[Self] MiniQuiz Lec14 [ALL CORRECT]

Q1 [CORRECT]

Evaluate the following integral: $I = \int_{-1}^{1} e^x dx$

a) analytically

b) using Romberg integration to an accuracy of ε_s = 0.5%

c) using the two-point Gauss quadrature formula

d) using the three-point Gauss quadrature formula

e) using the adaptive quadrature formula to an accuracy of ε_s = 0.5%

f) using the MATLAB integral function

Rounding a decimal number to four decimal places.

a) I_{true} =

b) $I_{romberg}$ =

c) $I_{2points}$ =

d) $I_{3points}$ =

e) $I_{quadadapt}$ =

f) $I_{integral}$ =

a

```
f = @(x) exp(x);
a = -1;
b = 1;
Itrue = integral(f, a, b)
```

```
Itrue =
2.3504
```

b

```
Irb = romberg(f, a, b, 0.5)
```

```
Irb =
2.3505
```

```
% [Irb, ea, iter] = romberg(f, a, b, 0.5)
```

c

```
a1 = (b+a)/2;
a2 = (b-a)/2;
f2 = @(xd) exp(a1 + a2*xd) * a2;

c0 = 1; c1 = 1;
x0 = -1/sqrt(3); x1 = 1/sqrt(3);
Ig2p = c0*f2(x0) + c1*f2(x1)
```

```
Ig2p =
2.3427
```

d

```
c0 = 5/9;
c1 = 8/9;
c2 = 5/9;
x0 = -sqrt(3/5);
x1 = 0;
x2 = sqrt(3/5);
Ig3p = c0*f2(x0) + c1*f2(x1) + c2*f2(x2)
```

```
Ig3p =
2.3503
```

e

```
Iqa = quadadapt(f, a, b, 0.5)
```

```
Iqa =
2.3505
```

f

```
Iint = integral(f, a, b) % basically the same as Itrue
```

```
Iint =
2.3504
```

Q2 [CORRECT]

Let's assume that we have the following data

x = [1 2 3 4 5 6]

y = [2 3 1 2 3 2]

Perform the 5th-order polynomial interpolation to get the interpolated polynomial function P.

Evaluate the following integral: $I = \int_1^6 P dx$

a) analytically

b) using Romberg integration to an accuracy of ε_s = 0.5%

c) using the two-point Gauss quadrature formula

d) using the three-point Gauss quadrature formula

e) using the adaptive quadrature formula to an accuracy of ε_s = 0.5%

f) using the MATLAB integral function

Rounding a decimal number to four decimal places.

a) I_{true} =

b) $I_{romberg}$ =

c) $I_{2points}$ =

d) $I_{3points}$ =

e) $I_{quadadapt}$ =

f) $I_{integral}$ =

```
x  = [1 2 3 4 5 6];
y  = [2 3 1 2 3 2];
p5 = polyfit(x, y, 5)
```

```
p5 = 1×6
    0.0833   -1.6250   11.8333  -39.3750   58.0833  -27.0000
```

```
f  = @(x) p5(1).*x.^5 + p5(2).*x.^4 + p5(3).*x.^3 + p5(4).*x.^2 + p5(5).*x + p5(6);
```

a

```
a = 1;
b = 6;
Itrue = integral(f, a, b)
```

```
Itrue =
11.7361
```

b

```
Irb = romberg(f, a, b, 0.5)
```

```
Irb =
11.7361
```

```
% [Irb, ea, iter] = romberg(f, a, b, 0.5)
```

c

```
a1 = (b+a)/2;
a2 = (b-a)/2;
f2 = @(xd) ( p5(1).*(a1 + a2*xd).^5 + ...
            p5(2).*(a1 + a2*xd).^4 + ...
            p5(3).*(a1 + a2*xd).^3 + ...
            p5(4).*(a1 + a2*xd).^2 + ...
            p5(5).*(a1 + a2*xd) + ...
            p5(6) ) * a2;

c0 = 1; c1 = 1;
x0 = -1/sqrt(3); x1 = 1/sqrt(3);
Ig2p = c0*f2(x0) + c1*f2(x1)
```

```
Ig2p =
14.6296
```

d

```
c0 = 5/9;
c1 = 8/9;
c2 = 5/9;
x0 = -sqrt(3/5);
x1 = 0;
x2 = sqrt(3/5);
Ig3p = c0*f2(x0) + c1*f2(x1) + c2*f2(x2)
```

```
Ig3p =
11.7361
```

e

```
Iqa = quadadapt(f, a, b, 0.5)
```

```
Iqa =
11.7361
```

f

```
Iint = integral(f, a, b) % basically the same as Itrue
```

```
Iint =
11.7361
```