Reinforcement Learning Training 2025

Model-Free Approach

Motivation

Recall in policy iteration

$$v_{k+1}(s) \leftarrow \sum_{a} \pi(a|s) \sum_{s',r} p(s', r | s, a) \left[r + \gamma \, v_k(s') \right]$$

- To make this work, we need to know the model dynamics or $p(s', r|s, a)$.
- However, we do now know p.
- Instead, we will resort to *sampling*.
    - Collecting experience by following some policy in the real world or running the agent through a policy in simulation.

Model-Free Learning

- Monte Carlo (MC) methods
- Temporal difference (TD) methods

Monte Carlo

- We use the law of large numbers (LLN) from statistics.
    - Average of samples is a good estimate for the actual unknown quantity.
    - This estimate becomes better and better as the number of trials of the experiment (samples) increases.

Monte Carlo

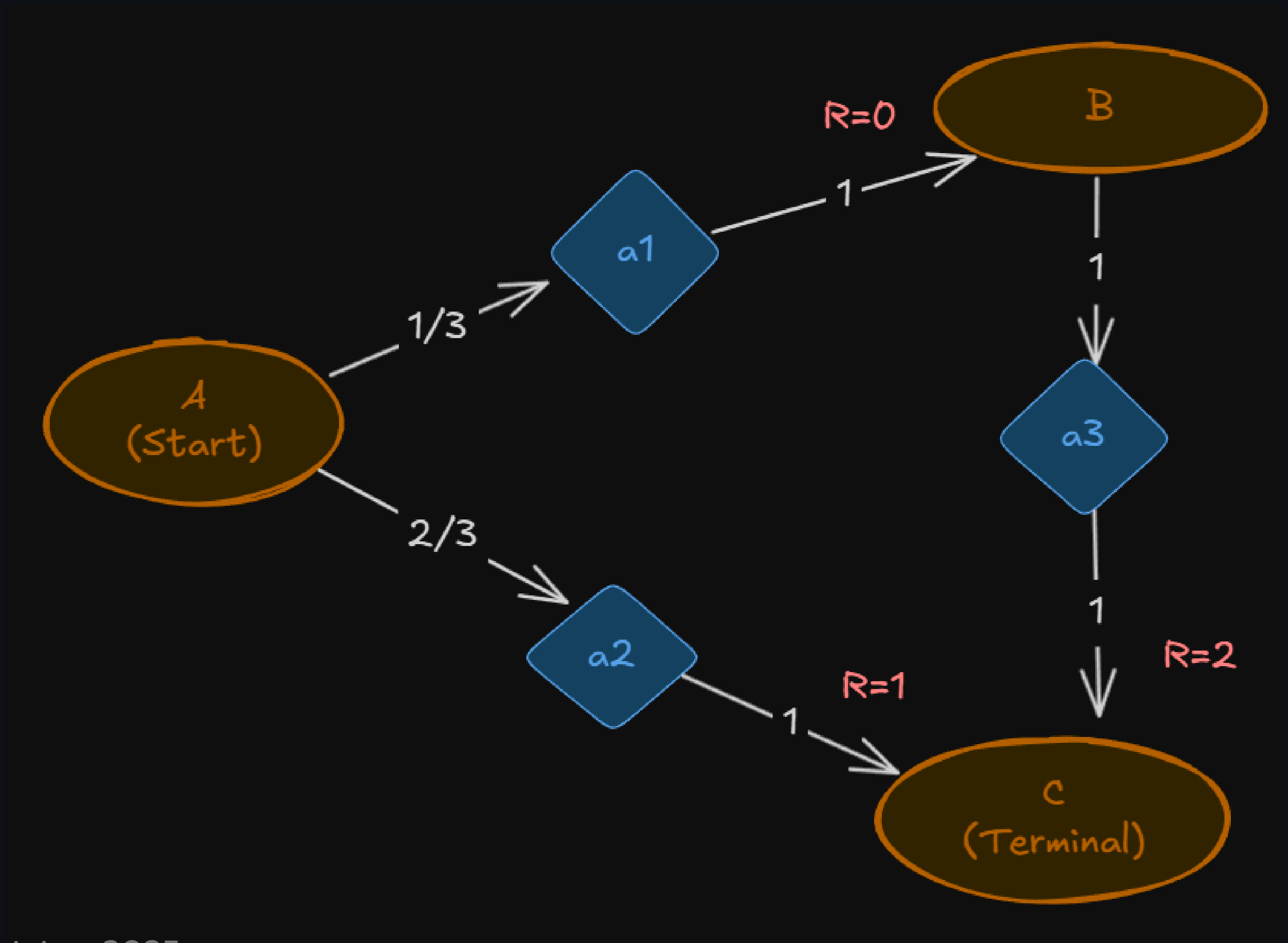
- Recall that We want to calculate

$$v_\pi(s) = \mathbf{E}_\pi[G_t | S_t = s]$$

- We let the agent start from this state $S_t = s$, follow the policy π to take actions, and keep doing so until termination.
  - We call one round of actions an **episode**.
- We record the total sum of rewards for each episode.
- We average the rewards to get an estimate of $v_\pi(s)$ for the policy π.

> MC methods replaces expected returns with the average of sample returns.

Worked Example



Solution v

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)\left[r + \gamma v_\pi(s')\right]$$

let $\gamma = 1$

$V(C) = \boxed{0}$ (Terminal)

$V(B) = 1\left[1 \times [2 + 1(0)]\right]$

$= \boxed{2}$

$V(A) = \frac{1}{3}\left[1 \times (0 + 1(2))\right]$

$+ \frac{2}{3}\left[1 \times (1 + 1(0))\right]$

$= \frac{1}{3}(2) + \frac{2}{3}(1) = \boxed{4/3}$

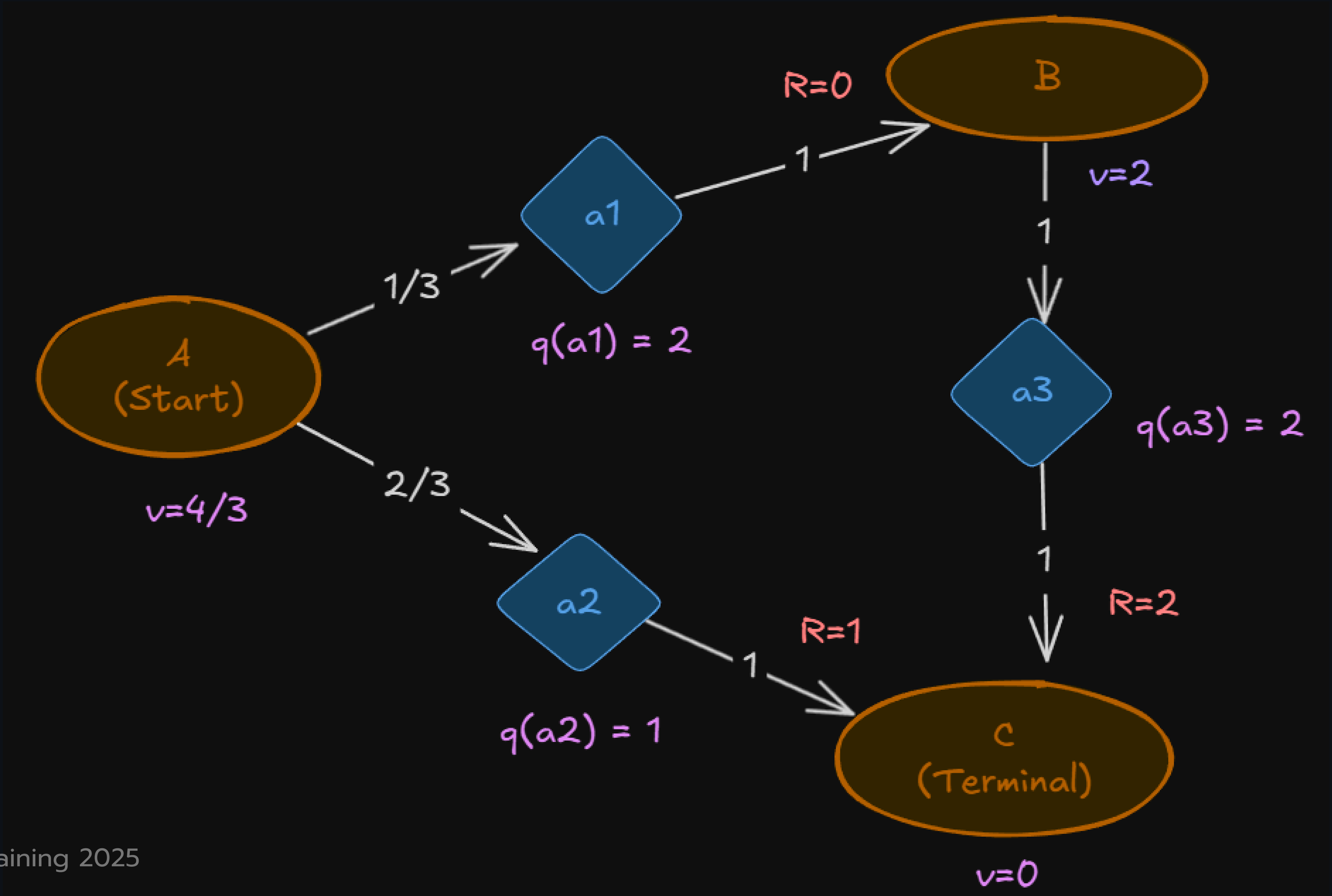
Solution q

$$q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\left[r + \gamma \sum_{a'} \pi(a'|s')\, q(s',a')\right]$$

$$q(a_3) = 1\left[2 + 1\cancelto{0}{\sum(\dots)}\right] = \boxed{2}$$

$$q(a_2) = 1\left[1 + 1\cancelto{0}{\sum(\dots)}\right] = \boxed{1}$$

$$q(a_1) = 1\left[0 + 1\left(1(2)\right)\right] = \boxed{2}$$


R=0
B
1
v=2
a1
1/3
1
A
(Start)
q(a1) = 2
a3
q(a3) = 2
v=4/3
2/3
1
a2
R=2
R=1
1
q(a2) = 1
C
(Terminal)
v=0

Estimate $v(A)$

- We simulate many episodes.

Episode	**Path**	**Reward from A**
1	A → C	G_1 = 1
2	A → B → C	G_2 = 0 + 2 = 2
3	A → B → C	G_3 = 0 + 2 = 2
4	A → C	G_4 = 1
…	…	G_n

Results

Monte Carlo estimates the value function $v(A)$ as the average return observed after visiting A.

$$v(A) = \frac{G_1 + G_2 + G_3 + G_4 + \ldots}{n} = \frac{1 + 2 + 2 + 1 + \ldots}{n} \to \frac{4}{3}$$

Online Method

- Instead of averaging all the returns at the end (the sample mean), we can use the incremental (update) method to estimate $v(A)$ as each new return is observed.
- This is also called the "sample-average" update and is given by:

$$v_{n+1} = v_n + \frac{1}{n}(G_n - v_n)$$

- Also, note the constant-α version

$$v_{n+1} = v_n + \alpha(G_n - v_n)$$

  where α is the learning rate.

Estimate $q(a_1)$ and $q(a_2)$

Episode	Path	Actions at A	Reward from Action at A
1	A → C	a_2	$G_1 = 1$
2	A → B → C	a_1	$G_2 = 0 + 2$
3	A → B → C	a_1	$G_3 = 0 + 2$
4	A → C	a_2	$G_4 = 1$
…	…	…	G_n

Estimate $q(a_1)$ and $q(a_2)$

$$q(a_1) = \frac{G_2 + G_3 + \dots}{n} = \frac{2 + 2 + \dots}{n} \rightarrow 2$$

$$q(a_2) = \frac{G_1 + G_4 + \dots}{n} = \frac{1 + 1 + \dots}{n} \rightarrow 1$$

Online update

$$q_{n+1} = q_n + \frac{1}{n}(G_n - q_n)$$

- Constant-α version

$$q_{n+1} = q_n + \alpha(G_n - q_n)$$

Comparing estimations of v and q

- Notice that the calculation of v and q is the same.
- This is because both functions are fundamentally estimates of an expected value—just over different types of returns.
  - $v(s)$ - average over all times you start at s
  - $q(s, a)$ - average over all times you start at s and pick a

Monte Carlo: First-Visit vs. Every-Visit

- *First-Visit MC*
  - Only the first time a state (or state-action pair) is visited in an episode, the return following that visit is used to update the estimate.
- *Every-Visit MC*
  - Every time a state (or state-action pair) is visited in an episode, the return following that visit is used to update the estimate (even if visited multiple times within the same episode).

Gridworld

- Calculating v using MC.

Control with Monte Carlo

- We will use generalized policy iteration (GPI).
  - Find q (not v)
  - Improve policy using greedy optimization.
  - Repeat …

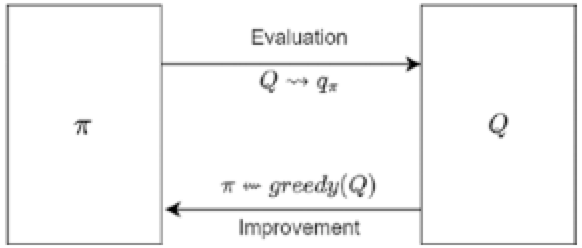


***Figure 4-5.** Iteration between the two steps. The first step evaluates to get the state-action values in sync with the policy being followed. The second step performs policy improvement to do a greedy maximization for actions*

Control with Monte Carlo

- However, in MC, the greedy opimization does not work very well.
    - To be greedy, we need to know all q.
    - But being greedy, we will never *explore* all q.
- To fix this, we will use **ε-greedy policy**.

Control with Monte Carlo

- The agent exploits the knowledge with probability 1-ε and explores with probability ε.

$$\pi(a|s) = \begin{cases} 1 - \varepsilon + \dfrac{\varepsilon}{|A|} & \text{for } a = \operatorname{argmax}_a Q(s,a) \\ \dfrac{\varepsilon}{|A|} & \text{otherwise} \end{cases}$$

Control with Monte Carlo

- Next, we will not run MC until values of q converges.
- We will run MC prediction followed by policy improvement on an episode-by-episode basis.
- This way, there is no need for a large number of iterations in the estimation/prediction step

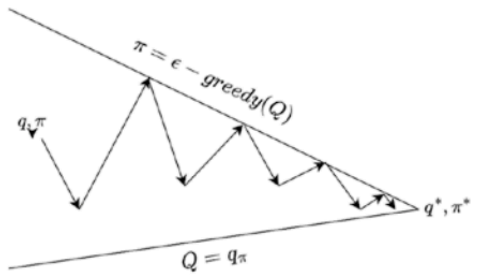


Figure 4-6. *Iteration between the two steps. The first step is the MC prediction/evaluation for a single step to move the q-values in the direction of the current policy. The second step is that of policy improvement to do a ε-greedy maximization for actions*

Control with Monte Carlo

- With the two tweaks, we have **Greedy in the Limit of Infinite Exploration (GLIE)**.

GLIE FOR POLICY OPTIMIZATION

Initialize:

State-action values $Q(s, a) = 0$ for all $s \in S$ and $a \in A$.

Visit count $N(s, a) = 0$ for all $s \in S$ and $a \in A$.

Policy π with enough exploration e.g., random policy

Loop:

sample episode(k) following policy π : $S_0, A_0, R_1, S_1, A_1, R_2, \ldots\ldots A_{t-1}, R_T, S_T$

$G \leftarrow 0$

Loop backwards for each step of episode, $t = T-1, T-2, \ldots, 1, 0$

$G \leftarrow \gamma.G + R_{t+1}$

$N(s, a) \leftarrow N(s, a) + 1$

$Q(s, a) \leftarrow Q(s, a) + 1/N(s, a) * [G - Q(s, a)]$

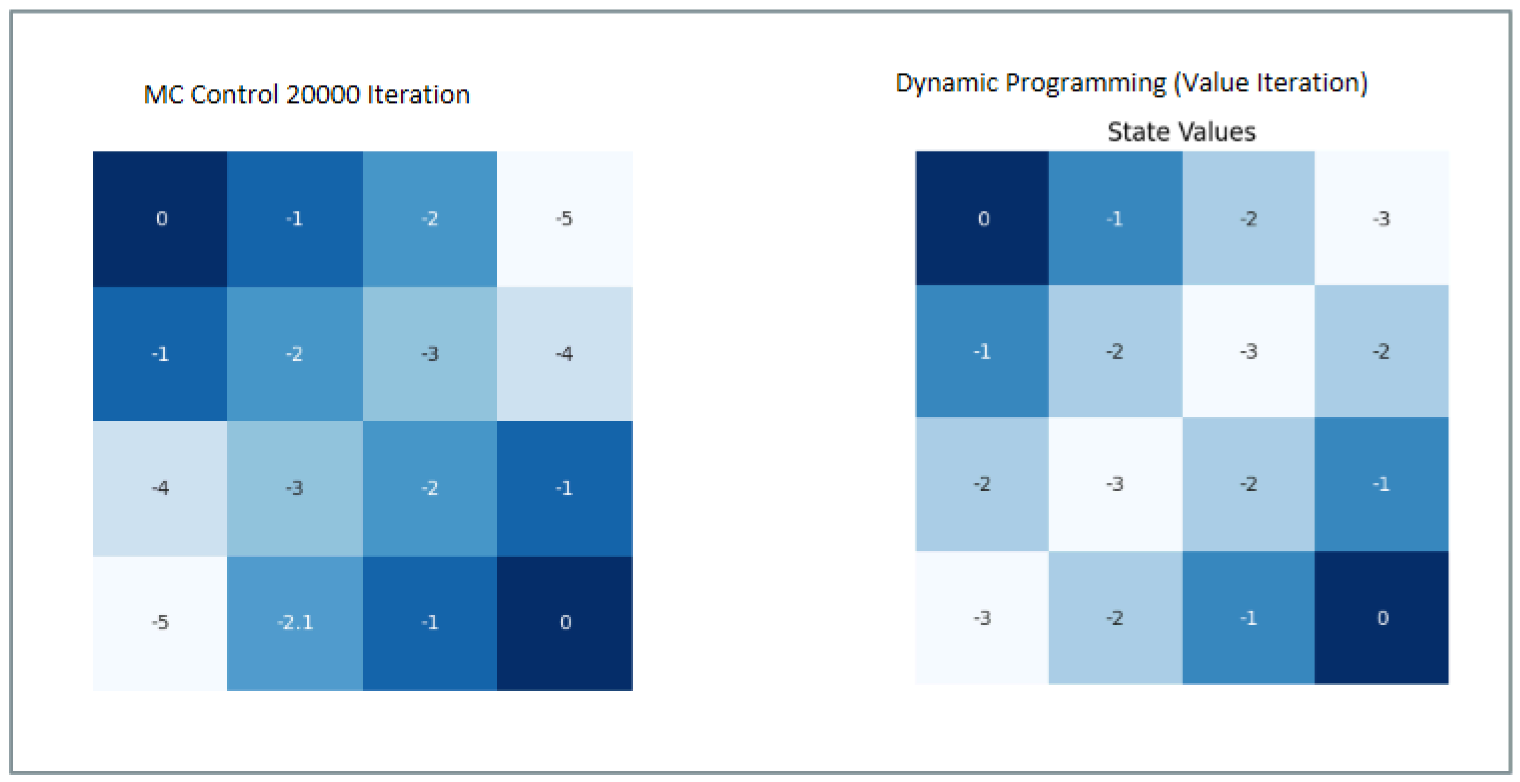
Reduce ε using $\varepsilon = 1/k$

Update policy with ε-greedy using revised $Q(s, a)$

Figure 4-7. *Every-visit (GLIE) MC control for policy optimization*

Note that we reduce ϵ to make policy more deterministic.

Simulation



(See python code)

On-Policy vs Off Policy

"On"-Policy MC Control

- In GLIE, we employ ϵ-greedy policy in two process.
  - Finding v or q.
  - Maximizing the policy.
- This is called an **on-policy** approach.
- However, the on-policy approach is not efficient.
  - The episode that you collect has to be thrown away.
  - You have no separate ability to control exploration.

"Off"-Policy MC Control

- Another approach is to use two policies.
    - More exploratory to generate samples.
    - Near determistic policy in policy maximization.
- This is called an **off-policy** approach.
    - **Behavior** policy (b) for generating samples
    - **Target** policy (π) in policy maximization

OFF POLICY MC CONTROL OPTIMIZATION

Initialize, for all $s \in S$ and $a \in A(s)$:

State-action values $Q(s,a) \in \mathbb{R}$ (arbitrarily)

$C(s,a) \leftarrow 0$

Policy $\pi = \arg\max_a Q(s,a)$

Loop for each episode:

$b \leftarrow$ a "behavior policy" with enough exploration

sample episode(k) following policy π : $S_0, A_0, R_1, S_1, A_1, R_2, \ldots\ldots A_{t-1}, R_T, S_T$

$G \leftarrow 0$

$W \leftarrow 1$

Loop backwards for each step of episode, $t = T-1, T-2, \ldots, 1, 0$

$G \leftarrow \gamma . G + R_{t+1}$

$C(S_t, A_t) \leftarrow C(S_t, A_t) + W$

$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t,A_t)}\left[G - Q(S_t, A_t)\right]$

$\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$

If $A_t \neq \pi(S_t)$ exit inner loop

$W \leftarrow W \frac{1}{b(A_t|S_t)}$

Temporal Difference Learning Methods

Problem with MC

- Only work for episodic environment.
- Only update q when episode ends.
- Temporal difference learning solve these problems.

Update Equation

- Monte Carlo (constant-α version)

$$V_{n+1} = V_n + \alpha\left(G - V_n\right)$$

- Temporal Difference, $TD(0)$

$$V(s) = V(s) + \alpha\left[R + \gamma \cdot V(s') - V(s)\right]$$

The value of v is estimated with the *estimate* of the successor state.
This is known as *bootstrapping*.

TD Error

$$\delta_t = R_{t+1} + \gamma \cdot V\left(S_{t+1}\right) - V\left(S_t\right)$$

- Error in the estimate of v based on reward and discounted next time-step state value.

Advantages of TD

- Compared with model-based approach, TD does not need knowledge of transition probabilities.
- Compared with Monte-Carlo appraoch, TD can update the value function at every step.
  - Faster convergence.

TD Control

- On-Policy SARSA
- Q-learning
- Expected SARSA

On-Policy SARSA

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot \left[R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \right]$$

$$\delta_t = R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)$$

- Note that terminate state -> q equals to zero.

SARSA on-policy TD control

Initialize:

State-action values $Q(s, a) = 0$ for all $s \in S$ and $a \in A$.

policy $\pi = \varepsilon$-greedy policy with some small $\varepsilon \in [0, 1]$

learning rate (step size) $\alpha \in [0, 1]$

discount factor $\gamma \in [0, 1]$

Loop for each episode:

Start state S, choose action A based on ε-greedy policy

Loop for each step till episode end:

Take action A and observe reward R and next state S'

Choose action A' using ε-greedy policy using current Q values

If S' not terminal:

$$Q(S, A) \leftarrow Q(S, A) + \alpha \cdot [R + \gamma \cdot Q(S', A') - Q(S, A)]$$

else:

$$Q(S, A) \leftarrow Q(S, A) + \alpha \cdot [R - Q(S, A)]$$

$$S \leftarrow S'; A \leftarrow A'$$

[optionally reduce ε periodically towards zero]

Return policy π based on final Q values.

Q-Learning

- SARSA

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot \left[R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \right]$$

- Q-learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot \left[R_{t+1} + \gamma \cdot \max_{A_{t+1}} Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \right]$$

Q-Learning

- You generate samples using ϵ-greedy policy.
- However, when you update q, you are using $\operatorname{argmax}_a q$, which is a different policy.
    - This is essentially an off-policy approach.

Q LEARNING OFF-POLICY TD CONTROL

Initialize:

State-action values $Q(s, a) = 0$ for all $s \in S$ and $a \in A$.

policy $\pi = \varepsilon$-greedy policy with some small $\varepsilon \in [0, 1]$

learning rate (step size) $\alpha \in [0, 1]$

discount factor $\gamma \in [0, 1]$

Loop for each episode:

Start state S

Loop for each step till episode end:

Choose action A based on ε-greedy policy

Take action A and observe reward R and next state S'

If S' not terminal:

$$Q(S, A) \leftarrow Q(S, A) + \alpha \cdot [R + \gamma \cdot \max_{A'} Q(S', A') - Q(S, A)]$$

else:

$$Q(S, A) \leftarrow Q(S, A) + \alpha \cdot [R - Q(S, A)]$$

$S \leftarrow S'$

Return policy π based on final Q values.

Expected SARSA

- SARSA

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot \left[R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

- Q-learning

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot \left[R_{t+1} + \gamma \cdot \max_{A_{t+1}} Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

- Expected SARSA

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \cdot \sum_{a \in A_{t+1}} \pi(a|S_{t+1}) \cdot Q(S_{t+1}, a) - Q(S_t, A_t)\right]$$

EXPECTED SARSA TD CONTROL

Initialize:

State-action values $Q(s, a) = 0$ for all $s \in S$ and $a \in A$.

policy $\pi = \varepsilon$-greedy policy with some small $\varepsilon \in [0, 1]$

learning rate (step size) $\alpha \in [0, 1]$

discount factor $\gamma \in [0, 1]$

Loop for each episode:

Start state S

Loop for each step till episode end:

Choose action A based on ε-greedy policy

Take action A and observe reward R and next state S'

If S' not terminal:

$$Q(S, A) \leftarrow Q(S, A) + \alpha \cdot [R + \gamma \cdot \sum_a \pi(a|S')Q(S', a) - Q(S, A)]$$

else:

$$Q(S, A) \leftarrow Q(S, A) + \alpha \cdot [R - Q(S, A)]$$

$S \leftarrow S'$

Return policy π based on final Q values.

Experience Replay

- Off-policy learning has two policies.
  - Behavior policy b
  - Target policy π
- Accordingly, we can use the samples generated by the behavior policy again and again to train the agent.
  - The approach makes the process sample efficient.
  - This is similar to supervised learning.
  - This is called *experience replay*.

Q Learning for continuous state spaces

- You will be applying the learning on a continuous environment, **CartPole**.
- It is done by
  - Extending the Gymnasium `ObservationWrapper`.
  - Implementing a function called `observation`, which takes in the continuous values from the CartPole environment and returns the discrete observation/state values.