Nonideal Measurement Locations in Planar Near-Field Antenna Metrology[1]

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1 Introduction

We introduce a near-field to far-field transformation method that relaxes the usual restriction that data points be located on a plane-rectangular grid [1]. It is not always practical or desirable to make uniformly spaced measurements; for example, the maintenance of positioning tolerances becomes more difficult as frequency is increased. Our method can (1) extend the frequency ranges of existing scanners, (2) make practical the use of portable scanners for on-site measurements, and (3) support schemes, such as plane-polar scanning, where data are collected on a nonrectangular grid.

Although "ideal" locations are not required, we assume that probe positions are known. (In practice, laser interferometry is often used for this purpose.) Our approach is based on a linear model of the form $\mathbf{A}\boldsymbol{\xi} = \mathbf{b}$ (see section 2). The conjugate gradient method is used to find the "unknown" $\boldsymbol{\xi}$ in terms of the "data" $\mathbf{b}$ (section 3). The operator $\mathbf{A}$ must be applied once per conjugate gradient iteration, and this is done efficiently using the recently developed unequally spaced fast Fourier transform [2], [3] and local interpolation (section 4). As implemented, each iteration requires $\mathcal{O}(N \log N)$ operations, where N is the number of measurements. The required number of iterations depends on desired computational accuracy and on conditioning. In section 5, we present a simulation that is based on actual near-field antenna data.

2 The Model

Consider a transmitting test antenna (located in the half space $z < 0$) and a receiving probe (translated without rotation). According to Kerns's theory [4], the probe response $w(\mathbf{r})$ may be modeled as

$$w(\mathbf{r}) = \sum_{\alpha} a_{\alpha} \exp(i\mathbf{k}_{\alpha} \cdot \mathbf{r}), \qquad z > 0 \tag{1}$$

[$\exp(-i\omega t)$ time convention], where a_{α} is the (normalized) coupling product and

$$\mathbf{k}_{\alpha} = \frac{\pi\alpha}{L_x}\hat{\mathbf{x}} + \frac{\pi\beta}{L_y}\hat{\mathbf{y}} + \gamma_{\alpha}\hat{\mathbf{z}}, \qquad \gamma_{\alpha} = \sqrt{k^2 - \left(\frac{\pi\alpha}{L_x}\right)^2 - \left(\frac{\pi\beta}{L_y}\right)^2}, \qquad k = \frac{2\pi}{\lambda}.$$

We assume that the probe response is negligible outside the interval $-L_x < x < L_x$, $-L_y < y < L_y$ for z values of interest. [That is, $w(\mathbf{r})$ is a periodic extension.] To improve conditioning (see below), we include only propagating plane waves (γ_{α} real) in the summation in (1). Evanescent waves (γ_{α} imaginary) are exponentially attenuated and are negligible in the far-field region. We must also ensure that evanescent waves are not important contributors to the measured probe response; this is usually accomplished by maintaining a probe-to-test-antenna separation of several wavelengths.

In matrix form (1) becomes

$$\mathbf{w} = \mathbf{Q}\boldsymbol{\xi} \tag{2}$$

where $\mathbf{w} = \{w(\mathbf{r}_n)\}$, $\mathbf{r}_n$ is the location of the nth measurement point, $\boldsymbol{\xi} = \{a_{\alpha}\}$, and $\mathbf{Q} = \{Q_{n,\alpha} = \exp(i\mathbf{k}_{\alpha} \cdot \mathbf{r}_n)\}$. The objective of near-field to far-field transformation is to

determine the coupling product ξ from measurements w made in a restricted region near the test antenna.

In practical situations, where the number of measurements often exceeds the number of unknowns, the system (2) is overdetermined and will generally not have a solution. We will actually solve the normal equations

$$A\xi = b, \tag{3}$$

where

$$A = Q^H Q, \qquad b = Q^H w.$$

The operator $Q^H = Q^H_{\theta_1;n} = \exp(-ik_{\theta_1}\hat{\mathbf{r}}_n \cdot \mathbf{a})$ is the Hermitian (conjugate) transpose of Q. The solution ξ of (3) minimizes $\|w - Q\xi\|$ (where $\|y\|^2 = y^H y$); that is, this ξ is the least-squares estimate. Most methods for processing planar near-field data [based on the model (1)] solve (3), either directly or indirectly. In the standard plane-rectangular grid algorithm, A is diagonal and Q^H and Q can be applied with fast Fourier transforms, giving a computational complexity of $O(N \log N)$. On the other hand, a direct solution using Gaussian elimination requires $O(N^3)$ operations. For typical problem sizes ($10^4 < N < 10^6$), the importance of computational efficiency is readily apparent.

3 Conjugate Gradient Solution

Because A is Hermitian and positive definite (assuming that Q is full rank) the conjugate gradient method is applicable. The algorithm is an iterative scheme which produces successive estimates $\xi^{(j)}$. Initial estimates are not critical and we use $\xi^{(0)} = 0$ for simplicity. The relative error (at the jth iteration) is bounded by the residual $r^{(j)} = b - A\xi^{(j)}$:

$$\frac{\|\xi^{(j)} - \xi\|}{\|\xi\|} \le c^2 \frac{\|r^{(j)}\|}{\|b\|}. \tag{4}$$

Rate of convergence can be estimated with

$$\|\xi^{(j)} - \xi\|_A \le 2\left(\frac{c-1}{c+1}\right)^j \|\xi^{(0)} - \xi\|_A. \tag{5}$$

Here $\|y\|_A^2 = y^H A y$ and the condition number c^2 is the ratio of the largest to smallest eigenvalue of A. (The condition number of Q is c, $c \ge 1$.) Thus, the conjugate gradient algorithm will always converge

When condition numbers are large (poor conditioning), equations (4) and (5) indicate potential problems with computational accuracy and/or convergence rate. Fortunately, it is often possible to improve conditioning by adding physically reasonable restrictions. For example, arbitrarily large condition numbers can arise when evanescent plane waves are included in the model (1). In the example of section 5, the exclusion of evanescent fields results in an acceptable condition.

4 Efficiency

In the conjugate gradient procedure it is necessary to apply the matrix $A = Q^H Q$ to a vector once each iteration. This can be done by a straightforward summation, but only in $O(N^2)$ operations. In order to reduce complexity to $O(N \log N)$ operations per iteration, we have developed a scheme that combines the unequally spaced fast Fourier transform with interpolation in z. For example, to apply Q to $\xi^{(j)}$, we use the unequally spaced fast Fourier transform to evaluate (2) [in $O(N \log N)$ operations] at the points $x_n\hat{\mathbf{x}} + y_n\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ for several fixed values of z. We then use local interpolation in z to reach the actual measurement locations $\mathbf{r}_n$. Since we are dealing with bandlimited functions, the numerical precision of the algorithm can be controlled and is specified as an input parameter. Computational time