HW09 [ALL CORRECT]

Q4

```
data = [0.90  1.42  1.30  1.55  1.63
        1.32  1.35  1.47  1.95  1.66
        1.96  1.47  1.92  1.35  1.05
        1.85  1.74  1.65  1.78  1.71
        2.29  1.82  2.06  2.14  1.27]
```

```
data = 5×5
    0.9000    1.4200    1.3000    1.5500    1.6300
    1.3200    1.3500    1.4700    1.9500    1.6600
    1.9600    1.4700    1.9200    1.3500    1.0500
    1.8500    1.7400    1.6500    1.7800    1.7100
    2.2900    1.8200    2.0600    2.1400    1.2700
```

```
format short

me = mean(data(:))
```

```
me =
1.6244
```

```
med = median(data(:))
```

```
med =
1.6500
```

```
mo = mode(data(:))
```

```
mo =
1.3500
```

```
range = max(data(:)) - min(data(:))
```

```
range =
1.3900
```

```
std_dev = std(data(:))
```

```
std_dev =
0.3394
```

```
variance = var(data(:))
```

```
variance =
0.1152
```

```
cov = (std_dev / me) * 100 % coefficient of variation
```

```
cov =
20.8931
```

Q1

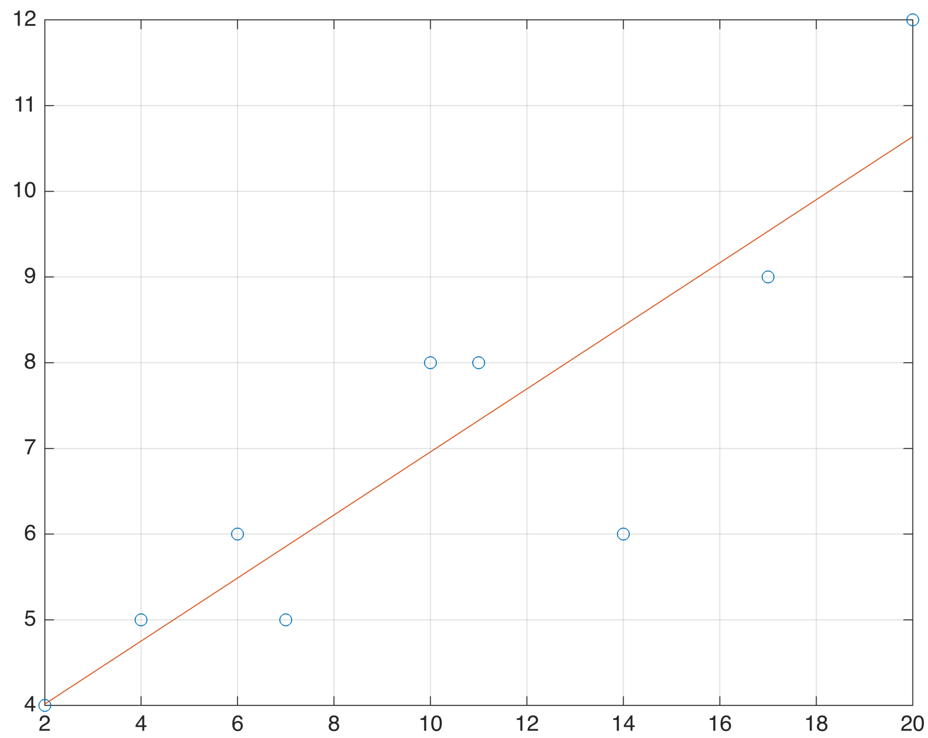
For the following data:

```
>>> x   2   4   6   7   10  11  14  17  20
>>> y   4   5   6   5   8   8   6   9   12
```

Fit a straight line to the above data and use the result to predict the value of y when x = 5.

```
x = [2 4 6 7 10 11 14 17 20];
y = [4 5 6 5 8 8 6 9 12];

% do linear regression
[a, ~] = linregr(x, y)
```




```
a = 1×2
    0.3678    3.2807
```

```
y_int = a(2)
```

```
y_int =
3.2807
```

```
slope = a(1)
```

```
slope =
0.3678
```

```
xtest1 = 5
```

```
xtest1 =
5
```

```
ytest1 = slope * xtest1 + y_int % y = mx + c
```

```
ytest1 =
5.1199
```

Derive the least-squares (basically the $y = a_0 + a_1 x + \text{error}$ from $y = m \cdot x + c + \text{error}$) fit of the following model: $y = a_1 x + e$.

That is, determine the slope that results in the least-squares fit for a straight line with a zero intercept ($a_0 = 0$).

Use the least-squares fit to predict the value of y when x = 5.

when $a_0 = 0$:

$$\cancel{(\sum x_i)(0)} + (\sum x_i^2) a_1 = \sum x_i y_i$$

$$a_1 = \frac{\sum x_i y_i}{\sum x_i^2}$$

```
y_int = 0
```

```
y_int =
0
```

```
slope = sum(x.*y) / sum(x.^2) % BASED OFF THE a1 FORMULA IN THE LINEAR REGRESSION SLIDES (one with
partial derivatives)
```

```
slope =
0.6144
```

```
xtest = 5
```

```
xtest =
5
```

```
ytest = slope * xtest + y_int
```

```
ytest =
3.0718
```

Q2

Table P17.33 lists the 2015 world record times and holders for outdoor running.

Note that all but the 100 m and the marathon (42,195 m) are run on oval tracks.

Fit a **power model** for each gender and use it to predict the record time for a half marathon (21,097.5 m).

TABLE P17.33 World record times and holders for outdoor running in 2015

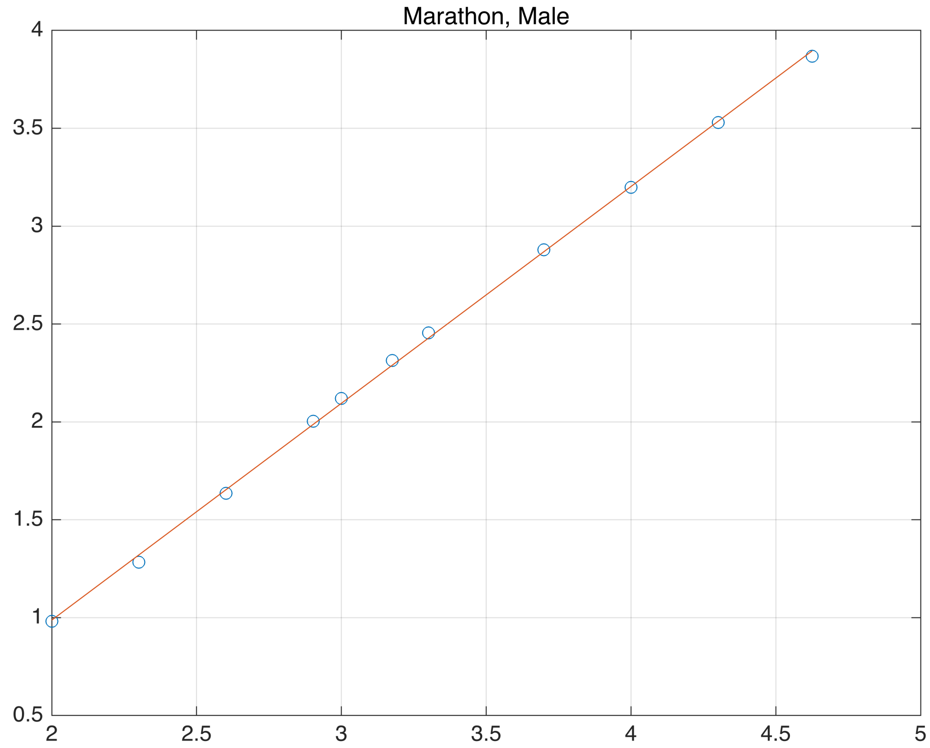
Event (m)	Time (s)	Men's record holder	Time (s)	Women's record holder
100	9.58	Bolt	10.49	Griffith-Joyner
200	19.19	Bolt	21.34	Griffith-Joyner
400	43.18	Johnson	47.60	Koch
800	100.90	Rudisha	113.28	Kratochvilova
1000	131.96	Ngeny	148.98	Masterkova
1500	206.00	El Guerrouj	230.07	Dibaba
2000	284.79	El Guerrouj	325.35	O'Sullivan
5000	757.40	Bekele	851.15	Dibaba
10,000	1577.53	Bekele	1771.78	Wang
20,000	3386.00	Gebrselassie	3926.60	Loroupe
42,195	7377.00	Kimetto	8125.00	Radcliffe

Note that the actual records for the half marathon are 3503 s (Tadese) and 3909 s (Kiplagat) for men and women, respectively. Rounding your calculation to **integers**.

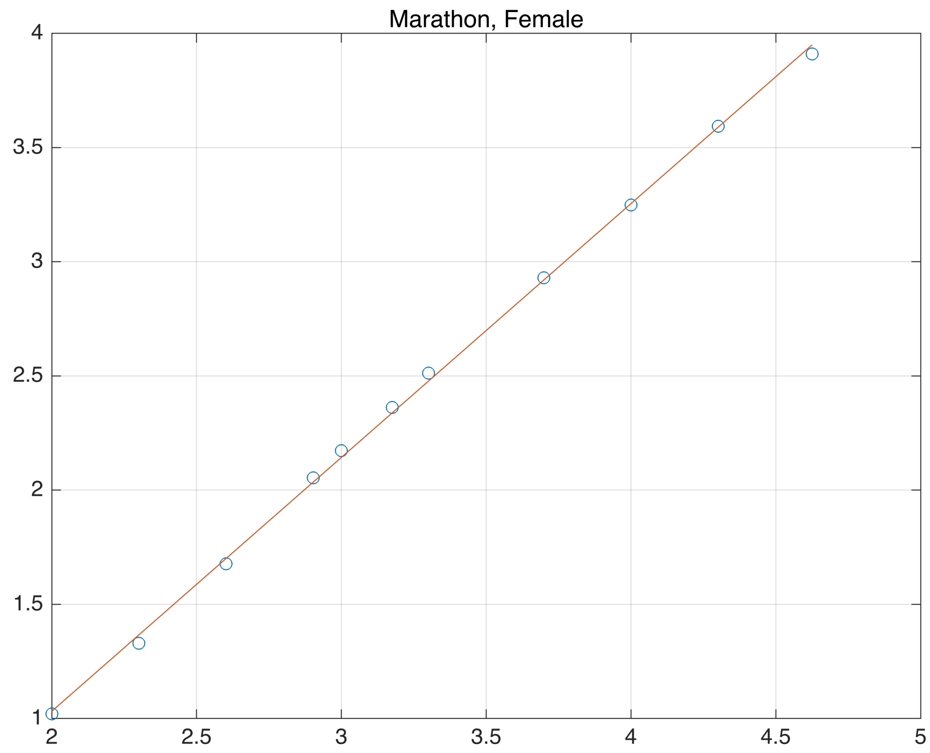
$$
\begin{aligned}
t &= \alpha d^{\beta} \\
\log(t) &= \log(\alpha) + \log(d^{\beta}) \\
\log(t) &= \log(\alpha) + \beta \log(d) \\
\because\ y &= mx + c \\
\therefore\ c &\rightarrow a_0 = \log_{10}\alpha \\
\therefore\ c &\rightarrow \alpha = 10^{a_0} && \text{(y-intercept)} \\
\therefore\ m &\rightarrow \beta && \text{(slope)}
\end{aligned}
$$

```
dist = [100 200 400 800 1000 1500 2000 5000 10000 20000 42195];
time_m = [9.58 19.19 43.18 100.90 131.96 206.00 284.79 757.40 1577.53 3386.00 7377.00];
time_f = [10.49 21.34 47.60 113.28 148.98 230.07 325.35 851.15 1771.78 3926.60 8125.00];

% `log()` = NATURAL log, `exp(k)` = e^k like in Python
[a_m, r2_m] = linregr(log10(dist), log10(time_m)); title('Marathon, Male');
```




```
[a_f, r2_f] = linregr(log10(dist), log10(time_f)); title('Marathon, Female');
```




```
% output: array `a`
% a(1) = vector of slope
beta_m = a_m(1);
beta_f = a_f(1);
% a(2) = y-intercept
alpha_m = 10^(a_m(2));
alpha_f = 10^(a_f(2));

dist_half = 21097.5; % half marathon
time_pred_m = alpha_m * (dist_half^beta_m);
time_pred_f = alpha_f * (dist_half^beta_f);

fprintf('time_pred_m = %d', round(time_pred_m));
```

```
time_pred_m = 3646
```

```
fprintf('time_pred_f = %d', round(time_pred_f));
```

```
time_pred_f = 4124
```

Q3

Use least-squares regression to fit a straight line to

```
>>> x   0   2   4   6   9   11   12   15   17   19
>>> y   5   6   7   6   9   8    8    10   12   12
```

Note that rounding a decimal number to four decimal places (`format short`).

Q3.1 Along with the slope and intercept, compute:

- the standard error of the estimate $\left(s_{y|x} = \sqrt{\frac{S_r}{n-2}} \text{ where } S_r = \sum (y_i - a_0 - a_1 x_i)^2\right)$
- the correlation coefficient

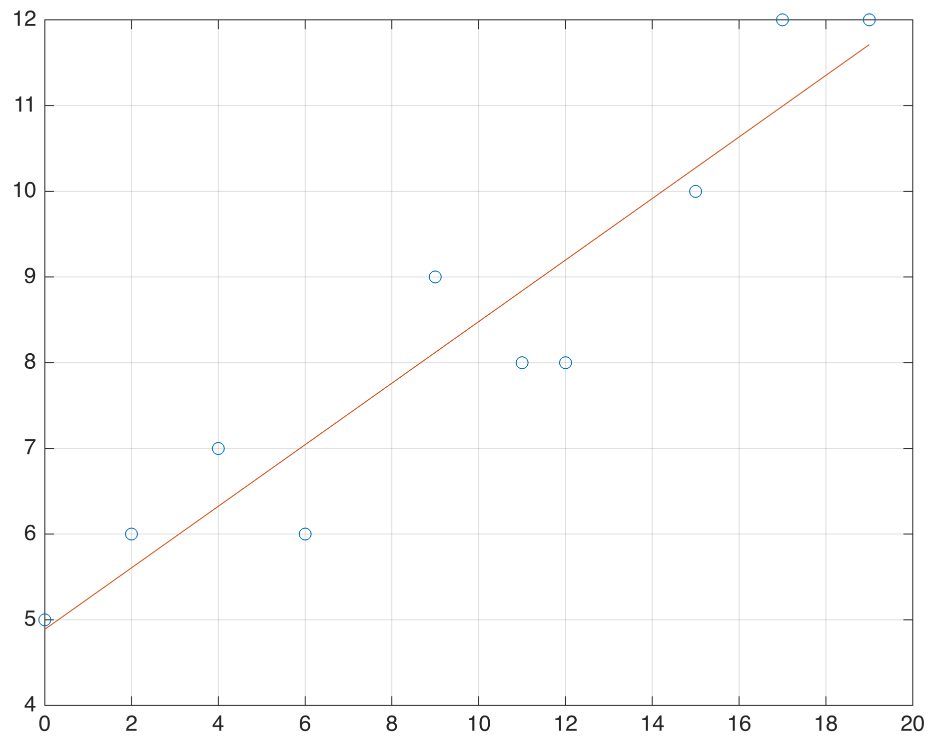
Q3.2 Then repeat the problem, but regress x versus y—that is, switch the variables. Then compute:

- the standard error of the estimate
- the correlation coefficient

```
format short

x = [0  2  4  6  9  11  12  15  17  19];
y = [5  6  7  6  9  8   8   10  12  12];

% X vs Y
[a2, r2_2] = linregr(x, y); % x is independent, y is dependent
```




```
% predicted y values
YPRED = a2(1) * x + a2(2); % (this is what they meant when they said "swap the variables")
RESIDUALS = y - YPRED;
SR = sum(RESIDUALS.^2);
SYX = sqrt(SR / (length(x) - 2))
```

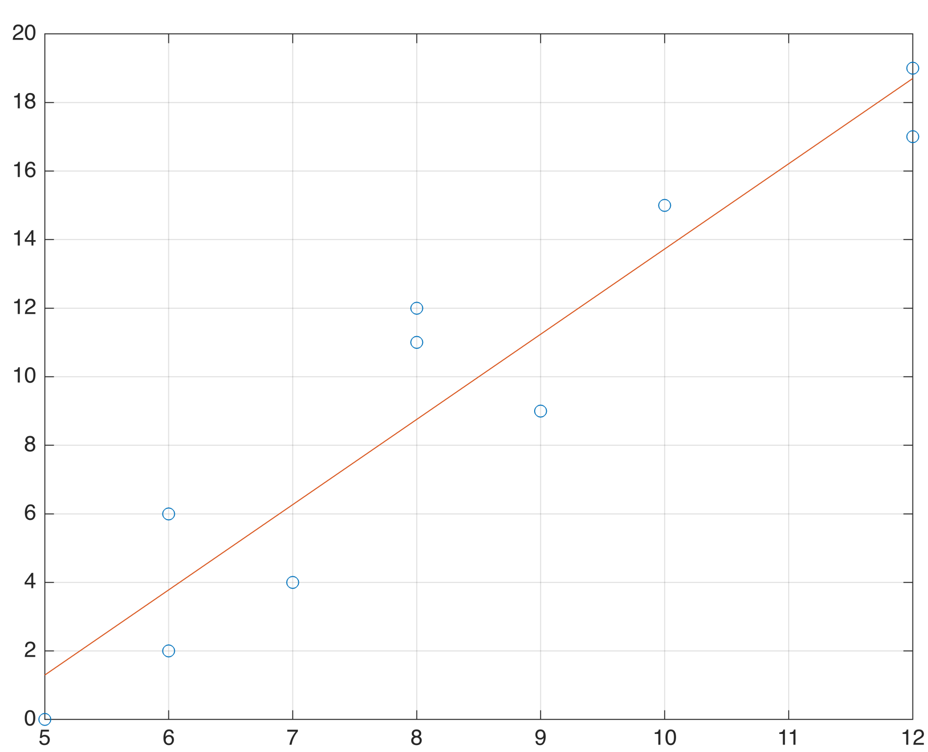
```
SYX =
0.8511
```

```
R2 = sqrt(r2_2)
```

```
R2 =
0.9449
```

```

% Y vs X (swapped)
[a1, r2_1] = linregr(y, x);
```




```
xpred = a1(1) * y + a1(2); % mx + c
residuals = x - xpred;
sr = sum(residuals.^2);
syx = sqrt(sr / (length(y) - 2))
```

```
syx =
2.2393
```

```
r2 = sqrt(r2_1)
```

```
r2 =
0.9449
```

Q5

There are other models that can be linearized using transformations. For example, $y = \alpha x \cdot e^{\beta x}$.

Linearize this model and use it to estimate α and β based on the following data.

```
x  0.1   0.2   0.4   0.6   0.9   1.3   1.5   1.7   1.8
y  0.75 1.25 1.45 1.25 0.85 0.55 0.35 0.28 0.18
```

Rounding a decimal number to four decimal places: α = ?, β = ?

Also use the model to predict the value of y when x = 1: x = 1, y = ?

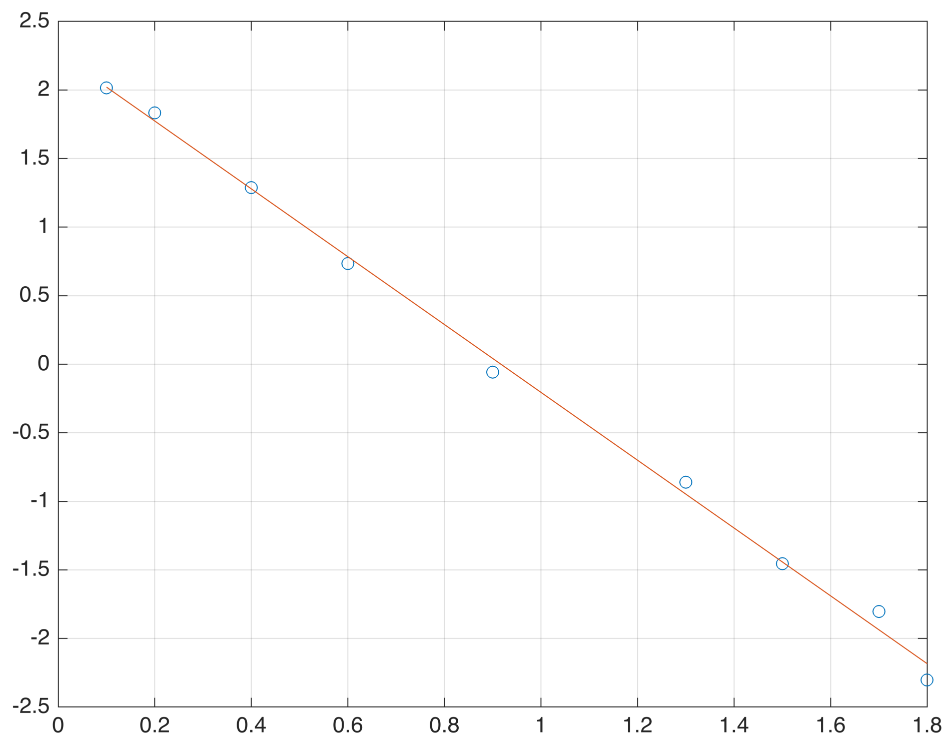
```
x = [0.1  0.2  0.4  0.6  0.9  1.3  1.5  1.7  1.8 ];
y = [0.75 1.25 1.45 1.25 0.85 0.55 0.35 0.28 0.18];

% [a, ~] = linregr(x, y);
% alpha = exp(a(2));
% beta = a(1);
% fprintf('alpha = %.4f', alpha);
% fprintf('beta = %.4f\n', beta);
%
% y_pred = alpha * exp(beta); % (x is 1)
% fprintf('y_pred = %.4f\n', y_pred);

yy = log(y./x)
```

```
yy = 1×9
    2.0149    1.8326    1.2879    0.7340   -0.0572   -0.8602   -1.4553   -1.8036 ···
```

```
[a, ~] = linregr(x, yy);
```




```
alpha = exp(a(2))
```

```
alpha =
9.6618
```

```
beta = a(1)
```

```
beta =
-2.4733
```

```
y_pred = alpha * exp(beta)
```

```
y_pred =
0.8145
```

Q6

The following data show the relationship between the viscosity of SAE 70 oil and temperature.

After taking the logarithm of the data, use linear regression to find the equation of the line that best fits the data.

```
>>> Temperature  26.67    93.33    148.89    315.56
>>> Viscosity    1.35     0.085    0.012     0.00075
```

Then use the fit model to **predict the viscosity (*y* value) when the temperature (*x* value) is 50**. Rounding a decimal number to *four decimal places*.

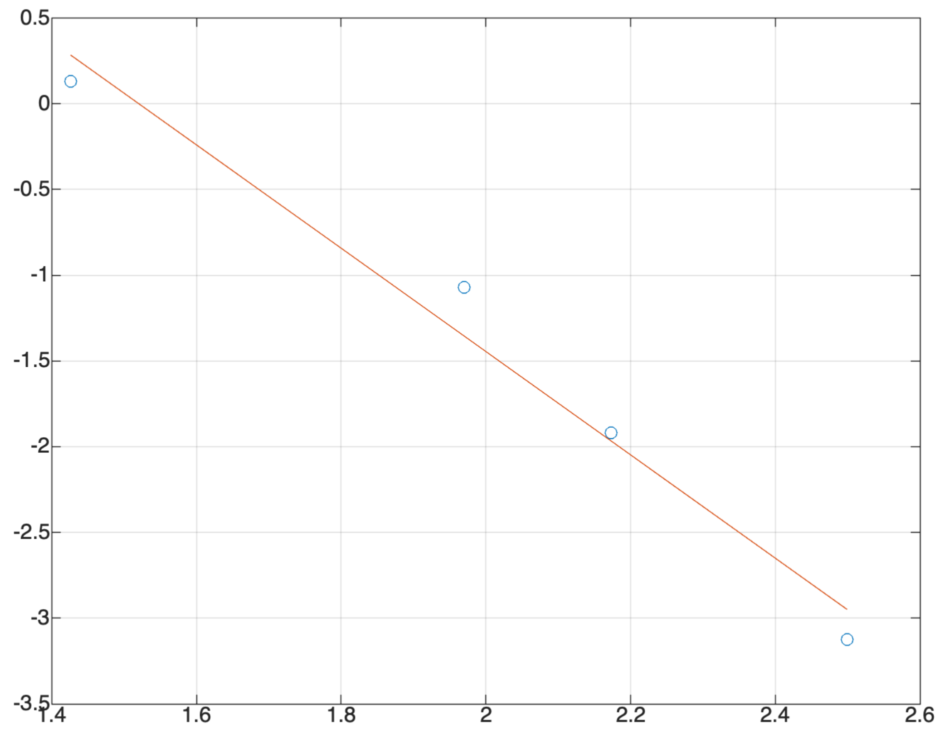
… Viscosity = ?

$$\begin{aligned} \text{From} \quad \log_{10} y &= \text{slope} \cdot \log_{10} \text{temp} + \text{intercept} \\ 10^{\log_{10} y} &= 10^{\text{slope} \cdot \log_{10} \text{temp} + \text{intercept}} \qquad \because n^{a+b} = n^a \cdot n^b \\ y &= 10^{\text{slope} \cdot \log_{10} \text{temp}} \cdot 10^{\text{intercept}} \\ y &= \left(10^{\log_{10} \text{temp}}\right)^{\text{slope}} \cdot 10^{\text{intercept}} \\ \therefore y &= \text{temp}^{\text{slope}} \cdot 10^{\text{intercept}} \end{aligned}$$

```
temp = [26.67    93.33    148.89    315.56];
visc = [1.35     0.085    0.012     0.00075];

temp_new = log10(temp);
visc_new = log10(visc);

[vals, r2] = linregr(temp_new, visc_new);
```




```
slope = vals(1);
intercept = vals(2);

y_pred = 50^slope * 10^intercept
```

```
y_pred =
0.2896
```

Q7

There are other models that can be linearized using transformations. For example, the following model applies to third-order chemical reactions in batch reactors: $c = c_0\left(\frac{1}{\sqrt{1 + 2k \cdot c_0^2 \cdot t}}\right)$.

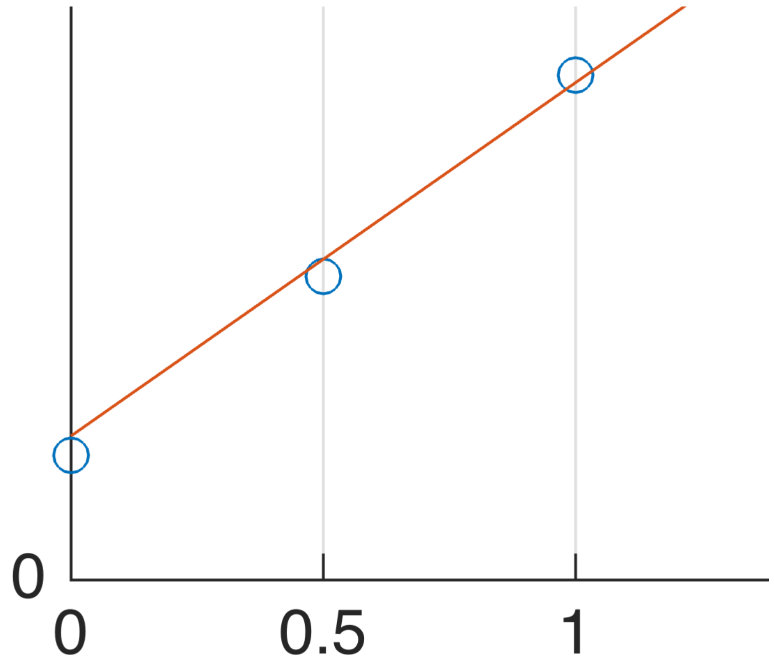
Where c = concentration, c_0 = initial concentration, k = reaction rate, and t = time.

Linearize this model and use it to **estimate k and** c_0 based on the following data.

```
>>> t = [0      0.5    1      1.5    2      3      4     5]
>>> c = [3.26   2.09   1.62   1.48   1.17   1.06   0.9   0.85]
```

```
t = [0      0.5    1      1.5    2      3      4     5];
c = [3.26   2.09   1.62   1.48   1.17   1.06   0.9   0.85];

[vals, r2] = linregr(t, 1./(c.^2));
```




```
c0 = sqrt(1/vals(2)); k = vals(1)/2;
fprintf('c0 = %.4f\n', c0); fprintf('k = %.4f', k);
```

```
c0 = 3.0324
k = 0.1332
```

Q8

Fitting the following data with the **exponential model ($y = \alpha e^{\beta x}$)**.

```
>>> x = [0     0.2000     0.4000     0.6000     0.8000     1.0000     1.2000     1.4000     1.6000     1.8000
2.0000]
```

```
>>> y = [0.1409     0.1651     0.1506     0.2174     0.2374     0.2517     0.3091     0.3803     0.4700     0.5536
0.6137]
```

Find *α* and *β*, and the prediction of y when x = 1.1.

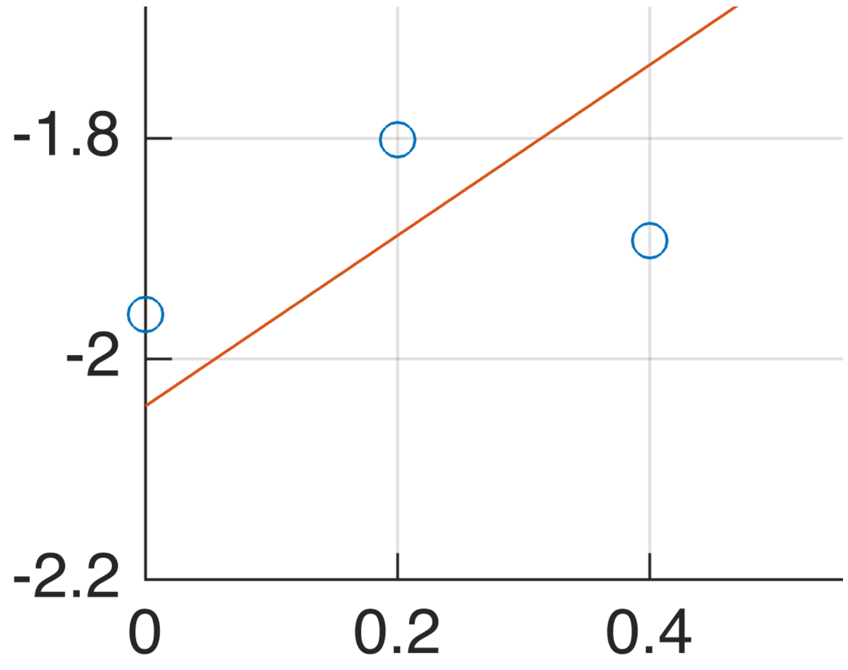
```
x = [0, 0.2000, 0.4000, 0.6000, 0.8000, 1.0000, 1.2000, 1.4000, 1.6000, 1.8000, 2.0000];
y = [0.1409, 0.1651, 0.1506, 0.2174, 0.2374, 0.2517, 0.3091, 0.3803, 0.4700, 0.5536, 0.6137];

% [vals, r2] = linregr(x, y);
% alpha = exp(vals(2)); beta = vals(1);
% y = log(alpha) + (beta * 1.1) % (x = 1.1)

yy = log(y)
```

```
yy = 1×11
   -1.9597   -1.8012   -1.8931   -1.5260   -1.4380   -1.3795   -1.1741   -0.9668 ···
```

```
[a, ~] = linregr(x, yy)
```




```
a = 1×2
    0.7724   -2.0427
```

```
% a(1) = slope, a(2) = y-intercept
```

```
alpha = exp(a(2))
```

```
alpha =
0.1297
```

```
beta = a(1)
```

```
beta =
0.7724
```

```
xtest = 1.1
```

```
xtest =
1.1000
```

```
y_pred = alpha * exp(beta * xtest)
```

```
y_pred =
0.3033
```