CSE 252A Computer Vision I Fall 2020 - Assignment 2

Instructor: David Kriegman

Assignment published on Sunday, November 1st, 2020

Due on: Thursday, November 12th, 2020 at 11:59pm Pacific Time

---

Instructions

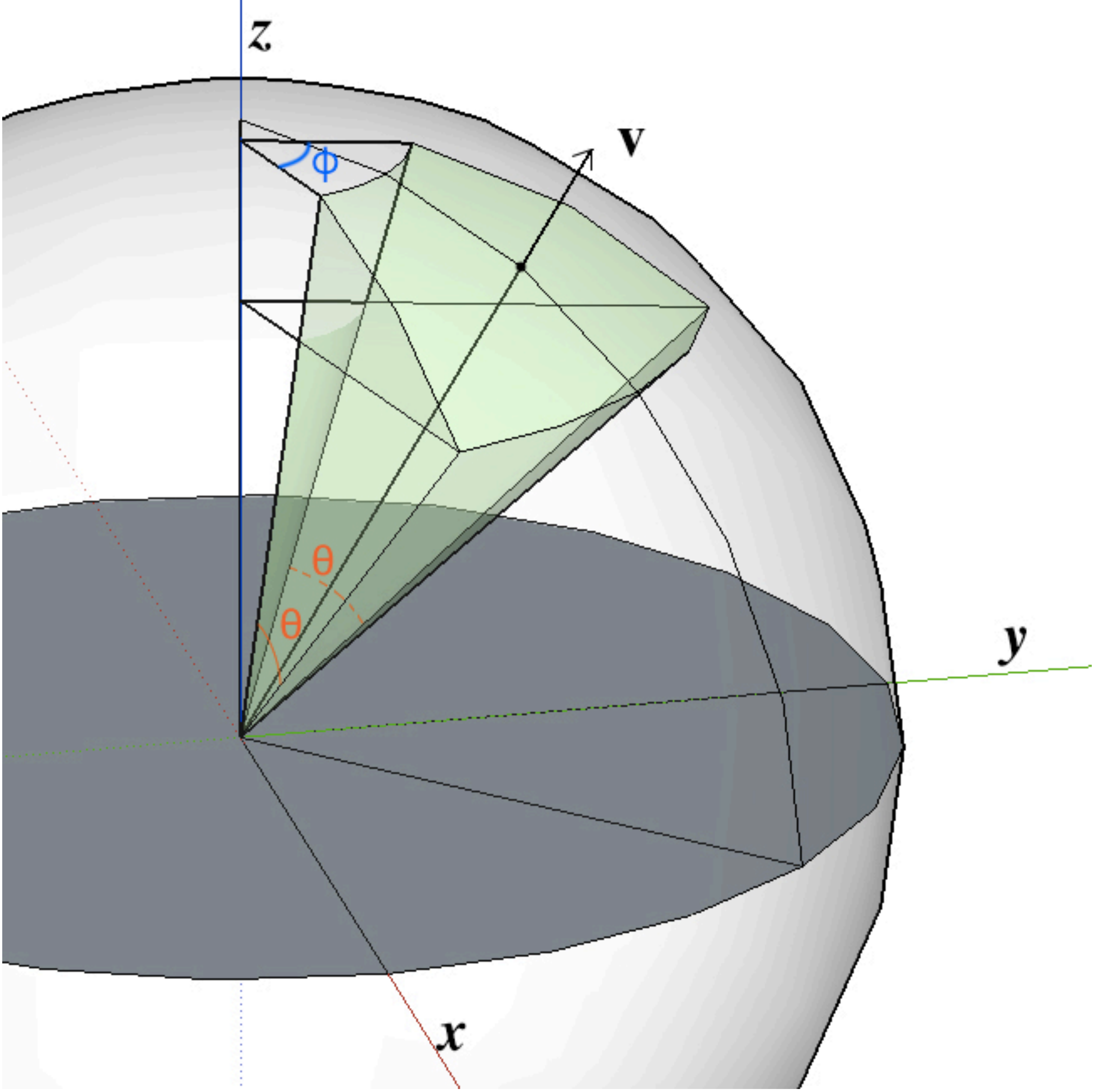
- Review the academic integrity and collaboration policies on the course website.
- This assignment must be completed individually.
- This assignment contains theoretical and programming exercises.If you plan to submit hand written answers for theoretical exercises, please be sure your writing is readable and merge those in order with the final pdf you create out of this notebook. You could fill the answers within the notebook iteself by creating a markdown cell.
- Programming aspects of this assignment must be completed using Python in this notebook.
- If you want to modify the skeleton code, you can do so. This has been provided just to provide you with a framework for the solution.
- You may use python packages for basic linear algebra (you can use numpy or scipy for basic operations), but you may not use packages that directly solve the problem.
- If you are unsure about using a specific package or function, then ask the instructor and teaching assistants for clarification.
- You must submit to Gradescope:
  - (1) This notebook exported as a `.pdf` (including any hand-written solutions scanned and merged into the PDF, if applicable).
  - (2) This notebook as an `.ipynb` file.
- You must mark each problem on Gradescope in the pdf.
- **Late policy:** Assignments submitted late will receive a 10% grade reduction for

each day late (e.g. an assignment submitted an hour after the due date will receive a 10% penalty, an assignment submitted 10 hours after the due date will receive a 10% penalty, and an assignment submitted 28 hours after the due date will receive a 20% penalty). Assignments will not be accepted 72 hours after the due date. If you require an extension (for personal reasons only), you must request one as far in advance as possible. Extensions requested close to or after the due date will only be granted for clear emergencies or clearly unforeseeable circumstances.

Problem 1: Steradians

Part 1 [2 pts]

Calculate the number of steradians contained in a spherical wedge with radius *r* = 1, defined by $\theta = \frac{\pi}{6}, \phi = \frac{\pi}{6}$ and centered around vector $\left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{2}\right)$.



Part 2 [1 pt]

How many steradians are in a quarter sphere?

In [51]:

```
import matplotlib.pyplot as plt

WrittenOne = plt.imread('CSE252A_HW2_1.jpg')             # read
a JPEG image
WrittenTwo = plt.imread('CSE252A_HW2_2.jpg')
WrittenThree = plt.imread('CSE252A_HW2_3.jpg')
WrittenFour = plt.imread('CSE252A_HW2_4.jpg')
plt.figure(figsize=(20, 20))
plt.imshow(WrittenOne)
plt.show()
```

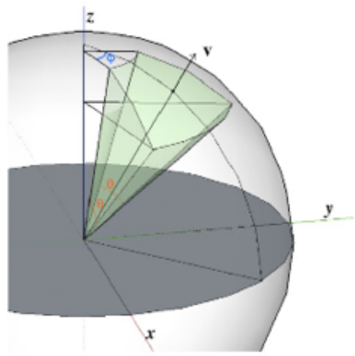
Homework 2 Written

Problem 1:

Problem 1: Steradians

Part 1 [2 pts]

Calculate the number of steradians contained in a spherical wedge with radius $r = 2$, defined by $\theta = \frac{\pi}{6}$, $\phi = \frac{\pi}{6}$ and centered around vector $\left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{2}\right)$.



Part 2 [1 pt]

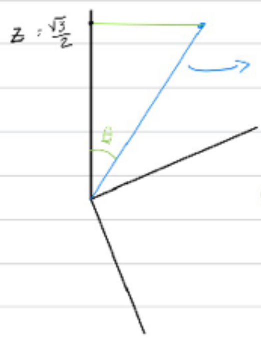
How many steradians are in a quarter sphere?

Part 1:

$$S = \int_\phi \int_\theta \sin\theta \, d\theta \, d\phi$$

need bounding on angles for integration

↳ find angle between z-axis & vector $\left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{2}\right)$



$$d = \sqrt{\left(\frac{\sqrt{2}}{4}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\cos\theta = \frac{\sqrt{3}/2}{d} = 0.866$$

$$\theta = \cos^{-1}(0.866) = \pi/6$$

Therefore θ goes $\left(\pi/6 - \left(\frac{1}{2}\right)\frac{\pi}{6}, \ \pi/6 + \left(\frac{1}{2}\right)\frac{\pi}{6}\right)$

ϕ goes $(0, \pi/6)$

$$S = \int_0^{\pi/6} \int_{\pi/12}^{\pi/4} \sin\theta \, d\theta \, d\phi = \int_0^{\pi/6} -\cos(\pi/4) + \cos(\pi/12) \, d\phi$$

$$= 0.2588\left(\pi/6\right) = \boxed{0.1355 \text{ Str}}$$

Part 2

quarter sphere?

$\phi : (0, \pi) \quad \theta : (0, \pi/2)$

$$S = \int_0^{\pi} \int_0^{\pi/2} \sin\theta \, d\theta \, d\phi = \int_0^{\pi} \underbrace{-\cos\left(\frac{\pi}{2}\right)}_{=0} + \underbrace{\cos(0)}_{=1} \, d\phi$$

$\boxed{S = \pi \text{ str}}$ (makes sense b/c (sphere $S = 4\pi$)

check: $\phi : (0, \pi/2)$, $\theta : (0, \pi)$

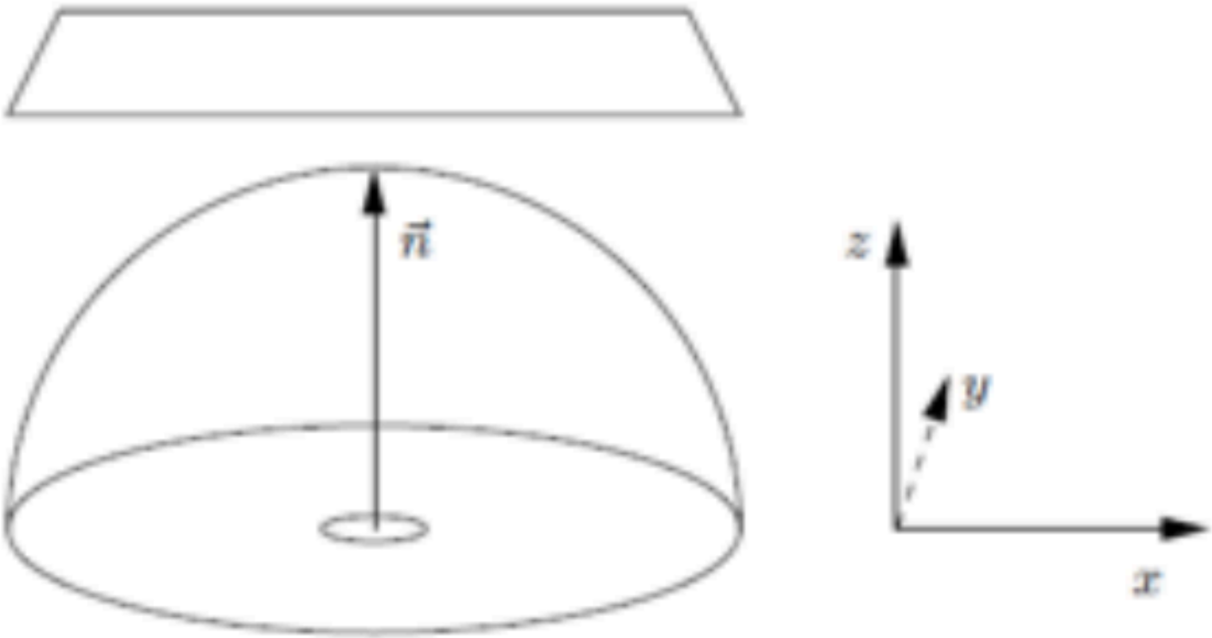
$$S = \int_0^{\pi/2} \int_0^{\pi} \sin\theta \, d\theta \, d\phi = \int_0^{\pi/2} -\cos(\pi) + \cos(0) \, d\phi$$

$= \pi/2 \, (+1 + 1) = \frac{\pi}{2}(2)$

$= \pi$ ✓✓✓

Problem 2: Irradiance [3 pts]

Consider a rectangular surface with vertices $(-4, -3, 1)$, $(4, -3, 1)$, $(-4, 3, 1)$, and $(4, 3, 1)$. If the radiance on the surface is equal in all directions and is given by $L \cdot (x^2 + y^2 + 3)$ (with L constant) what is the irradiance arriving at position $(0, 0, 0)$ with normal vector $(0, 0, 1)$? (Note: You do not need to perform the integration, just set up the integral.)



In [49]:

```
plt.figure(figsize=(20, 20))
plt.imshow(WrittenTwo)
plt.show()
```

Problem 2:

irradiance: $\iint L(x, \theta, \phi) \cos\theta \, d\omega$

find r and $\cos\alpha$ in terms of surface variables to find the irradiance from the whole surface by integration

$$d\omega = \frac{dA}{r^2} \cos\alpha$$

r = distance from point $(0,0,0)$ to surface,

$= \sqrt{x^2+y^2+z^2}$, z is const @ 1 over whole surface

$$r = \sqrt{x^2+y^2+1}$$

$$\cos\theta = \frac{|(0,0,1)|}{\sqrt{x^2+y^2+z^2}} = \frac{1}{\sqrt{x^2+y^2+1}}$$

$$\text{irradiance} = \iint L(x, \theta, \phi) \cos\theta \left(\cos\alpha\right) \frac{dA}{r^2}$$

$$= \iint L(x, \theta, \phi) \frac{\cos^2\theta}{r^2} \, dA$$

$$\boxed{= \int_{x=-4}^{4} \int_{y=-3}^{3} L\left(x^2+y^2+3\right) \frac{1}{(x^2+y^2+1)^2} \, dy\, dx}$$

Problem 3: Superposition of Light Sources and Lambertian Surfaces [2 pts]

A Lambertian surface is illuminated by two distant light sources. The first has intensity s_1 and direction $\hat{\mathbf{s}}_1$,and the second has intensity s_2 and direction $\hat{\mathbf{s}}_2$. Consider the surface to be shaped such that no part of it is in shadow from either individual light source.

For every pixel in the image where the surface is illuminated by both sources, there is a single effective distant light source which will produce the same irradiance at that pixel. What is the intensity and direction of that light source?

In [52]:

```
plt.figure(figsize=(20, 20))
plt.imshow(WrittenThree)
plt.show()
```

Problem 3 :

$s_1 \;\; 3 \;\; \hat{s}_1 \quad , \quad s_2 \;\; 3 \;\; \hat{s}_2$

single light source equivalent ?

$$I_{total} = I_1 + I_2$$

$$= a(u,v) \cdot \hat{n}(u,v) \cdot \underbrace{(s_1 \hat{s}_1 + s_2 \hat{s}_2)}_{s_1(x_1, y_1, z_1) + s_2(x_2, y_2, z_2)}$$

$$S_{total} = \sqrt{(s_1 x_1 + s_2 x_2)^2 + (s_1 y_1 + s_2 y_2)^2 + (s_1 z_1 + s_2 z_2)^2}$$

$\hat{S}_{total}$ direction is $\dfrac{(s_1 x_1 + s_2 x_2)\hat{i}}{S_{total}} + \dfrac{(s_1 y_1 + s_2 y_2)\hat{j}}{S_{total}} + \dfrac{(s_1 z_1 + s_2 z_2)\hat{k}}{S_{total}}$

Problem 4: Occlusion, Umbra and Penumbra [2 pts]

We have a square area source and a square occluder, both parallel to a plane.

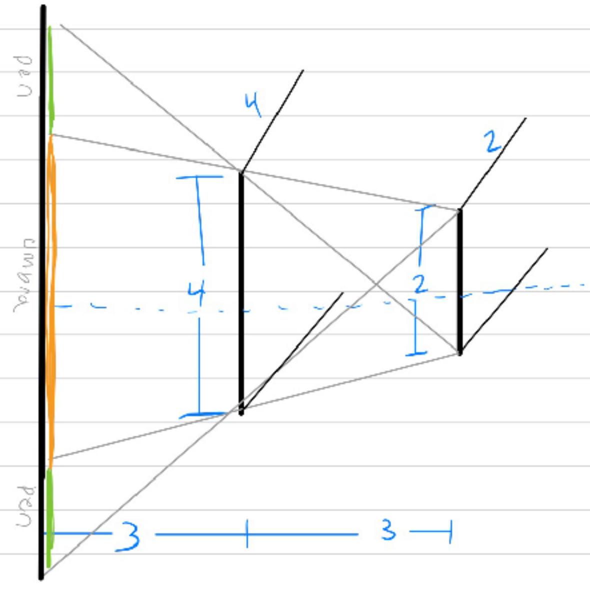
The edge lengths of the source and occluder are 2 and 4, respectively, and they are vertically above one another with their centers aligned. The distances from the occluder to the source and plane are both 3.

1. What is the area of the umbra on the plane?
2. What is the area of the penumbra on the plane?

In [53]:

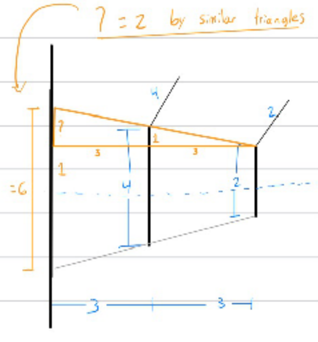
```
plt.figure(figsize=(20, 20))
plt.imshow(WrittenFour)
plt.show()
```

Problem 4) source : 2, occluder : 4, centers aligned



Umbra

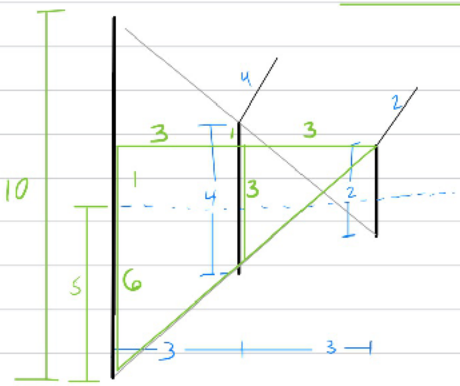
? = 2 by similar triangles



Since source & occluder are squares, so is penumbra. therefore is 6×6

Area of umbra 36

Penumbra



Penumbra area = 100 - Umbra area = 100 - 36

∴ Penumbra area = 64

Problem 5: Photometric Stereo [6 pts]

The goal of this problem is to implement a couple of different algorithms that reconstruct a surface using the concept of photometric stereo.

Your program will take in multiple images as input along with the light source direction (and color when necessary) for each image.

Data

Synthetic Images: Available in `*.pickle` files (graciously provided by Satya Mallick) which contain

- `im1` , `im2` , `im3` , `im4` ... images.
- `l1` , `l2` , `l3` , `l4` ... light source directions.

Implement the photometric stereo technique described in Forsyth and Ponce 2.2.4 (*Photometric Stereo: Shape from Multiple Shaded Images*) and the lecture notes.

Your program should have two parts:

1. Read in the images and corresponding light source directions, and estimate the surface normals and albedo map.
2. Reconstruct the depth map from the surface normals. You can first try the naive scanline-based shape by integration method described in the book. If this does not work well on real images, you can use the implementation of the Horn integration technique given below in `horn_integrate` function.

Try using only `im1` , `im2` and `im4` first. Display your outputs as mentioned below.

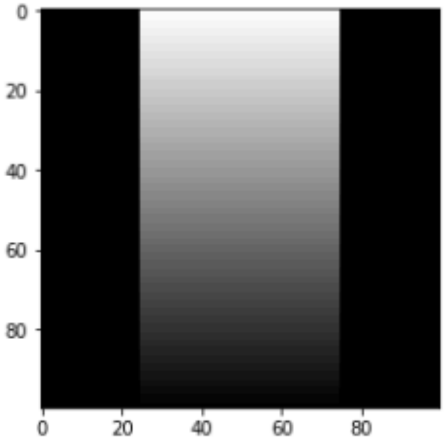
Then use all four images. (Most accurate).

For each of the above cases you must output:

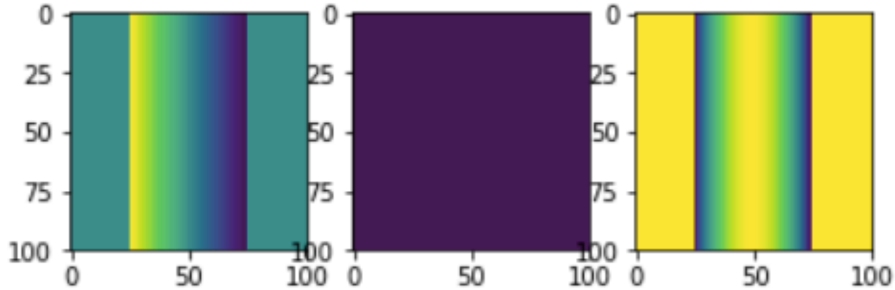
1. The estimated albedo map.
2. The estimated surface normals by showing both
   A. Needle map, and
   B. Three images showing components of surface normal.

3. A wireframe of depth map.
4. Horn output

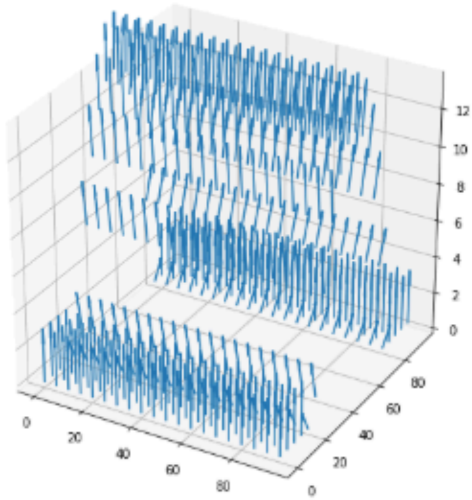
An example of outputs is shown below.



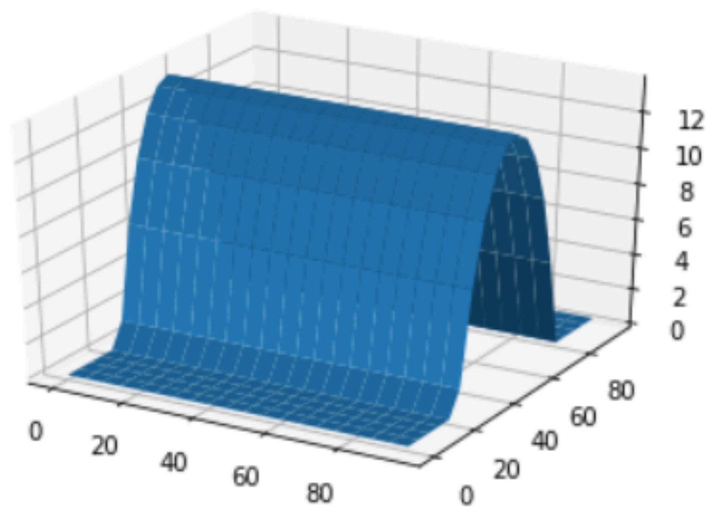
Albedo map



Normals as three separate channels



Needle map



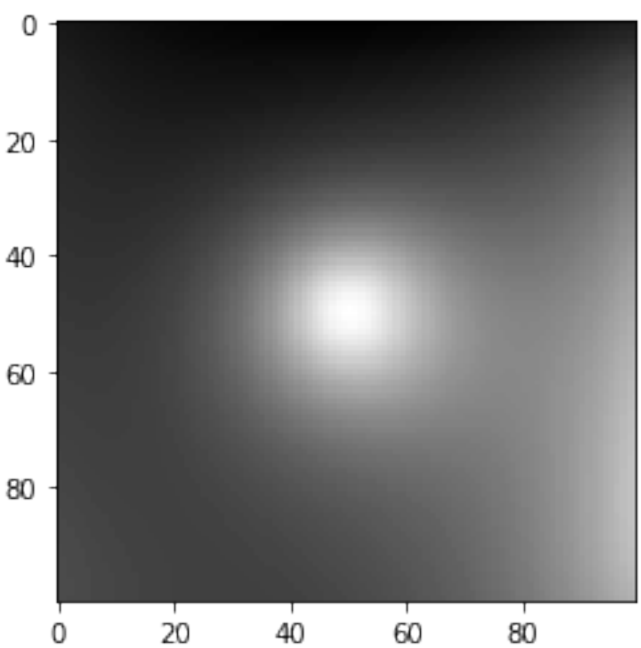
Wireframe of depth map

Note: You will find all the data for this part in `synthetic_data.pickle`.

In [8]:

```python
## Example: How to read and access data from a pickle
import pickle
import numpy as np
from time import time
from skimage import io
%matplotlib inline
import matplotlib.pyplot as plt

pickle_in = open('synthetic_data.pickle', 'rb')
data = pickle.load(pickle_in, encoding='latin1')

# data is a dict which stores each element as a key-value pair.
print('Keys: ' + str(data.keys()))

# To access the value of an entity, refer it by its key.
print('Image:')
plt.imshow(data['im1'], cmap = 'gray')
plt.show()

print('Light source direction: ' + str(data['l1']))
```

```
Keys: dict_keys(['__version__', 'l4', '__header__',
'im1', 'im3', 'im2', 'l2', 'im4', 'l1', '__globals__
', 'l3'])
Image:
```




```
Light source direction: [[0 0 1]]
```

In [9]:

```
from scipy.signal import convolve
from numpy import linalg

def horn_integrate(gx, gy, mask, niter):
    """ horn_integrate recovers the function g from its partial derivatives gx and gy.
        mask is a binary image which tells which pixels are involved in integration.
        niter is the number of iterations (typically 100,000 or 200,000,
        although the trend can be seen even after 1000 iterations).
    """
    g = np.ones(np.shape(gx))

    gx = np.multiply(gx, mask)
    gy = np.multiply(gy, mask)

    A = np.array([[0,1,0],[0,0,0],[0,0,0]]) #y-1
    B = np.array([[0,0,0],[1,0,0],[0,0,0]]) #x-1
    C = np.array([[0,0,0],[0,0,1],[0,0,0]]) #x+1
    D = np.array([[0,0,0],[0,0,0],[0,1,0]]) #y+1

    d_mask = A + B + C + D

    den = np.multiply(convolve(mask, d_mask, mode='same'), mask)
    den[den == 0] = 1
    rden = 1.0 / den
    mask2 = np.multiply(rden, mask)

    m_a = convolve(mask, A, mode='same')
    m_b = convolve(mask, B, mode='same')
    m_c = convolve(mask, C, mode='same')
    m_d = convolve(mask, D, mode='same')

    term_right = np.multiply(m_c, gx) + np.multiply(m_d, gy)
    t_a = -1.0 * convolve(gx, B, mode='same')
    t_b = -1.0 * convolve(gy, A, mode='same')
    term_right = term_right + t_a + t_b
    term_right = np.multiply(mask2, term_right)

    for k in range(niter):
```

```
        g = np.multiply(mask2, convolve(g, d_mask, mode='same'))
+ term_right

    return g
```

In [10]:

```
def photometric_stereo(images, lights, mask, horn_niter=25000):

    """ ==========YOUR CODE HERE=========="""
    '''Version to handle 4 images and lights'''
    sz = np.shape(images)
    eMatrix = np.zeros((sz[1]*sz[2], sz[0]))
    for i in range(0,sz[0]):
        eMatrix[:,i] = np.reshape(images[i,:,:], (sz[1]*sz[2],1)
)[:,0]
    eMatrix = eMatrix.T
    s = lights
    sFind = np.matmul(np.linalg.inv(np.matmul(s.T, s)), s.T)
    B = np.matmul(sFind, eMatrix).T
    # note:
    # images : (n_ims, h, w)
    # lights : (n_ims, 3)
    albedo = np.ones(images[0].shape)
    normals = np.dstack((np.zeros(images[0].shape),
                         np.zeros(images[0].shape),
                         np.ones(images[0].shape)))

    albedoIntermediate = np.reshape(albedo, (sz[1]*sz[2],1))
    albedoIntermediate[:,0] = B[:,0]*B[:,0] + B[:,1]*B[:,1] + B[
:,2]*B[:,2]
    albedoVect = np.sqrt(albedoIntermediate)
    albedo = np.reshape(albedoVect, (sz[1],sz[2]))

    normals[:,:,0] = np.reshape(B[:,0]/albedoVect[:,0], (sz[1],
sz[2]))
    normals[:,:,1] = np.reshape(B[:,1]/albedoVect[:,0], (sz[1],
sz[2]))
    normals[:,:,2] = np.reshape(B[:,2]/albedoVect[:,0], (sz[1],
sz[2]))

    slant = (-normals[:,:,0]/normals[:,:,2])*mask[:,:]
    tilt = (-normals[:,:,1]/normals[:,:,2])*mask[:,:]

    H = np.ones(images[0].shape)
```

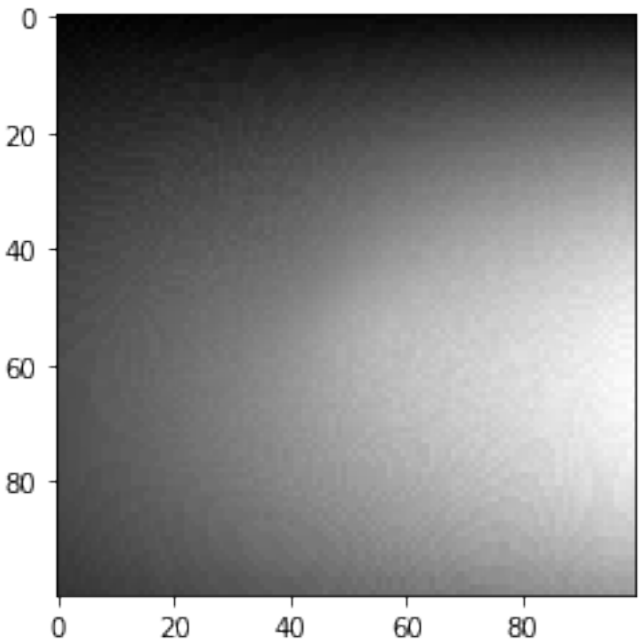
```
    H[0,0] = slant[0,0]

    for j in range(1,sz[2]):
        H[0,j] = H[0,j-1] - slant[0,j]

    for i in range(1,sz[1]):
        for j in range(0, sz[2]):
            H[i,j] = H[i-1,j] - tilt[i,j]

    H_horn = np.ones(images[0].shape)
    H_horn = horn_integrate(-slant, -tilt, mask, horn_niter)
    return albedo, normals, H, H_horn
```

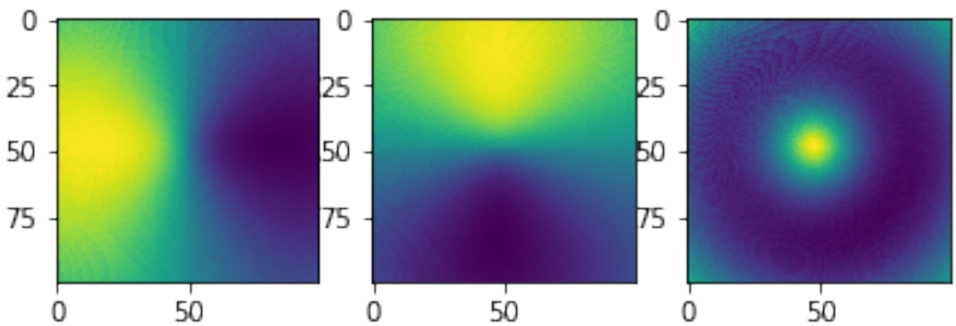
In [11]:

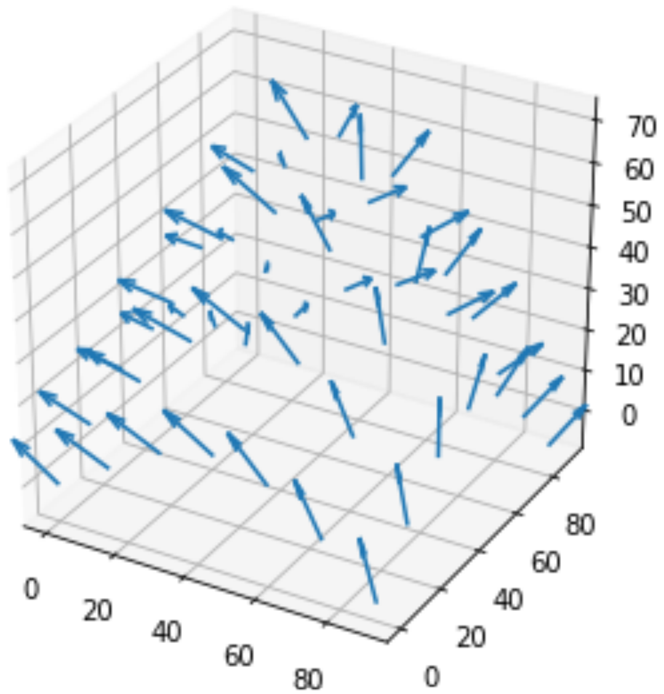
```
from mpl_toolkits.mplot3d import Axes3D

pickle_in = open('synthetic_data.pickle', 'rb')
data = pickle.load(pickle_in, encoding='latin1')

lights = np.vstack((data['l1'], data['l2'], data['l4']))

images = []
images.append(data['im1'])
images.append(data['im2'])
# images.append(data['im3'])
images.append(data['im4'])
images = np.array(images)

mask = np.ones(data['im1'].shape)

albedo, normals, depth, horn = photometric_stereo(images, lights
, mask)

# ---------------------------------------------------------------
------------
# The following code is just a working example so you don't get
stuck with any
# of the graphs required. You may want to write your own code to
align the
# results in a better layout.
# ---------------------------------------------------------------
------------

def visualize(albedo, normals, depth, horn):
```

```
    # Stride in the plot, you may want to adjust it to different
images
    stride = 15

    # showing albedo map
    fig = plt.figure()
    albedo_max = albedo.max()
    albedo = albedo / albedo_max
    plt.imshow(albedo, cmap='gray')
    plt.show()

    # showing normals as three separate channels
    figure = plt.figure()
    ax1 = figure.add_subplot(131)
    ax1.imshow(normals[..., 0])
    ax2 = figure.add_subplot(132)
    ax2.imshow(normals[..., 1])
    ax3 = figure.add_subplot(133)
    ax3.imshow(normals[..., 2])
    plt.show()

    # showing normals as quiver
    X, Y, _ = np.meshgrid(np.arange(0,np.shape(normals)[0], 15),
                          np.arange(0,np.shape(normals)[1], 15),
                          np.arange(1))
    X = X[..., 0]
    Y = Y[..., 0]
    Z = depth[::stride,::stride].T
    NX = normals[..., 0][::stride,::-stride].T
    NY = normals[..., 1][::-stride,::stride].T
    NZ = normals[..., 2][::stride,::stride].T
    fig = plt.figure(figsize=(5, 5))
    ax = fig.gca(projection='3d')
    plt.quiver(X,Y,Z,NX,NY,NZ, length=15)
    plt.show()

    # plotting wireframe depth map
    H = depth[::stride,::stride]
    fig = plt.figure()
    ax = fig.gca(projection='3d')
    ax.plot_surface(X,Y, H.T)
    plt.show()
```

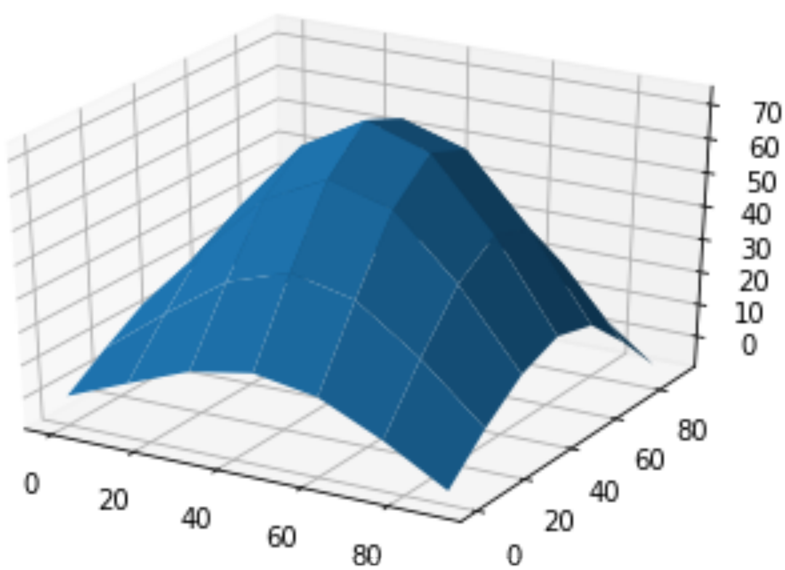
```
    # plot horn output
    H = horn[::stride,::stride]
    fig = plt.figure()
    ax = fig.gca(projection='3d')
    ax.plot_surface(X,Y, H.T)
    plt.show()

visualize(albedo, normals, depth, horn)
```



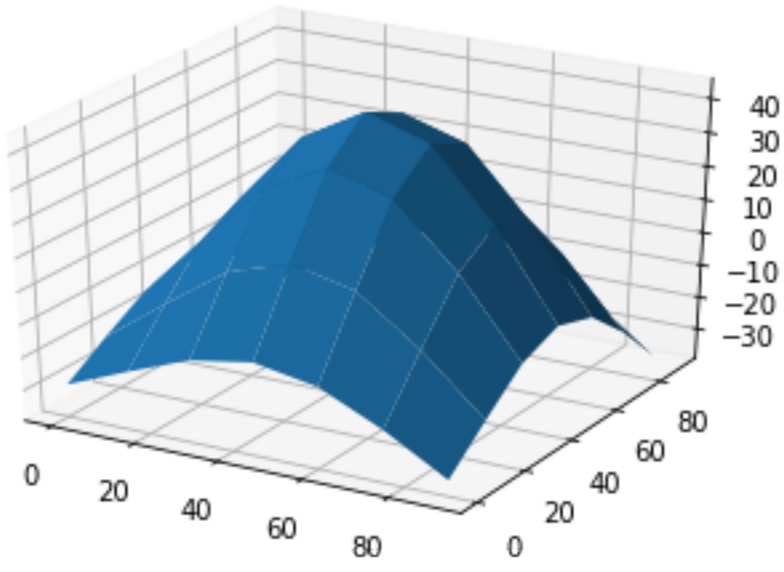




70
60
50
40
30
20
10
0
0
20
40
60
80
0
20
40
60
80

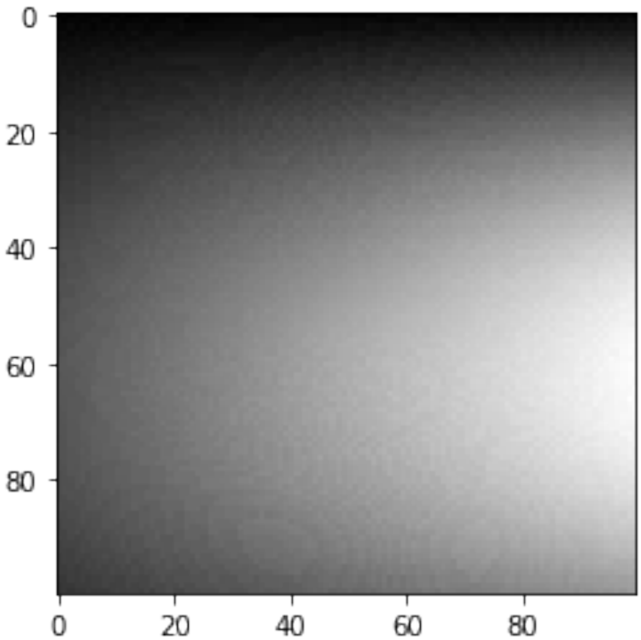


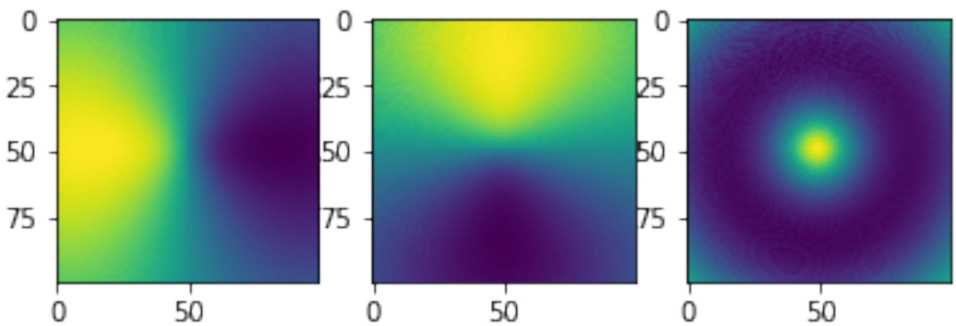
70
60
50
40
30
20
10
0
0
20
40
60
80
0
20
40
60
80

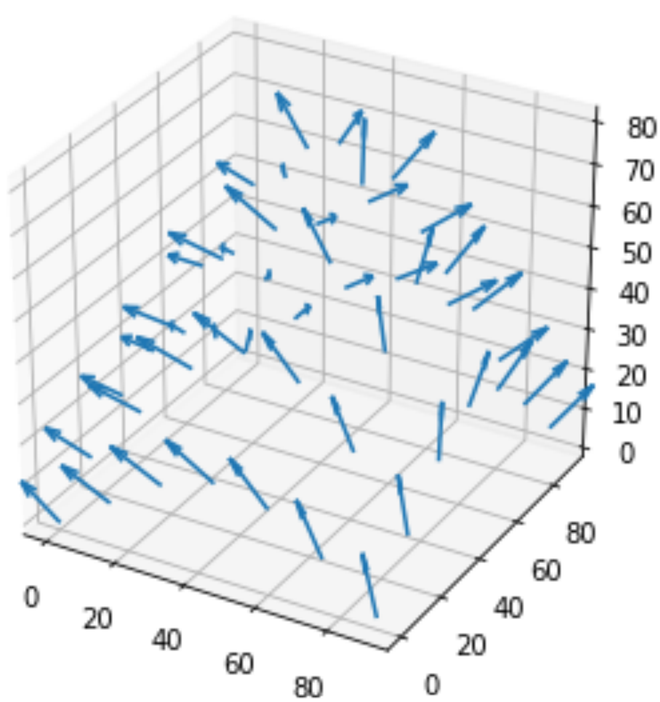


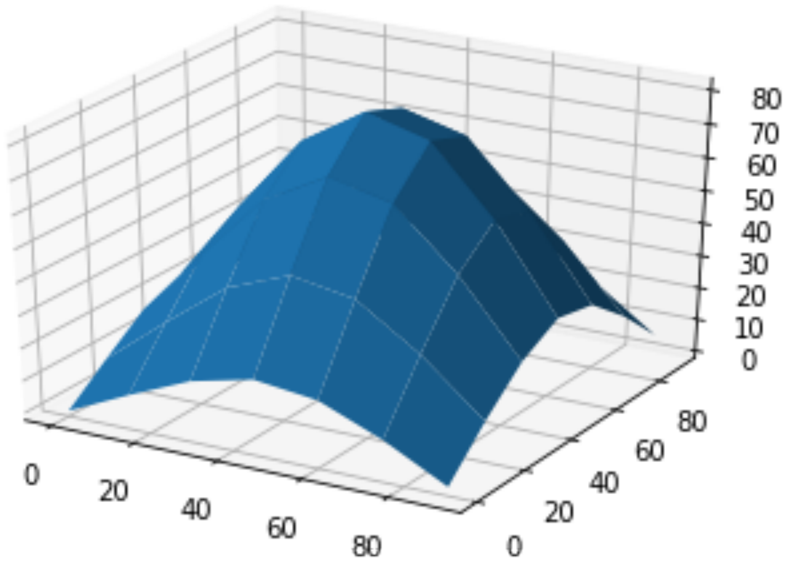
In [12]:

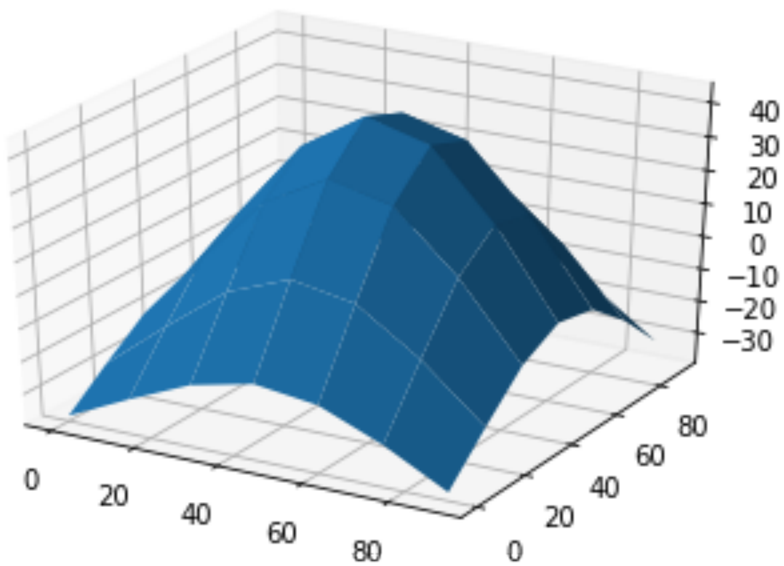
```
pickle_in = open('synthetic_data.pickle', 'rb')
data = pickle.load(pickle_in, encoding='latin1')

lights = np.vstack((data['l1'], data['l2'], data['l3'], data['l4
']))

images = []
images.append(data['im1'])
images.append(data['im2'])
images.append(data['im3'])
images.append(data['im4'])
images = np.array(images)

mask = np.ones(data['im1'].shape)

albedo, normals, depth, horn = photometric_stereo(images, lights
, mask)
visualize(albedo, normals, depth, horn)
```


0
20
40
60
80
0 20 40 60 80


0
25
50
75
0 50
0
25
50
75
0 50
0
25
50
75
0 50


0 20 40 60 80
0 20 40 60 80
0 10 20 30 40 50 60 70 80





Problem 6: Image Filtering [13 pts]

Part 1: Warmup [1.5 pts]

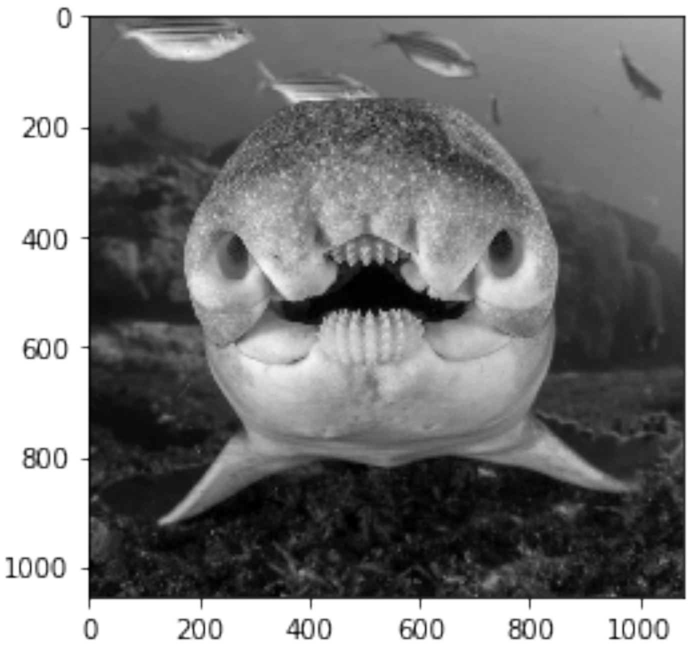
In this problem, we expect you to use convolution to filter the provided image with three different types of kernels:

1. A 5x5 Gaussian filter with σ = 5.
2. A 31x31 Gaussian filter with σ = 5.
3. A sharpening filter.

This is the image you will be using:

In [15]:

```
import numpy as np
from skimage import io
import matplotlib.pyplot as plt
import matplotlib.cm as cm

# Open image as grayscale
shark_img = io.imread('shark.png', as_gray=True)

# Show image
plt.imshow(shark_img, cmap=cm.gray)
plt.show()
```



For convenience, we have provided a helper function for creating a square isotropic Gaussian kernel. We have also provided the sharpening kernel that you should use. Finally, we have provided a function to help you plot the original and filtered results side-by-side. Take a look at each of these before you move on.

In [16]:

```
def gaussian2d(filter_size=5, sig=1.0):
    """Creates a 2D Gaussian kernel with side length 'filter_siz
e' and a sigma of 'sig'."""
    ax = np.arange(-filter_size // 2 + 1., filter_size // 2 + 1.
)
    xx, yy = np.meshgrid(ax, ax)
    kernel = np.exp(-0.5 * (np.square(xx) + np.square(yy)) / np.
square(sig))
    return kernel / np.sum(kernel)

sharpening_kernel = np.array([[1, 4,     6,  4, 1],
                              [4, 16,   24, 16, 4],
                              [6, 24, -476, 24, 6],
                              [4, 16,   24, 16, 4],
                              [1,  4,    6,  4, 1],
                            ]) * -1.0 / 256.0

def plot_results(original, filtered):
    # Plot original image
    plt.subplot(2,2,1)
    plt.imshow(original, vmin=0.0, vmax=1.0)
    plt.title('Original')
    plt.axis('off')

    # Plot filtered image
    plt.subplot(2,2,2)
    plt.imshow(filtered, vmin=0.0, vmax=1.0)
    plt.title('Filtered')
    plt.axis('off')
    plt.show()
```

Now fill in the functions below and display outputs for each of the filtering results. There should be three sets of (original, filtered) outputs in total. You are allowed to use the imported convolve function.

In [19]:

```python
from scipy.signal import convolve
def filter1(img):
    """Convolve the image with a 5x5 Gaussian filter with sigma=5."""
    kernel = gaussian2d(5, 5)
    return convolve(img, kernel)
def filter2(img):
    """Convolve the image with a 31x31 Gaussian filter with sigma=5."""
    kernel = gaussian2d(31, 5)
    return convolve(img, kernel)
def filter3(img):
    """Convolve the image with the provided sharpening filter."""
    return convolve(img, sharpening_kernel)
for filter_name, filter_fn in [
    ('5x5 Gaussian filter, sigma=5', filter1),
    ('31x31 Gaussian filter, sigma=5', filter2),
    ('sharpening filter', filter3),
]:
    filtered = filter_fn(shark_img)
    print(filter_name)
    plot_results(shark_img, filtered)
```

```
5x5 Gaussian filter, sigma=5
```


```
31x31 Gaussian filter, sigma=5
```


```
sharpening filter
```

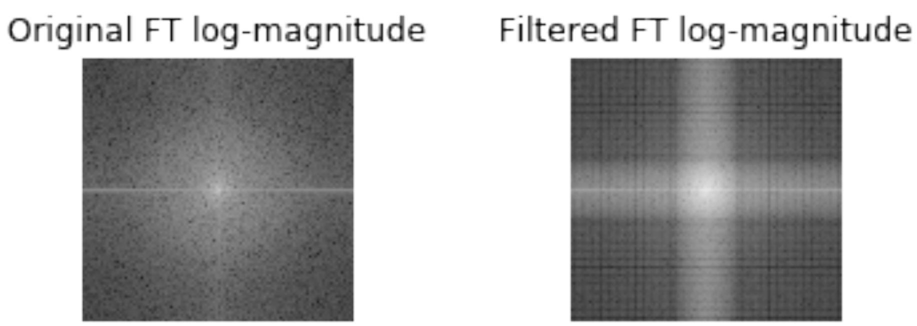
Part 2.1 [1 pt]

Display the Fourier log-magnitude transform image for the (original image, 31x31 Gaussian-filtered image) pair. (No need to include the others.) We have provided the code to compute the Fourier log-magnitude image.

In [20]:

```
# Visualize the frequency domain images
def plot_ft_results(img1, img2):
    plt.subplot(2,2,1)
    plt.imshow(img1, cmap='gray')
    plt.title('Original FT log-magnitude')
    plt.axis('off')
    plt.subplot(2,2,2)
    plt.imshow(img2, cmap='gray')
    plt.title('Filtered FT log-magnitude')
    plt.axis('off')
    plt.show()

def ft_log_magnitude(img_gray):
    return np.log(np.abs(np.fft.fftshift(np.fft.fft2(img_gray)))
)

print('31x31 Gaussian filter, sigma=5')
plot_ft_results(ft_log_magnitude(shark_img), ft_log_magnitude(fi
lter2(shark_img)))
```

```
31x31 Gaussian filter, sigma=5
```



Part 2.2 [1 pt]

Explain the differences you see between the original frequency domain image and the 31x31 Gaussian-filtered frequency domain image. In particular, be sure to address the following points: - Why is most of the frequency visualization dark after applying the Gaussian filter, and what does this signify? - What is an example of one of these dark regions in the spatial domain (original image)? - What do the remaining bright regions in the magnitude image represent? - What is an example of one of these bright regions in the spatial domain (original image)?

=== Write your answer here ===

- most of the frequency vizualtion is dark after applying the gaussian filter because the guassian filter smoothes the image, meaning high frequency components are filtered out. The fact that most of the frequency visualization is dark after smoothing means that there was a lot of high frequency components in the image that have now been filtered out by the gaussian filter.
- An example of one of these dark regions in the spatial domain are the speckles on the top of the shark's body, as these speckles are blurred after filtering with the 31x31 gaussian filter. Since the scales are blurred, this indicates that there arent as many high frequency changes after filtering.
- The remaining dark regions in the magnitude image represent the low frequency components of the image that were not blurred/removed by the smoothing gaussian filter.
- An example of one of these bright regions in the original image is separation of the sharks teeth and black of its mouth. This remains after filtering but is then a large change in the filtered image.

Part 3 [3 pts]

Consider (1) smoothing an image with a 3x3 box filter and then computing the derivative in the y-direction (use the derivative filter from Lecture 7). Also consider (2) computing the derivative first, then smoothing. What is a single convolution kernel that will simultaneously implement both (1) and (2)?

In [41]:

```python
print("3x3 box filter \n")
print("(1/9)*[1 1 1 \n       1 1 1 \n       1 1 1]\n")
box = (1/9)*np.ones((3,3))
print(box)

print("\n\nDerivative of 3x3 box filter in y-direction \n")
print("transpose of [-1/2  0  1/2] \n")
yDeriv = np.array([[-.5], [0], [.5]])
print(yDeriv)
```

```
3x3 box filter

(1/9)*[1 1 1
       1 1 1
       1 1 1]

[[0.11111111 0.11111111 0.11111111]
 [0.11111111 0.11111111 0.11111111]
 [0.11111111 0.11111111 0.11111111]]


Derivative of 3x3 box filter in y-direction

transpose of [-1/2  0  1/2]

[[-0.5]
 [ 0. ]
 [ 0.5]]
```

In [42]:

```python
print("Compute kernal for computing the 3x3 box filter followed by the derivative filter \n and kernal for computing the derivative filter followed by the 3x3 box filter \n\n")

boxFirst = convolve(box, yDeriv)
yDerivFirst = convolve(yDeriv, box)
print('Kernal for Box followed by yDeriv')
print(boxFirst)
print('\nKernal for yDeriv followed by Box')
print(yDerivFirst)
```

```
Compute kernal for computing the 3x3 box filter foll
owed by the derivative filter
 and kernal for computing the derivative filter foll
owed by the 3x3 box filter


Kernal for Box followed by yDeriv
[[-0.05555556 -0.05555556 -0.05555556]
 [-0.05555556 -0.05555556 -0.05555556]
 [ 0.          0.          0.        ]
 [ 0.05555556  0.05555556  0.05555556]
 [ 0.05555556  0.05555556  0.05555556]]

Kernal for yDeriv followed by Box
[[-0.05555556 -0.05555556 -0.05555556]
 [-0.05555556 -0.05555556 -0.05555556]
 [ 0.          0.          0.        ]
 [ 0.05555556  0.05555556  0.05555556]
 [ 0.05555556  0.05555556  0.05555556]]
```

=== Write your answer here ===

As seen above, both the Kernals are the same, meaning that the kernal shown for both isntances is the single convolutional kernal that will simultaneously implement (1) and (2) (box filter and derivative filter respectively). This makes sense since the operation of convolution is communative so as seen above, the order in which they are computed does not matter.

Part 4 [3 pts]

Give an example of a 3x3 separable filter and compare the number of arithmetic operations it takes to convolve using that filter on an $n \times n$ image before and after separation.

=== Write your answer here ===

In [56]:

```
import matplotlib.pyplot as plt

WrittenFive = plt.imread('CSE252A_HW2_5.jpg')
WrittenSix = plt.imread('CSE252A_HW2_6.jpg')
plt.figure(figsize=(20, 20))
plt.imshow(WrittenFive)
plt.show()
plt.figure(figsize=(20, 20))
plt.imshow(WrittenSix)
plt.show()
```

Problem 6

Part 4:

3x3 separable filter: box, $k=3$ $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

Since $k = v h^T$, then:

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix}1&1&1\end{bmatrix}$$

first, for 3x3 box filter.

This will be applied on an $n \times n$ image, which after padding will be $(n+2) \times (n+2)$ to ensure output image is $n \times n$.

total number of pixels: $(n+2) \times (n+2) = n^2 + 4n + 4 = p$

each pixel: 9 multiplications, one at each matrix entry.
8 additions to add the values
1 division
per pixel

meaning $9n^2 + 36n + 36$ multiplications
$8n^2 + 32n + 36$ additions
$n^2 + 4n + 4$ divisions

for separable:

1D column vector: applied to $(n+2) \times n$ pixels b/c only top & bottom need be padded, $n^2 + 2n$ pixels

This requires 3 multiplications & 2 additions, and 1 division per pixel

Meaning $3n^2 + 6n$ mults, $2n^2 + 6n$ additions, $n^2 + 2n$ divisions

1D row vector: same number of pixels as padding is just needed on right and left sides. so n^2+2n pixels
This requires 3 multiplications & 2 additions, and 1 division per pixel
Meaning $3n^2+6n$ mults, $2n^2+6n$ additions, n^2+2n divisions

In total, separable filter needs

$3n^2+6n$	$+3n^2+6n$	mults.	$6n^2+12n$
$2n^2+6n$	$+2n^2+6n$	additions.	$4n^2+12n$
n^2+2n	$+n^2+2n$	divisions	$2n^2+4n$

for the 2D Kernal: $9n^2+36n+36$ multiplications
$8n^2+32n+36$ additions
n^2+4n+4 divisions

Clearly it requires less operations when separated, specifically, applying the 2D box all at once requires an extra:

$3n^2+24n+36$ multiplications,
$4n^2+20n+36$ additions,
but less divisions by n^2-4.

however, the total number of operations is clearly more, meaning separating is a better approach.

Part 5: Filters as Templates [3.5 pts]

Suppose that you are a traveling ornithologist. You are trying to find a rare bird specimen in a museum collection. Because the museum curators are highly disorganized, you decide to build a computer vision system for finding specific birds in the museum's extensive collection.

Luckily, you have learned in CSE 252A (or are learning right now) that convolution can be used for template matching: a template g is multiplied with regions of a larger image f to measure how similar each region is to the template. Note that you will want to flip the filter before giving it to your convolution function, so that it is overall not flipped when making comparisons. You will also want to subtract off the mean value of the image or template (whichever you choose, subtract the same value from both the image and template) so that your solution is not biased toward higher-intensity (white) regions.

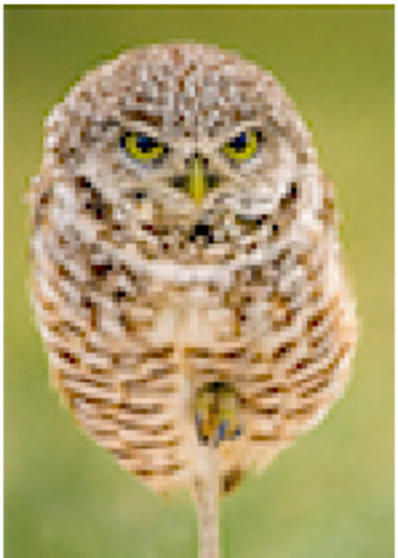
The template of a bird (template.jpg) and the image of the collection (bird_collection.jpg) is provided. We will use convolution to find the correct bird in the collection.

In [46]:

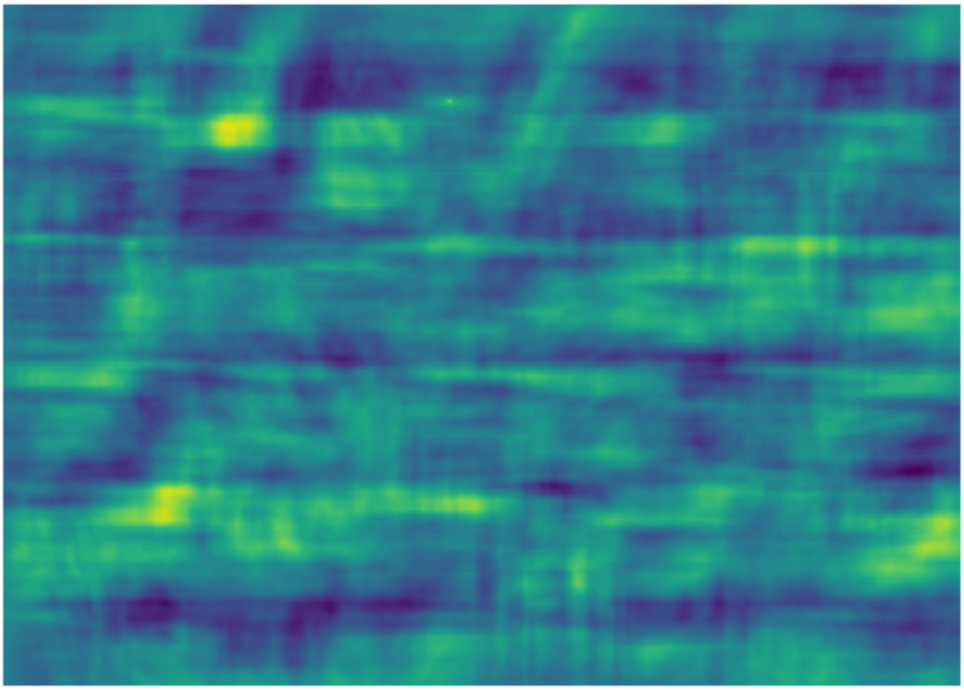
```
import numpy as np
from skimage import io
import matplotlib.pyplot as plt
from scipy.signal import convolve
%matplotlib inline

# Load template and image in grayscale
bird_img = io.imread('bird_collection.jpg')
img_gray = io.imread('bird_collection.jpg', as_gray=True)
temp_img = io.imread('template.jpg')
temp_gray = io.imread('template.jpg', as_gray=True)

# Perform a convolution between the image and the template
""" ========== YOUR CODE HERE ========== """
tempMean = np.mean(temp_gray)
subImageGray = img_gray - tempMean
subTempGray = temp_gray - tempMean
```

```python
flipTemp = subTempGray[::-1,::-1]
result = convolve(subImageGray, flipTemp, mode='same')
out = result

# Find the location with maximum similarity
y, x = (np.unravel_index(out.argmax(), out.shape))

# Display bird template
plt.figure(figsize=(20,16))
plt.subplot(3, 1, 1)
plt.imshow(temp_img)
plt.title('Template')
plt.axis('off')

# Display convolution output
plt.subplot(3, 1, 2)
plt.imshow(out)
plt.title('Convolution output (white means more correlated)')
plt.axis('off')

# Display image
plt.subplot(3, 1, 3)
plt.imshow(bird_img)
plt.title('Result (blue marker on the detected location)')
plt.axis('off')

# Draw marker at detected location
plt.plot(x, y, 'bx', ms=40, mew=10)
plt.show()
```

Template



Convolution output (white means more correlated)



Result (blue marker on the detected location)

In [ ]: