The Effect of Minimum Wage on Steady-state Unemployment through Labour Supply:
Based on a Search Model

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Abstract

Unemployment in European countries has been relatively high since 1970, which indicates an increase in steady-state unemployment. One institutional factor that may affect steady-state unemployment is the minimum wage. This paper focuses on the labour supply side and studies how minimum wages can affect unemployment through labour supply. In this paper, I reproduce a search model and add constraints on minimum wages. By analysing the numerical solutions under different minimum wage settings, I find that a relatively low minimum wage can decrease the labour supply and increase unemployment. In contrast, a relatively high minimum wage can lead to a higher labour supply and reduce unemployment. These results suggest that the minimum wage can be one factor that causes the high and persistent increase in unemployment in some European countries, as minimum wages in these European countries are relatively low, leading to a decrease in labour supply.

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1. Introduction

The unemployment rates of some European countries have been high and persistent since the 1970s, which indicates that the steady-state unemployment of these countries may be higher (Rogerson and Shimer, 2010). A potential reason for this European unemployment dilemma is that many countries encountered the oil shock in the 1970s, in which the oil price had risen dramatically (Blanchard and Wolfers, 2000; Ordonez *et al.* ,2019). Some economists claim that this shock led to a persistent increase in unemployment in these countries (Blanchard and Summers, 1987). Such thought is known as hysteresis, according to which unemployment is affected by its history. However, the shock alone cannot explain the heterogeneity in the unemployment experience of countries, as the same shock may have a different impact on countries with different national welfare systems (Ljungqvist and Sargent, 1998; Akdogan, 2016; Xie *et al*., 2018). This suggests that other factors, such as institutional factors, may also play an important role (Saint-Paul, 2004).

One institutional factor that may affect the unemployment rate is the minimum wage. The effect of minimum wage on unemployment is bewildering and worth consideration. On the one hand, many empirical studies show that setting a minimum wage can lead to higher unemployment (Linneman, 1982; Krueger, 1994; Dube *et al*., 2016). For example, the minimum wage in some European countries has been steadily increasing, and with the rising minimum wage, the unemployment in these countries has increased (Funk and Lesch, 2006; Christl *et al*., 2018). On the other hand, some empirical researches find that the rising minimum wage has no effect, or even a positive effect on employment level, meaning that the rise in minimum wages may occur with no loss in employment level (Card and Krueger, 1995). In this sense, the overall effect of minimum wage on unemployment is ambiguous, which can be considered the myth of minimum wages.

A critical indication provided by the myth of minimum wage is that an increase in the minimum wage may have different effects on labour supply and labour demand, leading to an uncertain overall effect on unemployment in different situations. For example, an increase in the minimum wage may raise the labour supply by a small amount but lead to a remarkable decrease in labour demand, which results in higher unemployment overall. This indication points out a direction to figure out the role of minimum wages in high and persistent unemployment in some European countries, which is to study the effects of minimum wage on labour supply and labour demand separately. In doing so, the overall effect of minimum wage on steady-state unemployment can be deduced once the effects on labour supply and demand are known.

This paper focuses on the labour supply side, and there are two goals. The first and most important goal is to study how the minimum wage influences steady-state unemployment through labour supply. To achieve this goal, I reproduce a search model in which the workers play a dominant role in determining employment level, and then add different constraints on the minimum wage. By solving this model with different minimum wages, one can know how workers' optimal choices change with minimum wages. I then simulate three economies with different minimum wages. In all three economies, unemployment is dominated by labour supply, and the labour demand is assumed to be always sufficient. By comparing the performance of these economies, one can understand the effect of minimum wage on steady-state unemployment through labour supply.

The second goal of this paper is to study whether the minimum wage can affect the economy's ability to handle shocks. This goal is achieved by imposing a transient shock into all simulated economies. By comparing how these economies respond to the same transient shock, one can understand the influence of transient shocks on economies' steady-state unemployment and how minimum wages affect the economy's ability to handle shocks.

This paper offers two key findings. The first is that different minimum wages have a different effect on labour supply and unemployment. When the minimum wage is relatively low, the labour supply will decrease, leading to higher unemployment than with no minimum wage. When the minimum wage is relatively high, the labour supply will increase, and the employment level will increase. In this sense, raising minimum wages has a non-monotonic effect on labour supply and unemployment.

The second finding is that minimum wages can affect an economy's response to shocks. This paper shows that when the three simulated economies encounter the same temporal shock of increasing separation rate, their unemployment rates will first soar, and then gradually decline to the original level. With different minimum wages, the time it takes for these economies to recover from the shock varies. When the minimum wage is relatively high, the economy recovers more quickly from the temporal shock, because the labour market is dominated by labour supply and a high minimum wage can encourage workers to work.

These findings provide three significances. Firstly, the findings point out that minimum wages can be one cause of European countries' high and persistent unemployment. Although minimum wages in some European countries, such as France, were steadily increasing, the rise in the minimum wage is relatively small, and it cannot render workers to search for their jobs more hardly. This is to say that the relatively low minimum wage cannot increase the labour supply, and it may even decrease the labour supply. Consequently, raising the minimum wage can only lead to a higher unemployment level. In this sense, the finding points out a potential solution for the role minimum wage played in the European unemployment dilemma.

Secondly, it helps to explain why the increase in minimum wage occurs with no loss in employment level in some situations. A potential reason for these phenomena is that the minimum wage is raised to a relatively high level in these cases, and the labour supply increases, leading to a positive effect on the employment level. This positive effect offsets or even exceeds the other negative effect of increasing the minimum wage on employment, which causes the phenomenon that increasing the minimum wage does not lead to a higher unemployment level.

The third significance is that our findings provide an objection to the thesis of hysteresis. According to the hysteresis thesis, a temporal shock has highly persistent effects on unemployment. This claim contradicts the simulation results in this paper. The simulation results show that although the temporal shock does lead to a spike in unemployment in the short term, it cannot affect the steady-state unemployment since the unemployment in all economies will go back to the previous steady-state level.

The paper will proceed as follows. Section 2 reviews literature works related to the European unemployment problem and the minimum wage. Section 3 describes the search model. Section 4 explains the optimal policies of workers at the stationary equilibrium. Section 5 displays the calibrations. Section 6 explains how workers' optimal choices change with different minimum wages. Section 7 simulates three economies with different minimum wages and their performances when a temporal shock appears. Section 8 concludes the research and its limitations.

2. Literature review

This paper relates to several important themes in the literature. Firstly, it related to a series of models characterised by search frictions and matching technology. Mortensen and Pissarides (1994, 1999) realize that job-matching is time-consuming process between workers and vacancies, and thereby construct models involving frictions of search and matching. One feature of these models is that they are constructed based on a micro-foundation, since the choices of firms and workers are considered. Different from Mortensen and Pissarides's general equilibrium models, McCall (1970, 1991, 2008) focuses more on the labour supply side. In McCall's search models, workers' choices are emphasized, and the labour demand is exogenous. The model constructed by Ljungqvist and Sargent (1998) is an extension of the McCall search model, and it also contains an exogenous separation rate. In Ljungqvist and Sargent's search model, workers are assigned idiosyncratic labour productivity. Therefore, to study the effect of minimum wages from the labour supply side, I choose to reproduce Ljungqvist and Sargent's search model.

Secondly, this paper is related to the debate on the effect of minimum wages. Many economists argue that raising minimum wages can lead to a higher unemployment (Brown *et al*., 1982). Nevertheless, there are also some economists find that the increase in minimum wage does not necessarily lead to higher unemployment, and it can even decrease the unemployment (Christl *et al*., 2018). Some theoretical works also predict that there exist minimum wages that can reduce the unemployment level (Cristopher, 2006). Basu, and Felkey (2009) construct a theoretical model based on efficiency wage theory and find that setting a minimum wage can raise the employment level. The result is consistent with this paper's finding that setting a high minimum wage can increase labour supply and thereby reduce unemployment.

Finally, this paper is related to the debate on the reason for the high and persistent increase in European countries. Blanchard and Summers (1986, 2006) claim that the history of unemployment can affect its current states, and the persistently high unemployment in European countries is caused by the oil crisis in 1970s. However, in many empirical studies, the hysteresis hypothesis is rejected (Bianchi and Zoega, 1994; Srinivasan and Mitra, 2013). In contrast to the hysteresis hypothesis, many empirical investigations show that the move in unemployment and its remaining persistence are correlated with institutional differences. The results in these empirical studies are consistent with predictions of some theoretical models (Ljungqvist and Sargent, 1995, 1998, 2008). This paper also supports the latter argument, since the simulation results show that the transient shocks cannot affect steady state unemployment. Instead, settings of minimum wages can affect steady-state unemployment.

3. The model

This paper reproduces the search model of Ljungqvist and Sargent (1998). To study the effect of minimum wages, I will extend this framework by adding constraints on the minimum wage. The model focuses on the labour supply side and has three features. Firstly, finding a job is a time-consuming process in this model, which is the same as the setting of traditional search theories. Secondly, on-the-job search is not allowed. This means workers in the model cannot search for another job if they still work in their current jobs. Employed workers have to quit their current job to search for new jobs. Thirdly, this model assumes that the job-finding rate of workers is dominantly determined by workers' search efforts. This means firms play a limited role in the job matching process, which is distinct from the traditional search and matching models. With this feature, the labour market is dominated by labour supply, and the demand or vacancies are assumed to be always sufficient. To ensure the mobility of the labour market, the model assumes an exogenous separation rate, which keeps the inflows of unemployment.

With the emphasis on labour supply, the set-up describes the behaviour of a continuum of workers indexed by the unit interval [0, 1]. Workers can be divided into three states according to their employment status and unemployment benefits: being employed, unemployed without unemployment benefits, and unemployed with unemployment benefits. The goal of every worker is to maximizing their expected lifetime utility, which can only come from earnings. All workers' lifespan is infinite, but they are subjected to a fixed probability of death, α, and a constant discount factor, β. Moreover, every worker is endowed with idiosyncratic labour productivity h. The set of all possible labour productivity is denoted by H. Workers' labour productivity may change over periods according to specific transitional probabilities.

3.1. Employed worker

At the beginning of each period t, an employed worker can choose between two options according to her labour productivity and the wage of the current job. The first option is to work on the current job. For employed workers who choose to keep the job, they can obtain earnings at the end of this period. The earning of a specific worker is the product of this worker's wage w and labour productivity h. Workers' wage cannot change if they keep working on the same job. Workers are also required to pay income tax with a tax rate τ. At the end of periods, there is a probability of λ for a worker to be laid off. If a worker is laid off, she will still be paid for the job in the current period t, but she will become an unemployed worker entitled to unemployment benefit in the next period $t+1$. Through a period of working, workers' labour productivity may change. For workers who were not laid off, their labour productivity may change from h to some h_{t+1} in the set H at the beginning of the next period, according to the transitional probability $\mu_e(h, h_{t+1})$. For workers who are laid off, the transitional probability for their labour productivity is $\mu_l(h, h_{t+1})$.

The second option is to give up the current job. If an employed worker chooses to quit, then this worker will become an unemployed worker, and she cannot obtain unemployment compensations or wages for the current period. This is to say that once the worker chooses to give up the current job, she will become an unemployed worker without unemployment benefits.

Based on given information, the Bellman equation for an employed worker is:

$$V(w, h) = \max_{accept, reject}\{V_1(w, h), V_0(h)\} \quad . \tag{1}$$

Given the wage w and labour productivity h at the current period, $V_1(w, h)$ is the expected lifetime utility of keeping the job, while $V_0(h)$ is the expected utility of quitting and being an unemployed worker without unemployment benefits. The maximum between these two options is the worker's choice, $V(w, h)$, which is the value of optimized expected lifetime utility for this worker.

Specifically,

$$\begin{aligned} V_1(w, h) = {} & (1-\tau)wh \\ & + (1-\alpha)\beta\left[(1-\lambda)\sum_{h_{t+1}\in H}\mu_e(h, h_{t+1})V(w, h_{t+1})\right. \\ & \left. + \lambda\sum_{h_{t+1}\in H}\mu_l(h, h_{t+1})V_b(I, h_{t+1})\right] \end{aligned} \tag{2}$$

where V_b denotes the optimized expected lifetime utility of unemployed workers entitled to unemployment benefits.

In this equation, the first term is the net earnings for working on the job during the current period. The second term is the discounted expected utility of all remaining periods. A worker has a $(1-\lambda)$ chance of not being laid off. With the probability $\mu_e(h, h_{t+1})$, the worker's labour productivity will become h_{t+1} at the beginning of the next period, and thereby at the next period, the optimized lifetime utility for this employed worker will be $V(w, h_{t+1})$. Consequently, for a given h_{t+1}, the product of $\mu_e(h, h_{t+1})$ and $V(w, h_{t+1})$ means there is a $\mu_e(h, h_{t+1})$ chance that the optimized expected utility of all remaining periods is $V(w, h_{t+1})$. Summing up such products for all possible h_{t+1}, we can obtain the expected utility of not being laid off. The expected utility of being laid off follows similar reasoning.

3.2. Unemployed worker with unemployment benefit

During each period t, an unemployed worker entitled to unemployment benefits can receive unemployment benefits for this period according to her last one-period earning. The last one-period earning I is the product of the worker's wage w_k and labour productivity h_k at the last period of working k. The unemployment benefits $b(I)$ is a function of the last earnings I. Unemployed workers who receive unemployment benefits are also required to pay income tax, with the tax rate τ. The experience of being unemployed can change workers' labour productivity, and the transitional probability is $\mu_u(h, h_{t+1})$.

At each period, an unemployed worker can choose her search intensity s for finding a job. Searching is a costly process, and the cost is a function of search intensity, $c(s)$. The probability of finding a job is also a function of search intensity: $\pi(s)$. With a higher search intensity, the worker is more likely to find a job. If the unemployed worker does not find a job during the current period, she will still be qualified with the same amount of unemployment benefits in the next period. If the unemployed worker successfully finds a job, she will randomly receive one wage offer from the cumulative distribution $F(w) = Prob(w_{t+1} \leq w)$. The worker needs to decide whether to worker or not at the beginning of the next period $t + 1$. If she chooses to work, she will start to work in the next period, and she cannot work immediately in the current period.

At the beginning of the period $t + 1$, for an unemployed worker who has unemployment benefits and receives a wage offer w, her earnings for this period are wh_{t+1} if she chooses to accept the job. For every unemployed worker who has unemployment benefits and finds a job, the government sets a "suitable earnings" threshold. If an unemployed worker receives a wage offer that can make her earnings higher than the threshold, then the wage offer is considered as being able to achieve suitable earnings. In this case, if the worker chooses to reject the offer, then she will still be an unemployed worker; moreover, she will lose her unemployment benefits and get nothing. In contrast, if the wage offer cannot make the worker's earning higher than the threshold, the worker will not lose unemployment benefit even if she rejects the offer. The threshold for suitable earnings is denoted by $I_g(I)$, which is a function of the worker's last earnings.

Based on the information, an unemployed worker with unemployment benefits need to choose an optimal search intensity s that maximizes her lifetime utility according to her current labour productivity and last earnings. The corresponding Bellman equation is:

$$V_b(I,h) = \max_s \left\{ -c(s) + (1-\tau)b(I) + (1-\alpha)\beta \sum_{h_{t+1}\in H} \mu_u(h,h_{t+1}) \times \left[[1-\pi(s)]\, V_b(I,h_{t+1}) + \pi(s)\left(\int_{w\geq\frac{I_g(I)}{h_{t+1}}} V(w,\ h_{t+1})dF(w) + \int_{w<\frac{I_g(I)}{h_{t+1}}} V_2(w,h_{t+1},\ I)dF(w) \right) \right] \right\} \tag{3}$$

where V_b is the optimized lifetime utility for the unemployed worker with unemployment benefits. In this equation, the first term is the cost for search, and the second term is the net income.

The third term is the discounted expected utility of remaining life. At the period $t+1$, for each possible h_{t+1}, there is a probability of $1-\pi(s)$ that the worker cannot find a job, and her optimized utility for remaining life is $V_b(I,h_{t+1})$. With the probability $\pi(s)$, the worker may randomly receive a wage offer. The probability of obtaining a wage w is just the differential of the cumulative distribution at w, which is $dF(w)$. Summing up the expected values for all possible labour productivity h_{t+1}, we obtain the undiscounted expected value for the remaining lifespan.

There are two possible situations for the wage offer. Firstly, the wage can achieve suitable earnings, which means $wh_{t+1} \geq I_g(I)$. In this case, the worker faces the same choices as an employed worker with the same wage and labour productivity, and she needs to make a decision between working or being unemployed without unemployment benefits. Therefore, the expected value of the remaining lifetime is the sum of the product of $dF(w)$ and $V(w,h_{t+1})$ for every given wage that satisfies $w \geq \frac{I_g(I)}{h_{t+1}}$. The second situation is that the wage does not achieve suitable earnings, which means $wh_{t+1} < I_g(I)$. In this case, $V_2(w, h_{t+1}, I)$ is the optimized lifetime utility given the wage w, labour productivity h_{t+1}, and the last earnings I. Therefore, the last integration is the optimized expected value for the second situation.

Specifically,

$$V_2(w, h_{t+1}, I) = \max_{accept,reject}\{V_1(w, h_{t+1}), V_b(I, h_{t+1})\}. \tag{4}$$

This means when encountering the second situation, the worker needs to choose the most prospective option between working and staying unemployed according to the given wage, labour productivity, and last earnings.

3.3. Unemployed worker without unemployment benefits

The set-up for unemployed workers without unemployment benefits is almost the same as those entitled to unemployment benefits, except that these workers have no unemployment benefits and thereby no government-determined threshold for suitable earnings. For these workers, they also need to choose an optimal search intensity to maximize lifetime utility V_0. The Bellman equation for unemployed workers without unemployment compensation is:

$$\begin{aligned} V_0(h) = \max_s \Bigg\{ & -c(s) + (1-\alpha)\beta \sum_{h_{t+1} \in H} \mu_u(h, h_{t+1}) \\ & \times \left[[1 - \pi(s)]\, V_0(h_{t+1}) + \pi(s) \int V(w, h_{t+1}) dF(w) \right] \Bigg\}. \end{aligned} \tag{5}$$

Compared with equation (3), there are three changes in equation (5). Firstly, the term representing net incomes disappears. This is because unemployed workers without unemployment benefits have no income. Secondly, if the worker does not find a job during this period, her remaining lifetime utility will become $V_0(h_{t+1})$ instead of $V_b(I, h_{t+1})$. This is because only workers who are laid off can have unemployment benefits. Therefore, even though the worker cannot find a job during the current period, she will still have no unemployment benefits in the next period. Thirdly, once the worker finds a job, there is no threshold of suitable earnings for the worker, and the worker's choice is simply the choice of an employed worker with the same wage and labour productivity.

4. Stationary equilibrium

At the equilibrium, given the labour productivity, wage offer, and unemployment benefits, the optimal expected lifetime utility is stationary for any worker. Therefore, workers' optimal policies are also stationary. I will now present optimal policies for workers at the equilibrium.

4.1. Employed worker

There is one policy for employed workers. Employed workers should choose between accepting and rejecting the current wage offer. The former gives them V_1, while the latter gives them V_0. An important equation associated with the workers' reservation wages is:

$$V_1(w, h) = V_0(h)\,. \tag{6}$$

By solving this equation, we can obtain a function $\bar{w}_0(h)$ that defines reservation wages of employed workers with given labour productivity h. If a worker with labour productivity h receives a wage offer that is higher than her reservation wage, the worker will accept the offer, since in this case V_1 is higher than V_0, and accept gives her higher lifetime utility. Otherwise, she will choose to reject it. Therefore, the policy for employed workers is the following function:

$$\sigma_e(w, h) = \mathbf{1}\{w \geq \bar{w}_0(h)\}\,. \tag{7}$$

where σ_e defines the policy of employed workers, and $\mathbf{1}\{P\} = 1$ if the statement P is true and equals zero otherwise. This means given the current states of wage and labour productivity, an employed worker will choose to accept the offer only if accepting the offer gives her a higher expected lifespan utility.

4.2. Unemployed worker with unemployment benefits

There are two policies for unemployed workers with unemployment benefits. Firstly, these workers need to choose their optimal search intensity given their current state. A function $\bar{s}_b(I, h)$ that defines the optimal search intensity needs to satisfy the following first-order condition:

$$\left.\frac{\partial v_b(I, h, s_b)}{\partial s_b}\right|_{s_b = s_b(I,h)} = 0 \tag{8}$$

where

$$v_b(I,h,s_b) = -c(s_b) + (1-\tau)b(I) + (1-\alpha)\beta \sum_{h_{t+1}\in H} \mu_u(h,h_{t+1}) \times \left[[1-\pi(s_b)]\, V_b(I,h_{t+1}) + \pi(s_b)\left(\int_{w\geq \frac{I_g(I)}{h_{t+1}}} V(w,\, h_{t+1}) dF(w) + \int_{w<\frac{I_g(I)}{h_{t+1}}} V_2(w,h_{t+1},\, I) dF(w) \right) \right]. \tag{9}$$

$v_b(I,h,s_b)$ is the lifetime expected utility for the unemployed worker, given her current labour productivity h, the last earnings I, and the search intensity she chooses. Provided h and I, her optimal search intensity is the one that satisfies the first-order condition (8), which maximizes the lifetime expected utility $v_b(I,h,\ \cdot)$.

Secondly, when an unemployed worker finds a job, she needs to choose between accepting and rejecting it. According to the wage offer, there are two possible situations for her reservation wage. The first one is that the wage is lower than the government-determined threshold. In this case, accepting gives V_1, while rejecting gives V_b. Consequently, by solving the following equation:

$$V_1(w,h) = V_b(I,h), \tag{10}$$

we can obtain the reservation wage $\overline{w}_{b1}(I,h)$ that makes the equation established.

The second situation is that the wage offer is equal to or higher than the threshold. In this case, accepting gives V_1, and rejecting gives V_0 since rejecting makes the worker loses unemployment benefits. Therefore, in the second case, the reservation wage $\overline{w}_{b2}(I,h)$ needs to satisfy the equation:

$$V_1(w,h) = V_0(h). \tag{11}$$

In each case, given the worker's wage offer, last earnings and labour productivity, if the wage is higher than the reservation wage, the worker will choose to accept the job. The worker's policy can be concluded as:

$$\sigma_b(w,I,h) = \begin{cases} \mathbf{1}\{w \geq \overline{w}_{b1}(I,h)\} & w < \dfrac{I_g(I)}{h} \\ \mathbf{1}\{w \geq \overline{w}_{b2}(I,h)\} & w \geq \dfrac{I_g(I)}{h} \end{cases} \tag{12}$$

where σ_b denotes the policy of unemployed workers with unemployment benefits.

4.3. Unemployed worker without unemployment benefits

For unemployed workers without unemployment benefits, their policy for search intensity is the function $\bar{s}_0(h)$ that satisfies the following first-order condition:

$$\left.\frac{\partial v_0(h, s_0)}{\partial s_0}\right|_{s_0 = s_0(h)} = 0 \tag{13}$$

where $v_0(h, s_0)$ is the worker's lifetime expected utility given her labour productivity h and her choice of search intensity s_0:

$$\begin{aligned} v_0(h, s_0) = -c(s_0) + (1-\alpha)\beta \sum_{h_{t+1} \in H} \mu_u\,(h, h_{t+1}) \\ \times \left[[1 - \pi(s_0)]\, V_0(h_{t+1}) + \pi(s_0) \int V(w,\, h_{t+1}) dF(w) \right]. \end{aligned} \tag{14}$$

For given level of labour productivity h, we need to find the search intensity that maximizes $v_0(h, \cdot)$. If a search intensity s_0 satisfies the first-order condition (13), then it is the optimal search intensity since $v_0(h, \cdot)$ is maximized.

When these workers receive wage offers, they also need to choose between accepting and rejecting. Their options are identical to those who are employed, since accepting gives them V_1while rejecting gives them V_0. This means that given the worker's labour productivity h, the reservation wage of unemployed workers without unemployment benefits also needs to satisfy the equation (6). Therefore, these unemployed workers' reservation wage is identical to the reservation wage of employed workers with the same labour productivity, which is $\bar{w}_0(h)$. The policy for unemployed workers without compensation is also identical to employed workers with the same labour productivity, which is:

$$\sigma_0(w, h) = \sigma_e(w, h) = \mathbf{1}\{w \geq \bar{w}_0(h)\} \tag{15}$$

where σ_0 denotes the policy for unemployed workers without compensation.

There is no closed-form solution for this model's value functions and policies. Instead, this paper will present numerical solutions after calibration.

5. Calibration

I adopt the calibration of Ljungqvist and Sargent for this model. The calibration is displayed in table 1.

Table 1. Calibration

Variable	Note	Calibration
α	Death rate	$\alpha = 0.0009$
β	Discount rate	$\beta = 0.9985$
τ	Tax rate	$\tau = 0.0285$
λ	Separation rate	$\lambda = 0.009$
H	Set of labour productivity	$H = \left\{h \,\middle|\, h \in [1,2],\ \frac{h}{0.05} \in \mathbb{Z}\right\}$
$\mu_u(h_t, h_{t+1})$	Transition probability for the unemployed	$\mu_u = \begin{cases} 0.9 & for\ h_{t+1} = h_t \\ 0.1 & for\ h_{t+1} = h_t - 0.05 \end{cases}$
$\mu_e(h_t, h_{t+1})$	Transition probability for the employed	$\mu_e = \begin{cases} 0.8 & for\ h_{t+1} = h_t \\ 0.2 & for\ h_{t+1} = h_t + 0.05 \end{cases}$
$\mu_l(h_t, h_{t+1})$	Transition probability for the laid-off	$\mu_l = 1\ for\ h_{t+1} = h_t$
s	Search intensity	$s \in [0,1]$
$c(s)$	Function for search cost	$c(s) = 0.5s$
$\pi(s)$	Function for probability of finding a job	$\pi(s) = s^{0.3}$
w	Wage	$w \in [0,1]$
$F(w)$	Wage distribution	Normal distribution with mean of 0.5 and variance of 0.1
I	Last earnings	$I = wh \in [0,2]$
$b(I)$	Function for unemployment benefit	$b(I) = 0.7 \times \frac{2}{15}\left\lceil \frac{I}{2/15} \right\rceil$
$I_g(I)$	Function for suitable earnings	$I_g(I) = 0.7 \times \frac{2}{15}\left\lceil \frac{I}{2/15} \right\rceil$

Each one period in the model is two weeks. In particular, there are 21 levels of labour productivity in total, which evenly partitions the internal [1, 2]. The calibration for transition probability $\mu_u(h_t, h_{t+1})$ means that for unemployed workers, there is a chance of 0.9 that their labour productivity will not change in the next period, and a chance of 0.1 that their labour productivity will drop one level. The calibration for transition probabilities for employed and laid-off workers follows similar reasoning.

For workers entitled to unemployment compensation, their last earnings can be evenly divided into 15 classes. For example, the first class of last earnings ranges from 0 to 0.13 ($\frac{2}{15}$), which is the lowest earning class. Unemployed workers obtain compensation according to the earning class they belong to. The unemployment compensation is equivalent to 70% of the upper limit of the worker's earning class. For example, a worker whose last earning is 0.05 belongs to the first earning class, and the upper limit of the first earning class is 0.13. Given this information, her compensation is equal to 70% of the upper limit of the first class, which is 0.091 (= 0.13 × 0.7). In this sense, the unemployment compensation can be higher than workers' last earnings. In addition, the government-determined suitable earnings equals to the unemployment benefits.

Given the calibration, I use Julia to produce a numerical solution for this model. This is achieved by value function iteration. Firstly, I generate random initial matrixes for V, V_b, V_0, V_1, and V_2. Secondly, I run the model using these initial values, and obtain a new set of matrixes representing the optimal expected lifetime utilities. Thirdly, I calculate the deviation for each term in each matrix, and check if any deviation is larger than the fixed tolerance I set (which is 10). The second step will be repeated until there is no deviation is larger than the tolerance. According to the fixed-point theorem, this algorithm will return a good approximation to the true solution of the model, which I can use to represent the value functions.

I also produce numerical solutions for two extensions of the model. These extensions have different settings of minimum wages. In the first extension, the minimum wage is 0.2. In the second extension, the minimum wage is 0.3. These two extensions are otherwise indistinguishable from the original model. The wage distributions in these two extensions are also identical to the original model, except that the minimum wage is truncated at 0.2 and 0.3, respectively.

I set these two minimum wages for extensions according to the minimum wage level in France and the Netherland in 1970. In France, the minimum wage as a percentage of the mean wage is 40% (0.4), while in the Netherland, the minimum wage is 0.6 of the mean wage. Since the mean wage of the exogenous wage distribution in these two extensions is 0.5, the minimum wages are set as 0.2 (= 0.5 × 0.4) and 0.3 (= 0.5 × 0.6), respectively.

6. Numerical solutions

I will now present the numerical solutions for the original model and its two extensions.

6.1. Original model: minimum wage = 0

In the original model, there is no constraint on minimum wage.

Reservation wage

Table 2

Worker's reservation wage	Minimum	Maximum	Average
Workers without compensation	0.709	0.800	0.741
Workers with compensation	0.709	0.914	0.763

Table 2 displays statistics of workers' reservation wage. The averages are calculated by grouping workers according to their labour productivity, current wage and last earnings, and then assigning each group of workers equal weight. The statistics show that workers entitled to unemployment benefits have higher reservation wage.

Figure 6.1.1 shows the reservation wage $\overline{w}_0(h)$ for workers who are not entitled to unemployment benefits as a function of labour productivity. For these workers, their reservation wage will first decrease and then go up as the labour productivity grows. The group of workers whose labour productivity is 1.35 has the lowest reservation wage, which is 0.709 in table 3. The intuition behind these findings is that one period of unemployment can lead to one level lower in labour productivity, and workers with labour productivity of 1.35 care about their loss in labour productivity the most. This renders them accept to low wage offers to save their loss in labour productivity.

Figure 6.1.1

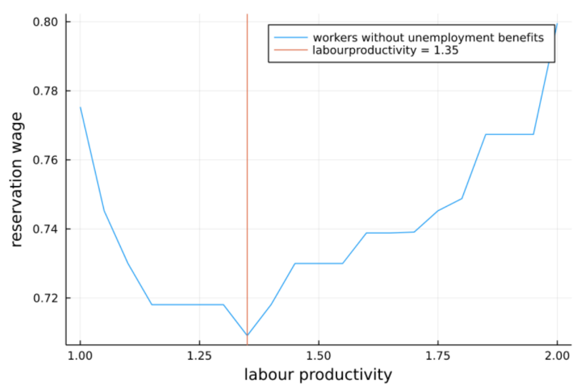


Figure 6.1.2

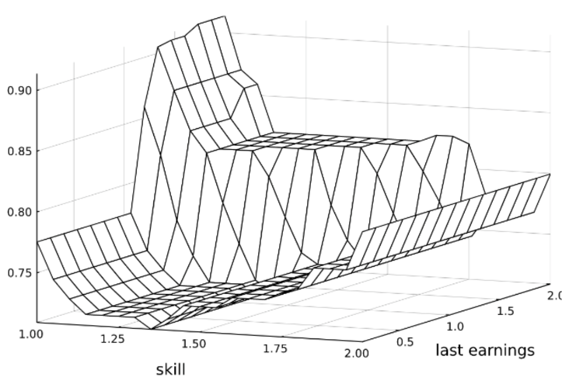


Figure 6.1.2 demonstrates the reservation wage as a function of labour productivity and the class of last earnings for workers entitled to unemployment compensation. There are three findings. Firstly, for a given level of labour productivity, workers belonging to higher earning classes tend to have higher reservation wages. Secondly, the group with the lowest reservation wage is those workers who belong to the first ten classes ($I < 1.33$) and have labour productivity of 1.35. Thirdly, in each earning class, the labour productivity of workers with the lowest reservation in this class may change. For example, as figure 6.1.3 shows, for workers who belong to the highest earning class, workers with labour productivity from 1.85 to 1.95 have the lowest reservation wage in this earning class. This is different from the first 10 earning classes, in which workers with labour productivity of 1.35 have the lowest reservation wage.

Figure 6.1.3

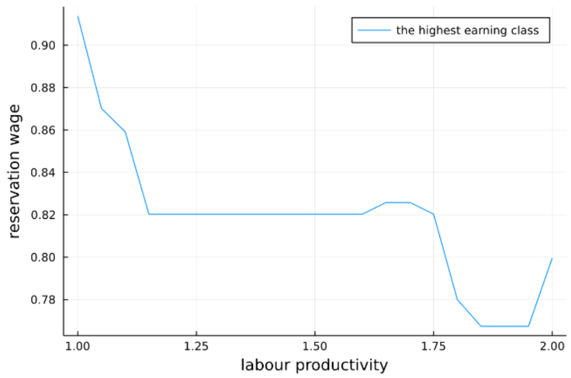


Figure 6.1.4

Search intensity

Figure 6.1.4 shows the search intensity of unemployed workers entitled to compensation. There are two findings. Firstly, as the figure shows, for workers belonging to the same earning class, those with moderate labour productivity tend to put in the most effort to search for a job. This is linked to our previous finding. Workers with mid-level labour productivity care about their loss in labour productivity the most. For workers with mid-level labour productivity, the expected value of searching for jobs is much higher than the value of the current situation. This is because working can improve their labour productivity and lead to higher future earnings. Therefore, these workers are willing to choose the highest search intensity. For workers with high labour productivity, working is limited in improving their labour productivity, which means the expected value of finding jobs is not much higher than their current lifetime utility, making them choose a relatively low search intensity. For low-skilled workers, their loss in labour productivity during unemployment periods is small, so they also choose low search intensity.

Secondly, for a given level of labour productivity, last earnings are negatively correlated with search intensity. This is because workers with higher last earnings have better outside options since they can obtain higher compensations. By combining these two findings, one can understand why workers with the lowest search intensity (and highest reservation wage) have the highest level of last earnings and lowest labour productivity. Because for a pretty long time, none of these workers can be paid more than their unemployment benefits from their jobs.

The average search intensity for all groups of workers with unemployment benefits is 0.908, in which every group have the same weight. In addition, workers without compensation will always choose the highest search intensity, which is 1.

Lifetime utility

Table 3

Worker's optimal expected lifetime utility	Minimum	Maximum	Average
Unemployed workers without compensation	586.72	662.05	630.61
Unemployed workers with compensation	587.59	668.71	636.15
Employed workers	586.72	697.11	633.54

Table 3 displays statistics of workers' optimal expected lifetime utilities. It shows that when there is no minimum wage, workers with unemployment compensation have the highest average expected lifetime utility (636.15), which is 2.61 units higher than the average lifetime utility of employed workers.

Figures 6.1.5 and 6.1.6 demonstrate the value functions for unemployed workers without compensation and unemployed workers with compensation. Figure 6.1.5 shows that as labour productivity increases, the lifetime utility for unemployed workers without compensation increases. This is because the expected future earnings will be higher with higher labour productivity. Similarly, figure 6.1.6 shows that for unemployed workers with unemployment benefits, their lifetime utility increases with the increase in labour productivity and their last earnings. Higher last earnings give these workers higher unemployment benefits.

Figure 6.1.5

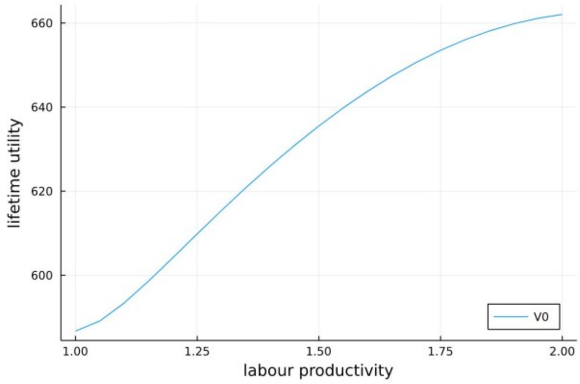
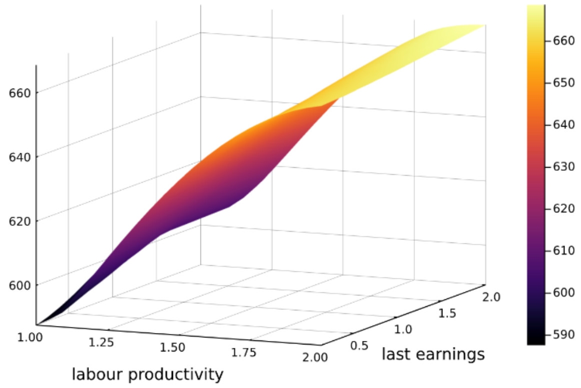


Figure 6.1.6



Figures 6.1.7 displays the optimal expected lifetime utility of employed workers as a function of the current wage and labour productivity. There are three findings. Firstly, the lifetime utility and labour productivity are positively correlated, and employed workers' lifetime utility increases with labour productivity. Secondly, taking the reservation wage as a dividing line, for given labour productivity, the current wage has a positive effect on lifetime utility only if it is higher than the worker's reservation wage. It does not affect the lifetime utility if it is lower than the reservation wage. One example of this is shown in figure 6.1.8. The intuition behind this is that if the current wage is lower than workers' reservation wage, workers will choose to reject the job, making the current wage offer does not affect utility.

Figure 6.1.7

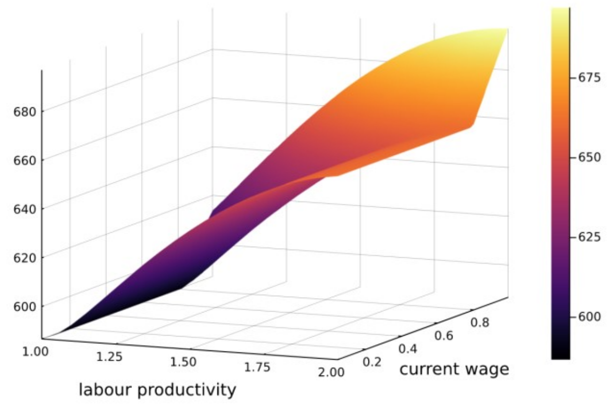
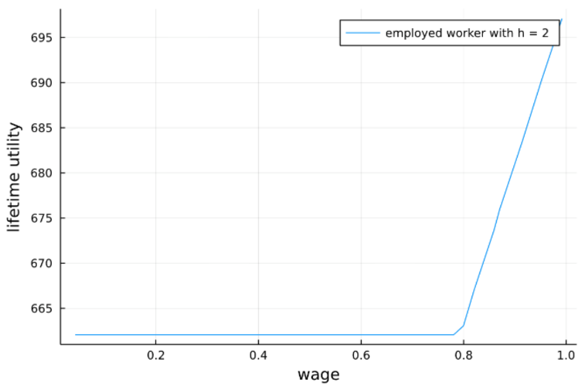


Figure 6.1.8



The third finding is that the effect of current wage on lifetime utility varies with labour productivity. An identical amount of rising in current wage brings different increases in utility for workers with different labour productivity. One example is shown in figures 6.1.9 and 6.1.10. Figure 6.1.9 illustrates the lifetime utility as a function of labour productivity when current wages are 1 and 0.74, respectively. Figure 6.1.10 shows the difference between these two functions, which is how the difference in lifetime utility (caused by wage) changes with labour productivity. It turns out that wage increases from 0.74 to 1 are most important to workers with middle-skilled workers, especially those whose labour productivity is around 1.7, because these workers benefit from this increase (0.74 to 1) most. For other workers, their utility does not rise as much as workers with labour productivity of 1.7, though they also benefit from this wage increasing. In this sense, wage rising has a different meaning for workers with different labour productivity.

Figure 6.1.9

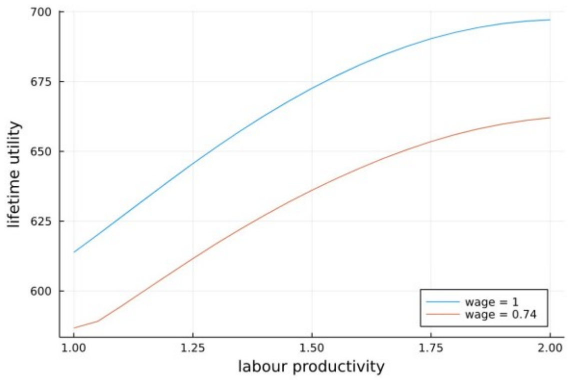
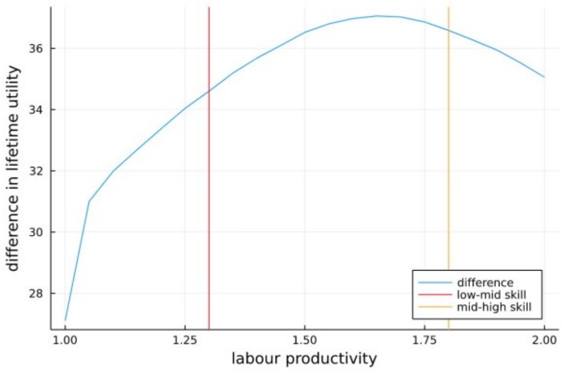


Figure 6.1.10



6.2. Extension 1: minimum wage = 0.2

In this extension, the minimum wage is 0.2. I will now present workers' reservation wage, search intensity, and optimal lifetime utility, and compare them with the statistics of the original model.

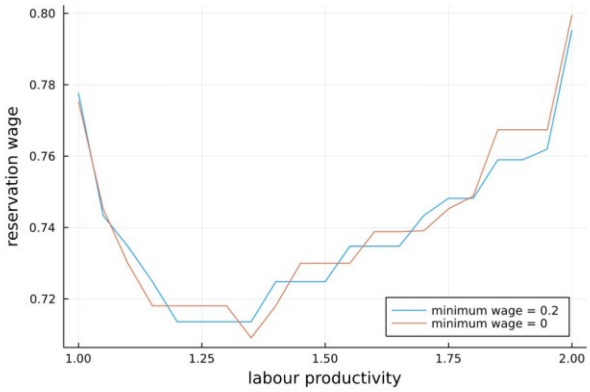
Reservation wage

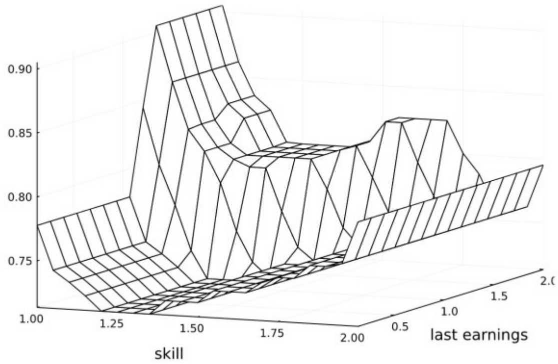
Table 4

Worker's reservation wage	Minimum	Maximum	Average
Workers without compensation	0.714	0.795	0.739
Workers with compensation	0.714	0.905	0.759

Table 4 shows workers' reservation wage when the minimum wage is 0.2. By comparing with table 2, one can have two findings. Firstly, with increasing the minimum wage to 0.2, the lowest reservation wage increases from 0.709 to 0.714. Secondly, the average reservation wage decreases, and becomes closer to the lowest reservation wage.

Figure 6.2.1 **Figure 6.2.2**





Figures 6.2.1 shows the reservation wages for workers without unemployment compensation in the original model and the first extension. Firstly, according to the figure, when the minimum wage increases from 0 to 0.2, the lowest reservation wage of workers without compensation increases, which is consistent with the statistics in table 4. Moreover, the group of workers with the lowest reservation wage expands, and it is no longer only those whose labour productivity is 1.35. Instead, workers whose labour productivity is from 1.20 to 1.35 have the lowest reservation wage when the minimum wage is 0.2. Secondly, although the lowest reservation wage increases, the average of workers' reservation wage decreases, as the blue line in which the minimum wage is 0.2 is generally lower than the red line. This is caused by the expansion of the group with the lowest reservation wage.

Similar findings also appear in the group of workers entitled to compensation. Figure 6.2.2 demonstrates the reservation wage for these workers. Firstly, when the minimum wage increases to 0.2, in most earning classes (13 out of 15), workers' lowest reservation wage is higher than before. The two exceptions are the eleventh earning class and the highest earning class. Secondly, the group with the lowest reservation wage changes. Workers who belong to the first nine earning classes and have labour productivity from 1.20 to 1.35 have the lowest reservation wage. This results in that, for all earning classes, the average reservation wage is generally lower than that in the original model, and becomes closer to the lowest reservation wage. In this sense, raising minimum wage to 0.2 will increase the lowest reservation wage, and make the average reservation wage closer to the lowest reservation wage.

A possible reason for the decrease in average reservation wage is that workers are more likely to receive relatively higher wages after raising the minimum wage. Therefore, working is more attractive than before. This leads to workers, especially those with no unemployment compensation willing to lower their reservation wage a bit in exchange for a job offer.

Search intensity

For workers without unemployment benefits, their search intensity is always 1, regardless of their labour productivity.

Figure 6.2.3

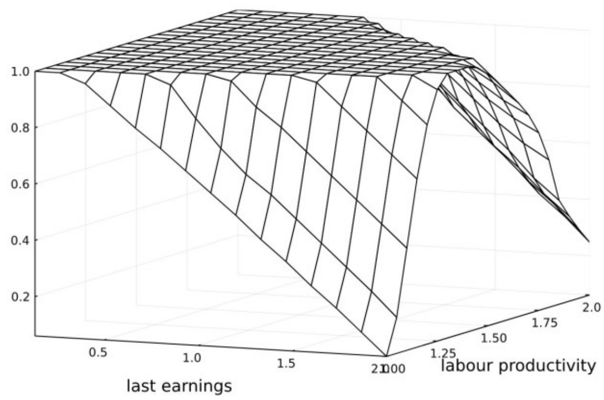
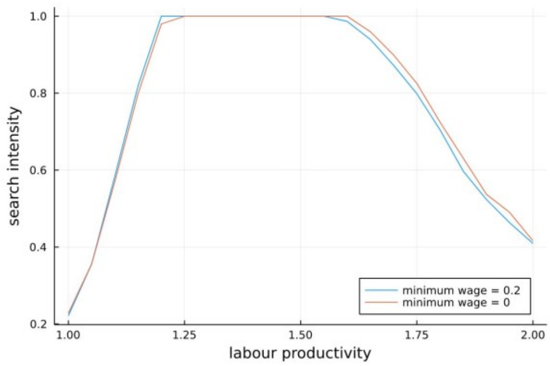


Figure 6.2.4



For workers who have unemployment compensation, their search intensity is shown in figure 6.2.3. The average search intensity of all groups is around 0.905, in which workers are grouped according to their labour productivity, and earning class and each group is assigned the same weight. This average search intensity is slightly lower than the average in the original model (0.908). Therefore, when the minimum wage increases to 0.2, workers' average search intensity decreases.

When raising minimum wage to 0.2, in each earning class, workers with low labour productivity will put more effort into searching for jobs, while workers with high labour productivity will search for jobs less intensively. An example is illustrated in figure 6.2.4. Figure 6.2.4 shows how workers' search intensity varies with labour productivity for the thirteenth earning class. It turns out that when the minimum wage increases to 0.2, workers whose labour productivity is less than 1.25 will choose higher search intensity, while workers with labour productivity higher than 1.55 will spend less effort on searching for jobs. The underlying reason is that for low-skilled workers, rising minimum wage means the expected value of searching increases, which attracts them to search more intensively. For high-skilled workers who cannot get a much higher increase in earnings from improving their labour productivity, the increase in expected value of searching allows them to choose lower search intensity and save their costs.

Lifetime utility

Table 5

Worker's optimal expected lifetime utility	Minimum	Maximum	Average
Unemployed workers without compensation	581.96	656.74	625.83
Unemployed workers with compensation	582.05	663.04	630.55
Employed workers	581.96	688.30	627.97

Table 5 displays workers' lifetime utility statistics when the minimum wage is 0.2. Figures 6.2.5, 6.2.6, and 6.2.7 show the value functions for unemployed workers without compensation, unemployed workers with compensation and employed workers, respectively. Overall, there is no significant change in the form of value functions.

Figure 6.2.5

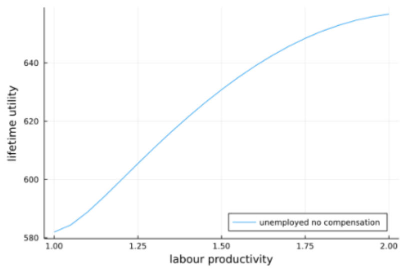


Figure 6.2.6

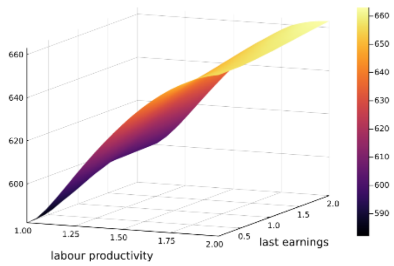
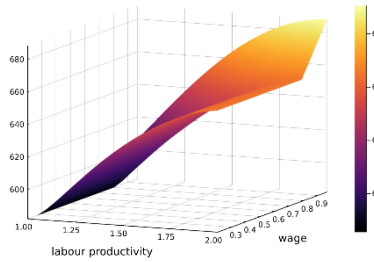


Figure 6.2.7



There are two crucial findings. Firstly, as table 5 shows, compared with the original model, the difference in lifetime utility between employed workers and unemployed workers with compensation becomes smaller. In the first extension, the lifetime utility of unemployed workers with compensation is 2.58 units higher than that of employed workers, which is smaller than the 2.61 of the original model. In this sense, the option of working becomes relatively more attractive than before when the minimum wage increases to 0.2.

Secondly, compared to table 2, increasing minimum wage to 0.2 lowers all workers' lifetime utility. This may be because of the decreases in workers' reservation wages. In this model, workers' utility comes from two sources, which are wages and unemployment benefits. Lower reservation wage means workers are more likely to accept lower wage offers, indirectly resulting in lower unemployment compensation when workers are fired. Consequently, the lower reservation wage decreases the optimal lifetime utility in two ways. Since increasing minimum wage to 0.2 decreases workers' reservation wage, it also leads to a decrease in workers' optimal lifetime utility.

6.3. Extension 2: minimum wage = 0.3

In the second extension, the minimum wage is 0.3.

Reservation wage

Table 6

Worker's reservation wage	Minimum	Maximum	Average
Unemployed workers without compensation	0.739	0.807	0.760
Unemployed workers with compensation	0.739	0.912	0.778

Table 6 shows the statistics of workers' reservation wages. Firstly, in this case, the lowest reservation wage is 0.739, which is higher than the previous two cases. This indicates that raising minimum wage does increase the lowest reservation wage. Secondly, when the minimum wage is 0.3, the difference between the lowest reservation wage and the average reservation for each type of workers decreases further, meaning that as the minimum wage increases, the average reservation wage becomes closer to the lowest reservation wage. Thirdly, when the minimum wage increases to 0.3, the average reservation wage for every type of workers becomes higher than the average in the original model, which is different from the case of increasing minimum wage to 0.2. This is because when the average reservation approaches the lowest reservation wage, the lowest reservation wage also keeps increasing.

Figure 6.3.1

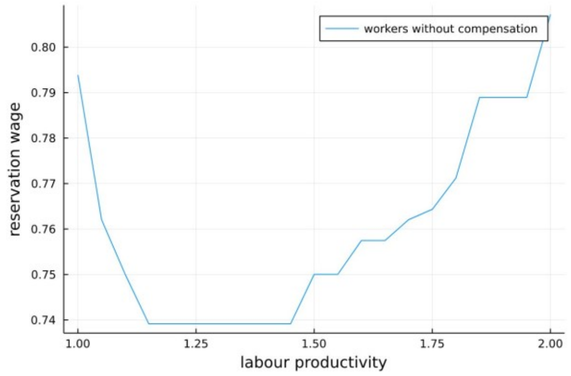
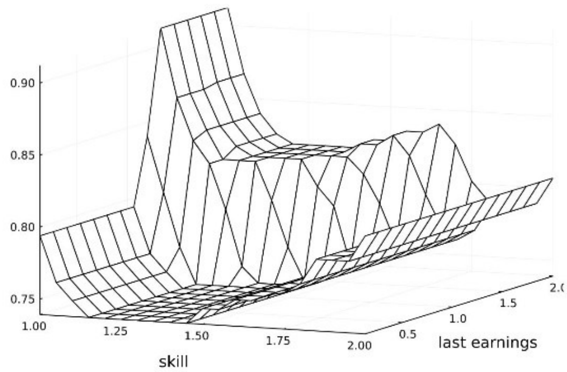


Figure 6.3.2



Figures 6.3.1 and 6.3.2 show the reservation wage of workers without unemployment compensation and workers entitled to unemployment compensation, respectively. From these

two figures, one can notice the group of the lowest reservation wage changes. When the minimum wage is 0.3, for workers who are not entitled to unemployment benefits, the group with the lowest reservation wage consists of those whose labour productivity is 1.15 to 1.45. For workers who have unemployment compensation, the lowest reservation group constitutes of workers who belong to the first nine earning classes and labour productivity from 1.15 to 1.45. Compared with previous cases, the range of labour productivity for the group with the lowest reservation wage increases. In the original model with no minimum wage, only workers with labour productivity of 1.35 may have the lowest reservation wage. In the first extension where the minimum wage is 0.2, labour productivity in the group of lowest reservation wage is from 1.20 to 1.35. When the minimum wage is 0.3, the range of labour productivity of the group with lowest reservation wage increases further, which is from 1.15 to 1.45. This indicates that the group of lowest reservation wage keeps expanding as the minimum wage increases.

A possible reason for the expansion in the group of lowest reservation wage is that with a high minimum wage (0.3), workers are more likely to receive high wage offers. In this case, even though the current wage offer is not high, working can provide accumulation in labour productivity and lead to higher earnings in the future. Therefore, more workers are willing to accept relatively low wages.

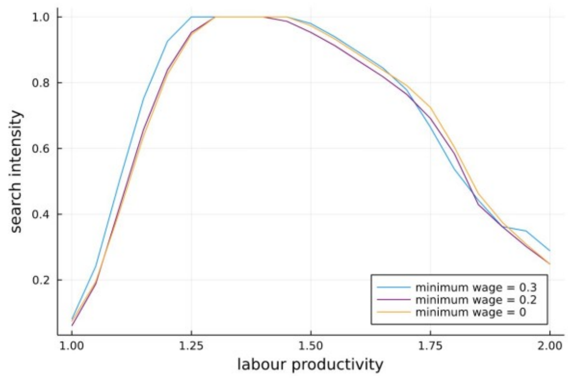
Search intensity

Workers without unemployment compensation always choose the highest search intensity, which is 1.

The search intensity of unemployed workers with compensation is shown in figure 6.3.3. In general, the average search intensity of all groups of workers is 0.914, which is higher than the average in the original model and the first extension.

Figure 6.3.3

Figure 6.3.4



Raising the minimum wage to 0.3 greatly increases the search intensity of workers with low labour productivity. One example is demonstrated in figure 6.3.4. As this figure shows, when the minimum wage increases to 0.3, these workers choose to put higher efforts into searching for jobs.

As mentioned before, workers increase their search intensity because working becomes much more attractive. When the minimum wage increases to 0.3, the low-skilled workers are more likely to have high wages than before when searching for jobs. Consequently, they are more willing to start a job. Moreover, working can raise these workers' labour productivity and allow them to obtain higher earnings. In this sense, the workers with low labour productivity will pay much more effort into searching for jobs.

Lifetime utility

Table 7

Worker's optimal expected lifetime utility	Minimum	Maximum	Average
Unemployed workers without compensation	592.33	668.38	637.21
Unemployed workers with compensation	592.42	673.59	641.21
Employed workers	592.33	699.77	641.31

The statistics for workers' lifetime utility are shown in table 7. Compared with previous settings, setting a high minimum wage (0.3) results in two changes. Firstly, the lifetime utility of unemployed workers with compensation is no longer higher than the lifetime utility of employed workers. This means that, on average, being employed is more attractive than receiving unemployment compensation. In this case, workers will be more willing to accept job offers.

Secondly, the high minimum wage increases all workers' optimal expected lifetime utility. Among the three settings of minimum wages, the setting with minimum wage of 0.3 has the highest minimum, maximum, and average of workers' lifetime utility. This may be caused by the increase in workers' reservation wages. The high reservation wage renders workers to accept relatively high wage offers, thereby leading to higher unemployment benefits.

7. Simulation and Robust Analysis

In this section, I simulate three economies according to the search model. The initial settings of these three economies are only different in minimum wages. The minimum wages in these economies are 0, 0.2, and 0.3, respectively. There exists an exogenous separation rate in all three economies, and vacancies are assumed to be always sufficient. This means all three economies are supply-dominated. In each economy, there are 10,000 ex-ante identical agents who are unemployed and with labour productivity at the lowest level in the first period. There are 1,500 periods in total, and a one-period transient shock happens in all three economies at the period $t = 1{,}000$. During the period of transient shock, the exogenous separation rate doubles in all three economies (0.018). After this period, the separation rate backs to the original level (0.009).

7.1. Steady State

Table 8 shows statistics of these three simulated economies at a steady state. These statistics are the averages from periods t=500 to t=800. There are four important findings.

Table 8

	Minimum wage = 0	Minimum wage = 0.2	Minimum wage = 0.3
Unemployment rate	6.68%	7.28%	4.76%
Average unemployment duration	11.40 weeks	12.04 weeks	7.37 weeks
Percent of Unemployed at a point of time with spells so far ≥ 6 months	14.18%	15.34%	5.29%
Percent of Unemployed at a point of time with spells so far ≥ 12 months	1.84%	2.40%	0.20%
Average labour productivity	1.875	1.871	1.890
Average wage of employed workers	0.88	0.86	0.90
Average earnings of employed workers	1.66	1.62	1.70
Average unemployment benefits of unemployed workers	0.92	0.69	0.90
Total number of quitting in a given period	8	12	47
Quitting as a percentage of all separation	8.62%	12.26%	50.00%

Firstly, the effect of increasing the minimum wage on employed workers' wages and earnings varies with the amount of increase. When the increase in the minimum wage is relatively low, the rise in minimum wage will negatively affect employed workers' wages and earnings. As table 8 shows, when the minimum wage increases from 0 to 0.2, the average wage of employed workers slightly decreases from 0.88 to 0.86, and the average earnings are reduced from 1.66 to 1.62. In contrast, when the minimum wage raises to a relatively high level, the increase in minimum wage positively affects workers' wages and earnings. In the economy with a minimum wage of 0.3, the average wage and earnings are 0.9 and 1.7, respectively.

These statistics are consistent with our findings in section 6. The increase in minimum wages reduces workers' wages and earnings in the second economy because workers' average reservation wage decreases to a level lower than the case without minimum wages. This renders workers to accept lower wage offers, which decreases workers' earnings for given labour productivity. When the minimum wage becomes relatively higher (0.3), workers' average reservation wage goes up, and becomes higher than the case with no minimum wage. This makes workers accept jobs with higher wages, and obtain higher earnings.

Secondly, raising minimum wages will lead to an increase in the number of workers choosing to quit. When there is no minimum wage, the average number of employed workers who choose to quit in a period is 8, and it is 8.62% of all workers who leave their jobs in the period. When the minimum wage increases to 0.2, the number of workers choosing to quit increases to 12. When the minimum wage becomes 0.3, on average, there are 47 employed workers choosing to quit in a period, which accounts for half of the total separation in the period. In this sense, raising the minimum wage can increase the number of quitting.

This phenomenon can also be explained by our previous findings. With the increase in minimum wages, the expected value of working becomes relatively higher, and the difference between being employed with and being unemployed with compensation $(V - V_b)$ is larger. This means working becomes more attractive to workers with the rise in minimum wages, since it can improve workers' labour productivity and lead to higher future earnings. Therefore, workers are willing to accept relatively low wage offers to improve their labour productivity through working, and quit their current job when they believe that their labour productivity is high and they can have higher earnings by finding other jobs.

Thirdly, according to the simulation results, minimum wages can lower the average unemployment benefits of unemployed workers. When there is no minimum wage, the average compensation for the unemployed is 0.92. With a minimum wage of 0.2, the average compensation declines significantly, which is 0.69. The average compensation in the economy with a minimum wage of 0.3 is 0.9, which is higher than the second economy but still lower than the first economy.

In general, the average unemployment benefits are determined by workers' average reservation wages. If workers' reservation wages are lower, the average unemployment benefits will also be lower. This is because workers are more likely to accept low wage offers before being laid off. After these workers are fired, their low wages will result in low unemployment benefits, since unemployment benefits are determined by earning classes. Consequently, the average reservation wage can explain why the economy with a minimum wage of 0.2 has the lowest unemployment benefits, since the second economy has the lowest average reservation wage.

Workers in the third economy actually have the highest average reservation wage, yet the average unemployment benefits per unemployed worker are not the highest. This is because the third economy has the highest quitting rate. If a worker chooses to quit, she cannot get compensation during unemployment. In this sense, the high quitting rate decreases the average compensation per unemployed worker, as many unemployed workers cannot get compensation. Therefore, the comparison between the first and the third economies does not disprove that the average unemployment benefits are determined by the average reservation wage.

Finally, different minimum wages have different effects on steady-state unemployment. When the minimum wage rises from 0 to 0.2, the steady-state unemployment increases from 6.68% to 7.68%, which means a relatively low increase in the minimum wage will lead to a higher steady-state unemployment rate. When the minimum wage rises to 0.3, the steady-state unemployment becomes 4.76%. In this sense, a relatively high increase in the minimum wage, will decrease the unemployment rate.

These economies' steady-state unemployment rates are negatively correlated with two factors. Firstly, there is a negative relationship between steady-state unemployment and workers' average lifetime utility in the economy. Recall the statistics in section 6, the second economy has the lowest average lifetime utility, and the third economy has the highest average lifetime utility. Therefore, higher steady-state unemployment is associated with lower lifetime utility. For example, the second economy has the lowest expected lifetime utility, and the highest steady-state unemployment. Secondly, steady-state unemployment is negatively correlated with the average search intensity of workers. The average search intensity in economies with minimum wages of 0, 0.2 and 0.3 are around 0.908, 0.905 and 0.914, respectively. Consequently, economies with higher average search intensity tend to have lower steady-state unemployment.

The underlying reason for these correlations is that all three economies are dominated by labour supply. The change in unemployment actually reflects the change in labour supply. Our simulation shows that rising minimum wage will first decrease employed workers' lifetime utility, and then increase it. With a low increase in the minimum wage, workers' lifetime utility is low, and thereby they tend to put less effort into searching to save the costs. When the minimum wage is high, the expected utility of being employed is high, which attracts workers to search more intensively to find jobs. Thus, raising the minimum wage will first reduce the labour supply, and then increase it. This leads to the steady-state unemployment in supply-dominated economies first increasing and then decreasing with minimum wage raises.

These findings indicate that minimum wages can be one of the causes of the high and persistent increase in some European countries. According to these findings, if there is a minimum wage and the minimum wage is not high enough, it cannot encourage workers to find jobs. Instead, workers with relatively high labour productivity may choose to lower their search intensity to save costs. Consequently, a relatively low minimum wage may decrease the labour supply and lead to a higher unemployment rate. This is consistent with empirical data. The minimum wage as a percentage of the mean wage in France was kept at a relatively low level before 1970, which is around 0.4. While in Netherland, the minimum wage is around 0.6 of the mean wage before 1970. The unemployment rates of France and the Netherland were 2.5 per cent and 0.9 per cent in 1970 (Saint-Paul, 2004). In this sense, the relatively low minimum wage in France may be why its unemployment rate is higher than in the Netherlands before 1970, because the labour supply in France may be lower than that in the Netherlands. After 1970, the minimum wage in France still remained relatively low, which may be one of the causes of its persistently high unemployment.

7.2. Temporal Shock

Figures 7.2.1 demonstrates how the unemployment rates in the first and second economies change when a one-period transient shock appears. 7.2.2 shows the unemployment rates in the first and third economies during the transient shock.

Figure 7.2.1

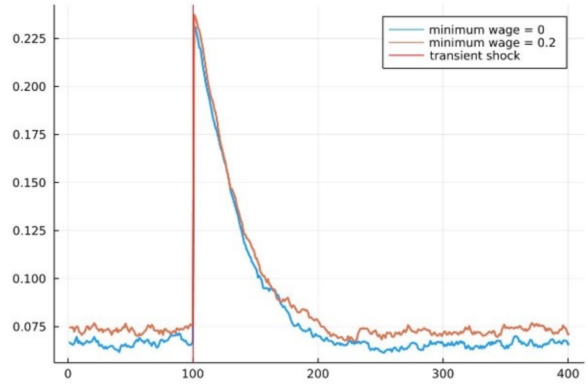
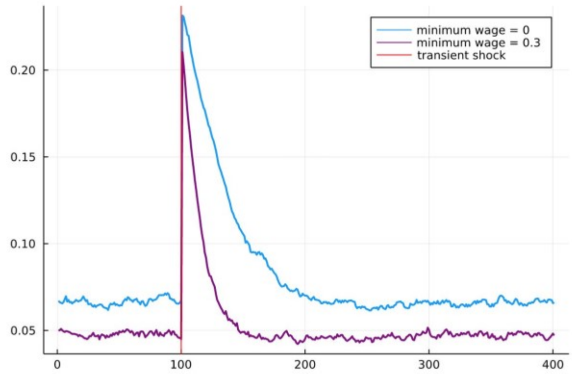


Figure 7.2.2



There are three findings. Firstly, the transient shock does not change the steady-state unemployment. In all three economies, the unemployment rates rise to a high level first and gradually decline to their original steady states. This suggests that the hysteresis according to which all history of unemployment can affect its current state is implausible.

Secondly, when the transient shock appears, the highest unemployment rates of these three economies are different. As the figures show, the second economy has the highest maximum unemployment rate (23.73%), which is slightly higher than that of the first economy (23.14%). The third economy has the lowest maximum unemployment, which is 21.04%. This indicates that the same transient shock has different negative impacts on economies with different minimum wages, and the economy with a relatively low minimum wage is the most negatively affected by the shock.

Thirdly, economies with different minimum wages take different times to recover from the transient shock. According to figure 7.2.1, the first and second economies recover from the shock at about the same speed, since their slopes of unemployment rates are similar. Figure 7.2.2 shows that the third economy in which the minimum wage is 0.3 takes the least amount of time to recover from the shock, which is around 45 periods. The other two economies spent around 120 periods recovering from the shock. Therefore, minimum wages can affect economies' ability to handle shocks.

8. Conclusions

In this paper, I reproduce a search model and add constraints on the minimum wage to study the effect of minimum wage on labour supply and unemployment. The focus of this paper is on the labour supply side. The numerical solutions and simulation results show that a relatively low minimum wage will reduce workers' optimal expected lifetime utility, thereby decreasing search intensity and employment level. In contrast, a high minimum wage will encourage workers to search for jobs and increase employment level, because the high minimum wage increases the lifetime utility of working. In addition, the results also show that minimum wages can affect economies' ability to handle shocks.

These results suggest that minimum wages may be one factor that causes the high and persistent increases in some European countries, because the minimum wage as a proportion of mean wage in some European countries has been maintained at a relatively low level. The relatively low minimum wage may lead to a decrease in labour supply, and thereby reduce unemployment.

The search model in this paper is formulated with an exogenous separation rate, and the job-finding rate is entirely determined by workers' search intensity. With exogenous settings of labour demand, the effect of raising the minimum wage on unemployment is not comprehensive, since minimum wage can also affect labour demand, and its overall effect on unemployment is uncertain. Therefore, developing a framework with endogenous labour demand remains an outstanding task for future research.

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