

# Cursul nr. 7

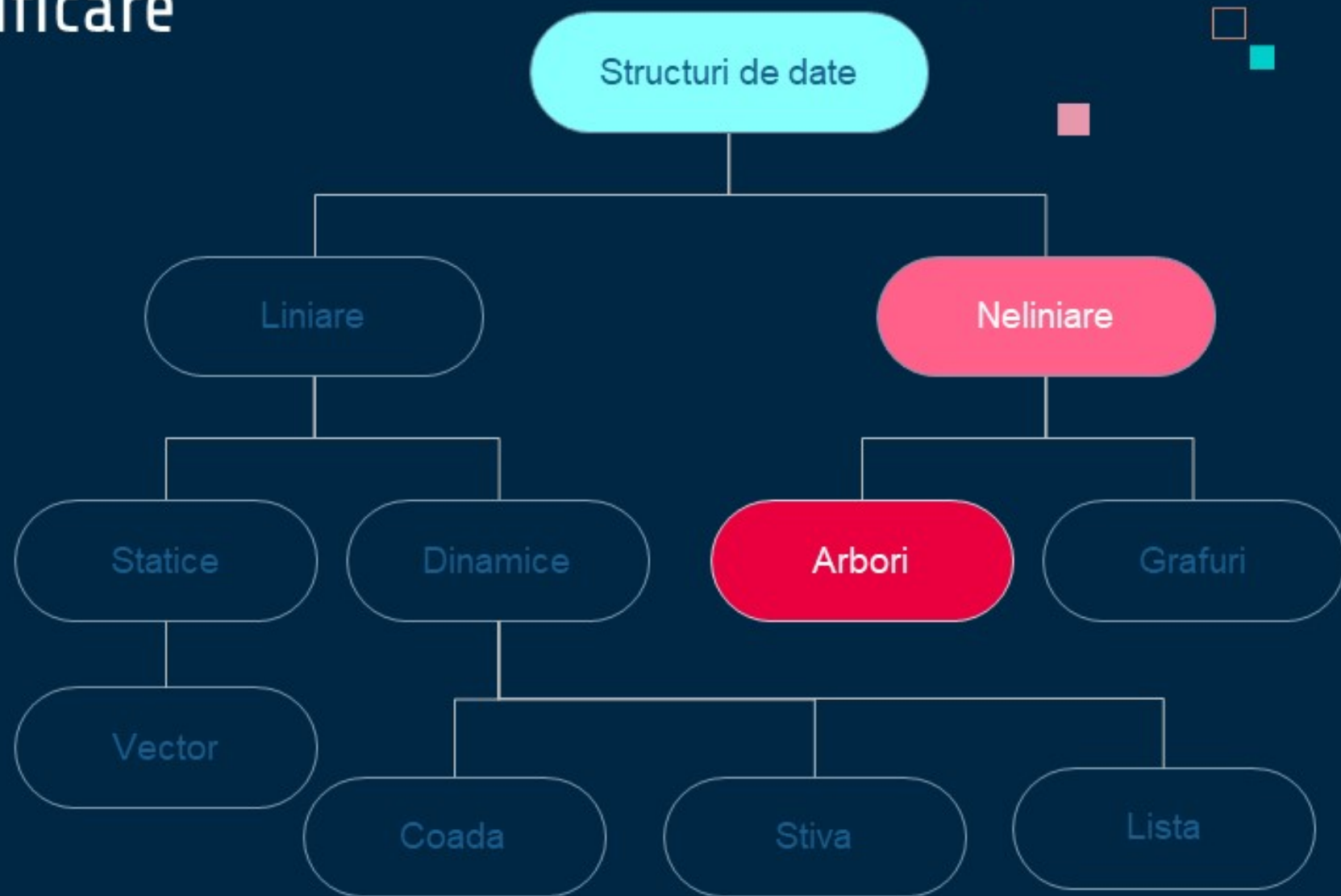
# STRUCTURI DE DATE

## neliniare

### Arbori

Lector dr. Dorin IORDACHE

# Clasificare



# Agenda



01

Arbori



02

Parcurgeri

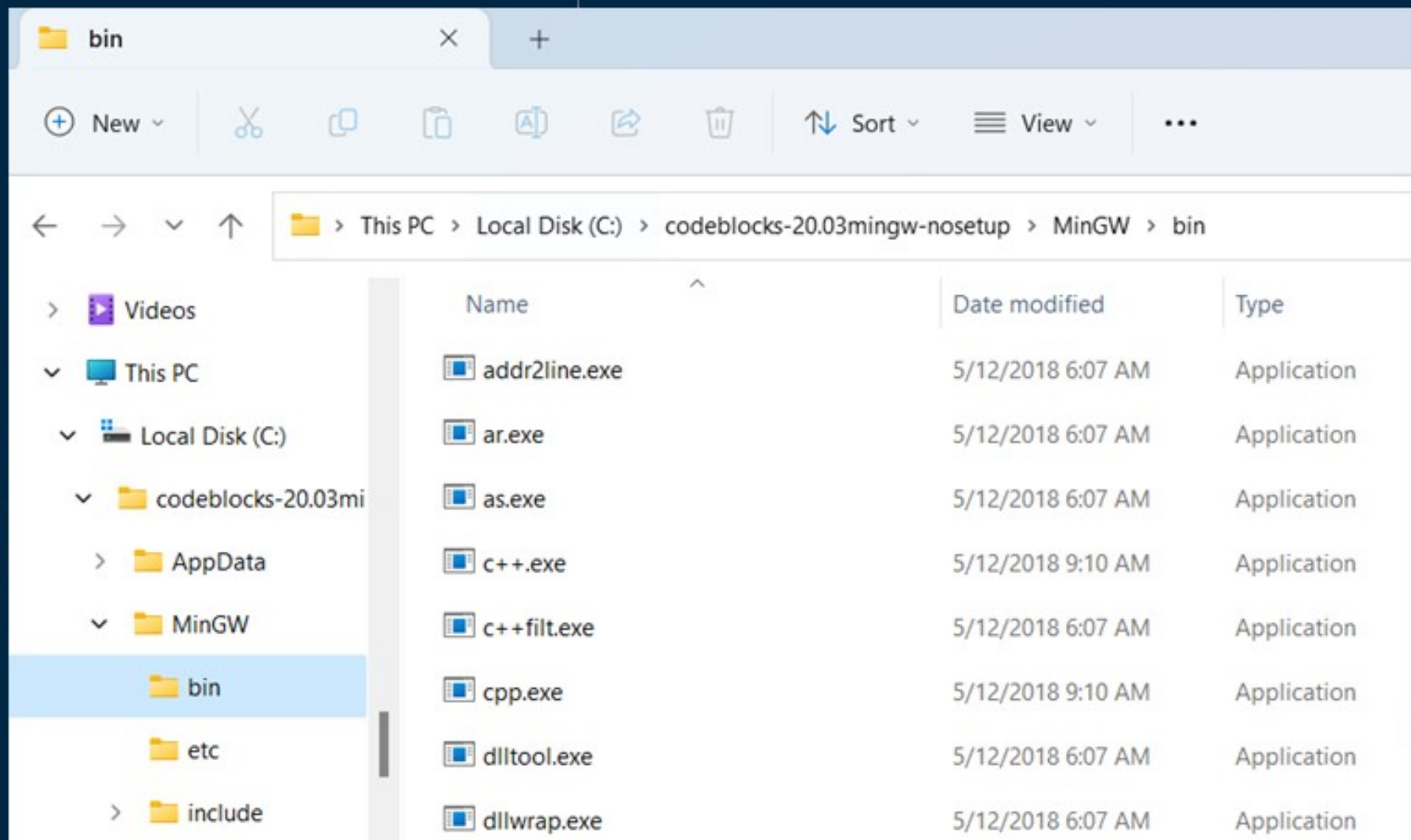


03

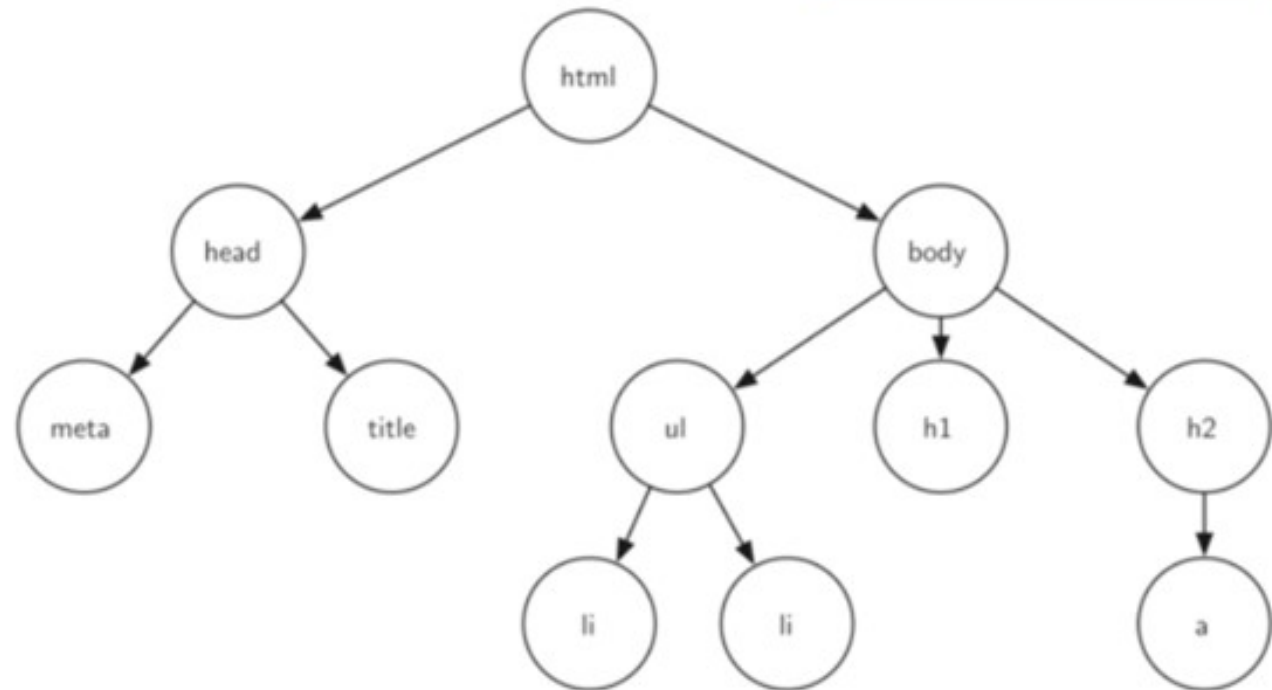
Arbori Binar

Arbori

01



```
<html xmlns="http://www.w3.org/1999/xhtml" xml:lang="en" lang="en">
<head>
<meta http-equiv="Content-Type" content="text/html; charset=utf-8" />
<title>HTML simplu</title>
</head>
<body>
<h1>Un exemplu structura arborescenta</h1>
<ul>
<li>Item lista 1</li>
<li>Item lista 2</li>
</ul>
<h2><a href="http://fmi.univ-ovidius.ro">fmi.univ-ovidius.ro</a></h2>
</body>
</html>
```



/etc

└─ acpi

| └─ asus-keyboard-backlight.sh

| └─ asus-wireless.sh

| └─ events

| | └─ asus-keyboard-backlight-down

| | └─ asus-keyboard-backlight-up

| | └─ asus-wireless-off

| | └─ asus-wireless-on

| | └─ ibm-wireless

| | └─ lenovo-undock

| | └─ thinkpad-cmos

| | └─ tosh-wireless

| └─ ibm-wireless.sh

| └─ tosh-wireless.sh

| └─ undock.sh

└─ adduser.conf

└─ aliases

└─ alsa

| └─ conf.d

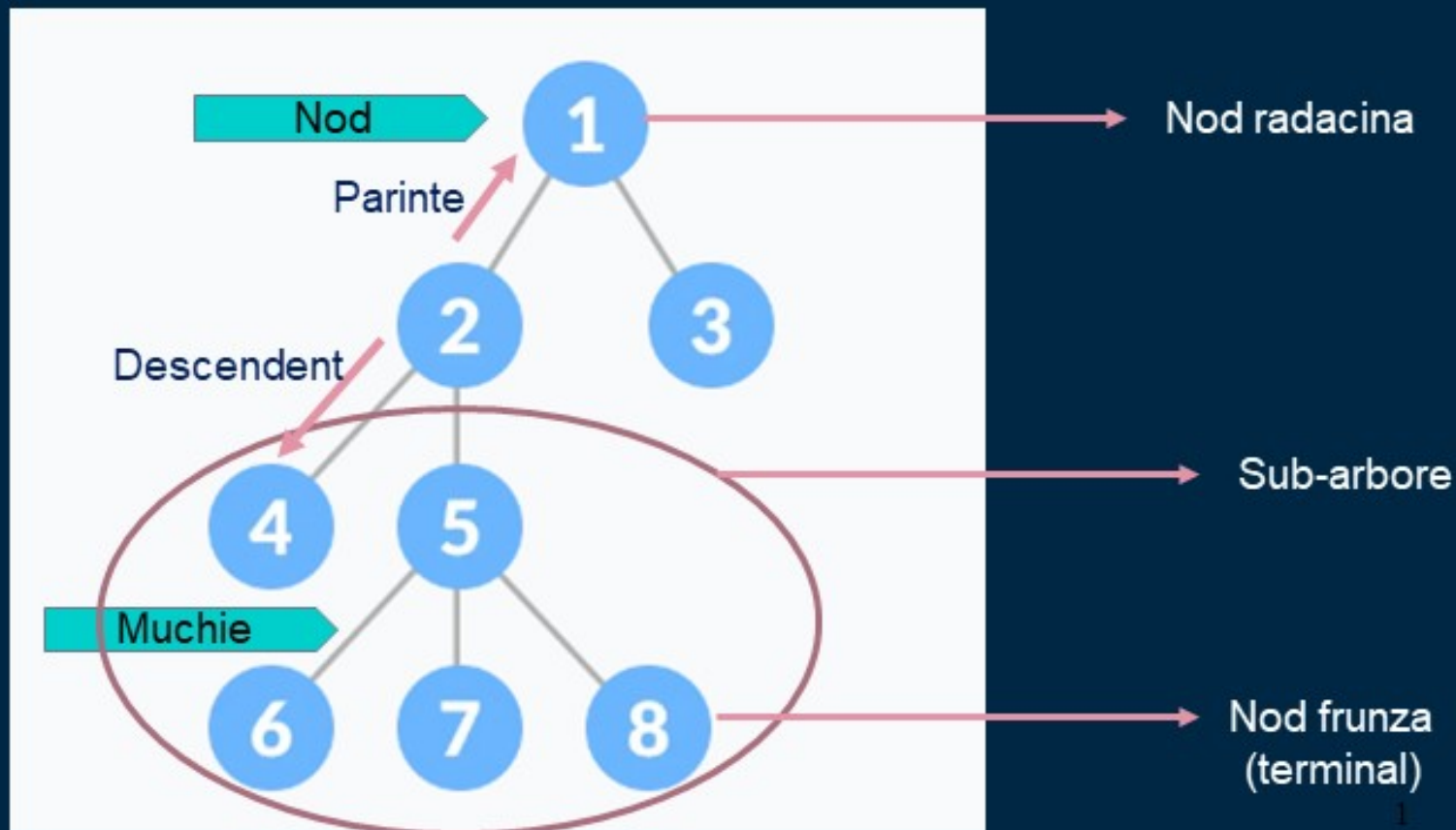


The screenshot displays the Oracle JDeveloper IDE interface with three main panels:

- Structure Panel (Left):** Shows the project structure for `FndDemoEmpVO.xml`. The `Attributes` folder is expanded, listing various attributes such as `CreatedBy`, `CreationDate`, `EffectiveEndDate`, `EffectiveStartDate`, `EmployeeId`, `FirstName` (highlighted), `LastName`, `LastUpdateDate`, `LastUpdateLogin`, `LastUpdatedBy`, `RowID`, and `Salary`.
- Console Panel (Middle):** Displays the output of the `Running: Embedded...` window. The log shows a timestamp `2008-06-03 10:49:10.528` and a message indicating the use of the `ProviderURI /adflib/oracle/apps/fnd/applcore/patterns/uishell/Empty.jsff` for processing requests. The bottom of the console indicates it is an `Embedded OC4J Server`.
- Property Inspector Panel (Right):** Titled `FirstName - Property Inspector`, it shows configuration options for the `FirstName` attribute. The `Applications` section includes settings for `Currency Type`, `Date Time Sensitive`, and `Hierarchy Display`, all set to `<default>`. The `General` and `Other` sections are also visible, with `EntityDiscrColumn` set to `<default>`.

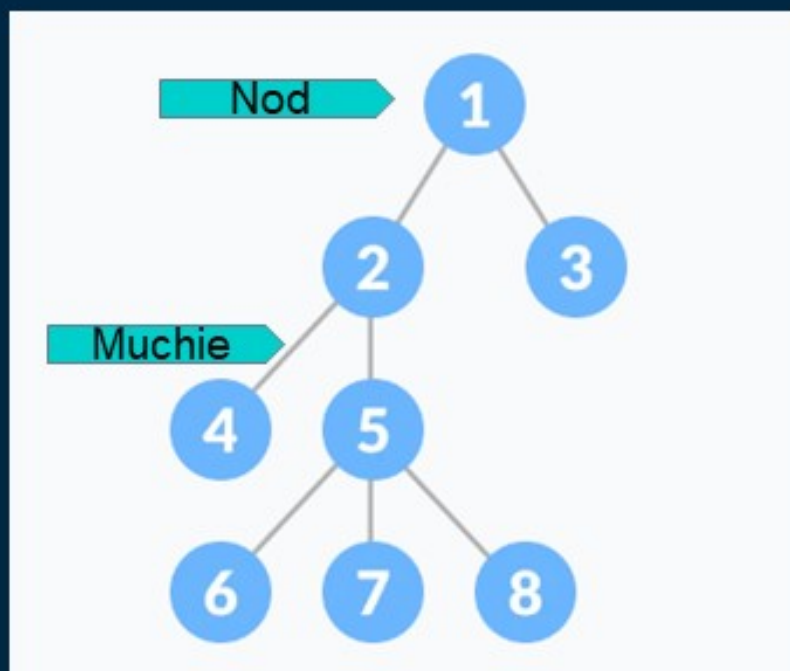
# Arbori

= structură de date ierarhică neliniară care constă din noduri conectate prin muchii.



# Arbori

= structură de date ierarhică neliniară care constă din noduri conectate prin muchii.



# Arbori

= structură de date ierarhică neliniară care constă din noduri conectate prin muchii.

Nod

= entitate care conține o cheie sau o valoare și legături către nodurile sale subordonate.

Ultimele noduri ale fiecărei căi sunt numite **noduri frunză** sau **noduri externe**.

Nodul care are cel puțin un nod copil se numește **nod intern**.

Muchie

Legătura între 2 noduri



# Arbori – notiuni descriptive

## Rădăcină ( root)

Este nodul cel mai de sus al arborelui. Este unic.

## Înălțimea unui nod

Înălțimea unui nod este numărul de muchii de la nod la cea mai adâncă frunză (adică cea mai lungă cale de la nod la un nod de frunză).

## Adâncimea unui nod

Adâncimea unui nod este numărul de muchii de la rădăcină la nod.

## Înălțimea arborelui

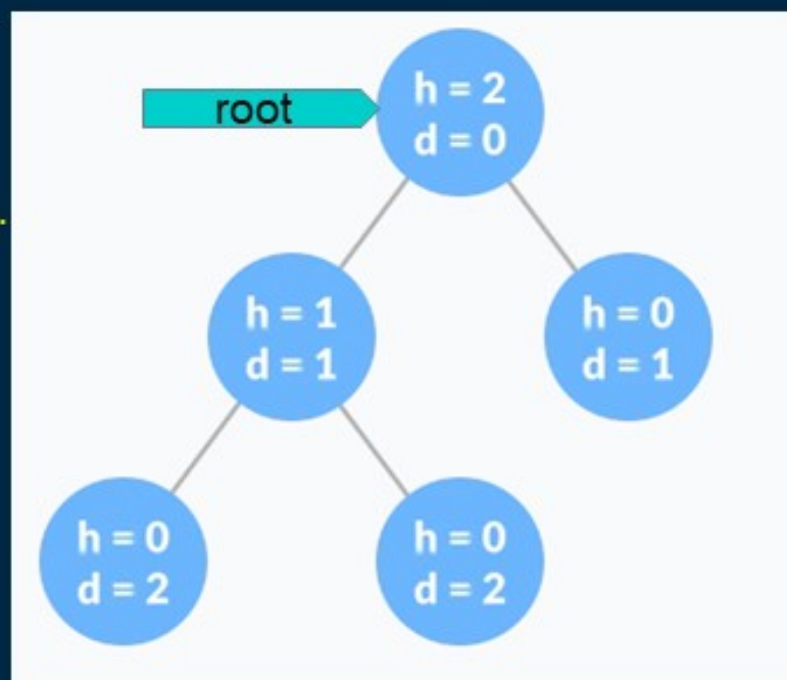
Înălțimea unui arbore este înălțimea nodului rădăcină sau adâncimea celui mai adânc nod.

## Gradul unui nod

Gradul unui nod este numărul total de ramuri ale acelui nod.

## Padure ( Forest)

O colecție de arbori disjuncti se numește pădure.



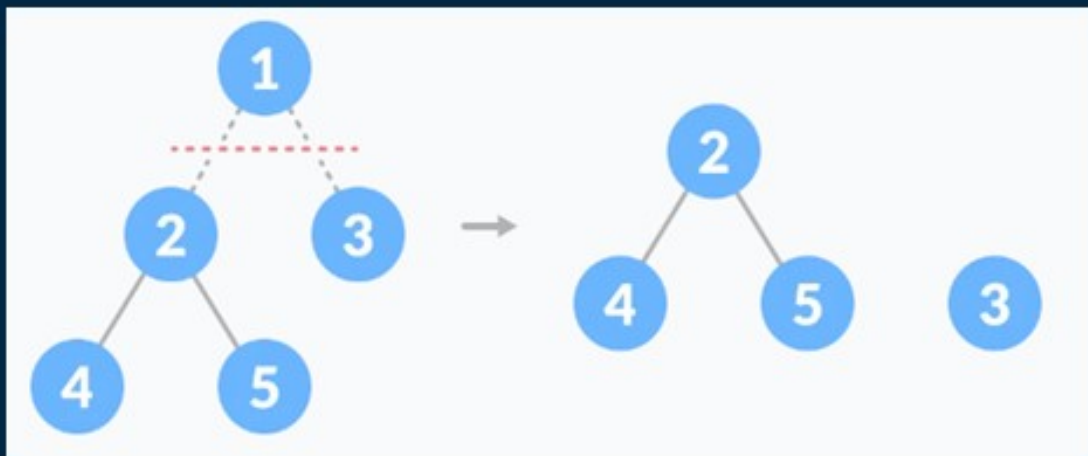
Inaltimea (h)

Adancimea (d)

# Arbori – notiuni descriptive

## Padure ( Forest)

O colecție de arbori disjuncti se numește pădure.



Parcurgeri

02

# Parcurgerea arborilor

Efectuarea oricărei operații pe un arbore, necesita accesarea unui nod specific.

Algoritmul de traversare a arborelui – **solutia**

Tipuri de parcurgeri:

- InOrdine
- PreOrdine
- PostOrdine



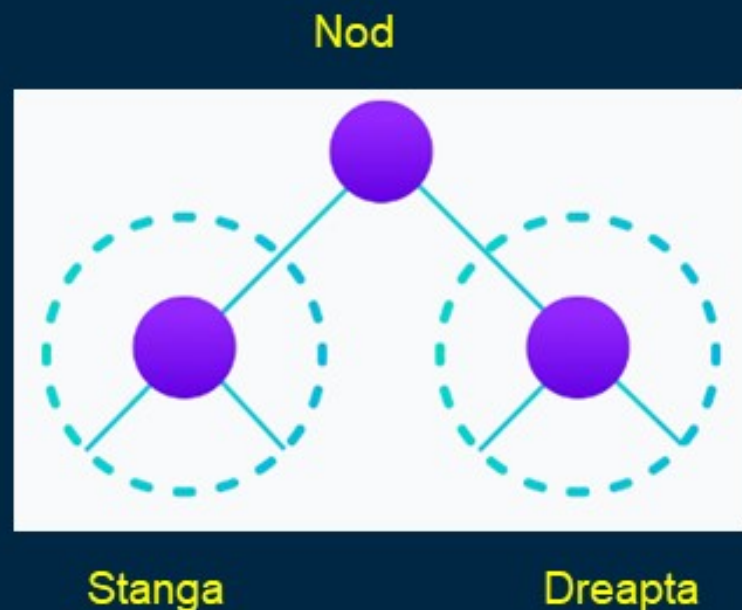
# InOrdine ( SND, LNR)

Pas 1: Se parcurg toate nodurile din subarborele din stânga

Pas 2: Apoi nodul rădăcină

Pas 3: Se parcurg toate nodurile din subarborele din dreapta

```
InOrdine(root->stanga)  
afisare(root->data)  
InOrdine(root->dreapta)
```



# PreOrdine ( NSD, NLR)

Pas 1: Se viziteaza nodul radacina

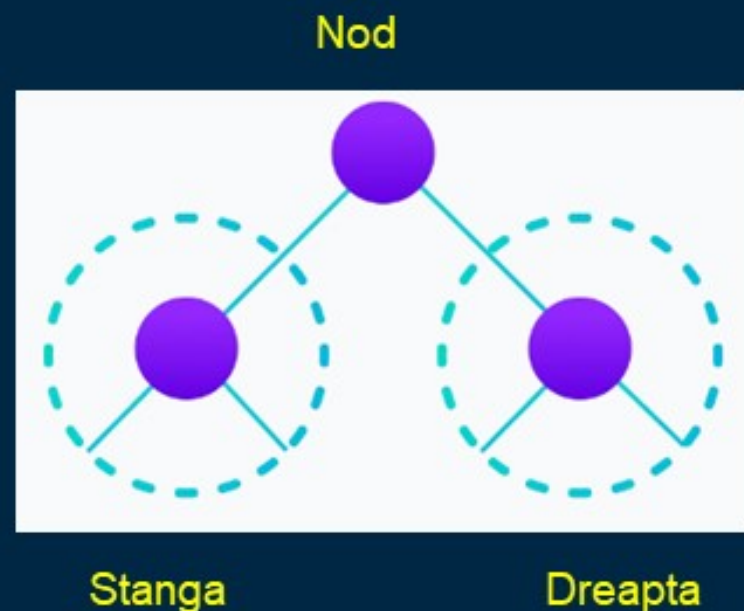
Pas 2: Se parcurg toate nodurile din subarborele din stânga

Pas 3: Se parcurg toate nodurile din subarborele din dreapta

```
afisare(root->data)
```

```
PreOrdine(root->stanga)
```

```
PreOrdine(root->dreapta)
```



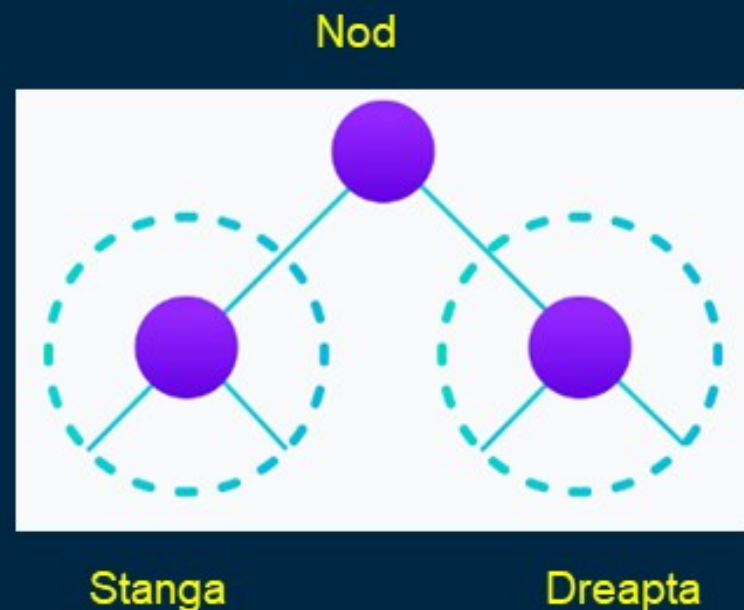
# PostOrdine ( SDN, LRN)

Pas 1: Se parcurg toate nodurile din subarborele din stânga

Pas 2: Se parcurg toate nodurile din subarborele din dreapta

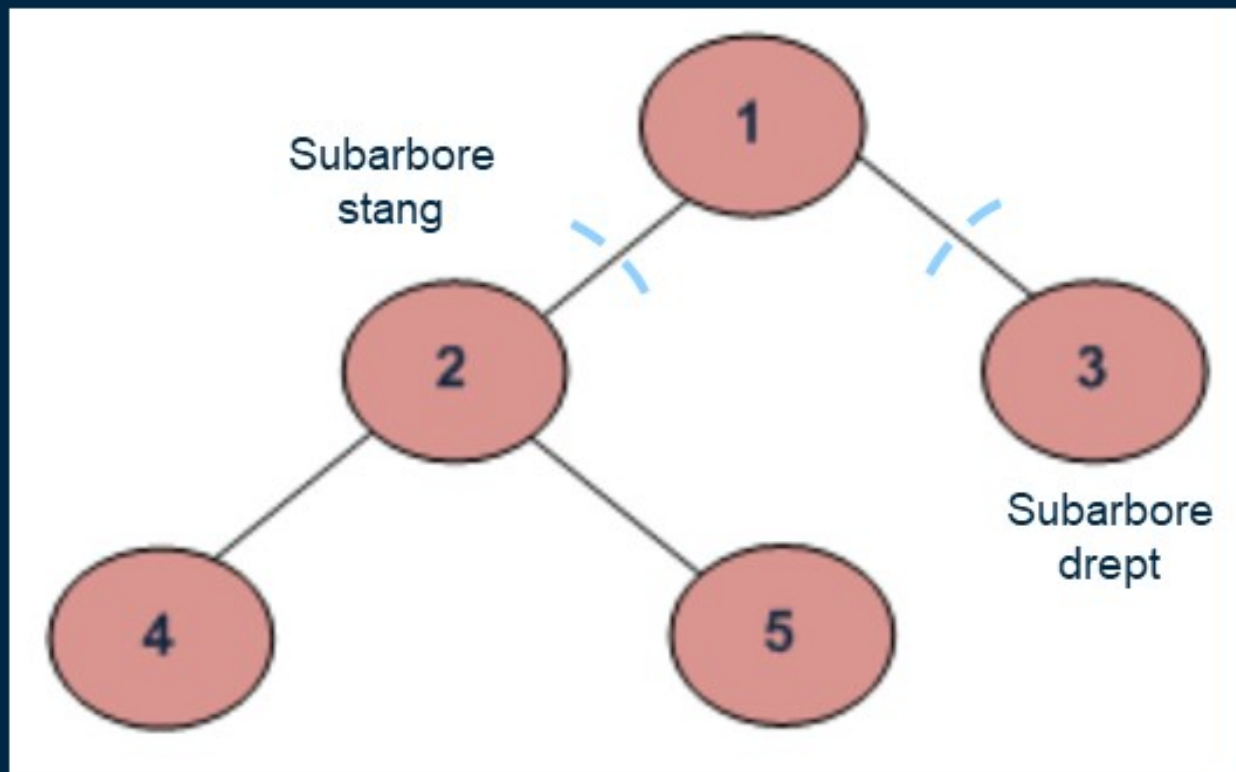
Pas 3: Se viziteaza nodul radacina

```
PostOrdine(root->stanga)  
PostOrdine(root->dreapta)  
afisare(root->data)
```

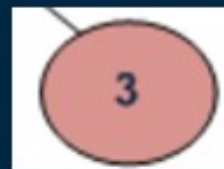
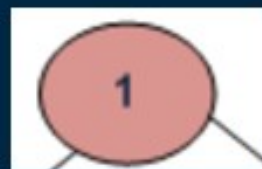
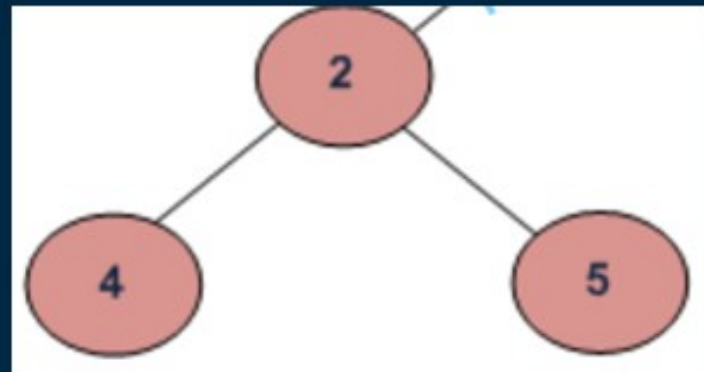


# Parcurgeri - exemplu

InOrdine (SND)



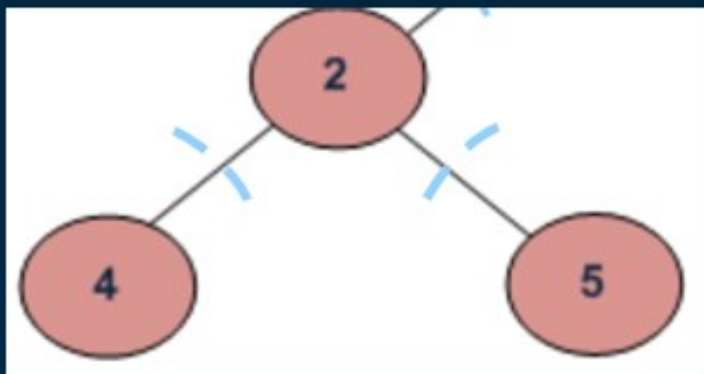
Stiva



# Parcurgeri - exemplu

InOrdine ( SND):

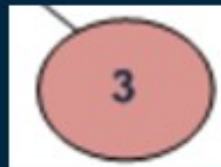
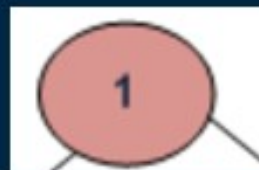
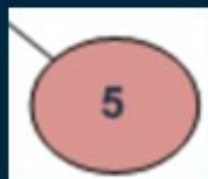
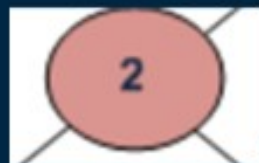
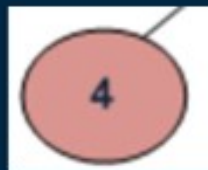
Root -> subarbore Stang -> subarbore Drept



Rezultat parcurgere InOrdine:

**4 -> 2 -> 5 -> 1 -> 3**

Stiva

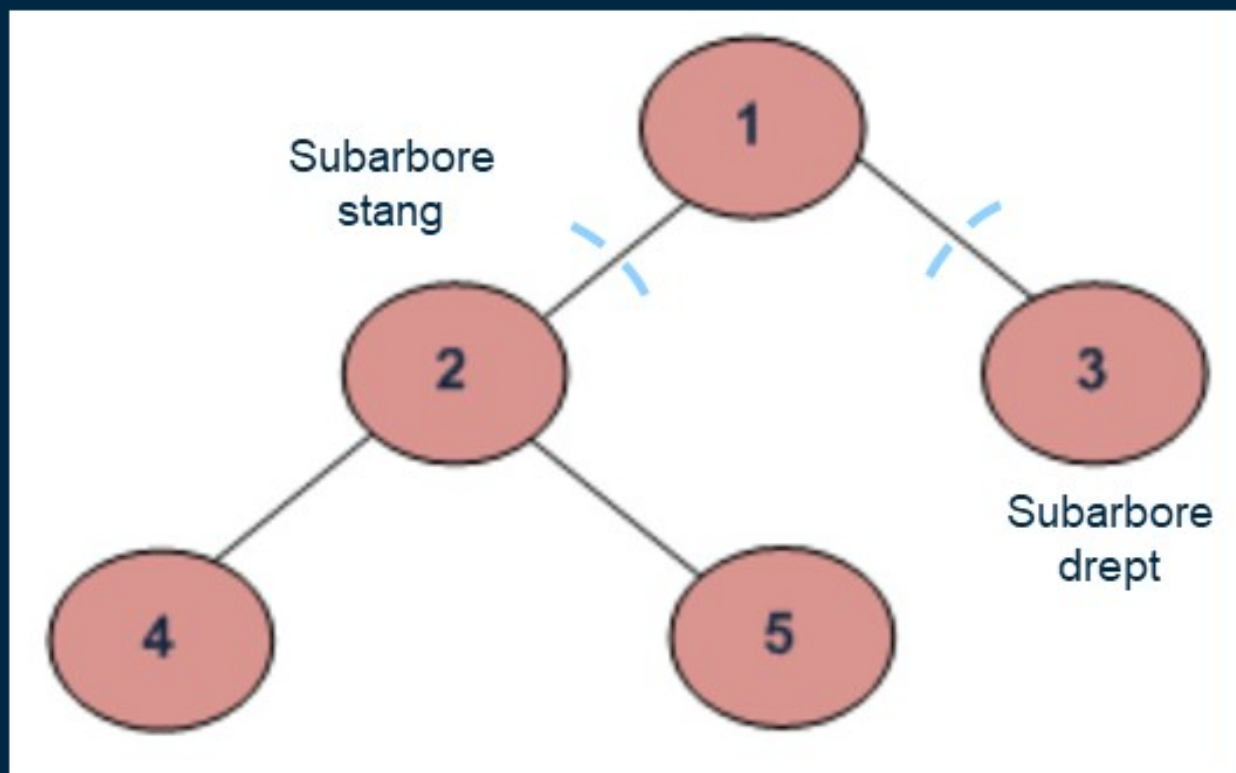


InOrdine ( SND)

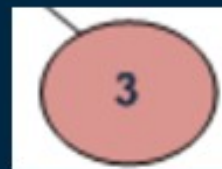
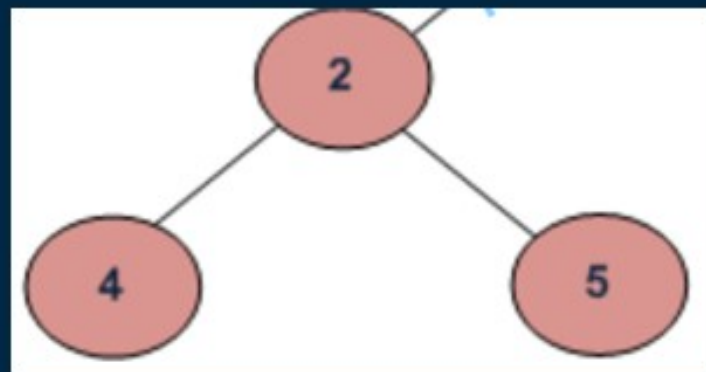
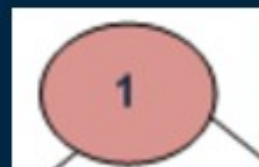


# Parcurgeri - exemplu

PreOrdine ( NSD )



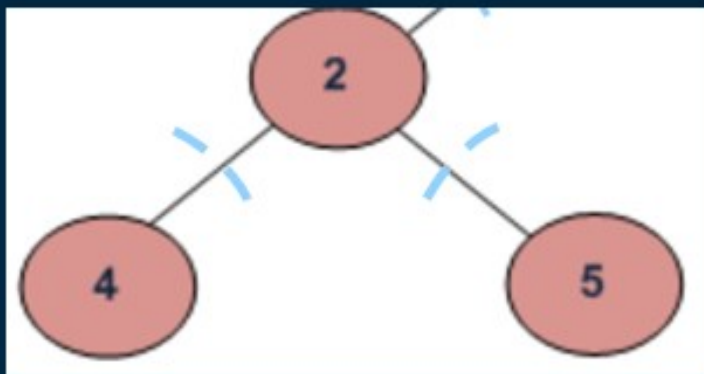
Stiva



# Parcurgeri - exemplu

PreOrdine ( NSD):

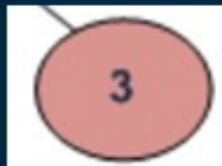
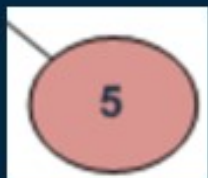
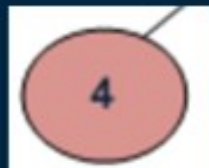
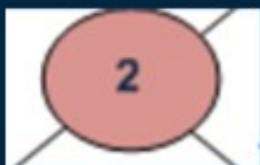
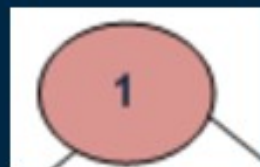
Root -> subarbore Stang -> subarbore Drept



Rezultat parcurgere PreOrdine:

**1 -> 2 -> 4 -> 5 -> 3**

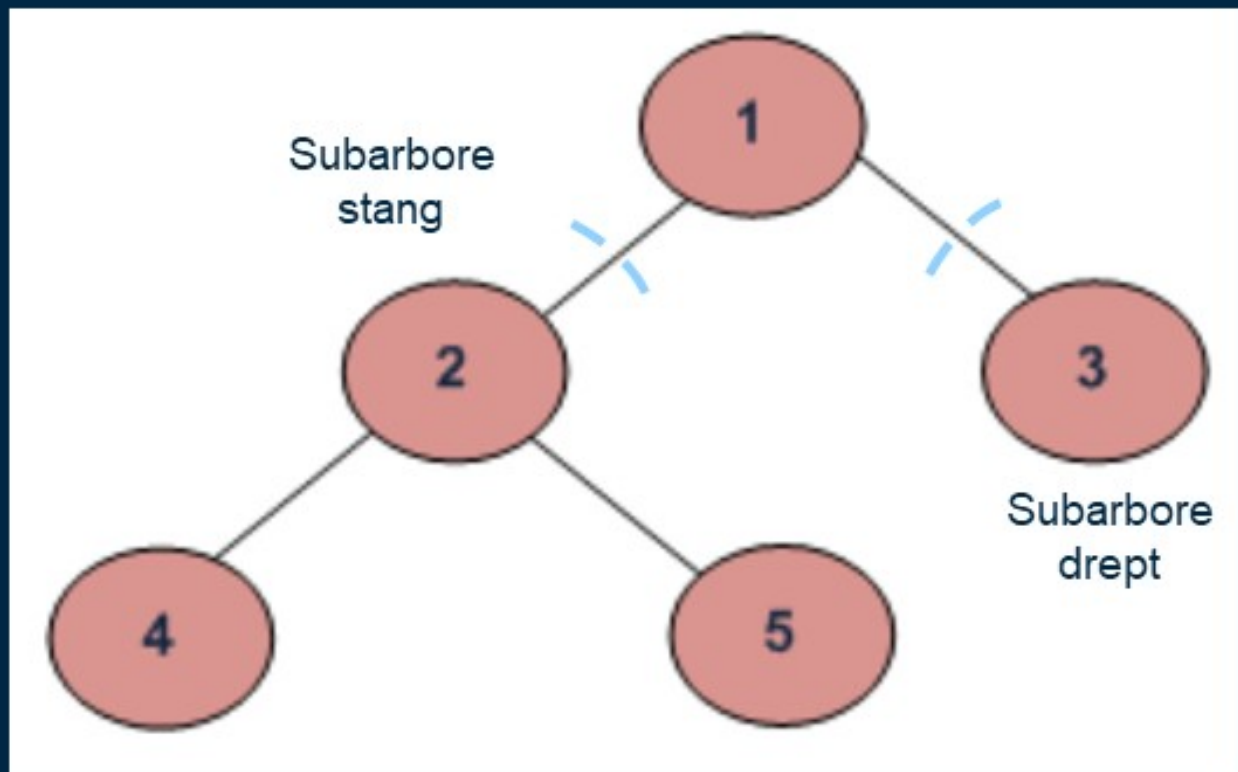
Stiva



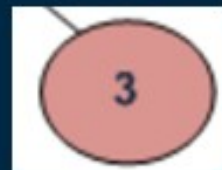
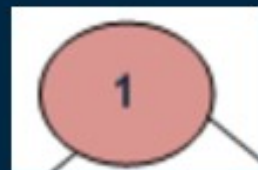
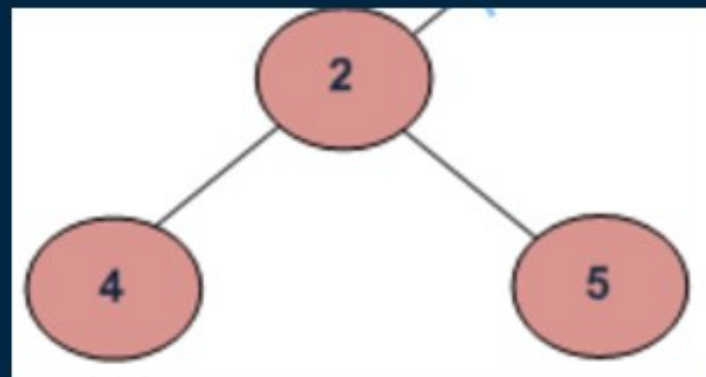
PreOrdine ( NSD)

# Parcurgeri - exemplu

PostOrdine( SDN)



Stiva

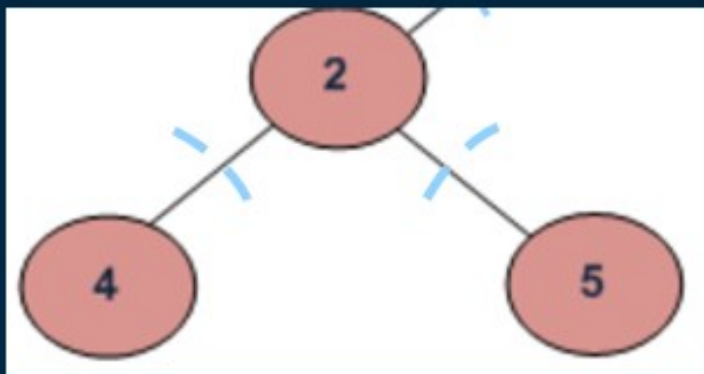




# Parcurgeri - exemplu

PostOrdine ( SDN):

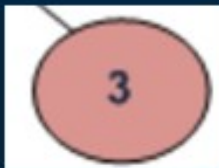
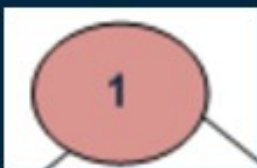
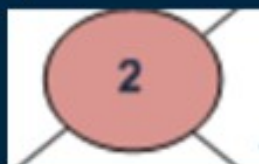
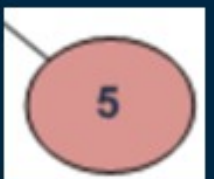
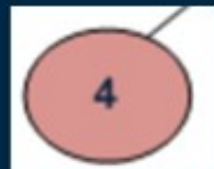
subarbore Stang -> subarbore Drept -> root



Rezultat parcurgere PostOrdine:

**4 -> 5 -> 2 -> 3 -> 1**

Stiva



PostOrdine ( SDN)

# Aplicații ale arborilor

- Arborele de căutare binar (BST) – operații de căutare.
- Heap - folosit pentru sortarea heap.
- A stoca informații de rutare in echipamente de retea.
- SGBD - B-Trees și T-Trees, pentru a stoca datele.
- Compilatoare - arbore de sintaxă pentru a valida sintaxa fiecărui program.

# Arbori Binar

03

# Arbori binari ( AB )

- Structura de date simplă și eficientă structură utilizată în majoritatea sistemelor software.
- AB poate fi implementat ca o matrice folosind idei de Binary Heap.
- Există o gamă largă de variante ale arborelui binar..

# Arbori binari ( AB )

## **Variante:**

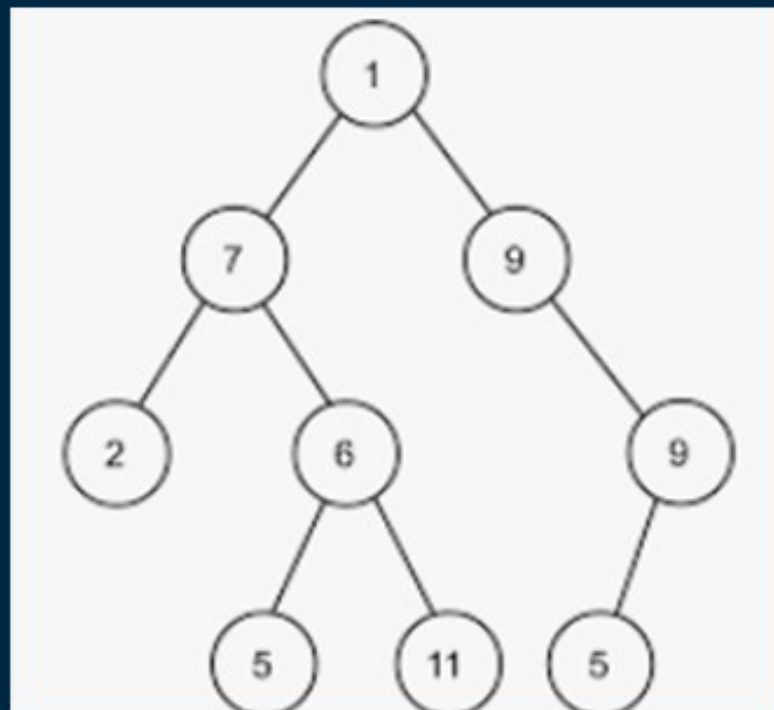
- B-Tree si B+ Tree: index BD
- Space Partitioning Tree: jocuri de mari dimensiuni
- Quadtree
- Tree pyramid (T-pyramid)
- Octree
- k-d (K dimensional) tree
- R-tree: pentru a determina calea ce amai scurtă sau obiectele apropiate în grafuri 3D

# Arbore binar

structură de date arborescentă în care fiecare nod părinte poate avea cel mult doi copii.

Fiecare nod al unui arbore binar este format din trei elemente:

- element de date
- adresa copilului stâng
- adresa copilului drept

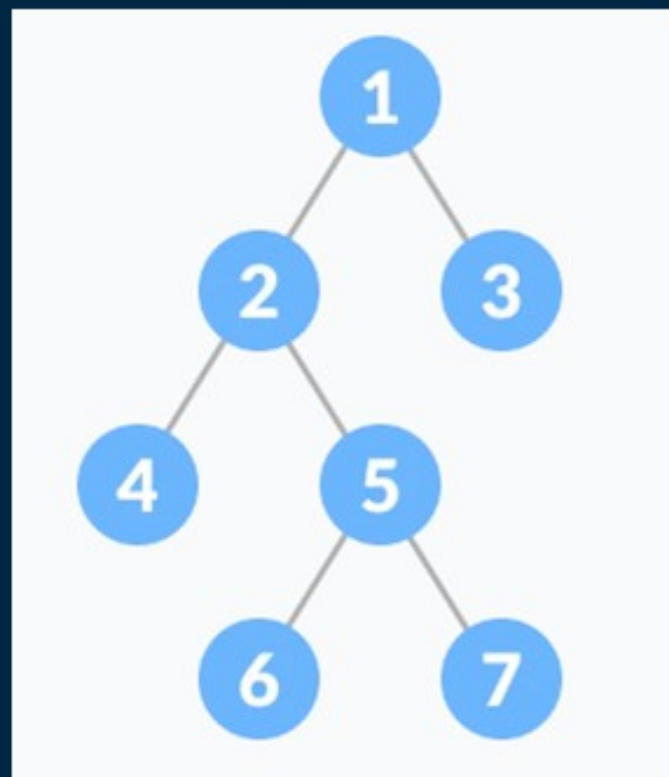




# Arbore binar - tipologii

## 1. Arbore Binar plin

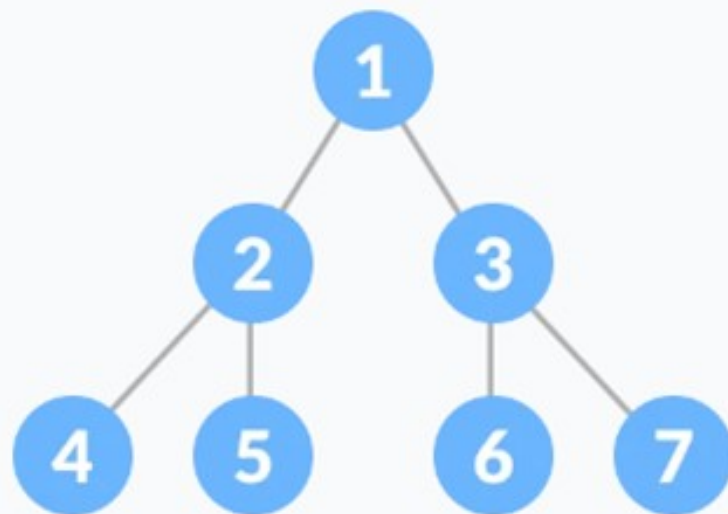
fiecare nod intern are doi sau  
niciun nod subordonat



# Arbore binar - tipologii

## 2. Arbore Binar perfect

fiecare nod intern are exact două noduri subordonate, iar nodurile externe sunt la același nivel

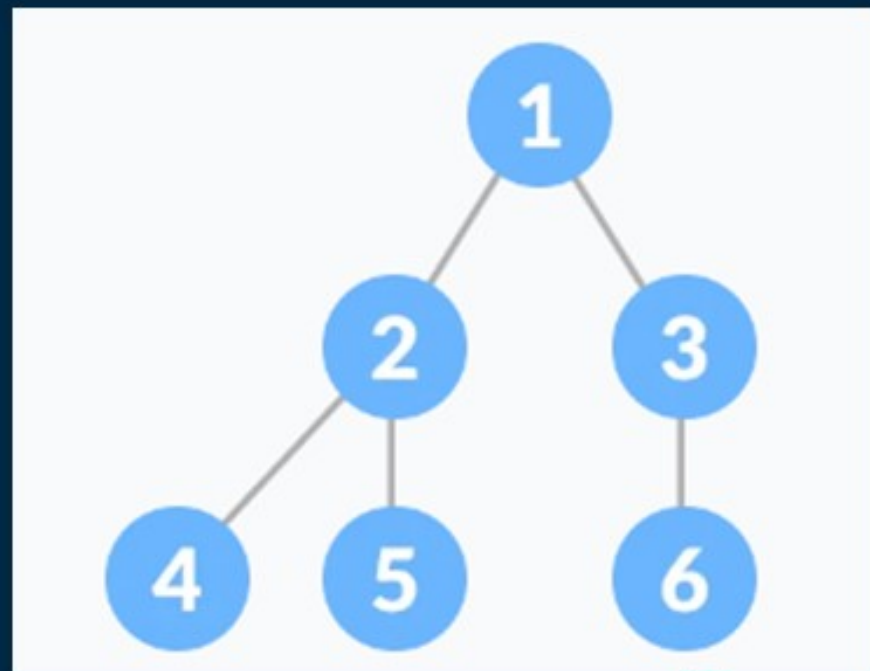




# Arbore binar - tipologii

## 3. Arbore Binar complet

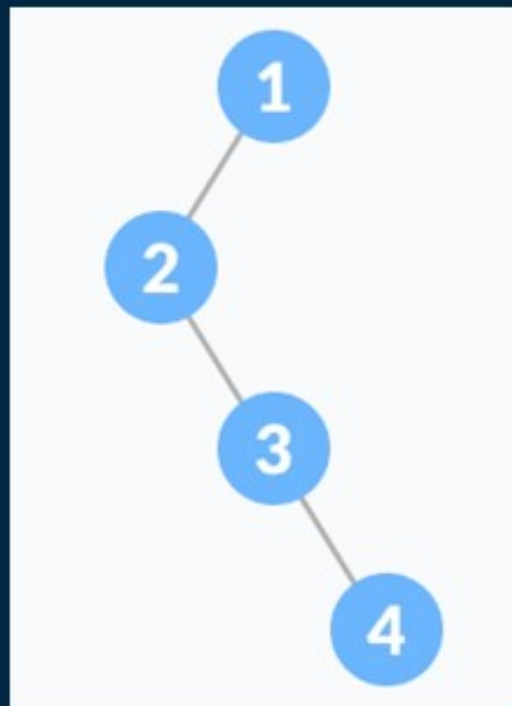
fiecare nod intern are exact două noduri subordonate, cu excepția celui mai din dreapta nod, iar nodurile externe sunt la același nivel



# Arbore binar - tipologii

## 4. Arbore Binar degenerat

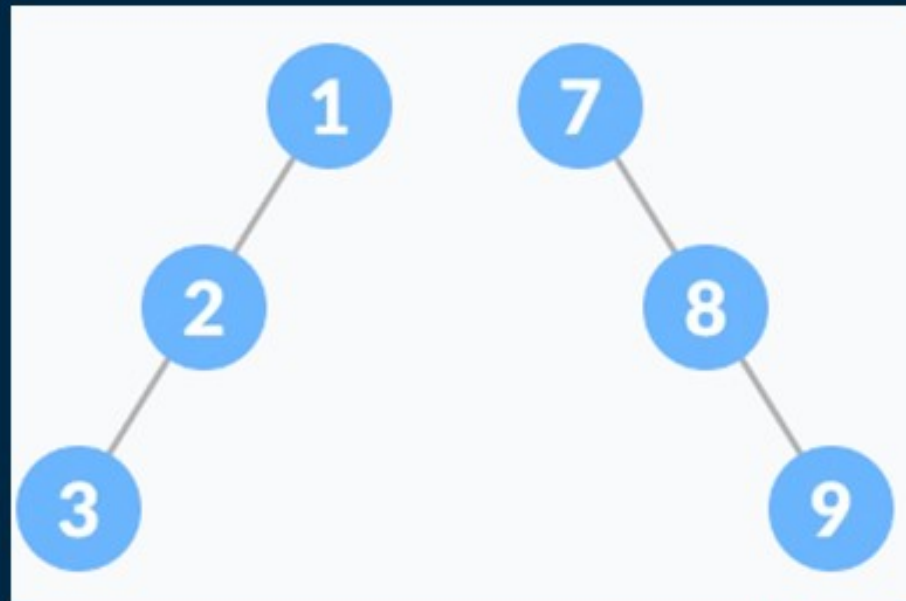
fiecare nod intern are cel mult un nod subordonat



# Arbore binar - tipologii

## 5. Arbore Binar debalansat

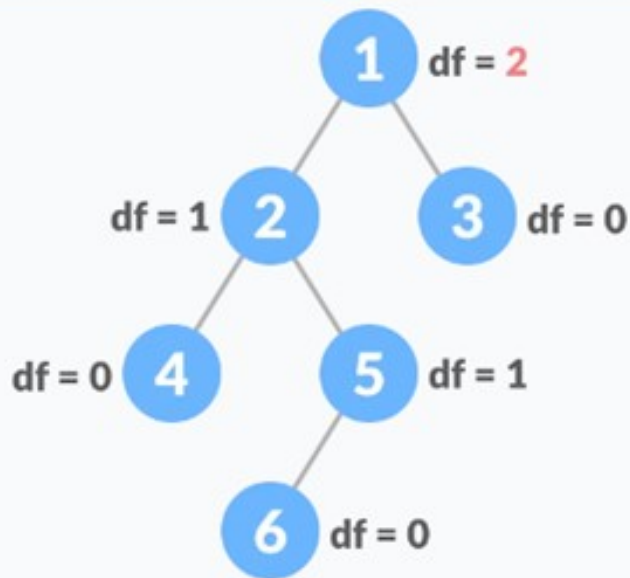
fiecare nod intern are cel mult un nod subordonat, fie doar stânga, fie doar dreapta



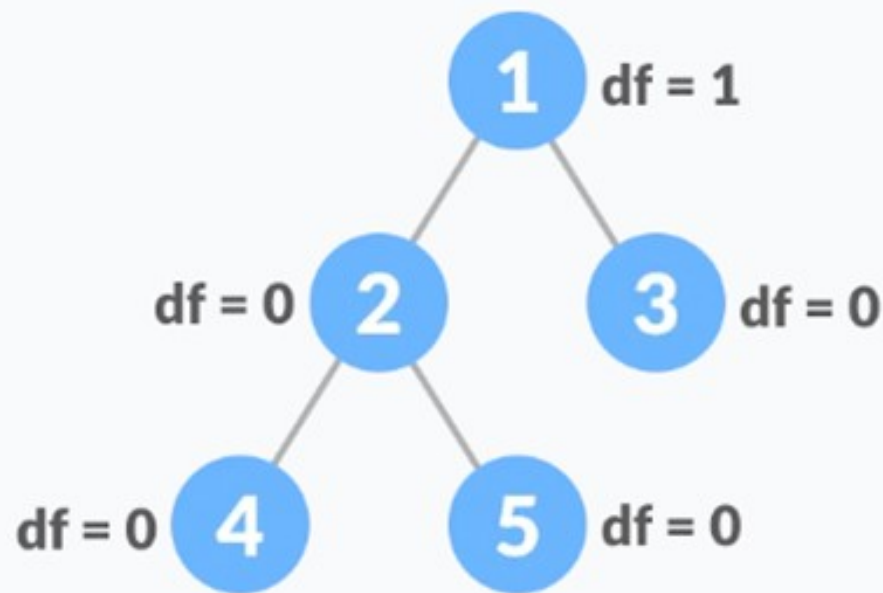
# Arbore binar - tipologii

## 6. Arbore Binar echilibrat

în care diferența dintre înălțimea subarborelui din stânga și din dreapta pentru fiecare nod este fie 0, fie 1

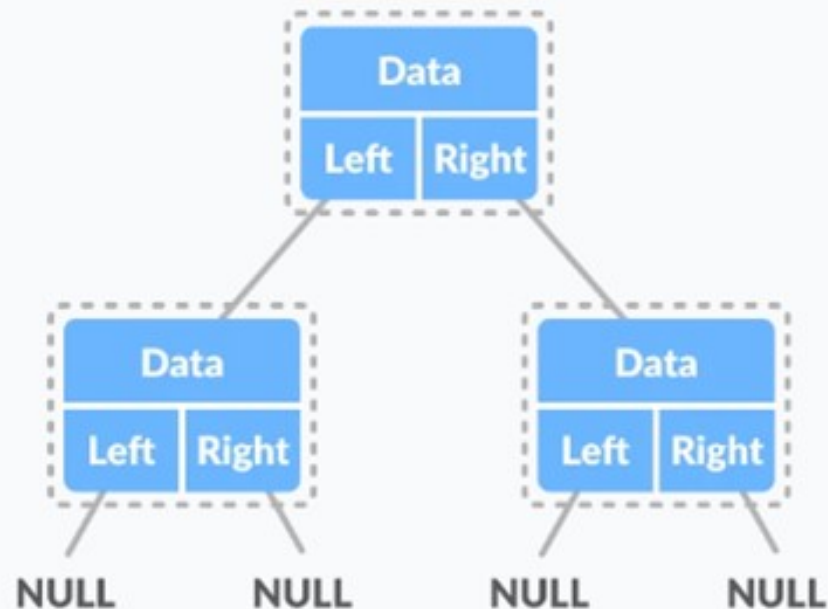


Neechilibrat



# Arbore binar - reprezentare

```
struct nod {  
    int data;  
    struct nod *stanga;  
    struct nod *dreapta;  
};
```



# Probleme

<https://medium.com/techie-delight/binary-tree-interview-questions-and-practice-problems-439df7e5ea1f>

<https://iq.opengenus.org/list-of-binary-tree-problems/>



# Aplicații ale arborilor binari

- Acces ușor și rapid la date.
- Algoritmi de rutare în rețele informatice.
- Structuri de date heap.
- Compilatoare - arbore de sintaxă pentru a valida sintaxa fiecărui program.
- Compresie – Huffman
- Merkle tree, blockchain
  - asigurarea integrității datelor identice în sistemele distribuite
  - verificarea inconsecvenței (modificarea datelor)
  - identificarea corectă a datelor modificate
  - sisteme de monedă virtuală

# Aplicații ale arborilor binari

- Generare functii de numere pseudoaleatoare - Goldreich, Goldwasser and Micali (GGM)
- Paul-Carole game model - Scapegoat tree
- Treap – un arbore binar care pastreaza proprietatea de ABC si Heap



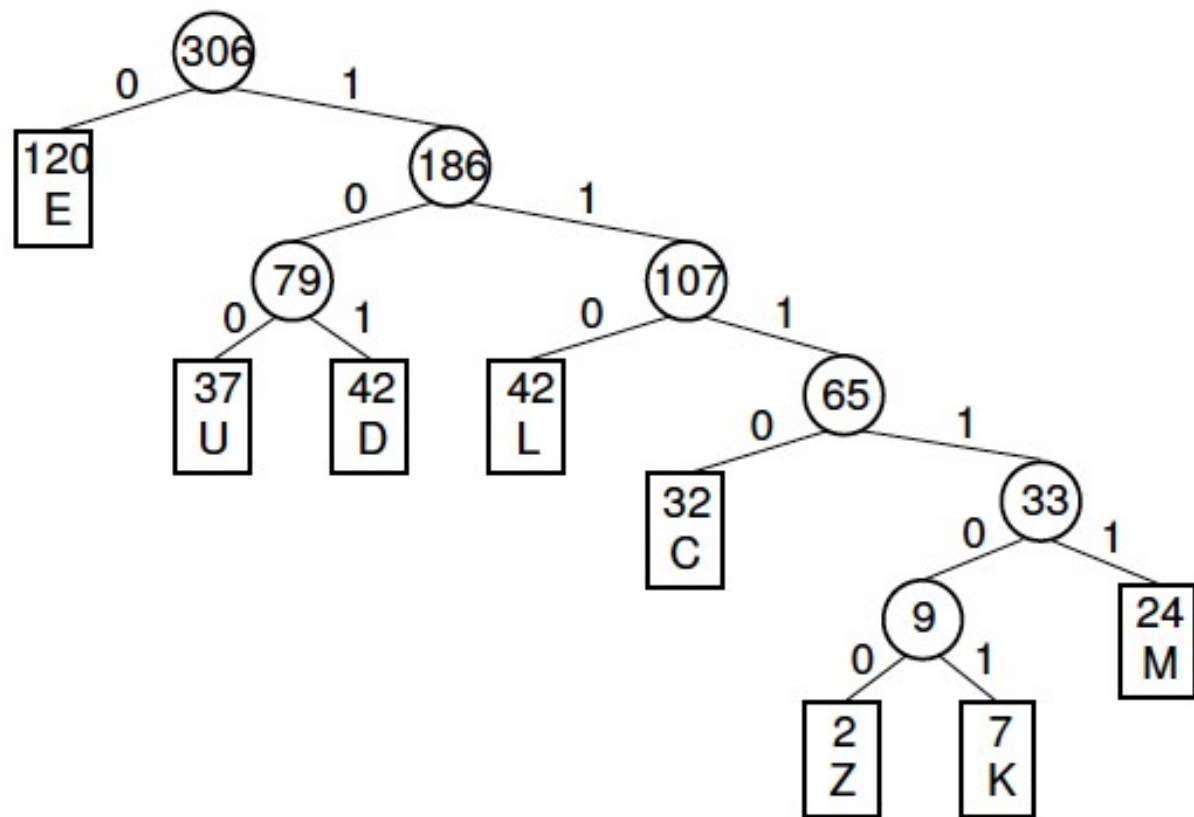
# Exemple

Letter	Z	K	M	C	U	D	L	E
Frequency	2	7	24	32	37	42	42	120

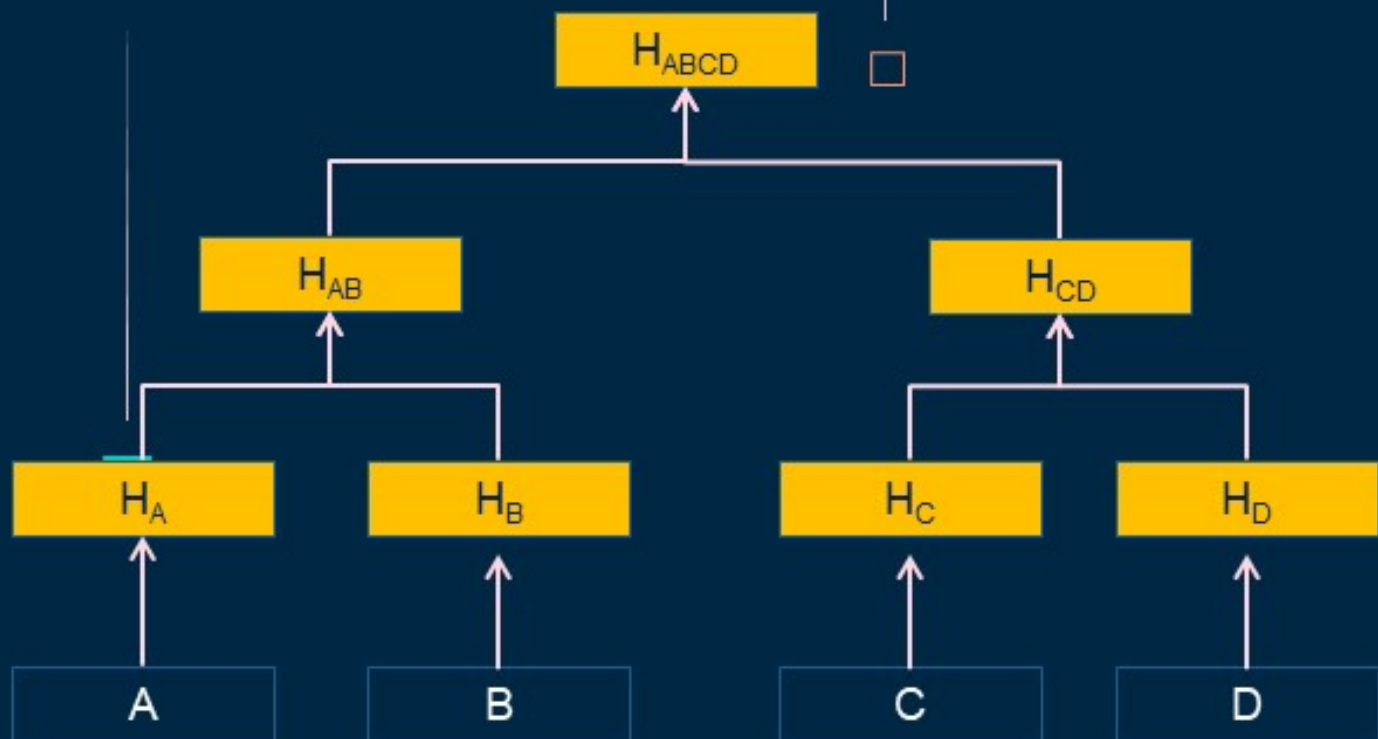
DEED 10100101 (8 bits)

MUCK 111111001110111101 (18 bits)

Huffman



# Example



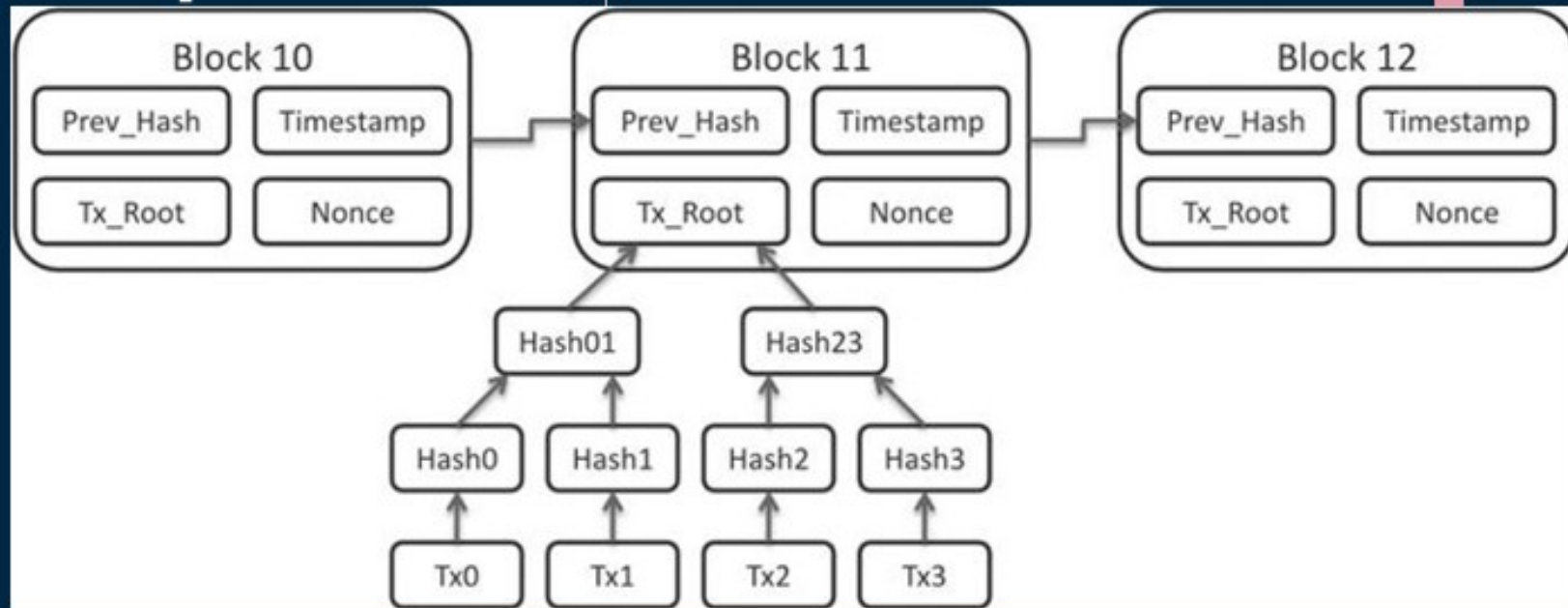
Merkle tree

$$H_{ABCD} = \text{hash}(H_{AB} + H_{CD})$$

$$H_{XY} = \text{hash}(\text{hash}(x) + \text{hash}(y))$$

$$H_X = \text{hash}(X)$$

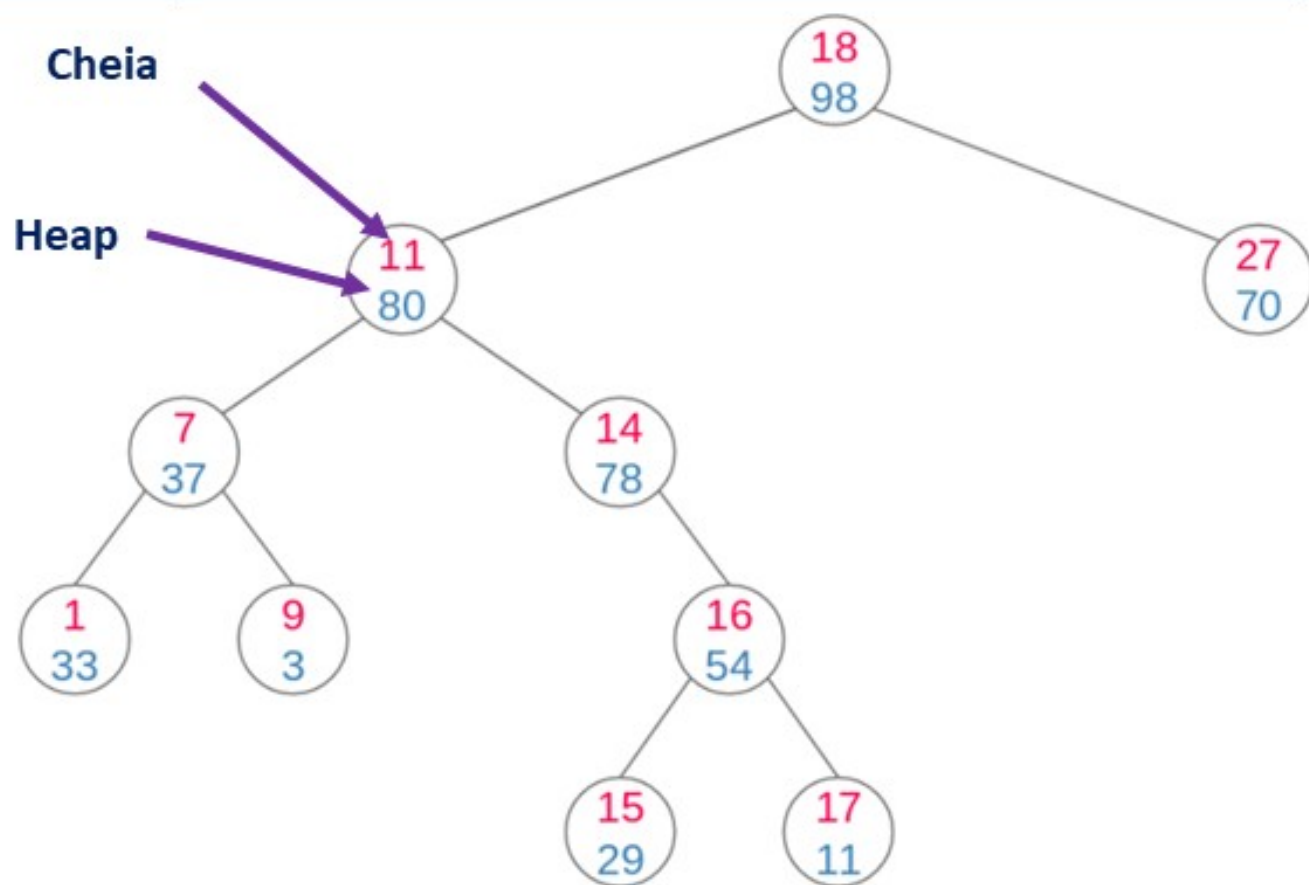
# Exemple



**Blockchain**

- **Integritate date**
- **Non – repudiere**
- **Disponibilitate**
- **Confidentialitate**

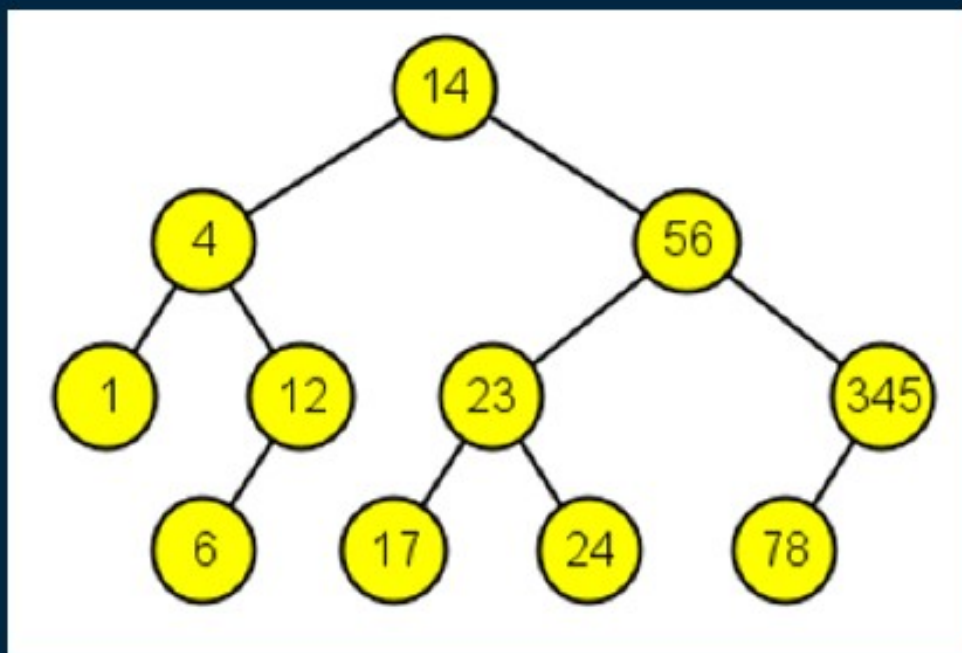
# Exemple



Treap

# Exemple

un arbore Scapegoat este un arbore de căutare binar cu auto-echilibrare

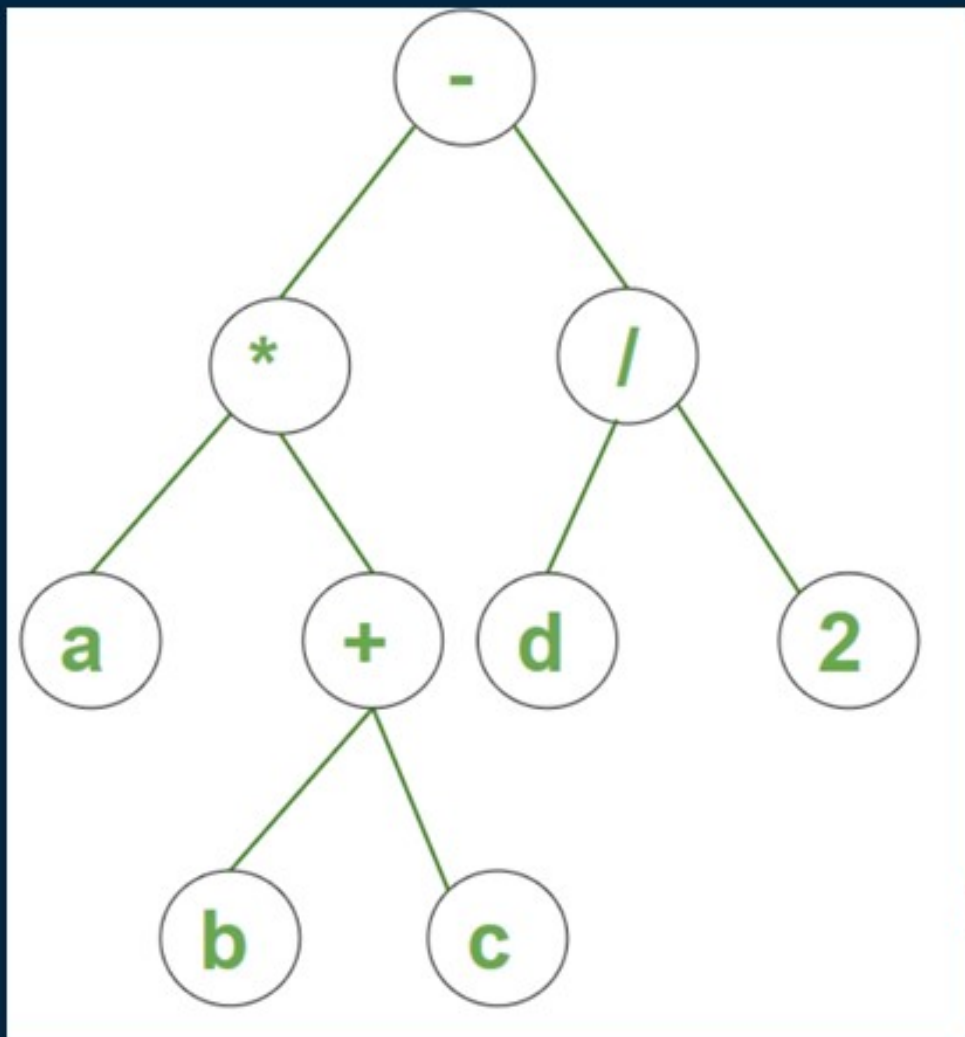


Scapegoat tree  
(Țap ispășitor)

# Exemple

$a * (b + c) - d / 2$

Arbore de sintaxa

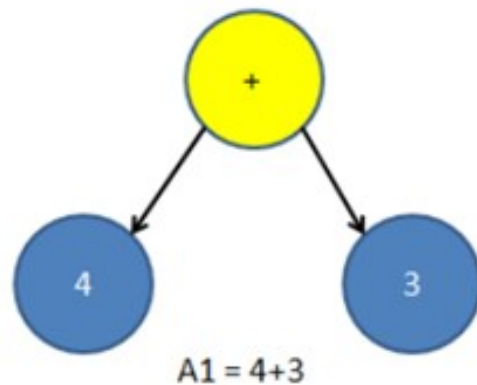
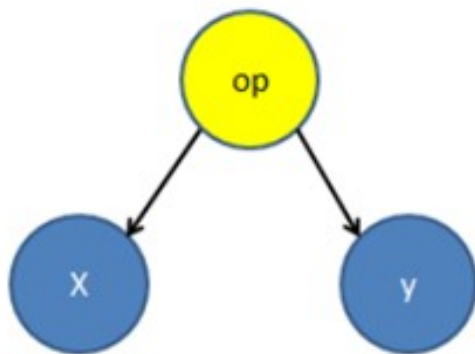




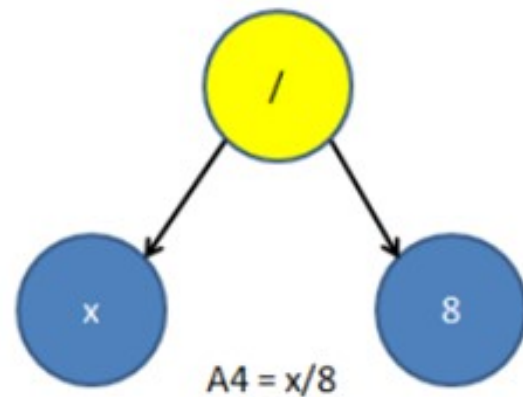
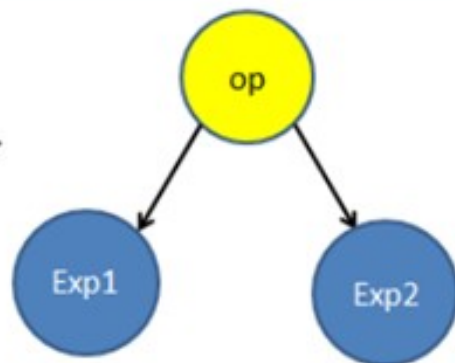
# Example

## Forma poloneza a expresiilor aritmetice

Expresie = x op y →



Expresie = Exp1 op Exp2 →

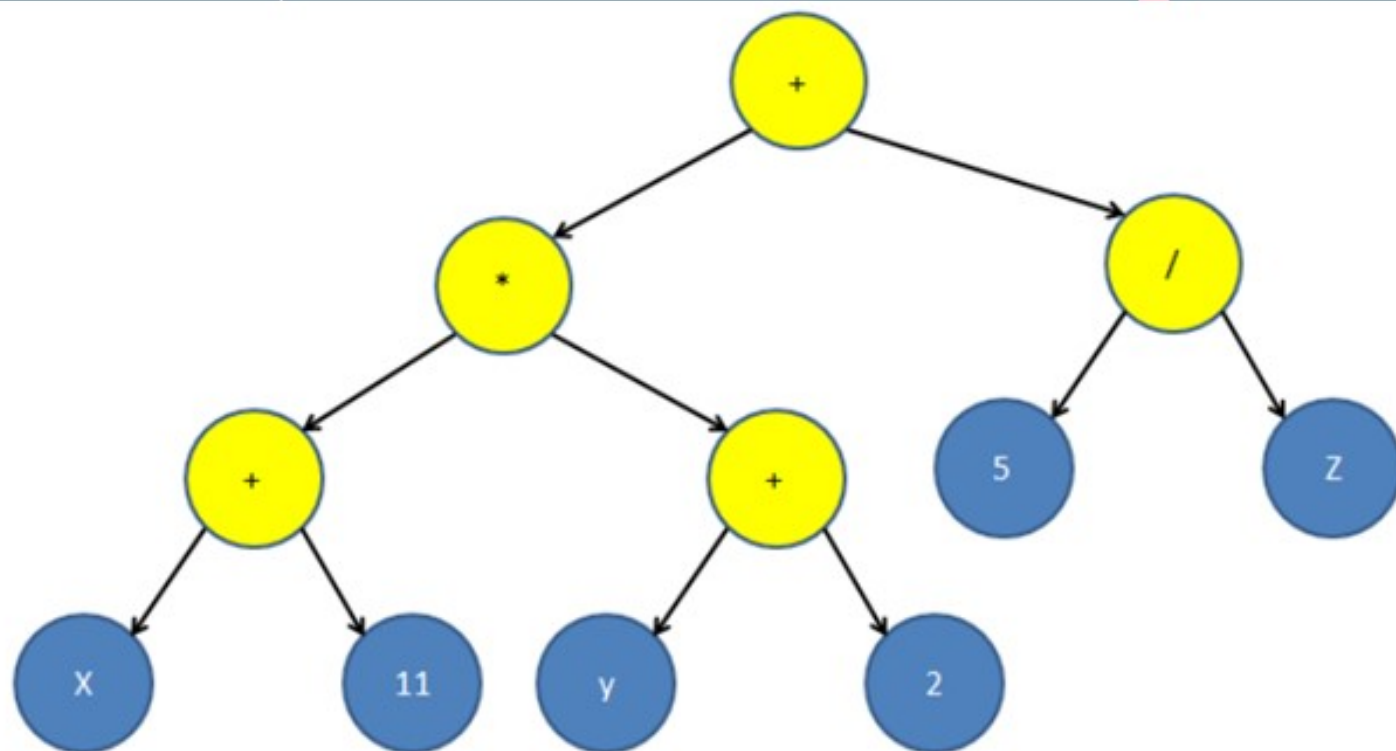


# Example

Forma poloneza a expresiilor aritmetice

$$E = (x+11)*(y+2)+5/z$$

Infix



# Problema

Care este adâncimea nodului E? dar a nodului A?

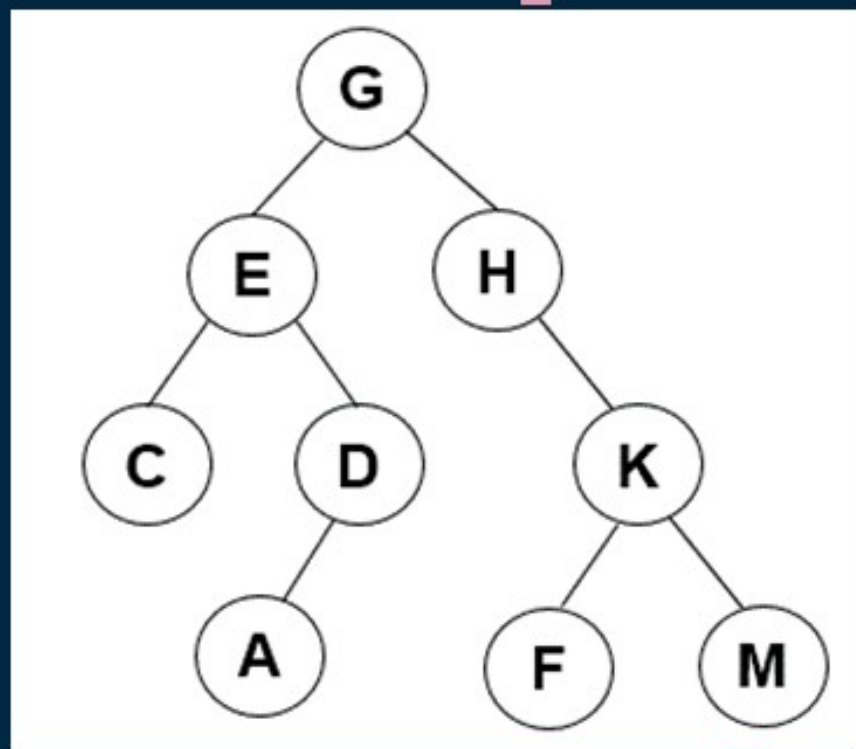
Care este înălțimea arborelui?

Care este ordinea cheilor pentru traversare PreOrdine ?

Care este ordinea cheilor pentru traversare InOrdine ?

Care este ordinea cheilor pentru traversare PostOrdine ?

Care ar fi rezultatul unei traversări în Latime?



Intrebari?

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# Multumesc

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