The Greeks in Derivatives – A Technical Overview

Purpose of the Greeks

The Greeks measure the sensitivity of an option's value to changes in underlying parameters. These sensitivities guide hedging, risk management, and pricing adjustments.

1. Delta (): Sensitivity to Underlying Price (S)

Definition:

$$\Delta = \frac{\partial V}{\partial S}$$

Where V is the option price and S is the underlying price.

Key Features:

Option Type	Long Position	Short Position
Call	$\Delta \in (0, 1)$	$\Delta \in (-1, 0)$
Put	$\Delta \in (-1, 0)$	$\Delta \in (0, 1)$

Extremes:

- ATM: Delta ± 0.5 (steepest slope).
- ITM: $|\Delta| \to 1$
- OTM: $|\Delta| \to 0$

Interactions:

- Gamma controls the curvature of Delta.
- Delta increases with time decay if ITM; decreases if OTM.

2. Gamma (): Sensitivity of Delta to S

Definition:

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

Key Features:

- Always positive for long options.
- Always negative for short options.

Extremes:

- ATM: Gamma is maximum.
- ITM/OTM: Gamma $\to 0$.

Interactions:

- Inversely proportional to time-to-expiry and volatility.
- High Gamma = Delta changes rapidly $\to$ riskier to hedge.
- Gamma drives convexity in P&L. $\to$ desirable in volatile markets if Long

3. Theta (): Sensitivity to Time Decay

Definition:

$$\Theta = \frac{\partial V}{\partial t}$$

Key Features:

Option Type	Long Position	Short Position
Call/Put	Negative ($\Theta < 0$)	Positive ($\Theta > 0$)

Extremes:

- ATM: Theta decay is steepest.
- Near expiry: Magnitude increases.
- OTM and deep ITM: Lower Theta.

Interactions:

- Theta-Gamma trade-off: high Gamma comes with high Theta decay.
- Hedgers often buy Gamma (risk protection) and short Theta (cost).

4. Vega (): Sensitivity to Volatility ()

Definition:

$$\nu = \frac{\partial V}{\partial \sigma}$$

Key Features:

- Vega is positive for long options.
- Vega is negative for short options.
- Vega is same for Call and Put with same strike and expiry (since both gain from vol).

Extremes:

- ATM options: Vega is maximum.
- Longer maturities: Higher Vega.
- Short expiry/Short-dated options: Vega $\to 0$.

Interactions:

- High Vega = sensitive to implied volatility (IV) spikes (earnings, events).
- Vega-Theta conflict: Gain from vol spike but suffer decay. You may gain from vol spike but suffer decay.

5. Rho (): Sensitivity to Interest Rate (r)

Definition:

$$\rho = \frac{\partial V}{\partial r}$$

Key Features:

Option Type	Long Rho	Short Rho
Call	Positive	Negative
Put	Negative	Positive

Extremes:

- Far ITM: Rho is highest.
- OTM: Rho ≈ 0.
- Short expiry/short term options: Low Rho.

Interactions:

- Often ignored in equities, critical in long-dated options and FX.
- Rho interacts with Delta via cost-of-carry models. Rho interacts with Delta through cost-of-carry models

6. Charm: $dDelta/d\theta$

Time decay of Delta.

- Relevant for managing intraday delta hedging.
- Large for short-dated options with high Gamma.

7. Vanna: $dDelta/d\sigma$ or $dVega/dS$

- Reflects how Vega changes with asset price.
- Key for managing exposure to volatility surfaces.

8. $Vomma : dVega/d\sigma$

- Vega of Vega.
- Important in volatility trading and vol-of-vol products.

Greek Interactions Matrix

Assuming long European positions:

Greek	**Increases With**	**Decreases With**
Delta	ITM, underlying ↑	OTM
Gamma	ATM, short time to expiry	ITM/OTM, long expiry
Theta	ATM, short expiry	OTM, long expiry
Vega	ATM, longer time to expiry	ITM/OTM, short expiry
Rho	ITM, long expiry	OTM, short expiry

Behavior Across Position Types

Position	**Delta**	**Gamma**	**Vega**	**Theta**	**Rho**
Long Call	+	+	+	−	+
Short Call	−	−	−	+	−
Long Put	−	+	+	−	−
Short Put	+	−	−	+	+

Practical Trade-offs

- Long Gamma vs. Theta: Costly to hedge Delta. You pay Theta to own Gamma (hedge Delta better)
- Vega vs. Theta: Vega strategies (e.g., straddles) decay quickly. Vega-rich strategies decay fast (calendar spreads, long straddles).
- Delta-Neutral: Market direction agnostic, but Gamma, Vega, Theta matter. Market direction agnostic, but Gamma/Theta/Vega matter.
- Vega-Neutral: Sensitive to realized vol (Gamma scalping, dispersion trades). Sensitive to realized vol (Gamma scalping or dispersion trades).
- Rho-Delta Hedging: Key in long-dated or FX options. For long-dated options or FX where rates matter.

Summary of Extremes

Greek	**Max When**	**Min When**
Delta	Deep ITM	Deep OTM
Gamma	ATM, near expiry	Deep ITM/OTM
Theta	ATM, near expiry	Far expiry, OTM
Vega	ATM, long expiry	Short expiry, ITM/OTM
Rho	Deep ITM, long expiry	Short expiry, OTM

Excellent observation — this contrast between **Gamma decreasing** and **Vega increasing** with **longer time to expiration** is not only **correct**, but also **critical** for understanding the risk and structure of option profiles.

Let's unpack **why** this happens.

Gamma vs. Vega: Time-to-Expiry Behavior

Greek	**Increases with Time?**	**Description**
Gamma	Decreases	Measures how quickly Delta changes with price.
Vega	Increases	Measures how much the option price changes with volatility.

Why Gamma Decreases with Longer Expiry

Gamma is:

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

- **Gamma is highest when the option is ATM and near expiration**.
- With more time to expiry, price movements have **less immediate effect** on whether an option ends ITM or not.
- Thus, the **slope of Delta vs. price becomes flatter**.
- Longer time = **Delta changes more gradually** → **lower Gamma**.

Intuition:

For a short-dated option, every small price move is a big deal — it might flip the option from OTM to ITM. That makes Delta very sensitive → High Gamma.

For a long-dated option, one price move doesn't matter as much — the option has plenty of time to come back. So Delta moves slowly → Low Gamma.

Why Vega Increases with Longer Expiry

Vega is:

$$\nu = \frac{\partial V}{\partial \sigma}$$

- Vega measures how much an option's value changes with a **1% change in volatility**.

- When expiration is far away, there's **more uncertainty** over where the underlying will end up.
- A higher volatility increases that uncertainty **more** when there is **more time left**.
- So: **Long-dated options are much more sensitive to volatility** changes → **Higher Vega**.

Intuition:
Volatility matters more when time is longer — more time for uncertainty to manifest → greater potential impact on option value.

Visual Example (Conceptual)

Imagine a 1-day vs. 1-year ATM option:

Option	**Gamma (Steepness of Delta)**	**Vega (Impact of Vol)**
1-day ATM	High Gamma	Low Vega
1-year ATM	Low Gamma	High Vega

Implications for Traders

- **Near-term options**:
  - High Gamma → Good for directional plays (quick gains/losses).
  - High Theta → Costly to hold.
- **Long-term options**:
  - High Vega → Good for volatility speculation.
  - Low Gamma → Poor for quick directional delta hedging.

This is why **Gamma and Vega are in tension**:

- Buying Gamma (e.g., short-dated ATM options) gives strong convexity but poor Vega.
- Buying Vega (e.g., long-dated options) gives strong vol exposure but poor convexity.

Mathematical Formulas Support This

In Black-Scholes:

$$\Gamma = \frac{e^{-qT}\phi(d_1)}{S\sigma\sqrt{T}}, \quad \text{Vega} = Se^{-qT}\phi(d_1)\sqrt{T}$$

Where $\phi(d_1)$ is the standard normal PDF.

Notice:

- **Gamma** $\propto \frac{1}{\sqrt{T}} \rightarrow$ Decreases with time.
- **Vega** $\propto \sqrt{T} \rightarrow$ Increases with time.

Certainly. The **Theta ()** of an option measures the **rate of change of the option price with respect to time** — essentially, how much value the option loses per day, all else held constant.

General Definition:

$$\Theta = \frac{\partial V}{\partial t}$$

In the **Black-Scholes model**, Theta is derived differently for **European Calls** and **Puts**.

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1. Theta for a European Call Option:

$$\Theta_{\text{call}} = -\frac{Se^{-qT}\phi(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2) + qSe^{-qT}N(d_1)$$

Components:

- S: Current stock price
- K: Strike price
- T: Time to expiration (in years)
- σ: Volatility
- r: Risk-free interest rate
- q: Dividend yield
- $N(d)$: Standard normal CDF
- $\phi(d)$: Standard normal PDF
- $d_1 = \frac{\ln(S/K)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

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2. Theta for a European Put Option:

$$\Theta_{\text{put}} = -\frac{Se^{-qT}\phi(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2) - qSe^{-qT}N(-d_1)$$

Interpretation:

- The first term (**negative**) is common to both and reflects **time decay due to uncertainty**.
- The other terms represent the effects of **interest rates** and **dividends**:
  - For calls: Interest rate reduces Theta (since higher r increases present value of payoff).
  - For puts: Interest rate increases Theta (since higher r increases put value).
  - Dividend yield q works in the opposite direction.

Approximate Behavior:

- **Theta is more negative (i.e., decays faster)**:
  - When the option is **ATM**
  - As **expiration approaches**
  - In **low interest rate environments**
- **Long options**: Theta is negative
- **Short options**: Theta is positive (you ”collect” time decay)

Let me know if you want to visualize the Theta curve or explore second-order time sensitivity (like **DTheta/Dt**).

Yes — **the Greeks are interconnected**, and **changes in one Greek often influence others**. This dynamic relationship is central to derivatives risk management and understanding the **non-linear behavior** of options.

Let's dive into these **Greek interactions**, how they evolve **dynamically**, and what that means in practice.

Interdependence and Dynamics of the Greeks

Summary Table: How a Change in One Greek Affects Others

Trigger (↑)	**Effect on Others**
Underlying Price (S) ↑	$\Delta \uparrow$ (call), $\downarrow$ (put) Γ changes (depends on moneyness) Θ, ν, ρ shift accordingly
$\Gamma \uparrow$	Implies Δ becomes more sensitive Implies Θ becomes more negative May imply $\nu \downarrow$ (if short-dated)
$\Theta \uparrow$ **(more negative)**	Often caused by increased Γ Can imply $\nu \downarrow$ if near expiry
$\nu \uparrow$	Often implies lower Γ (long-dated options) May reduce Θ (less decay per day)
Volatility (σ) ↑	$\nu \uparrow$, Option Value ↑ $\Gamma \downarrow$ for long-dated options Δ moves toward 0.5
Time (T) ↓	$\Gamma \uparrow$ (ATM) $\Theta \uparrow$ (more negative) $\nu \downarrow$ Δ accelerates toward 0 or 1
Interest Rate (r) ↑	$\rho \uparrow$ (Call), $\rho \downarrow$ (Put) Slight changes in Δ due to forward value shifts

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Deep Dive: How One Greek Influences the Rest

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1. $\Gamma \leftrightarrow \Delta, \Theta$

- High Γ means Δ is changing rapidly $\rightarrow$ hedging is more volatile.
- Also leads to higher Θ because ATM options with high Γ lose value quickly.

- In short-dated options, Γ and Θ are tightly coupled:

$$\text{Higher } \Gamma \Rightarrow \text{more negative } \Theta$$

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2. $\nu \leftrightarrow \Gamma, \Theta$

- ν and Γ are typically inversely related with respect to time:
  - Long-dated ATM options: High ν, low Γ
  - Short-dated ATM options: High Γ, low ν
- Increasing ν implies:
  - Greater sensitivity to implied volatility
  - Less sharp movement in $\Delta \to$ Lower Γ
  - Slower time decay $\to$ Less negative Θ

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3. Time to Expiry $\leftrightarrow$ All Greeks

- As time T decreases:
  - $\Gamma \uparrow$ (especially ATM)
  - Θ becomes more negative
  - $\nu \downarrow$
  - Δ accelerates toward 0 or 1
- This leads to a Gamma-Theta-Vega squeeze in short-dated options:
  - Great for scalping (fast moves in Δ)
  - Expensive to hold (high Θ loss)
  - Minimal ν exposure (less impact from vol changes)

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4. Volatility $\leftrightarrow \Delta, \Gamma, \Theta$

- When $\sigma \uparrow$:
  - Δ of ATM options moves toward 0.5
  - $\Gamma \downarrow$
  - Θ becomes less negative
- When $\sigma \downarrow$:
  - $\Gamma \uparrow$
  - Θ becomes more negative

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5. Interest Rates $(r) \leftrightarrow \Delta, \rho, \Theta$

- For calls: higher r increases option value $\rightarrow \Delta \uparrow$, $\rho \uparrow$
- For puts: higher r decreases option value $\rightarrow \Delta \downarrow$, $\rho \downarrow$
- ρ is most relevant in long-dated options
- Higher r can also make Θ less negative for calls

Practical Dynamics in Hedging

Hedging Action	**Greek Effect Cascade**
Delta-hedging (neutralizing Δ)	High $\Gamma \rightarrow$ frequent rebalancing $\rightarrow$ Theta bleed
Gamma-scaling (owning Γ)	Must short Θ to fund Gamma
Vega-neutral portfolio	Vol changes affect Δ, Γ; needs rebalancing
Rolling short-term ATM options	High Γ, high Θ; fast re-hedging required

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Feedback Loops in Practice

Example: Long Call ATM, 5 Days to Expiry

- Γ is high $\rightarrow$ frequent Δ-hedging
- Each hedge consumes Θ
- If volatility spikes: $\nu \uparrow \rightarrow$ Option value $\uparrow$
- But $\Gamma \downarrow \rightarrow$ Hedging becomes less frequent

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Meta-Rule: "You Can't Own Everything"

- Own $\Gamma \to$ Pay Θ
- Own $\nu \to$ Sacrifice Γ
- Own Θ (short options) $\to$ Exposed to Γ and ν

Conclusion: Dynamic Interactions Matter

Understanding the **dynamic system** of the Greeks allows you to:

- Anticipate evolving **risk exposure**
- Design **stable portfolios** (delta- and vega-neutral)
- Align **Greek profile** with your outlook (price, vol, time)

Let me know if you'd like a live Python simulation where we evolve all the Greeks as functions of time, price, and volatility — it makes these dynamics very clear.