HW14 [ALL CORRECT]

Q1 [CORRECT]

Evaluate $I = \int_0^8 (-0.055x^4 + 0.86x^3 - 4.2x^2 + 6.3x + 2)dx$

a) using analytically evaluation

b) using Romberg integration to an accuracy of ε_s = 0.5%

c) using the three-point Gauss quadrature formula

d) using MATLAB integral function

Rounding a decimal number to four decimal places.

a) I_{true} =

b) I_{approx} =

c) I_{approx} =

d) I_{approx} =

a & d

```
f = @(x) -0.055.*x.^4 + 0.86.*x.^3 - 4.2.*x.^2 + 6.3.*x + 2;
a = 0; b = 8;
Iint = integral(f, a, b)
```

```
Iint =
20.9920
```

b

```
Irb = romberg(f, a, b, 0.5)
```

```
Irb =
20.9920
```

c

```
a1 = (b+a)/2;
a2 = (b-a)/2;
f2 = @(xd) (-0.055.*(a1+a2.*xd).^4 + ...
  0.86.*(a1+a2.*xd).^3 - ...
  4.2.*(a1+a2.*xd).^2 + ...
  6.3.*(a1+a2.*xd) + ...
  2) .* a2;
c0 = 5/9;
c1 = 8/9;
c2 = 5/9;
x0 = -sqrt(3/5);
x1 = 0;
x2 = sqrt(3/5);

Ig3p = c0*f2(x0) + c1*f2(x1) + c2*f2(x2)
```

```
Ig3p =
20.9920
```

Q2 [CORRECT]

There is no closed form solution for the error function

$erf(a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-x^2} dx$

Estimate $erf(1.5)$ using

a) the two-point Gauss quadrature formula

b) the three-point Gauss quadrature formula

Rounding a decimal number to four decimal places.

a)

b)

a

```
a = 0; b = 1.5; k = 2/sqrt(pi);
f = @(x) exp(-(x.^2))
```

```
f = 値をもつ function_handle:
    @(x)exp(-(x.^2))
```

```
Itrue = k * integral(f, a, b)
```

```
Itrue =
0.9661
```

```
a1 = (b+a)/2; a2 = (b-a)/2;
f2 = @(xd) (exp(-(a1 + a2*xd).^2)) * a2;

c0 = 1; c1 = 1;
x0 = -1/sqrt(3); x1 = 1/sqrt(3);
Ig2p = (c0*f2(x0) + c1*f2(x1)) * k
```

```
Ig2p =
0.9742
```

b

```
c0 = 5/9; c1 = 8/9; c2 = 5/9;
x0 = -sqrt(3/5); x1 = 0; x2 = sqrt(3/5);
Ig3p = (c0*f2(x0) + c1*f2(x1) + c2*f2(x2)) * k
```

```
Ig3p =
0.9655
```

Q3 [CORRECT]

Compute $F = \int_0^{30} 200(\frac{x}{5+x})e^{-\frac{x}{15}}dx$

using

a) Romberg integration to a tolerance of e_s = 0.5%

b) the two-point Gauss-Legendre formula

c) the three-point Gauss-Legendre formula

d) the MATLAB integral function

Rounding a decimal number to four decimal places.

a)

b)

c)

d)

```
f = @(x) 200 .* (x./(5+x)) .* exp(-x/15);
a = 0; b = 30;
```

a

```
Irb = romberg(f, a, b, 0.5)
```

```
Irb =
1.4768e+03
```

```
fprintf("%.4f", Irb)
```

```
1476.7973
```

b

```
a1 = (b+a)/2; a2 = (b-a)/2;
c0 = 1; c1 = 1;
x0 = -1/sqrt(3); x1 = 1/sqrt(3);
f2 = @(xd) (200 * ((a1 + a2.*xd)/(5+(a1 + a2.*xd))) * exp(-(a1 + a2.*xd)/15)) * a2;
Ig2p = c0*f2(x0) + c1*f2(x1)
```

```
Ig2p =
1.6106e+03
```

```
fprintf("%.4f", Ig2p)
```

```
1610.5723
```

c

```
c0 = 5/9; c1 = 8/9; c2 = 5/9;
x0 = -sqrt(3/5); x1 = 0; x2 = sqrt(3/5);
Ig3p = c0*f2(x0) + c1*f2(x1) + c2*f2(x2)
```

```
Ig3p =
1.5103e+03
```

```
fprintf("%.4f", Ig3p)
```

```
1510.3329
```

d

```
Iint = integral(f, a, b)
```

```
Iint =
1.4806e+03
```

```
fprintf("%.4f", Iint)
```

```
1480.5685
```

Q4 [CORRECT]

Suppose we have the data

x	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35
F	0	10	28	46	63	82	110	130

Fit these data with a 6th-order polynomial.

Then use the MATLAB integral function to evaluate the integral between x = 0 and x = 0.35

Rounding a decimal number to four decimal places.

```
x = [0 0.05 0.1 0.15 0.2 0.25 0.3 0.35];
y = [0 10 28 46 63 82 110 130];
a = 0; b = 0.35;
p6 = polyfit(x, y, 6);
f = @(x) p6(1).*x.^6 + p6(2).*x.^5 + p6(3).*x.^4 + p6(4).*x.^3 + p6(5).*x.^2 + p6(6).*x + p6(7);
Iint = integral(f, a, b)
```

```
Iint =
20.2182
```

Q5 [CORRECT]

Suppose we have the data

x	0	0.2	0.4	0.6	0.8	1	1.2
y	0.2	0.3683	0.3819	0.2282	0.0486	0.0082	0.1441

Fit these data with a fifth-order polynomial.

Then use the MATLAB integral function to evaluate the integral between x = 0 and x = 1.2

Rounding a decimal number to four decimal places.

```
x = [0 0.2 0.4 0.6 0.8 1 1.2];
y = [0.2 0.3683 0.3819 0.2282 0.0486 0.0082 0.1441];
a = 0; b = 1.2;
p5 = polyfit(x, y, 5);
f = @(x) p5(1).*x.^5 + p5(2).*x.^4 + p5(3).*x.^3 + p5(4).*x.^2 + p5(5).*x + p5(6);
Iint = integral(f, a, b)
```

```
Iint =
0.2414
```