HW11

Q1 [CORRECT]

Employ inverse interpolation to determine the value of x that corresponds to f(x) = 0.93 for the following tabulated data:

x = [0 1 2 3 4 5]

f(x) = [0 0.5 0.8 0.9 0.941176 0.961538]

Note that the values in the table were generated with the function $f(x) = \frac{x^2}{1+x^2}$

(a) Determine the correct value analytically.

(b) Use quadratic interpolation and the quadratic formula to determine the value numerically. (use the last three points to fit quadratic polynomial)

Rounding a decimal number to four decimal places.

(a) x_{true} =

(b) x_{est} =

a)

```
xx = [0  1  2  3  4  5];
yy = [0  0.5  0.8  0.9  0.941176  0.961538];
ytest = 0.93
q1_xpredinv = sqrt(ytest / (1 - ytest))
```

b)

```
xx2 = [3 4 5];
yy2 = [0.9  0.941176  0.961538];
p2 = polyfit(xx2, yy2, 2)
a = p2(1)
b = p2(2)
c = p2(3) - ytest
x1 = (-b + sqrt(b^2 - (4*a*c))) / (2*a)
x2 = (-b - sqrt(b^2 - (4*a*c))) / (2*a)
% ans = x1 = 3.6730
```

Q2 [CORRECT]

2.1

For a function $f(x) = x^5 - 16x^4 + 99x^3 - 296x^2 + 428x - 240$,

1) Estimate $f_1(2.5)$ using linear interpolation (a first-order Newton polynomial) between $x_1 = 2$ and $x_2 = 3$ and determine constant b's in

$f_1(x) = b_1 + b_2(x - x_1)$

And compute the percent relative error (e_t) of $f_1(2.5)$. Note that e_t should be answered in positive number only.

Rounding a decimal number to four decimal places.

$f_1(2.5) =$

$b_1 =$

$b_2 =$

$e_t =$ % Note that e_t should be answered in positive number only.

- When $x = x_1$
- When $x = x_2$

$$b_1 = f(x_1)$$

$$b_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

NOTE: $f_1(2.5)$ is NOT `ftrue` (look at the expression given above)

```
xtest = 2.5;
f = @(x) x.^5 - (16.*x.^4) + (99.*x.^3) - (296.*x.^2) + (428.*x) - 240;
ftrue = f(xtest);
x = [2 3];
y = f(x);
y1newttest = Newtint(x, y, xtest)
```

```
b = 2×2
     0     0
     0     0
y1newttest =
0
```

```
et = abs((ftrue - y1newttest) / ftrue) * 100
```

```
et =
100
```

2.2

2) Estimate $f_2(2.5)$ using quadratic interpolation (a second-order Newton polynomial) between $x_1 = 2, x_2 = 3, x_3 = 3.5$ and determine constant b's in

$f_2(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2)$

And compute the percent relative error of $f_2(2.5)$

Rounding a decimal number to four decimal places.

$f_2(2.5) =$

$b_1 =$

$b_2 =$

$b_3 =$

$e_t =$ % Note that e_t should be answered in positive number only.

NOTE: $f_2(2.5)$ is NOT `ftrue` (look at the expression given above)

```
xtest = 2.5;
x = [2 3 3.5];
f = @(x) x.^5 - 16*x.^4 + 99*x.^3 - 296*x.^2 + 428*x - 240;
y = f(x);
y2newttest = Newtint(x, y, xtest)
```

```
b = 3x3
         0         0         0
         0         0         0
    0.8438         0         0
b = 3x3
         0         0         0
         0    1.6875         0
    0.8438         0         0
b = 3x3
         0         0    1.1250
         0    1.6875         0
    0.8438         0         0
y2newttest =
-0.2812
```

```
et2 = abs((ftrue - y2newttest) / ftrue) * 100
```

```
et2 =
40
```

2.3

3) Estimate $f_3(2.5)$ using cubic interpolation (a third-order Newton polynomial) between $x_1 = 2, x_2 = 3, x_3 = 3.5, x_4 = 4$ and determine constant b's in

$f_3(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2) + b_4(x - x_1)(x - x_2)(x - x_3)$

And compute the percent relative error of $f_3(2.5)$

Rounding a decimal number to four decimal places.

$f_3(2.5) =$

$b_1 =$

$b_2 =$

$b_3 =$

$b_4 =$

$e_t =$ % Note that e_t should be answered in positive number only.

NOTE: $f_3(2.5)$ is NOT `ftrue` (look at the expression given above)

```
x = [2 3 3.5 4];
y = f(x);
y3newttest = Newtint(x, y, xtest)
```

```
b = 4x4
         0         0         0         0
         0         0         0         0
    0.8438         0         0         0
```

```
         0         0         0         0
b = 4×4
         0         0         0         0
         0    1.6875         0         0
    0.8438         0         0         0
         0         0         0         0
b = 4×4
         0         0         0         0
         0    1.6875         0         0
    0.8438   -1.6875         0         0
         0         0         0         0
b = 4×4
         0         0    1.1250         0
         0    1.6875         0         0
    0.8438   -1.6875         0         0
         0         0         0         0
b = 4×4
         0         0    1.1250         0
         0    1.6875   -3.3750         0
    0.8438   -1.6875         0         0
         0         0         0         0
b = 4×4
         0         0    1.1250   -2.2500
         0    1.6875   -3.3750         0
    0.8438   -1.6875         0         0
         0         0         0         0
y3newttest =
-0.8438
```

```
et3 = abs((ftrue - y3newttest) / ftrue) * 100
```

```
et3 =
80
```

Q3 [CORRECT]

Edit this line only for answering the L_i.

```
for i = 1:n
    % product = y(i);
    product = 1;
    for j = 1:n
        if i ~= j
            product = product.*(xx-x(j))/(x(i)-x(j));
        end
    end
    fprintf("L%d = %.4f\n", i, product)
    s = s+product;
end
```

3.1

For a function $f(x) = x^5 - 16x^4 + 99x^3 - 296x^2 + 428x - 240$,

1) Estimate $f_1(2.5)$ using linear interpolation (a first-order Lagrange polynomial) between $x_1 = 2$ and $x_2 = 3$ and determine constant L's in

$f_1(x) = L_1 f(x_1) + L_2 f(x_2)$

And compute the percent relative error (e_t) of $f_1(2.5)$. Note that e_t should be answered in positive number only.

Rounding a decimal number to four decimal places.

$f_1(2.5) =$

$L_1 =$

$L_2 =$

$e_t =$ % Note that e_t should be answered in positive number only.

NOTE: $f_1(2.5)$ is NOT `ftrue` (look at the expression given above)

```
xtest = 2.5;
f = @(x) x.^5 - (16.*x.^4) + (99.*x.^3) - (296.*x.^2) + (428.*x) - 240;
x = [2 3];
y = f(x);
ftrue = f(xtest);
ftest = Lagrange(x, y, xtest)
```

```
L1 = 0.0000
L2 = 0.0000
ftest =
0
```

```
et = abs((ftrue - ftest) / ftrue) * 100
```

```
et =
100
```

3.2

2) Estimate $f_2(2.5)$ using quadratic interpolation (a second-order Lagrange polynomial) between $x_1 = 2, x_2 = 3, x_3 = 3.5$ and determine constant L's in

$f_2(x) = L_1 f(x_1) + L_2 f(x_2) + L_3 f(x_3)$

And compute the percent relative error of $f_2(2.5)$

NOTE: $f_2(2.5)$ is NOT `ftrue` (look at the expression given above)

```
xtest = 2.5;
x = [2 3 3.5];
f = @(x) x.^5 - 16*x.^4 + 99*x.^3 - 296*x.^2 + 428*x - 240;
y = f(x);
y2lagtest = round(Lagrange(x, y, xtest), 4)
```

```
L1 = 0.0000
L2 = 0.0000
L3 = -0.2812
y2lagtest =
-0.2813
```

```
et2 = abs((ftrue - y2lagtest) / ftrue) * 100
```

```
et2 =
39.9893
```

3.3

3) Estimate $f_3(2.5)$ using cubic interpolation (a third-order Lagrange polynomial) between $x_1 = 2, x_2 = 3, x_3 = 3.5, x_4 = 4$ and determine constant L's in

$f_3(x) = L_1 f(x_1) + L_2 f(x_2) + L_3 f(x_3) + L_4 f(x_4)$

And compute the percent relative error of $f_3(2.5)$

NOTE: $f_3(2.5)$ is NOT `ftrue` (look at the expression given above)

```
x = [2 3 3.5 4];
y = f(x);
y3lagtest = Lagrange(x, y, xtest)
```

```
L1 = 0.0000
L2 = 0.0000
L3 = -0.8438
L4 = 0.0000
y3lagtest =
-0.8438
```

```
et3 = abs((ftrue - y3lagtest) / ftrue) * 100
```

```
et3 =
80
```

Q4 [CORRECT]

Suppose that we have an exponential model $y = 5e^{0.25x}$.

We can calculate y_{true} when x=12.5.

a) Perform a quadratic polynomial interpolation to the three points with equal interval between 0 and 20. And determine the relative error at x = 12.5

b) Perform a cubic polynomial interpolation to the four points with equal interval between 0 and 20. And determine the relative error at x = 12.5

c) Perform a fourth-order polynomial interpolation to the five points with equal interval between 0 and 20. And determine the relative error at x = 12.5

Rounding a decimal number to four decimal places.

a) e = %

b) e = %

c) e = %

```
f = @(x) 5*exp(0.25*x);
xtest = 12.5;
ytrue = f(xtest);
```

4a

```
xxA = linspace(0, 20, 3);
ypredA = Newtint(xxA, f(xxA), xtest);
etA = abs((ytrue - ypredA) / ytrue) * 100
```

4b

```
xxB = linspace(0, 20, 4);
ypredB = Newtint(xxB, f(xxB), xtest);
```

```
etB = abs((ytrue - ypredB) / ytrue) * 100
```

4c

```
xxB = linspace(0, 20, 5);
ypredB = Newtint(xxB, f(xxB), xtest);
etB = abs((ytrue - ypredB) / ytrue) * 100
```

Q5 [WRONG]

A table of values derived for the unknown function

x	1	2	3	4	5	6	7
f(x)	1	0.5	0.33	0.25	0.2	0.17	0.14

For the inverse interpolation problem, estimate the value of x that corresponds to f(x) = 0.3 by fitting a quadratic polynomial to the three points: (2,0.5), (3,0.33), (4,0.25).

Rounding a decimal number to **two** decimal places.

x =

```
x = [1   2   3    4    5   6    7   ];
y = [1   0.5 0.33 0.25 0.2 0.17 0.14];
ytest = 0.3;
chosenx = [2   3    4];
choseny = [0.5 0.33 0.25];
q5_xpredinv = round(Newtint(choseny, chosenx, ytest), 2) % 3.3400
```