	-381.111	19404
2	30	200	-0.013	0.087	4311.505	127396.588	2377.454	70214
3	10	30	-0.052	0.245	0.031	0.230	-0.122	0
3	10	100	-0.064	0.134	0.024	0.127	-0.121	0
3	10	200	-0.051	0.096	0.039	0.089	-0.111	0
3	30	30	-0.024	0.206	0.032	0.202	-0.124	0
3	30	100	-0.030	0.108	0.028	0.108	-0.128	0
3	30	200	-0.023	0.080	0.038	0.078	-0.120	0

Table 2: Results for beta0 bias with Standarddeviation and success rate, 1000 replications,  
run: GM123-trio-1000reps

GM	T	N	MLM_bias	MLM_sd	GEE-Ex_bias	GEE-Ex_sd	GEE-AR1_bias	GEE-AR1_sd
3	10	200	-0.051	0.096	0.039	0.089	-0.111	0.096
3	30	200	-0.023	0.080	0.038	0.078	-0.120	0.080
3a	10	200	0.002	0.048	0.002	0.048	-0.146	0.048
3a	30	200	-0.000	0.028	-0.000	0.028	-0.154	0.028
3d	10	200	0.005	0.087	0.005	0.087	-0.195	0.087
3d	30	200	0.001	0.075	0.001	0.075	-0.207	0.075
3h	10	200	0.020	0.092	0.053	0.089	-0.046	0.092
3h	30	200	0.003	0.078	0.042	0.078	-0.053	0.078

Table 3: Results for beta0 bias with Standarddeviation, 1000 replications, run: GM3adh-trio-1000reps

## 4 Discussion

The first research question, pertained to the extent of bias in MLM estimates of generative model that were nested in GM3. Here, we found that bias in the treatment effect practically dissapeared when removing either (1) the dependency between the random intercept and covariate, (2) the random slope for treatment or (3) the interaction effect.

This

- Initially, it seemed that the issue of endogenous covariates meant that, for unbiased estimation of the treatment effect, we should rely on GEE with independence. However, it is important not to conflate the issues mentioned by Qian et al. (2020). The first issue pertains to model interpretation: should we expect covariates to be endogenous and be primarily interested in marginal interpretations of the parameters rather than the person-specific (conditional-on-the-random-effect) inter-

pretation, we should indeed employ GEE with working independence (OR STRUCTURAL MARGINAL MODELS??? CHECK THIS OUT IN ZOTERO). The second issue pertains to model fitting: once we conclude that person-specific interpretation aligns with our interest but we fear the presence of endogenous covariates, we have to assess the conditional independence assumption.

- Despite a violation of the conditional independence assumption in GM 3, 3A and 3B; biased MLM estimates were only found for GM3.
- Across all generative model, we generally found that the estimated fixed treatment effect differed more from the specified MLM effect for the analytical GEE models than for the analytical MLM model. This is rather unsurprising, considering that the MLM is analyzing the exact same model as was specified, thereby putting it at an advantage over the GEE.
- For GEE with independence we found the largest difference between the estimated marginal and specified conditional effect for GM1, indicating that a false interpretation of the MLM parameters as marginal, would potentially have the greatest inferential consequences compared to the other GMs. For GM3A/3B, such a mistake would not be consequential.
- While GEE with independence may indeed yield unbiased estimators of the marginal effect as mentioned by Qian et al. (2020), they do not reflect the value of the model parameter. And since the endogeneity of a covariate implies that it is determined by the random effect, thereby making the marginal relationship between any given  $X_{it}$  and  $Y_{it+1}$  different as shown by Qian et al. (2020) (section 2.2), it is unclear what utility the combined marginal effect may serve in this context.

- For GM2, we noticed extreme model fitting issues for the MLM, due to, among other things, a lack of convergence. It should be noted that unlike the script used here, Qian et al. (2020) deals only with errors of the `lmer()` function, but not with warnings (e.g., pertaining to non-convergence) in their script. This discrepancy may explain the slightly different estimates of MLM bias for GM2.

They set the same seed for every setting, potentially making the first dataset the same for every setting. Nevertheless, properly randomizing does not seem to drastically impact the results. The settings with GM2,  $T = 30$  and  $N \geq 100$  of Qian et al. (2020) results in very implausible values for the covariate  $X_{it}$  and outcome  $Y_{it+1}$ , often exceeding a million. This may imply the presence of a non-stationary process (e.g., unit root). In the case of GEE analytical models, these settings result in very large estimates.

- ...

## 5 To-do

- wasn't working exchangeability in GEE equivalent to MLM? then use this to argue why using this GEE variation. Because for the difference of GEE with independence from the actual specified conditional parameter estimates should indicate a difference between the marginal and conditional effect, but this is not bias. Can we speak of bias with exchangeability?

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## 7 Appendix

### 7.1 Original Section from Qian et al. (2020): “4. Simulation”

In the simulation, we considered three generative models (GMs), all of which have an endogenous covariate. In the first two GMs, the endogenous covariate  $X_{it}$  equals the

previous outcome  $Y_{it}$  plus some random noise, so the conditional independence assumption (10) is valid. In GM 3, the endogenous covariate depends directly on  $b_i$ , violating assumption (10). The details of the generative models are described below.

In GM1, we considered a simple case with only a random intercept and a random slope for  $A_{it}$ , so that  $Z_{i(t_0)} = Z_{i(t_2)} = 1$  in model (7). The outcome is generated as:

$$Y_{it+1} = \alpha_0 + \alpha_1 X_{it} + b_{i0} + A_{it}(\beta_0 + \beta_1 X_{it} + b_{i2}) + \epsilon_{it+1}.$$

The random effects  $b_{i0} \sim N(0, \sigma_{b_0}^2)$  and  $b_{i2} \sim N(0, \sigma_{b_2}^2)$  are independent of each other. The covariate is generated as  $X_{i1} \sim N(0, 1)$ , and for  $t \geq 2$ ,

$$X_{it} = Y_{it} + N(0, 1).$$

The randomization probability  $p_t$  is constant at  $1/2$ . The exogenous noise is  $\epsilon_{it+1} \sim N(0, \sigma_\epsilon^2)$ .

In GM2, we considered the case where  $Z_{i(t_0)} = Z_{i(t_2)} = 1$ , with time-varying randomization probability. The outcome is generated as:

$$Y_{it+1} = \alpha_0 + \alpha_1 X_{it} + b_{i0} + b_{i1} X_{it} + A_{it}(\beta_0 + \beta_1 X_{it} + b_{i2} + b_{i3} X_{it}) + \epsilon_{it+1}.$$

The random effects  $b_{ij} \sim N(0, \sigma_{b_j}^2)$ , for  $0 \leq j \leq 3$ , are independent of each other. The covariate is generated as  $X_{i1} \sim N(0, 1)$ , and for  $t \geq 2$ ,

$$X_{it} = Y_{it} + N(0, 1).$$

The randomization probability depends on  $X_{it}$ :

$$p_t = 0.7 \cdot 1(X_{it} > -1.27) + 0.3 \cdot 1(X_{it} \leq -1.27),$$

where  $1(\cdot)$  represents the indicator function, and the cutoff  $-1.27$  was chosen so that  $p_t$  equals 0.7 or 0.3 for about half of the time. The exogenous noise is  $\epsilon_{it+1} \sim N(0, \sigma_\epsilon^2)$ .

GM3 is the same as GM 1, except that the covariate  $X_{it}$  depends directly on  $b_i$ :

$$X_{i1} \sim N(b_{i0}, 1), \quad X_{it} = Y_{it} + N(b_{i0}, 1) \text{ for } t \geq 2.$$

We chose the following parameter values:

$$\alpha_0 = -2, \quad \alpha_1 = -0.3, \quad \beta_0 = 1, \quad \beta_1 = 0.3,$$

$$\sigma_{b0}^2 = 4, \quad \sigma_{b1}^2 = \frac{1}{4}, \quad \sigma_{b2}^2 = 1, \quad \sigma_{b3}^2 = \frac{1}{4}, \quad \sigma_\epsilon^2 = 1.$$



## 7.2 Overview of Variations on Generative Model 3

Table 4: Models with 1 Parameter Less

Generative	random slope	interactie $\beta_1$	fixed slope	bias
Model	treatment $b_{i2}$		covariate $\alpha_1$	
3	yes	yes	yes	yes, negative
3a	no	yes	yes	no
3d	yes	no	yes	no
3h	yes	yes	no	yes, positive

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Generative	random slope	interactie $\beta_1$	fixed slope	bias
Model	treatment $b_{i2}$		covariate $\alpha_1$	
3	yes	yes	yes	yes, negative
3b	no	no	yes	no
3i	no	yes	no	
3j	yes	no	no	

## 7.3 Simulation Plan Proposal

To uncover the undesirable effects of endogenous covariates and investigate robustness against these effects, we will carry out simulations in which data will be generated according to several increasingly complex scenarios. These scenarios will be visually represented using directed acyclic graphs and analyzed using GEE, MLM and DSEM. We will start out with a scenario of the basic MLM—where a time-varying outcome  $Y$  is regressed on a single time-varying predictor  $X$  and in the presence of stable between person differences

in the intercept—and increase the complexity until we reach the scenario that includes a time-varying endogenous covariate. The primary interest of this simulation study is the comparative performance of different specifications of the MLM and GEE in terms of bias in the estimation of the effect of  $X$  to  $Y$ . The secondary interest is the efficiency in mean squared error (MSE). We consider settings with timepoints  $T = 10, 30$  and sample size  $N = 30, 100, 200$ .

Statistical analyses pertaining to the GEE and basic MLM will be performed in R, version 4.4.2 ([Team, 2024](#)). To fit the GEE, the R-package `geepack` ([Halekoh et al., 2006](#)) will evaluate several different working correlation structures, including independent, exchangeable, AR(1) and unstructured. To fit the basic MLM, the R-package `lme4` ([Bates et al., 2015](#)) will be employed, where we will use restricted maximum likelihood estimation.

## 7.4 Trash

DAG for Generative Models