Reinforcement Learning Training 2025

Model-Free Approach

Motivation

Recall in policy iteration

$$v_{k+1}(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) \left[r + \gamma v_k(s') \right]$$

- To make this work, we need to know the model dynamics or $p(s', r|s, a)$.
- However, we do now know p.
- Instead, we will resort to *sampling*.
  - Collecting experience by following some policy in the real world or running the agent through a policy in simulation.

Model-Free Learning

- Monte Carlo (MC) methods
- Temporal difference (TD) methods

Monte Carlo

- We use the law of large numbers (LLN) from statistics.
  - Average of samples is a good estimate for the actual unknown quantity.
  - This estimate becomes better and better as the number of trials of the experiment (samples) increases.

Monte Carlo

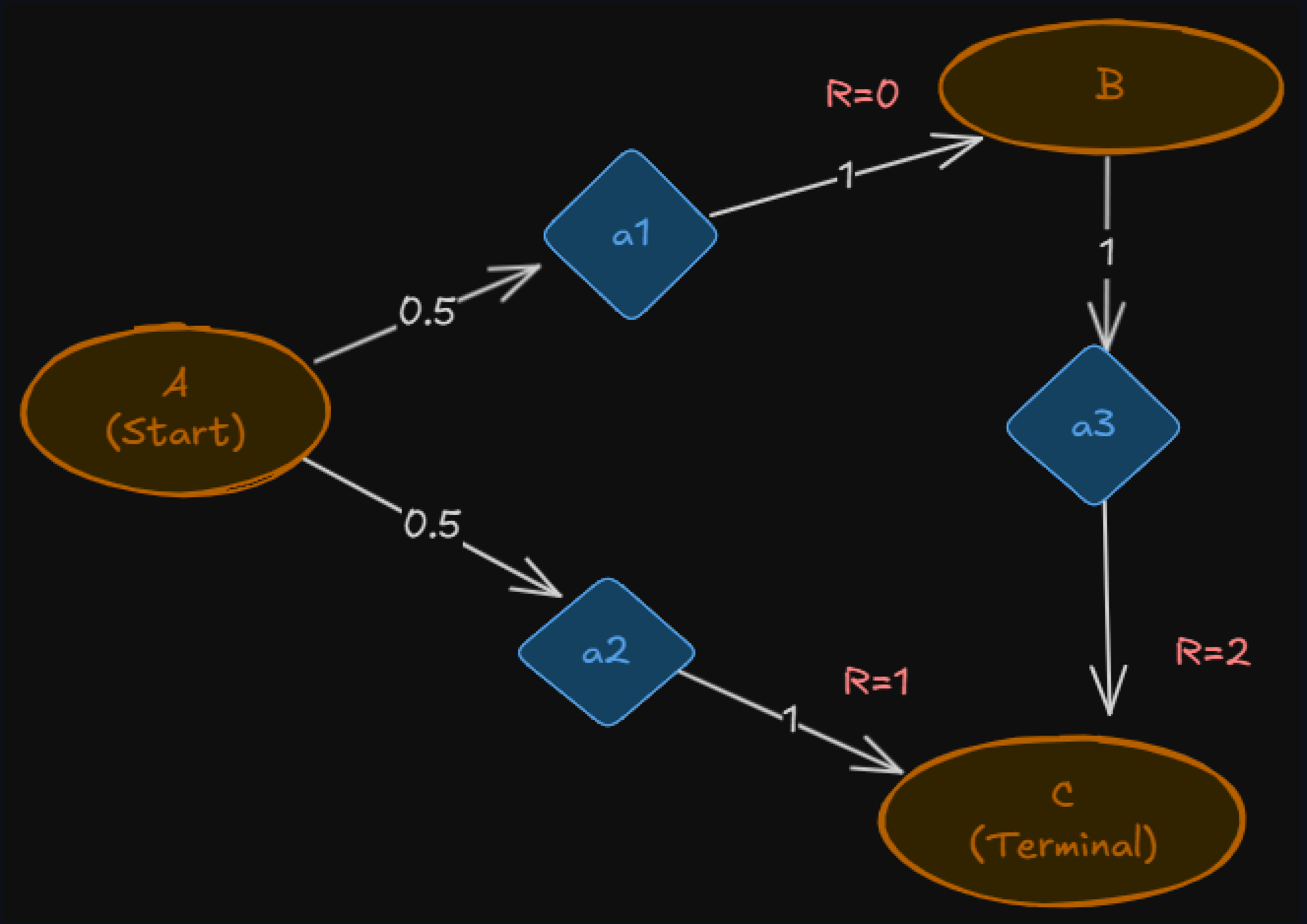
- Re call that We want to calculate

$$v_\pi(s) = \mathbf{E}_\pi[G_t | S_t = s]$$

- We let the agent start from this state $S_t = s$, follow the policy π to take actions, and keep doing so until termination.
  - We call one round of actions an **episode**.
- We record the total sum of rewards for each episode.
- We average the rewards to get an estimate of $v_\pi(s)$ for the policy π.

> MC methods replaces expected returns with the average of sample returns.

Worked Example



Solution v

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\left[r + \gamma v_\pi(s')\right]$$

let $\gamma = 1$

$$V(C) = \boxed{0} \quad (\text{Terminal})$$

$$V(B) = 1\left[1 \times [2 + 1(0)]\right]$$

$$= \boxed{2}$$

$$V(A) = \frac{1}{3}\left[1 \times (0 + 1(2))\right] + \frac{2}{3}\left[1 \times (1 + 1(0))\right]$$

$$= \frac{1}{3}(2) + \frac{2}{3}(1) = \boxed{4/3}$$

Solution q

$$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\left[r + \gamma \sum_{a'} \pi(a'|s')\, q(s',a')\right]$$

$$q(a_3) = 1\left[2 + 1\,\Sigma\,\cancelto{0}{(\ldots)}\right]$$

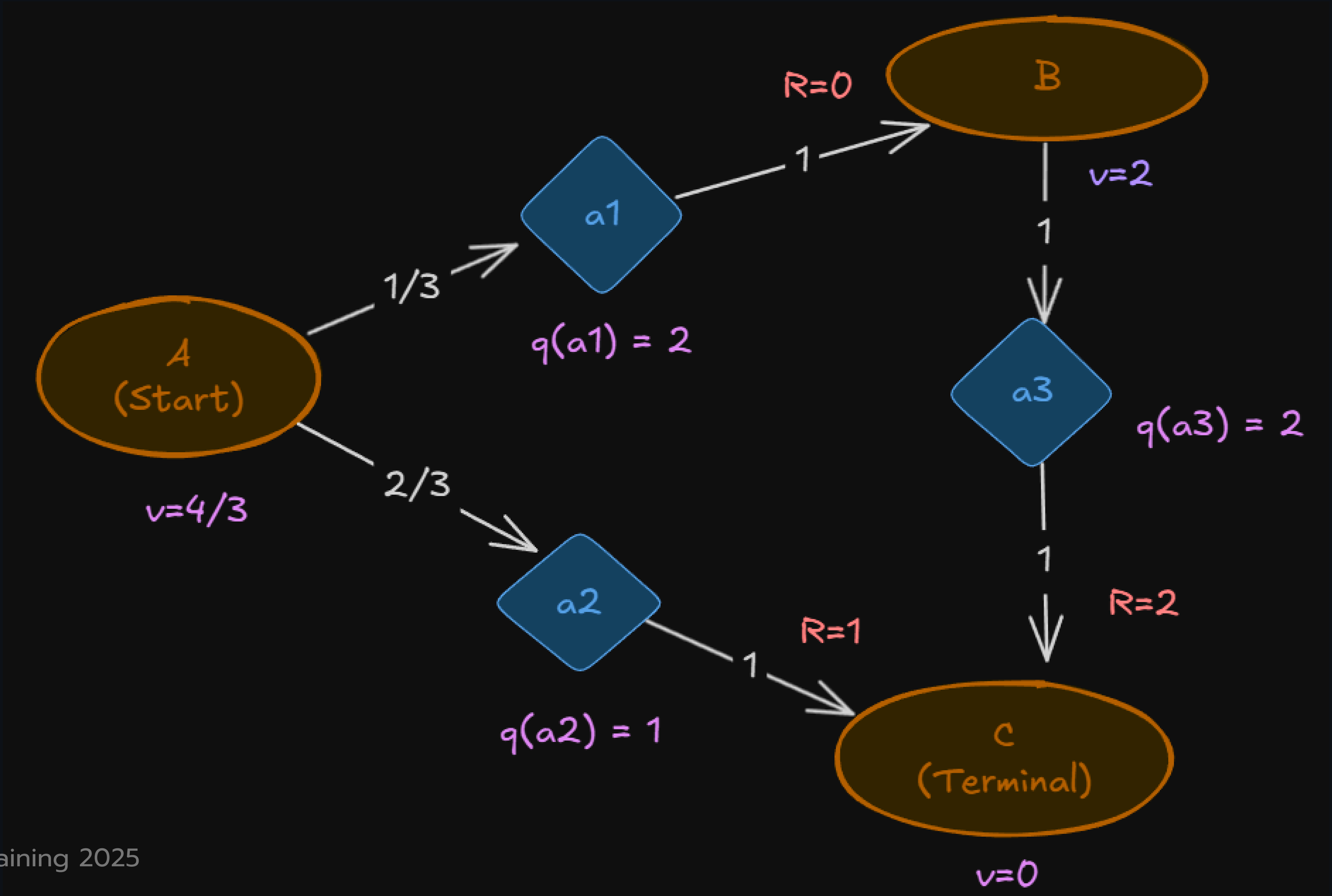
$$= \boxed{2}$$

$$q(a_2) = 1\left[1 + 1\,\Sigma\,\cancelto{0}{(\ldots)}\right]$$

$$= \boxed{1}$$

$$q(a_1) = 1\left[0 + 1\left(1\,(2)\right)\right]$$

$$= \boxed{2}$$


R=0
B
v=2
a1
1
1/3
q(a1) = 2
A
(Start)
a3
q(a3) = 2
v=4/3
2/3
1
R=2
a2
R=1
1
q(a2) = 1
C
(Terminal)
v=0

Estimate $v(A)$

- We simulate many episodes.

Episode	**Path**	**Reward from A**
1	A → C	G_1 = 1
2	A → B → C	G_2 = 0 + 2 = 2
3	A → B → C	G_3 = 0 + 2 = 2
4	A → C	G_4 = 1
…	…	G_n

Results

Monte Carlo estimates the value function $v(A)$ as the average return observed after visiting A.

$$v(A) = \frac{G_1 + G_2 + G_3 + G_4 + \ldots}{n} = \frac{1 + 2 + 2 + 1 + \ldots}{n} \to \frac{4}{3}$$

Online method

- Instead of averaging all the returns at the end (the sample mean), we can use the incremental (update) method to estimate $v(A)$ as each new return is observed.
- This is also called the "sample-average" update and is given by:

$$v_{n+1} = v_n + \frac{1}{n}(G_n - v_n)$$

Estimate $q(a_1)$ and $q(a_2)$

Episode	Path	Actions at A	Reward from Action at A
1	A → C	a_2	$G_1 = 1$
2	A → B → C	a_1	$G_2 = 0 + 2$
3	A → B → C	a_1	$G_3 = 0 + 2$
4	A → C	a_2	$G_4 = 1$
...	...	...	G_n

Estimate $q(a_1)$ and $q(a_2)$

$$q(a_1) = \frac{G_2 + G_3 + \ldots}{n} = \frac{2 + 2 + \ldots}{n} \to 2$$

$$q(a_2) = \frac{G_1 + G_4 + \ldots}{n} = \frac{1 + 1 + \ldots}{n} \to 1$$

Online update

$$q_{n+1} = q_n + \frac{1}{n}(G_n - q_n)$$

Comparing estimations of v and q

- Notice that the calculation of v and q is the same.
- This is because both functions are fundamentally estimates of an expected value—just over different types of returns.
  - $v(s)$ - average over all times you start at s
  - $q(s, a)$ - average over all times you start at s and pick a