HW13

Q1 [CORRECT]

Evaluate the following integral: $I = \int_0^{11} (5 + 0.25x^2)dx$

a) analytically

b) composite trapezoidal rule (use 1-unit increment)

c) composite Simpson's rule (4 segments of Simpson's 1/3 rules + 1 segment of Simpson's 3/8 rule) (use 1-unit increment)

Rounding a decimal number to four decimal places.

a) I_{true} =

b) I_{approx} =

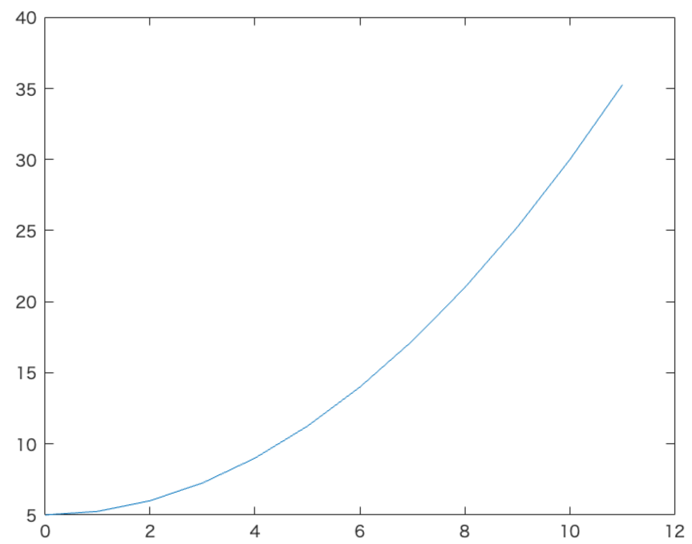
c) I_{approx} =

← ***Interesting (c)***

```
a = 0; b = 11;
x = 0:1:11;
f = @(x) 5 + 0.25.*x.^2;
Itrue = integral(f, a, b)
```

```
Itrue =
165.9167
```

```
plot(x, f(x), '-')
```



```
Iapprox_trap = trap(f, a, b, (b - a))
```

```
Iapprox_trap =
166.3750
```

```
% "1-unit increment" means h = 1
% "4 segments" means ns = 4

% Simpson's 1/3
%  ns #1      ns #2      ns #3      ns #4
%  0—1—2      2—3—4      4—5—6      6—7—8
% |——2h——|   |——2h——|   |——2h——|   |——2h——|
Isimpson13 = 0;
h1 = 1;
for i = [0 2 4 6] % the 4 left-ends are 0, 2, 4, 6 (+2h because it makes up 1 ns)
    x0 = a + i; % keep adding from the lower bound
    simpson13 = (h1/3) * ...
        ( f(x0) + ...
        (4*f(x0 + h1)) + ...
        (f(x0 + (2*h1))) );
    Isimpson13 = Isimpson13 + simpson13;
end

% Simpson's 3/8
%    ns #1
%  8—9—10—11
% |———3h————|
Isimpson38 = 0;
h2 = 1;
```

```
% no loop because there's 1 ns
x0 = 8;
simpson38 = ((3*h2)/8) * ...
    ( f(x0) + ...
    (3*f(x0 + h2)) + ...
    (3*f(x0 + (2*h2))) + ...
    (f(x0 + (3*h2))) );
Isimpson38 = Isimpson38 + simpson38;

Iapprox_simpsons = Isimpson13 + Isimpson38
```

```
Iapprox_simpsons =
165.9167
```

```

% xs = linspace(0,11,400);  plot(xs,f(xs)), hold on
% for v=[0 2 4 6 8 11],  xline(v,'--'); end
% title({'Integration intervals', ...
%        'Simpson 1/3 on [0,8]  (four 2-unit panels)', ...
%        'Simpson 3/8 on [8,11] (one 3-unit panel)'})
% axis([0, 11.5, 5, 40])
% xlabel x, ylabel f(x), hold off
```

Q2 [MOSTLY CORRECT]

Evaluate the following integral: $I = \int_{-2}^{4}(1 - x - 4x^3 + 2x^5)dx$

a) analytically

b) single application of the trapezoidal rule

c) composite trapezoidal rule with 2 segments (ns = 2)

d) composite trapezoidal rule with 4 segments (ns = 4)

e) single application of Simpson's 1/3 rule

f) composite Simpson's 1/3 rule with 4 segments (ns = 4)

g) single application of Simpson's 3/8 rule

h) composite Simpson's rule, with 2 segments (the first segment to apply Simpson's 1/3 rule, the second segment to apply Simpson's 3/8 rule)

For each of the numerical estimates b) through h), determine the absolute of the true percent relative error based on a).

Rounding a decimal number to four decimal places.

a) I_{true} = 1104 ✓

b) I_{approx} = 5280 ✓ , $|e_t|$ = 378.2609 ✓

c) I_{approx} = 2634 ✓ , $|e_t|$ = 138.5870 ✓

d) I_{approx} = 1516.8750 ✓ , $|e_t|$ = 37.3981 ✓

e) I_{approx} = 1752 ✓ , $|e_t|$ = 58.6957 ✓

f) I_{approx} = 1106.5312 ✗ , $|e_t|$ = 0.2293 ✓

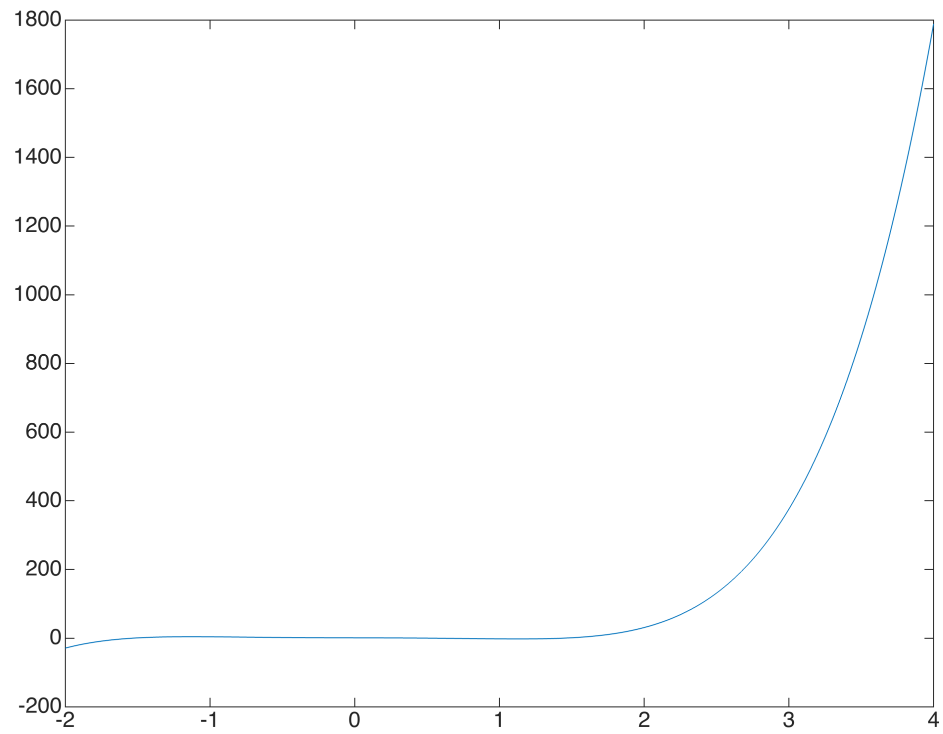
g) I_{approx} = 1392 ✓ , $|e_t|$ = 26.0870 ✓

h) I_{approx} = 14301.6691 ✗ , $|e_t|$ = 1195.4410 ✗

NOTE: H is now correct

Data & a. analytically

```
format short
f = @(x) 1 - x - (4.*x.^3) + (2.*x.^5);
a = -2;
b = 4;
xx = a : 0.01 : b;
yy = f(xx);
plot(xx, yy, '-')
```




```
Itrue = integral(f, a, b)
```

```
Itrue =
1104
```

b. single application of the trapezoidal rule

```
ns = 1
```

```
ns =
1
```

```
I_trap_ns1 = trap(f, a, b, ns)
```

```
I_trap_ns1 =
5280
```

```
abs_err_I_trap_ns1 = abs(((Itrue - I_trap_ns1) / Itrue) * 100)
```

```
abs_err_I_trap_ns1 =
378.2609
```

c. composite trapezoidal rule with 2 segments (ns = 2)

```
ns = 2
```

```
ns =
2
```

```
I_trap_ns2 = trap(f, a, b, ns)
```

```
I_trap_ns2 =
2634
```

```
abs_err_I_trap_ns2 = abs(((Itrue - I_trap_ns2) / Itrue) * 100)
```

```
abs_err_I_trap_ns2 =
138.5870
```

d. composite trapezoidal rule with 4 segments (ns = 4)

```
ns = 4
```

```
ns =
4
```

```
I_trap_ns4 = trap(f, a, b, ns)
```

```
I_trap_ns4 =
1.5169e+03
```

```
fprintf("%.4f", I_trap_ns4)
```

```
1516.8750
```

```
abs_err_I_trap_ns4 = abs(((Itrue - I_trap_ns4) / Itrue) * 100)
```

```
abs_err_I_trap_ns4 =
37.3981
```

e. single application of Simpson’s 1/3 rule

```
ns = 1
```

```
ns =
1
```

```
simpson13_ns1 = zeros(1, ns);
for i = 1:ns
    h = (b - a) / (2 * ns);
    left = (2*(i-1)*h) + a;
    simpson13_ns1 = (h/3) * ...
        ( f(left) + ...
        (4*f(left + h)) + ...
        f(left + (2*h)) );
end
I_simpson13_ns1 = sum(simpson13_ns1)
```

```
I_simpson13_ns1 =
1752
```

```
abs_err_I_simpson13_ns1 = abs(((Itrue - I_simpson13_ns1) / Itrue) * 100)
```

```
abs_err_I_simpson13_ns1 =
58.6957
```

f. composite Simpson's 1/3 rule with 4 segments (ns = 4)

```
a = -2;
b = 4;
ns = 4;
I_simpson13_ns4 = 0;
for i = 1:ns
    h = (b - a) / (2 * ns);
    left = (2*(i-1)*h) + a;
    simpson13_ns4 = (h/3) * ...
        ( f(left) + ...
        (4*f(left + h)) + ...
        f(left + (2*h)) );
    I_simpson13_ns4 = I_simpson13_ns4 + simpson13_ns4;
end
fprintf("%.4f", I_simpson13_ns4)
```

```
1106.5312
```

```
abs_err_I_simpson13_ns4 = abs(((Itrue - I_simpson13_ns4) / Itrue) * 100)
```

```
abs_err_I_simpson13_ns4 =
0.2293
```

g. single application of Simpson's 3/8 rule

```
ns = 1
```

```
ns =
1
```

```
I_simpson38_ns1 = 0;
for i = 1:ns
    h = (b - a) / (3 * ns);
    left = (3*(i-1)*h) + a;
    simpson38_ns1 = ((3*h)/8) * ...
        ( f(left) + ...
        (3*f(left + h)) + ...
        (3*f(left + (2*h))) + ...
        f(left + (3*h)) );
    I_simpson38_ns1 = I_simpson38_ns1 + simpson38_ns1;
end
I_simpson38_ns1
```

```
I_simpson38_ns1 =
1392
```

```
abs_err_I_simpson38_ns1 = abs(((Itrue - I_simpson38_ns1) / Itrue) * 100)
```

```
abs_err_I_simpson38_ns1 =
26.0870
```

h. composite Simpson's rule, with 2 segments (the first segment to apply Simpson's 1/3 rule, the second segment to apply Simpson's 3/8 rule) *[the step size is shared over the 2 segments]*

```
format default

a = -2;
b = 4;

h = (b - a) / 5;

% the first 2 panels
% the left-end is already the starting point (a)
simpson13 = (h/3) * ...
    ( f(a) + ...
    (4*f(a + h)) + ...
    f(a + (2*h)) );
```

```
% the following 3 panels
% the left-end is at two step sizes ahead of a
left = a + (2*h)
```

```
left =
0.4000
```

```
simpson38 = ((3*h)/8) * ...
    ( f(left) + ...
    (3*f(left + h)) + ...
    (3*f(left + (2*h))) + ...
    f(left + (3*h)) );

combined = simpson13 + simpson38;
abs_err_combined = abs(((Itrue - combined) / Itrue) * 100);

fprintf("%.4f", combined)
```

```
1147.9603
```

```
fprintf("%.4f", abs_err_combined)
```

```
3.9819
```

Q3 [CORRECT]

Evaluate the following integral: $I = \int_0^4 (1 - e^{-x})dx$

Let's assume the true value is 3.018316

Using

a) single application of the trapezoidal rule

b) composite trapezoidal rule with 2 segments (ns = 2)

c) composite trapezoidal rule with 4 segments (ns = 4)

d) single application of Simpson's 1/3 rule

e) composite Simpson's 1/3 rule with 4 segments (ns = 4)

f) single application of Simpson's 3/8 rule

g) composite Simpson's rule, with 2 segments (the first segment to apply Simpson's 1/3 rule, the second segment to apply Simpson's 3/8 rule)

For each of the numerical estimates a) through g), determine the absolute of the true percent relative error based on the given true value.

Rounding a decimal number to four decimal places.

a) I_{approx} = ____ , $|e_t|$ = ____

b) I_{approx} = ____ , $|e_t|$ = ____

c) I_{approx} = ____ , $|e_t|$ = ____

d) I_{approx} = ____ , $|e_t|$ = ____

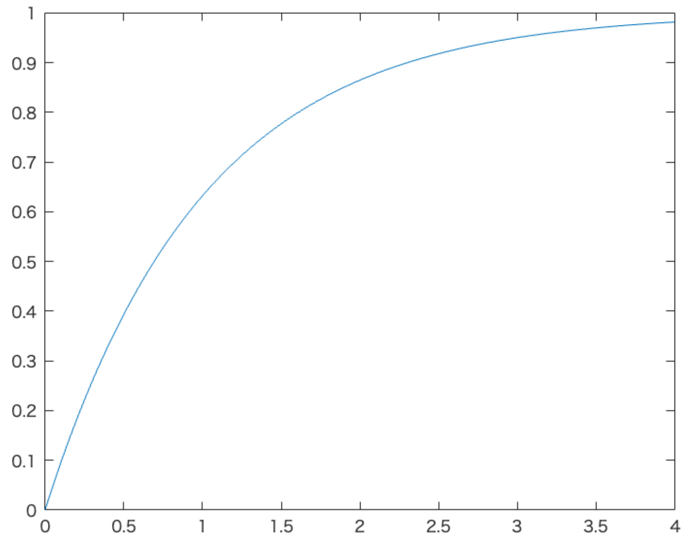
e) I_{approx} = ____ , $|e_t|$ = ____

f) I_{approx} = ____ , $|e_t|$ = ____

g) I_{approx} = ____ , $|e_t|$ = ____

Data

```
f = @(x) 1 - exp(-x);
xx = 0 : 0.01 : 4;
yy = f(xx);
Itrue = 3.018316;
a = 0;
b = 4;
plot(xx, yy, '-')
```



a) single application of the trapezoidal rule

```
ns = 1
```

```
ns =
1
```

```
I_trap_ns1 = trap(f, a, b, ns)
```

```
I_trap_ns1 =
1.9634
```

```
abs_err_I_trap_ns1 = abs(((Itrue - I_trap_ns1) / Itrue) * 100)
```

```
abs_err_I_trap_ns1 =
34.9515
```

b) composite trapezoidal rule with 2 segments (ns = 2)

```
ns = 2
```

```
ns =
2
```

```
I_trap_ns2 = trap(f, a, b, ns)
```

```
I_trap_ns2 =
2.7110
```

```
abs_err_I_trap_ns2 = abs(((Itrue - I_trap_ns2) / Itrue) * 100)
```

```
abs_err_I_trap_ns2 =
10.1812
```

c) composite trapezoidal rule with 4 segments (ns = 4)

```
ns = 4
```

```
ns =
4
```

```
I_trap_ns4 = trap(f, a, b, ns)
```

```
I_trap_ns4 =
2.9378
```

```
abs_err_I_trap_ns4 = abs(((Itrue - I_trap_ns4) / Itrue) * 100)
```

```
abs_err_I_trap_ns4 =
2.6662
```

d) single application of Simpson’s 1/3 rule

```
ns = 1
```

```
ns =
1
```

```
simpson13_ns1 = zeros(1, ns);
for i = 1:ns
    h = (b - a) / (2 * ns);
    left = (2*(i-1)*h) + a;
    simpson13_ns1 = (h/3) * ...
        ( f(left) + ...
        (4*f(left + h)) + ...
        f(left + (2*h)) );
end
I_simpson13_ns1 = sum(simpson13_ns1)
```

```
I_simpson13_ns1 =
2.9602
```

```
abs_err_I_simpson13_ns1 = abs(((Itrue - I_simpson13_ns1) / Itrue) * 100)
```

```
abs_err_I_simpson13_ns1 =
1.9245
```

e) composite Simpson's 1/3 rule with 4 segments (ns = 4)

```
ns = 4
```

```
ns =
4
```

```
I_simpson13_ns4 = 0;
for i = 1:ns
    h = (b - a) / (2 * ns);
    left = (2*(i-1)*h) + a;
    simpson13_ns4 = (h/3) * ...
        ( f(left) + ...
        (4*f(left + h)) + ...
        f(left + (2*h)) );
    I_simpson13_ns4 = I_simpson13_ns4 + simpson13_ns4;
end
I_simpson13_ns4
```

```
I_simpson13_ns4 =
3.0180
```

```
abs_err_I_simpson13_ns4 = abs(((Itrue - I_simpson13_ns4) / Itrue) * 100)
```

```
abs_err_I_simpson13_ns4 =
0.0110
```

f) single application of Simpson's 3/8 rule

```
ns = 1
```

```
ns =
1
```

```
I_simpson38_ns1 = 0;
for i = 1:ns
    h = (b - a) / (3 * ns);
    left = (3*(i-1)*h) + a;
    simpson38_ns1 = ((3*h)/8) * ...
        ( f(left) + ...
        (3*f(left + h)) + ...
        (3*f(left + (2*h))) + ...
        f(left + (3*h)) );
    I_simpson38_ns1 = I_simpson38_ns1 + simpson38_ns1;
end
I_simpson38_ns1
```

```
I_simpson38_ns1 =
2.9912
```

```
abs_err_I_simpson38_ns1 = abs(((Itrue - I_simpson38_ns1) / Itrue) * 100)
```

```
abs_err_I_simpson38_ns1 =
0.8977
```

g) composite Simpson's rule, with 2 segments (the first segment to apply Simpson's 1/3 rule, the second segment to apply Simpson's 3/8 rule) *[the step size is shared over the 2 segments]*

```
format short

% segment 1 (left-end)
a1 = 0; % a2 will be later determined when h is defined
% segment 2 (right-end)
b2 = 4; % b2 will be later determined when h is defined

% shared h
% (Simpson's 1/3 has 2 subsegments
%  Simpson's 3/8 has 3 subsegments
%  2 + 3 = 5 segments in total)
h = (b2 - a1) / 5;
a2 = 2*h % segment 1 (right-end)
```

```
a2 =
1.6000
```

```
b1 = 2*h % segment 2 (left-end)
```

```
b1 =
1.6000
```

```
simpson13 = (h/3) * ...
    ( f(a1) + ...
    (4*f(a1 + h)) + ...
    f(a1 + (2*h)) );

simpson38 = ((3*h)/8) * ...
    ( f(a2) + ...
    (3*f(a2 + h)) + ...
    (3*f(a2 + (2*h))) + ...
    f(a2 + (3*h)) );

combined = simpson13 + simpson38
```

```
combined =
3.0158
```

```
abs_err_combined = abs(((Itrue - combined) / Itrue) * 100)
```

```
abs_err_combined =
0.0829
```

Q4 [MOSTLY CORRECT]

The function $f(x) = e^{-x}$ can be used to generate the following table of unequally spaced data:

x	0	0.1	0.3	0.5	0.7	0.95	1.2
f(x)	1	0.9048	0.7408	0.6065	0.4966	0.3867	0.3012

Evaluate the integral from a = 0 to b = 1.2.

Assume the true value is 0.69880579.

Now using the following methods to compute the integral.

a) the trapezoidal rule

b) a combination of the trapezoidal rule + Simpson's 3/8 rule + Simpson's 1/3 rule

For each of the numerical estimates a) and b), determine the absolute of the true percent relative error based on the given true value.

Rounding a decimal number to four decimal places.

a) I_{approx} = 0.7012 ✓ , $|e_t|$ = 0.3483 ✓

b) I_{approx} = 0.6989 ✓ , $|e_t|$ = 0.0128 ✗

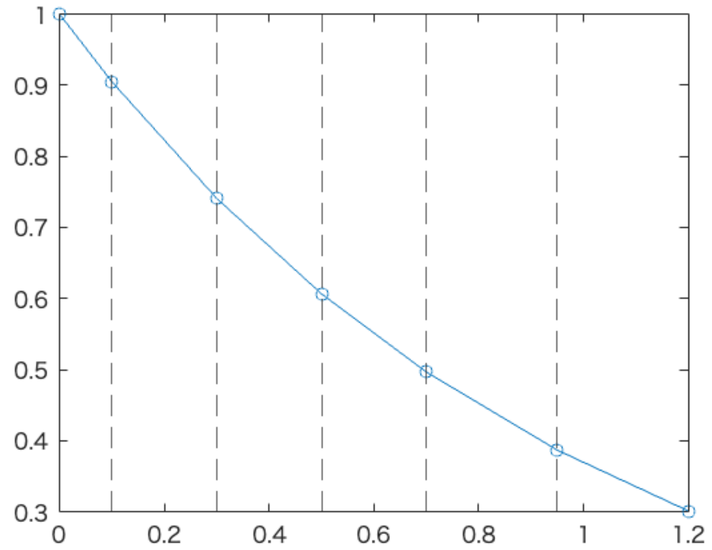
a

```
x = [0       0.1       0.3       0.5       0.7       0.95       1.2];
y = [1     0.9048     0.7408     0.6065     0.4966     0.3867     0.3012];
a = 0; b = 1.2; Itrue = 0.69880579;

f = @(x) exp(-x)
```

```
f = 値をもつ function_handle:
    @(x)exp(-x)
```

```
figure
plot(x, y, 'o-')
hold on
for i=x
    xline(i,'--')
end
hold off
```



```
Itrap = trapuneq(x, y)
```

```
Itrap =
0.7012
```

```
errtrap = round(abs((Itrue - Itrap) / Itrue) * 100, 4)
```

```
errtrap =
0.3483
```

b

```
x2 = [0 0.1];
y2 = [1 0.9048];
Itrap2 = trapuneq(x2, y2)
```

```
Itrap2 =
0.0952
```

```
errtrap2 = round(abs((Itrue - Itrap2) / Itrue) * 100, 4)
```

```
errtrap2 =
86.3710
```

```
% [0.1 0.3 0.5 0.7] (no loop; already covered whole)
h38 = 0.2;
left = 0.1;
simpson38 = ((3*h38)/8) * ...
    ( f(left) + ...
    (3*f(left + h38)) + ...
    (3*f(left + (2*h38))) + ...
    f(left + (3*h38)) );

% [0.7 0.95 1.2] (no loop; already covered whole)
h13 = 0.25;
left = 0.7;
simpson13 = (h13/3) * ...
    ( f(left) + ...
    (4*f(left + h13)) + ...
    f(left + (2*h13)) );

Icombined = Itrap2 + simpson38 + simpson13
```

```
Icombined =
0.6989
```

```
errcombined = round(abs((Itrue - Icombined) / Itrue) * 100, 4)
```

```
errcombined =
0.0128
```

Q5 [CORRECT]

Evaluate the following integral: $I = \int_0^{\pi/2} (8 + 4cos(x))dx$

Let's assume the true value is 16.56637

Using

a) single application of the trapezoidal rule

b) composite trapezoidal rule with 2 segments (ns = 2)

c) composite trapezoidal rule with 4 segments (ns = 4)

d) single application of Simpson's 1/3 rule

e) composite Simpson's 1/3 rule with 4 segments (ns = 4)

f) single application of Simpson's 3/8 rule

g) composite Simpson's rule, with 2 segments (the first segment to apply Simpson's 1/3 rule, the second segment to apply Simpson's 3/8 rule)

For each of the numerical estimates a) through g), determine the absolute of the true percent relative error based on the given true value.

Rounding a decimal number to four decimal places.

a) I_{approx} = ___, $|e_t|$ = ___

b) I_{approx} = ___, $|e_t|$ = ___

c) I_{approx} = ___, $|e_t|$ = ___

d) I_{approx} = ___, $|e_t|$ = ___

e) I_{approx} = ___, $|e_t|$ = ___

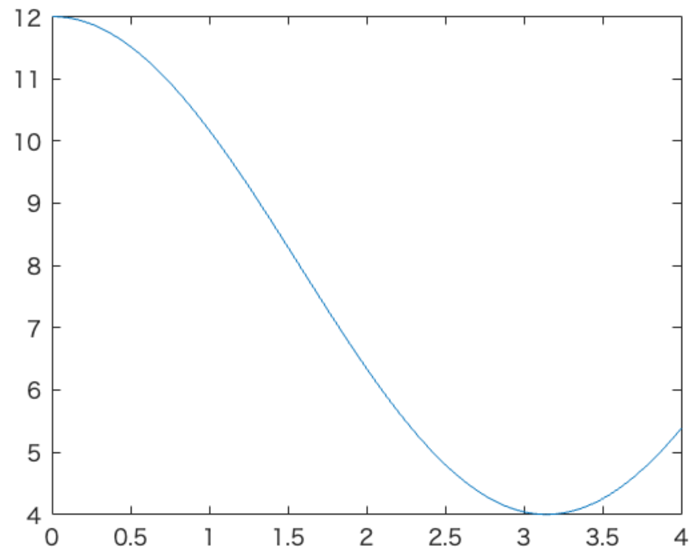
f) I_{approx} = ___, $|e_t|$ = ___

g) I_{approx} = ___, $|e_t|$ = ___

Data

```
format short
```

```
f = @(x) 8 + (4*cos(x));
xx = 0 : 0.01 : 4;
yy = f(xx);
Itrue = 16.56637;
a = 0;
b = pi/2;
plot(xx, yy, '-')
```



a) single application of the trapezoidal rule

```
ns = 1
```

```
ns =
1
```

```
I_trap_ns1 = trap(f, a, b, ns)
```

```
I_trap_ns1 =
15.7080
```

```
abs_err_I_trap_ns1 = abs(((Itrue - I_trap_ns1) / Itrue) * 100)
```

```
abs_err_I_trap_ns1 =
5.1816
```

b) composite trapezoidal rule with 2 segments (ns = 2)

```
ns = 2
```

```
ns =
2
```

```
I_trap_ns2 = trap(f, a, b, ns)
```

```
I_trap_ns2 =
16.3586
```

```
abs_err_I_trap_ns2 = abs(((Itrue - I_trap_ns2) / Itrue) * 100)
```

```
abs_err_I_trap_ns2 =
1.2541
```

c) composite trapezoidal rule with 4 segments (ns = 4)

```
ns = 4
```

```
ns =
4
```

```
I_trap_ns4 = trap(f, a, b, ns)
```

```
I_trap_ns4 =
16.5148
```

```
abs_err_I_trap_ns4 = abs(((Itrue - I_trap_ns4) / Itrue) * 100)
```

```
abs_err_I_trap_ns4 =
0.3111
```

d) single application of Simpson's 1/3 rule

```
ns = 1
```

```
ns =
1
```

```
simpson13_ns1 = zeros(1, ns);
for i = 1:ns
    h = (b - a) / (2 * ns);
```

```
    left = (2*(i-1)*h) + a;
    simpson13_ns1 = (h/3) * ...
        ( f(left) + ...
        (4*f(left + h)) + ...
        f(left + (2*h)) );
end
I_simpson13_ns1 = sum(simpson13_ns1)
```

```
I_simpson13_ns1 =
16.5755
```

```
abs_err_I_simpson13_ns1 = abs(((Itrue - I_simpson13_ns1) / Itrue) * 100)
```

```
abs_err_I_simpson13_ns1 =
0.0551
```

e) composite Simpson's 1/3 rule with 4 segments (ns = 4)

```
format default
ns = 4
```

```
ns =
4
```

```
I_simpson13_ns4 = 0;
for i = 1:ns
    h = (b - a) / (2 * ns);
    left = (2*(i-1)*h) + a;
    simpson13_ns4 = (h/3) * ...
        ( f(left) + ...
        (4*f(left + h)) + ...
        f(left + (2*h)) );
    I_simpson13_ns4 = I_simpson13_ns4 + simpson13_ns4;
end
I_simpson13_ns4
```

```
I_simpson13_ns4 =
16.5664
```

```
abs_err_I_simpson13_ns4 = abs((Itrue - I_simpson13_ns4) / Itrue) * 100
```

```
abs_err_I_simpson13_ns4 =
2.0401e-04
```

```
fprintf("%.4f", abs_err_I_simpson13_ns4)
```

```
0.0002
```

f) single application of Simpson's 3/8 rule

```
ns = 1
```

```
ns =
1
```

```
I_simpson38_ns1 = 0;
for i = 1:ns
    h = (b - a) / (3 * ns);
    left = (3*(i-1)*h) + a;
    simpson38_ns1 = ((3*h)/8) * ...
        ( f(left) + ...
        (3*f(left + h)) + ...
        (3*f(left + (2*h))) + ...
        f(left + (3*h)) );
    I_simpson38_ns1 = I_simpson38_ns1 + simpson38_ns1;
end
I_simpson38_ns1
```

```
I_simpson38_ns1 =
16.5704
```

```
abs_err_I_simpson38_ns1 = abs(((Itrue - I_simpson38_ns1) / Itrue) * 100)
```

```
abs_err_I_simpson38_ns1 =
0.0243
```

g) composite Simpson's rule, with 2 segments (the first segment to apply Simpson's 1/3 rule, the second segment to apply Simpson's 3/8 rule) ***[the step size is shared over the 2 segments]***

```
format short

% segment 1 (left-end)
a1 = 0; % a2 will be later determined when h is defined
% segment 2 (right-end)
b2 = pi/2; % b2 will be later determined when h is defined
```

```
% shared h
% (Simpson's 1/3 has 2 subsegments
%  Simpson's 3/8 has 3 subsegments
%  2 + 3 = 5 segments in total)
h = (b2 - a1) / 5;
a2 = 2*h % segment 1 (right-end)
```

```
a2 =
0.6283
```

```
b1 = 2*h % segment 2 (left-end)
```

```
b1 =
0.6283
```

```

simpson13 = (h/3) * ...
    ( f(a1) + ...
    (4*f(a1 + h)) + ...
    f(a1 + (2*h)) );

simpson38 = ((3*h)/8) * ...
    ( f(a2) + ...
    (3*f(a2 + h)) + ...
    (3*f(a2 + (2*h))) + ...
    f(a2 + (3*h)) );

combined = simpson13 + simpson38
```

```
combined =
16.5667
```

```
abs_err_combined = abs(((Itrue - combined) / Itrue) * 100)
```

```
abs_err_combined =
0.0020
```