HW10 [ALL CORRECT]

Q1

For the following data.

x = [10 20 30 40 50 60 70 80]

y = [25 70 380 550 610 1220 830 1450]

Derive the least-squares fit of the following model:

$y = a_1 x + a_2 x^2 + e$

That is, determine the coefficients that results in the least-squares fit for a second-order polynomial with a zero intercept.

Determine the r^2.

Use the least-squares fit to predict the value of y when x = 45.

Rounding a decimal number to four decimal places.

r^2 =

x = 45, y =

```
% CORRECT

x = [10 20 30  40  50  60  70  80]';
y = [25 70 380 550 610 1220 830 1450]';

% GENERAL APPROACH
% y = Za + e
Z = [x x.^2] % no ones() because there's no a_0 (raised to 0) terms in the given expression
```

```
Z = 8×2
          10         100
          20         400
          30         900
          40        1600
          50        2500
          60        3600
          70        4900
          80        6400
```

```
a = (Z'*Z) \ (Z'*y)
```

```
a = 2×1
    7.7710
    0.1191
```

```
xpred = 45
```

```
xpred =
45
```

```
ypred = (a(1)*xpred) + (a(2)*xpred.^2)
```

```
ypred =
590.8228
```

```
ytrue = (a(1).*x) + (a(2).*x.^2)
```

```
ytrue = 8×1
10^3 ×
    0.0896
    0.2031
    0.3403
    0.5014
    0.6862
    0.8949
    1.1274
    1.3838
```

```
st = sum((y - mean(y)) .^ 2)
```

```
st =
1.8083e+06
```

```
sr = sum((y - ytrue) .^ 2)
```

```
sr =
2.3016e+05
```

```
r2 = 1 - (sr/st)
```

```
r2 =
0.8727
```

Q2 (similar to HW09)

Given the data

x = [5 10 15 20 25 30 35 40 45 50]

y = [17 24 31 33 37 37 40 40 42 41]

use least-squares regression to fit

(a) a straight line,

(b) a power equation,

(c) a saturation-growth-rate equation, and

(d) a parabola.

For (b) and (c), employ transformations to linearize the data.

Rounding a decimal number to four decimal places.

Use the least-squares fits to predict the value of y when x = 22

(a) y =

(b) y =

(c) y =

(d) y =

```
% dataset
x = [5  10  15  20  25  30  35  40  45  50];
y = [17  24  31  33  37  37  40  40  42  41];
xplot = linspace(min(x), max(x), 100)
```

```
xplot = 1×100
    5.0000    5.4545    5.9091    6.3636    6.8182    7.2727    7.7273    8.1818 ···
```

```
xtest = 22
```

```
xtest =
22
```

```
% Linear (straight line) fit
Lp1 = polyfit(x, y, 1)
```

```
Lp1 = 1×2
    0.4945   20.6000
```

```
Lyy1 = polyval(Lp1, x)
```

```
Lyy1 = 1×10
   23.0727   25.5455   28.0182   30.4909   32.9636   35.4364   37.9091   40.3818 ···
```

```
Lytest1 = polyval(Lp1, xtest)
```

```
Lytest1 =
31.4800
```

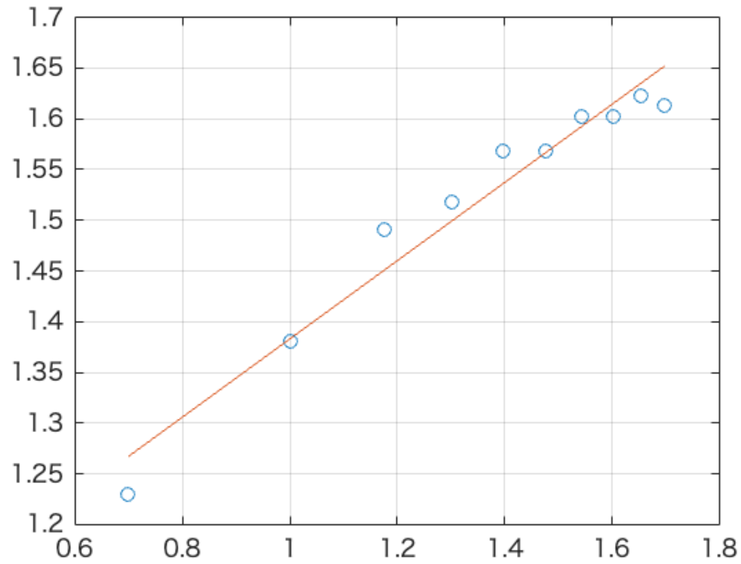
```
% Power equation linearization [can't use polyfit/polyval; no degrees]
Pnew_x = log10(x)
```

```
Pnew_x = 1×10
    0.6990    1.0000    1.1761    1.3010    1.3979    1.4771    1.5441    1.6021 ···
```

```
Pnew_y = log10(y)
```

```
Pnew_y = 1×10
    1.2304    1.3802    1.4914    1.5185    1.5682    1.5682    1.6021    1.6021 ···
```

```
[Pa, ~] = linregr(Pnew_x, Pnew_y)
```



```
Pa = 1×2
    0.3851    0.9980
```

```
Pslp = Pa(1)
```

```
Pslp =
0.3851
```

```
Pint = Pa(2)
```

```
Pint =
0.9980
```

```
Palpha = 10^Pint
```

```
Palpha =
9.9529
```

```
Pbeta = Pslp
```

```
Pbeta =
0.3851
```

```
Pytest = Palpha .* (xtest .^ Pbeta)
```

```
Pytest =
32.7255
```

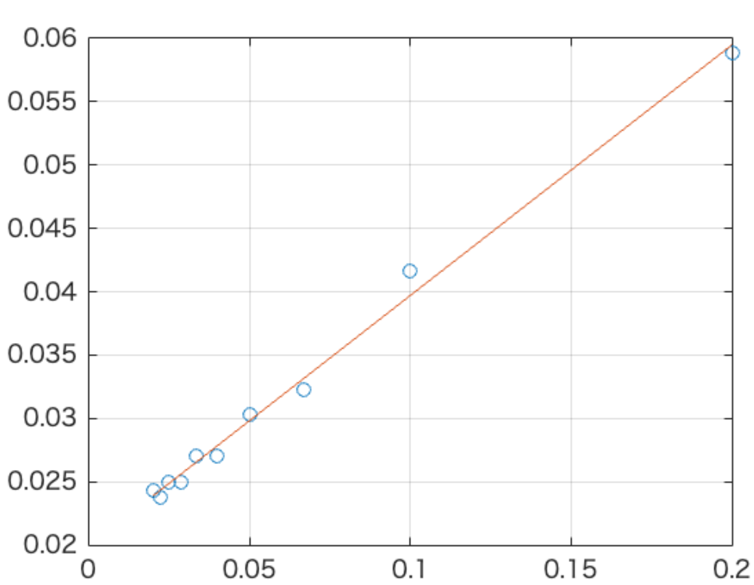
```
% Pyplot = Palpha .* (xplot .^ Pbeta)
% plot(x, y, 'o', xplot, Pyplot)

% Saturation growth rate linearization [can't use polyfit/polyval; no degrees]
Snew_x = 1 ./ x
```

```
Snew_x = 1×10
    0.2000    0.1000    0.0667    0.0500    0.0400    0.0333    0.0286    0.0250 ⋯
```

```
Snew_y = 1 ./ y
```

```
Snew_y = 1×10
    0.0588    0.0417    0.0323    0.0303    0.0270    0.0270    0.0250    0.0250 ⋯
```

```
[Sa, ~] = linregr(Snew_x, Snew_y)
```




```
Sa = 1×2
    0.1975    0.0200
```

```
Sslp = Sa(1)
```

```
Sslp =
0.1975
```

```
Sint = Sa(2)
```

```
Sint =
0.0200
```

```
Salpha = 1 / Sint
```

```
Salpha =
50.0921
```

```
Sbeta = Salpha * Sslp
```

```
Sbeta =
9.8914
```

```
% Syplot = Salpha .* (xplot ./ (Sbeta + xplot))
Sytest = Salpha .* (xtest ./ (Sbeta + xtest))
```

```
Sytest =
34.5556
```

```
% plot(x, y, 'o', xplot, Syplot, '-')
```

```
% parabola (Quadratic) fit
Qp2 = polyfit(x, y, 2)
```

```
Qp2 = 1×3
   -0.0161    1.3779   11.7667
```

```
Qyy2 = polyval(Qp2, x)
```

```
Qyy2 = 1×10
   18.2545   23.9394   28.8212   32.9000   36.1758   38.6485   40.3182   41.1848 ···
```

```
Qytest2 = polyval(Qp2, xtest)
```

```
Qytest2 = 
34.3067
```

Q3

Three disease-carrying organisms decay exponentially in seawater according to the following model:

$p(t) = Ae^{-1.5t} + Be^{-0.3t} + Ce^{-0.05t}$

Estimate the initial concentration of each organism (A, B, and C) given the following measurements:

t	0.5	1	2	3	4	5	6	7	9
p(t)	6	4.4	3.2	2.7	2	1.9	1.7	1.4	1.1

Rounding a decimal number to four decimal places.

Determine

A =

B =

C =

Also use the least-squares fit to predict the value of p(t) when t = 8.

p(8) =

```
% CORRECT

format short

t     = [0.5  1  2  3  4  5  6  7  9]';
p     = [6  4.4  3.2  2.7  2  1.9  1.7  1.4  1.1]';
ttest = 8
```

```
ttest = 
8
```

```
Z = [exp(-1.5*t) exp(-0.3*t) exp(-0.05*t)]
```

```
Z = 9×3
    0.4724    0.8607    0.9753
    0.2231    0.7408    0.9512
    0.0498    0.5488    0.9048
    0.0111    0.4066    0.8607
    0.0025    0.3012    0.8187
    0.0006    0.2231    0.7788
    0.0001    0.1653    0.7408
    0.0000    0.1225    0.7047
    0.0000    0.0672    0.6376
```

```
a = (Z' * Z) \ (Z' * p)
```

```
a = 3×1
    4.1375
    2.8959
    1.5349
```

```
ypred = a(1)*exp(-1.5*ttest) + a(2)*exp(-0.3*ttest) + a(3)*exp(-0.05*ttest)
```

```
ypred = 
1.2916
```

Q4

Use multiple linear regression to fit

x_1 = [0 1 1 2 2 3 3 4 4]

x_2 = [0 1 2 1 2 1 2 1 2]

y = [15.1 17.9 12.7 25.6 20.5 35.1 29.7 45.4 40.2]

Determine the r^2 and $s_{y/x}$.

Use the least-squares fit to predict the value of y when x_1 = 3 and x_2 = 3.

Rounding a decimal number to four decimal places.

$s_{y/x}$ =

r^2 =

x_1 = 3, x_2 = 3, y =

```
x1 = [0 1 1 2 2 3 3 4 4]';
x2 = [0 1 2 1 2 1 2 1 2]';
y = [15.1 17.9 12.7 25.6 20.5 35.1 29.7 45.4 40.2]';
Z = [ones(size(y)), x1, x2]
```

```
Z = 9×3
     1     0     0
     1     1     1
     1     1     2
     1     2     1
     1     2     2
     1     3     1
     1     3     2
     1     4     1
     1     4     2
```

```
a = (Z' * Z) \ (Z' * y)
```

```
a = 3×1
   14.4609
    9.0252
   -5.7043
```

```
sr = sum((y - (Z*a)).^2)
```

```
sr =
4.7397
```

```
st = sum((y - mean(y)).^2)
```

```
st =
1.0587e+03
```

```
r2 = 1 - (sr/st)
```

```
r2 =
0.9955
```

```
syx = sqrt(sr/(length(y)-3))
```

```
syx =
0.8888
```

```
x1test = 3;
x2test = 3;

ypred = a(1) + (a(2) * x1test) + (a(3) * x2test)
```

```
ypred =
24.4235
```

Q5

For the following data.

x = [10 20 30 40 50 60 70 80]

y = [25 70 380 550 610 1220 830 1450]

Fit a parabola to the data and determine the r^2.

Rounding a decimal number to four decimal places.

r^2 =

```
x = [10 20 30 40 50 60 70 80];
y = [25 70 380 550 610 1220 830 1450];
p = polyfit(x, y, 2)
```

```
p = 1×3
    0.0372   16.1220 -178.4821
```

```
sr = sum((y - polyval(p, x)) .^ 2)
```

```
sr =
2.1379e+05
```

```
st = sum((y - mean(y)) .^2)
```

```
st =
1.8083e+06
```

```
r2 = 1 - (sr/st)
```

```
r2 =
0.8818
```

`linregr` function

```
function [a, r2] = linregr(x,y)
  % linregr: linear regression curve fitting
  % [a, r2] = linregr(x,y): Least squares fit of straight
  % line to data by solving the normal equations
  % input:
  % x = independent variable
  % y = dependent variable
  % output:
  % a = vector of slope, a(1), and intercept, a(2)
  % r2 = coefficient of determination
  n = length(x);
  if length(y)~=n, error('x and y must be same length'); end
  x = x(:); y = y(:); % convert to column vectors
  sx = sum(x); sy = sum(y);
  sx2 = sum(x.*x); sxy = sum(x.*y); sy2 = sum(y.*y);
  a(1) = (n*sxy-sx*sy)/(n*sx2-sx^2);
  a(2) = sy/n-a(1)*sx/n;
  r2 = ((n*sxy-sx*sy)/sqrt(n*sx2-sx^2)/sqrt(n*sy2-sy^2))^2;
  % create plot of data and best fit line
  xp = linspace(min(x),max(x),2);
  yp = a(1)*xp+a(2);
  plot(x,y,'o',xp,yp)
  grid on
end
```