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The Intelligent Driver Model: Analysis and Application to Adaptive Cruise Control

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The Intelligent Driver Model: Analysis and Application To Adaptive Cruise Control

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Abstract

There are a large number of models that can be used to describe traffic flow. Although some were initially theoretically derived, there are many that were constructed with utility alone in mind. The Intelligent Driver Model (IDM) is a microscopic model that can be used to examine traffic behavior on an individual level with emphasis on the relation to an ahead vehicle. One application for this model is that it is easily molded to performing the operations for an Adaptive Cruise Control (ACC) system. Although it is clear that the IDM holds a number of convenient properties, like easily interpreted parameters, there is yet to be any rigorous examining of this model from a mathematical standpoint. This paper will place this model into the form of a vector-valued time-autonomous ODE system and analytically examine it. Additionally, the parameter estimation problem will be formulated. Simulations will demonstrate the model in practice.

Dedication

To JPF, with me in every endeavor.

Acknowledgments

"It's still magic even when you know how it's done"

-Terry Pratchett

I've been lucky to count on the support and help of a great many people. This would never have been possible without you.
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Thank you to my brothers, for making me look like the sibling who has their life together. Thank you to my mom and my dad, who provide me with so many benefits that it would be silly to enumerate them all.Thank you to my office-mates: we're a grand mathematical forest and I'm so proud of you all. Brownie points to Tony, for being my go-to computer guy.
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Symbols Used

$\alpha_i \ i \in \{0, ..., 8\}$	coefficients of the IDM
δ	Acceleration Exponent
Δv_α	The velocity differential between $v_{\alpha-1}$ and v_α
a	acceleration
b	comfortable deceleration
l_α	Length of car α
q	Vector of Parameters
s_0	Linear Jam Distance
s_1	Nonlinear Jame Distance
s_α	The bumper-to-bumper gap between $x_{\alpha-1}$ and x_α
t_0	Beginning of time interval
t_f	End of time interval
T	Safe Time headway
T'	Psychological Reaction Time
T_r	Reaction Time
$v_{\alpha-1}$	The velocity of the car ahead
$\hat{v}$	Initial Velocity of the car of interest
v_α	The velocity of the car of interest
v_0	The Desired Speed
x_α	The position of the car of interest
$x_{\alpha+1}$	The position of the car ahead
$\hat{x}$	Initial Position of the car of interest

Table 1: Symbols

λ_T	T ACC multiplier
λ_a	a ACC multiplier
λ_b	b ACC multiplier
τ	Relaxation Parameter for EMA
v_{free}	Free Traffic Threshold
v_{cong}	Jam Traffic Threshold
v_{up}	Upstream Traffic Threshold
v_{down}	Downstream Traffic Threshold
$v_{\alpha,EMA}$	Exponential Moving Average of measured velocity
x_{begin}	location of the start of a bottleneck
x_{end}	location of the end of a bottleneck
$x_{\alpha,EMA}$	Exponential Moving Average of measured position

Table 2: Symbols specific to the ACC

Chapter 1

Introduction

Congested automotive traffic has become an extremely common situation in the last century, leading to higher levels of frustration and unhappiness among the general public, but sparking interest in the applied mathematical community. By the 1950's there were three necessary objects to make this research effective: a significantly large number of cars spread over the population, the technology to observe and analyze their movements, and a number of similar freeways where results could be replicated. This decade gave rise to the primary theories of traffic flow modeling, which are mostly delineated between macroscopic and microscopic. Early attempts at modeling were effectively restricted to the macroscopic, which mimicked continuous physical patterns, particularly waves. However, improvements in technology have more recently allowed for the tracking of individual drivers' behavior in thorough detail. This has opened up the possibility of improving the overall state of traffic conditions by controlling individual decisions, and has greatly increased the number of microscopic models being explored.

1.1 Motivation

Nearly all current drivers are familiar with the system of cruise control, which allows a vehicle to independently reach and maintain a speed chosen by the driver. Adaptive Cruise Control (ACC) is a system that allows for an automated driving style that can adapt to traffic conditions and situations. It extends control of the vehicle over decisions traditionally left to the driver by additionally setting and maintaining a time gap (safe headway). The car is equipped with radar or infrared sensors to detect and track the vehicle immediately ahead in order to control braking and thereby the deceleration necessary to avoid any collisions. Future ACC models will contain the ability to avoid rear-end collisions as well [9]. However, ACC has no authority over lane-changing decisions and is limited in control in terms of velocity, acceleration, and deceleration. Currently, some cars have a partly automated version of this system already in place that allows for the user to choose when it is activated. However, these systems are only sophisticated enough to improve driver comfort, without the ability to improve the capacity of a road network.

The next generation of the ACC presents an opportunity to decrease road congestion without requiring special infrastructure or dedicated lanes, as needed by automated highway systems [9]. It will also operate effectively in all speed ranges, including stop and go traffic. The ACC system can also allow for further automation by involving a decision-making layer that assesses the current or predicted traffic state (for example, by taking the knowledge of an on-ramp's position from a satellite to predict a bottleneck ahead). The ACC is thereby made up of two steps: The strategic layer and the operational layer. The strategic layer determines the current traffic state and the corresponding multiplier on the parameters, and the operational layer calculates the correct acceleration or deceleration based on a given microscopic model. The Intelligent Driver Model(IDM) is one such model that can be integrated into the ACC system.

1.1.1 How the ACC works

First, the user picks their desired speed v_0 and safe headway T. The sensors in the car can determine where the car is and how fast the car is going, x_α and v_α, respectively. It then uses other tools within the car to find the same information about the car in front of it, given by $x_{\alpha-1}$ and $v_{\alpha-1}$. s_α, the bumper-to-bumper distance from the car in front, and Δv_α, the velocity difference from the car in front, are calculated from these measurements. Then, the ACC takes data from satellites, reported traffic conditions, and position/velocity data from the car to decide what kind of traffic it is in. The options are Free, Upstream, Jam, Bottleneck, or Down. Each state has an associated multiplier $(\lambda_T, \lambda_a, \lambda_b)$ as given in table 1.1. ACC takes all of this information and decides what T, a, b should be input as parameters into the model. The model takes all this information and outputs the needed acceleration to maintain v_0 and T as consistently as possible.

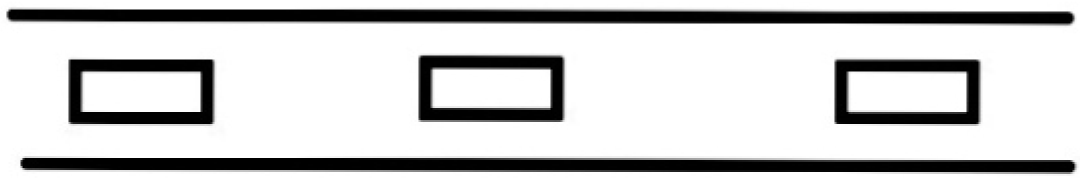

Figure 1.1: Free Traffic flows easily as each car attains its desired speed

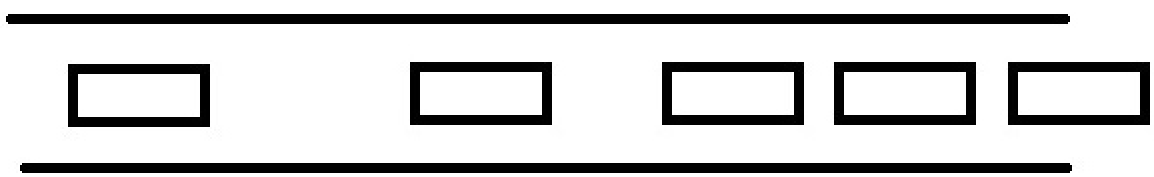

Figure 1.2: Upstream Traffic slows down as traffic becomes congested

The ACC determines which state of traffic the car is in independently of the model. The states of traffic are defined by the type of inflow, outflow, and the characterization of the inhomogeneity of the traffic. Each of the five states presents different challenges to the driver.

1. Free: All cars are spaced out enough that they can reach v_0 easily. The drivers are "free" to make all decisions. This state does not require anything more complicated than cruise control. However, using ACC can greatly enhance the drivers' comfort, since less attention is required to avoid collisions, except in extreme cases.

2. Upstream: This traffic state occurs when the driver is headed into congested traffic. "The objective is to increase safety by reducing velocity gradients [9]." Earlier and heavier braking is necessary since any ahead vehicle will likely be vastly slower.

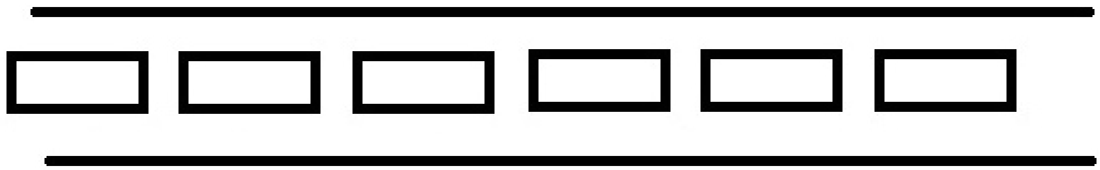

Figure 1.3: Jam Traffic prevents any car from attaining its desired speed

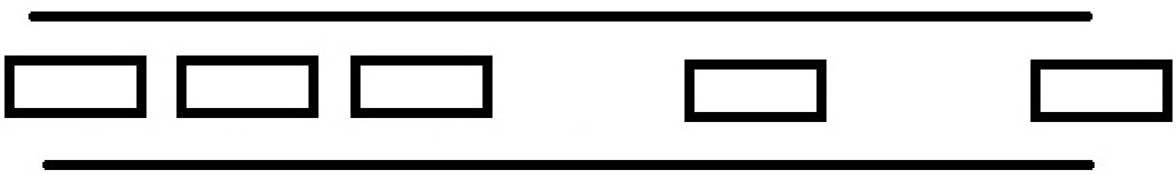

Figure 1.4: Downstream Traffic speeds up as cars leave the congested area

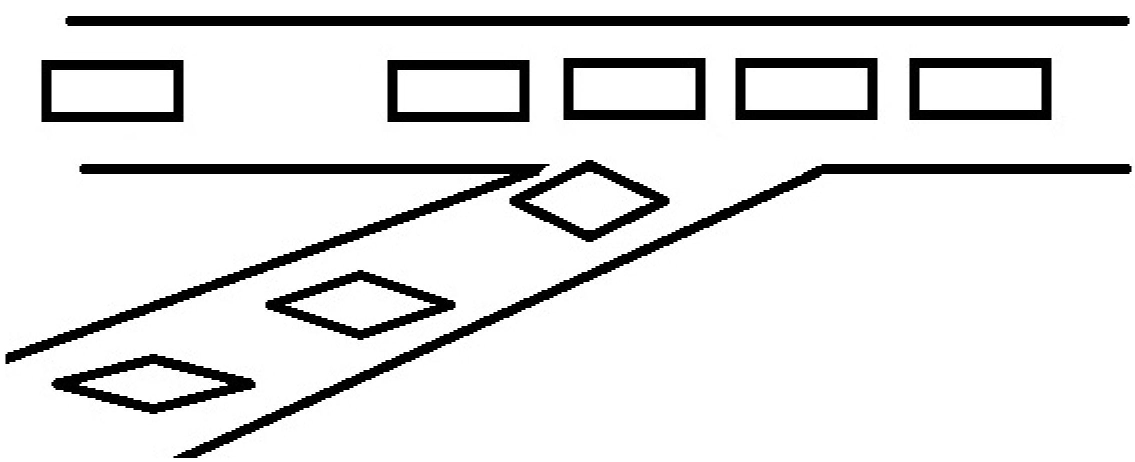

Figure 1.5: Bottleneck Traffic develops jam-like conditions as a response to physical differences on the road network

Traffic State	λ_T	λ_a	λ_b
Free	1	1	1
Upstream	1	1	1.7
Jam	1	1	1
Downstream	.5	1	2
Bottleneck	.5	1.5	1

Table 1.1: The ACC multipliers

3. Jam: Jam conditions are characterized by extremely low velocities. The driver is greatly restricted, and is moving in a stop-and-go fashion. Drivers, when given the option, tend to not use the ACC system in this type of traffic [11].

4. Downstream: This traffic state occurs when the driver is headed out of congested traffic. The goal is to decrease the time that any car has to spend in the congested area. It has been demonstrated that this will improve traffic efficiency. To meet this goal, time gaps are neglected since it can be assumed that the cars ahead will also be increasing their speed in the attempt to remove themselves from the jam situation and will not suddenly slow or stop. However, braking ability is also increased in the case that the jam condition is not truly over.

5. Bottleneck: This traffic state appears in particular locations along the road where there are physical reasons for decreases in capacity. Examples include lane closings or on-ramps. Here, drivers (without ACC) are more comfortable with leaving a large gap, wary of sudden lane or speed changes. The advantage for cars using ACC is that smaller time gaps than a human driver is willing to achieve are left intact, while still maintaining safety. Acceleration capability is also increased to get through the bottleneck area.

After the ACC strategic layer identifies the traffic state, the default measurements of T, a, and b are multiplied by the associated multiplier in order to attain the new value for the parameter before using it in the application of the model. As the table of multipliers indicates, the deviation from the default measurements only occurs a small portion of the time. However, when the nonstandard multipliers are used, the solutions have the possibility of losing their smoothness. In order to solve this problem, a smoothing function with respect to time was applied to the measured quantity of $x(t)$ in the simulations performed by the IDM researchers [9].

This smoothing function is given by an exponential moving average (EMA), where

$$x_{\alpha,EMA}(t) = \frac{1}{\tau}\int_{-\infty}^{t} e^{-(t-t')/\tau} x(t')dt' \tag{1.1}$$

which solvers the ODE

$$v_{\alpha,EMA}(t) = \frac{\partial x_{EMA}}{\partial t} = \frac{x_{\alpha}(t) - x_{\alpha,EMA}(t)}{\tau} \tag{1.2}$$

where $\tau = 5$ seconds is the relaxation time. This smoothing function provides a criteria for determining what traffic state the ACC should choose. The traffic state is chosen using the following rules:

1. Consider v_{free} as the threshold to enter the free state. This is determined by a high velocity. When

   $$v_{\alpha,EMA}(t) > v_{free},$$

   the ACC will choose "free."

2. Similarly, let v_{cong} be the threshold (determined by low velocity) that indicates entrance to jam conditions. Then ACC with choose "jam" when

   $$v_{\alpha,EMA}(t) < v_{cong}.$$

   Whereas these states are demonstrated by the value of the velocities, entering into an upstream or downstream jam front is characterized by the change in velocity compared to past time steps.

3. The ACC will choose "upstream" when the deceleration

$$v_\alpha(t) - v_{\alpha,EMA}(t) < -\Delta v_{up}$$

is detected and "downstream" when the acceleration

$$v_\alpha(t) - v_{\alpha,EMA}(t) > \Delta v_{down}$$

is detected.

4. Finally, since the bottleneck state is usually determined ahead of time by foreknowledge of the freeway design or information that can be found from the integration of satellite information, "bottleneck" is only chosen when both of the spatial criteria

$$x_\alpha(t) > x_{begin}$$

and

$$x_\alpha(t) < x_{end}$$

are met, where (x_{begin}, x_{end}) is the location any particular bottleneck.

After the decision layer has identified the traffic state and used the corresponding multipliers to adjust the parameters if needed, the operational layer begins. The output of this layer is the acceleration found by the chosen model.

Chapter 2

The Intelligent Driver Model

2.1 Literature Review

When approaching the problem of traffic congestion, there are two major classes of models. The first is macroscopic, which uses aggregate measures like density or flow to describe large-scale patterns, and is the classical approach. Two examples of macroscopic models are the gas-kinetic (GKT) model and the Kühne-Kerner-Konhäuser-Lee (KKKL) model [5, 10]. The second class is microscopic, which has benefited greatly from advancements in technology and applies the notion that individual drivers make separate (although likely related) decisions. A layer of complication can be implemented to either type through the use of stochasticity or meta-models.

Microscopic can be understood in the sense of cellular automata, but the primary form these models take are those with the aspect of time-continuity, the car-following models. Car following theory, the events that occur in which a car follows another without passing it, is used to "mimic the interaction between adjacent vehicles in a traffic stream [4]." Well known car following models include the Gazis-Herman-Rothery (GHR) model, the Optional Velocity (OV) model family, and the Collision Avoidance model [3, 2, 4].

The Intelligent Driver Model (IDM) is a deterministic car-following (time-continuous and autonomous) model in the OV family, with additional clauses to make it accident-free. The advantages that result from using this model are as follows [14]:

1. It is constructed to be accident-free by particular dependence on Δv_α.
2. All model parameters can be interpreted, are known to be relevant, are empirically measurable, and are within the expected order of magnitude.
3. The stability of the model can be easily calibrated to empirical data.
4. It can be quickly numerically simulated.
5. An equivalent macroscopic model is known [6].

2.2 Derivation

Car-following models are defined in acceleration functions. In early models [3, 14], the acceleration of a particular car (denoted α) is described by

$$v'_\alpha(t + T_r) = \frac{-\lambda v_\alpha^{m_1} \Delta v_\alpha}{s_\alpha^{m_2}}$$

where T_r is a reaction time, λ is a constant coefficient, m_1 and m_2 are constant exponents that determine the order of the ODE, and s_α is the bumper-to-bumper gap to the car ahead defined as

$$s_\alpha = x_{\alpha-1} - x_\alpha - l_\alpha$$

where l_α is the length of car α.

The deceleration is assumed to be proportional to the approach rate to the car ahead $v_{\alpha-1}$ which is given by

$$\Delta v_\alpha(t) = v_{\alpha-1}(t) - v_\alpha(t). \tag{2.1}$$

These types of models have a number of problems. Eqn (2.1) makes it clear that the acceleration is directly dependent upon another vehicle's behavior, which makes these models not applicable to free-traffic or low density situations. However, they are not particularly accurate to driver behavior in dense traffic situations since the gap s_α doesn't necessarily relax to an equilibrium value. In particular, if Δv_α is zero, even small values of s_α may not result in deceleration, which results in an accident. This general model also neglects to include the idea that the driver should not accelerate without limit when faced with a free road.

To solve this problem, we move into the optimal velocity model proposed by Newell [12]. This model is expressed by

$$v'_\alpha(t+T_r) = V(s_\alpha(t)) = v_0 \left[1 - e^{\dfrac{-(s_\alpha - s_0)}{v_0 T}} \right] \tag{2.2}$$

which includes a desired velocity v_0 that will allow vehicles to increase their speed when there is not another car in the way as well as a safe time headway T. By instituting a jam distance s_0 to keep cars separate, the model can be classified as collision-free. However, the added dependence on density results in unrealistically high accelerations on the order of $\dfrac{v_0}{T_r}$.

The final OV model proposed by Bando et al. [1] is given by

$$v'_\alpha = \frac{V(s_\alpha) - v_\alpha}{\tau} \tag{2.3}$$

which is very similar to the Newell model given that T_r is comparable to the velocity relaxation time τ, and is widely used based on its simplicity. However, keeping the equation collision-free requires $\tau < .9$ and acceleration is still problematically high. The reason this occurs for both OV models is that the vehicle interactions are not heavily considered, which is a stabilizing factor of real-life traffic situations.

The IDM is a highly complicated system in the same family. The fundamental idea behind this particular system is to combine the ability to reach the desired speed limit in a free traffic situation with the ability to identify how much braking is necessary to steer clear of any collision situations.

$$v'_\alpha(s_\alpha, v_\alpha, \Delta v_\alpha) = a_\alpha \left[1 - \left(\frac{v_\alpha}{v_{0,\alpha}} \right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2 \right] \tag{2.4}$$

where the desired gap s^* is determined by

$$s^*(v_\alpha, \Delta v_\alpha) = s_{0,\alpha} + s_{1,\alpha} \sqrt{\frac{v_\alpha}{v_{0,\alpha}}} + T_\alpha v_\alpha + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{a_\alpha b_\alpha}}. \tag{2.5}$$

and s_α is determined by equation (2.2). The parameters in this model are described below, where

Parameter	Symbol	Realistic Bounds	A Realistic Value
Desired Velocity	v_0	$[0, 11]$	3.33 (m/s)
Safe Time Headway	T	$[1, 3]$	1.6 (s)
Maximum Acceleration	a	$[.5, 2]$	.73 (m/s^2)
Comfortable Deceleration*	b	$[.5, 2]$	1.67 (m/s^2)
Acceleration Exponent	δ	—	4
Length of car	l	$4, 5$	4 (m)
Linear Jam Distance	s_0	$[0, 5]$	2 (m)
Non-linear Jam Distance	s_1	$[0, 5]$	3 (m)

Table 2.1: A summary of the Parameters Used

* The IDM will brake stronger than b if required by in an emergency situation.

each car α can have it's own individual parameter set.

2.3 Parameters Explained

The first variable $x_\alpha(t)$ is the position of a particular vehicle whose properties are known. Decisions made by the operational layer of the ACC are on the order of seconds. The update of time steps for the IDM are typically below .5 seconds. For simplification, each of the parameters will be judged to be equivalent for all vehicles. (i.e. $v_{0,\alpha} = v_0$).

The other variable $v_\alpha(t)$ is the velocity of the vehicle. The approach rate, defined as

$$\Delta v_\alpha = v_{\alpha-1} - v_\alpha.$$

The Jam distance is the bumper-to-bumper distance present in congested traffic situations that is enforced in order to avoid collisions. It has been split into two parameters: a linear and a nonlinear term. In an effort to keep the model from becoming too complex, the nonlinear jam distance was set to 0 in the model's proposal [14]. However, due to the presence of the safe time headway, T, and the maintenance of the gap $s_\alpha = x_{\alpha_1} - x_\alpha - l$, the model will remain collision-free even if s_1 is not 0.

2.4 The Model

The General IDM for any car α is described by

$$v'_\alpha(s_\alpha, v_\alpha, \Delta v_\alpha) = a\left[1 - \left(\frac{v_\alpha}{v_0}\right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)^2\right] \tag{2.6}$$

with

$$s^*(v_\alpha, \Delta v_\alpha) = s_{0,\alpha} + s_{1,\alpha}\sqrt{\frac{v_\alpha}{v_{0,\alpha}}} + T_\alpha v_\alpha + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{a_\alpha b_\alpha}}. \tag{2.7}$$

In order to examine how each component of the model controls the results, this function was reordered as directly dependent on v_α and transformed to an ODE system. Note that each term also includes position inside the distance function s_α. In order to simplify the examination of this system, one individual car α was specified. Here, only one other car is considered, $\alpha - 1$.

Let $x_1 = x_\alpha$ and $x_2 = x'_\alpha$=v_α

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\mathbf{f}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ g(x_1, x_2; q) \end{pmatrix} \tag{2.8}$$

$$q = (T, a, b, v_0, \delta, l, x_{\alpha-1}, v_{\alpha-1}, s_0, s_1)$$

$$\mathbf{x}_0 = \begin{pmatrix} x_1(0) = \hat{x} \\ x_2(0) = \hat{v} \end{pmatrix}$$

With $\hat{x}$ is the initial position, $\hat{v}$ is the initial velocity, and

$$g(x_1, x_2; q) = \alpha_0 x_2^\delta + \alpha_1 x_2^4 + \alpha_2 x_2^3 + \alpha_3 x_2^{5/2} + \alpha_4 x_2^2 + \alpha_5 x_2^{3/2} + \alpha_6 x_2 + \alpha_7 x_2^{1/2} + \alpha_8 \tag{2.9}$$

where

$$
\begin{aligned}
\alpha_0 &= \frac{-a}{v_0^\delta} \\
\alpha_1 &= -\frac{1}{4bs^2} \\
\alpha_2 &= \frac{\bar{v}}{4bs^2} - \frac{\sqrt{a}T}{\sqrt{b}s^2} \\
\alpha_3 &= \frac{\sqrt{a}s_1}{\sqrt{bv_0}s^2} \\
\alpha_4 &= \frac{\sqrt{a}s_0}{\sqrt{b}s^2} - \frac{aT^2}{s^2} - \frac{\bar{v}^2}{4bs^2} - \frac{\sqrt{a}\bar{v}T}{\sqrt{b}s^2} \\
\alpha_5 &= \frac{-2\sqrt{a}s_0 s_1 \bar{v}}{\sqrt{bv_0}s^2} - \frac{2as_1T}{\sqrt{v_0}s^2} \\
\alpha_6 &= \frac{-2as_1^2}{v_0 s^2} - \frac{\sqrt{a}s_0\bar{v}}{\sqrt{b}s^2} - \frac{2as_0T}{s^2} \\
\alpha_7 &= \frac{-2as_0s_1}{\sqrt{v_0}s^2} \\
\alpha_8 &= \frac{-as_0^2}{s^2} + a
\end{aligned}
$$

2.5 Parameter Estimation Problem

When running a simulation, the values in q are chosen ahead of time. Then the function calculates the parameterized coefficients, and an ODE solver can interpolate the function to find the position and velocity at any time. However, the values in q are only chosen in a way that is reasonable and reflects certain specified behavior. In practice, there is no rule to determine what q accurately describes an individual driver's behavior. The only information that is available is data about the position and velocity of each car in a study, so the inverse problem must be formulated in order to find an effective estimation of q.

Suppose N cars are observed over a time period from t_0 to t_f. For any vehicle j at any time $t \in [t_0, t_f]$, there is an associated $\mathbf{x_j}$ which consists of the simulated estimate of the position and the velocity of j-th vehicle, and $\hat{\mathbf{x}}_\mathbf{j}$ is the actual observed data that represents the position and velocity of the j-th vehicle.

The inverse problem to find the best estimate, denoted q_{opt}, is now formulated as a minimization of least squares:

$q_{opt} = \min J(q)$ where $J(q)$ is defined as follows:

$J(q) = \sum_{j=1}^{N} \|\mathbf{\hat{x}_j - x_j}\|_{\mathbf{inv}}$

where the $\|\cdot\|_{inv}$ norm is given by,

where $\|\mathbf{x} - \mathbf{y}\|_{\mathbf{inv}} = \sqrt{\int_{\mathbf{t_0}}^{\mathbf{t_f}} |\mathbf{x_1(t) - y_1(t)}|^{\mathbf{2}} + \ldots |\mathbf{x_N(t) - y_N(t)}|^{\mathbf{2}} \mathbf{dt}}$.

Starting at a reasonable guess q_0, as long as the function used to simulate data is continuous with regard to parameter, a minimum will always be found for this new function $J(q)$. This can be tested using the matlab function lsqnonlin.m.

Chapter 3

Analysis

The Intelligent Driver Model has now been expressed as a vector-valued autonomous ordinary differential equation. Under a few simple and practical assumptions, it can be shown that a unique solution for $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_\alpha \\ v_\alpha \end{pmatrix}$ exists.

First, it is useful to define the space that $\mathbf{x}$ exists in. Both x_1 and x_2 are in $\mathbb{R}$. Neither the position nor the velocity will ever be less than 0. Velocity is upper bounded by the fact that cars cannot go faster than a certain speed. Certainly any cars we are interested in will not go faster than 11 m/s (250 mph, 400 km/h). Position will only be upper bounded under the constraints of a particular problem. For example, for a two minute simulation, a car starting in the $\hat{x}$ position will not be able to move further than $\hat{x} + 1320$ meters away based on the above velocity bound.

Let the $|| \cdot ||$ norm be given by

$\|\mathbf{x} - \mathbf{y}\| = \max_{\mathbf{t_0 \leq t \leq t_f}} |\mathbf{x} - \mathbf{y}|$,

where $|\mathbf{x} - \mathbf{y}| = \sqrt{|\mathbf{x_1(t) - y_1(t)|^2 + |x_2(t) - y_2(t)|^2}}$.

3.1 Lipshitz Continuity

Definition 1. *Let* $\mathbf{f}$ *be a vector valued function on* E*, an open subset of* $\mathbb{R}^n$*.* $\mathbf{f}$ *satisfies a* ***Lipshitz condition*** *on* E *if there is a positive constant* K *such that for all* $\mathbf{x}, \mathbf{y} \in E$*,*

$$|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y})| \leq K|\mathbf{x} - \mathbf{y}| \tag{3.1}$$

Lemma 1. *[7, 13] If* $\mathbf{f}(\mathbf{x}, t)$ *has continuous partial derivatives on a compact domain* D*, then it satisfies a Lipshitz condition on* E*.*

Notation 1. *When* $\mathbf{f}$ *has continuous partial derivatives, we conclude that* $\mathbf{f} \in C^1(E)$*.*

Definition 2. $\mathbf{f}$ *is* ***Locally Lipshitz*** *on* E *if for every point* $\mathbf{x_0} \in E$*, there is an* ϵ*-neighborhood of* $\hat{\mathbf{x}}$*,* $N_\epsilon(\hat{\mathbf{x}}) \subset E$ *and a constant* $K_0 > 0$ *such that for all* $\mathbf{x}, \mathbf{y} \in N_\epsilon(\hat{\mathbf{x}})$*,*

$$|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y})| \leq K_0|\mathbf{x} - \mathbf{y}|. \tag{3.2}$$

Lemma 2. *[13] Let* E *be an open subset of* $\mathbb{R}^n$ *and let* $\mathbf{f} : E \to \mathbb{R}^n$*. Then if* $\mathbf{f} \in C^1(E)$*,* $\mathbf{f}$ *is locally Lipshitz on E.*

3.2 Existence and Uniqueness of the Solution

Theorem 1 (The Fundamental Existence-Uniqueness Theorem [13])**.** *Let* E *be an open subset of* $\mathbb{R}^n$ *containing* $\hat{\mathbf{x}}$ *and assume that* $\mathbf{f} \in C^1(E)$*. Then there exists an* $m > 0$ *such that the initial value problem*

$$\mathbf{x}' = \mathbf{f}(\mathbf{x})$$

$$\mathbf{x}(0) = \hat{\mathbf{x}}$$

has a unique solution $\mathbf{x}(t)$ *on the interval* $[-m, m]$*.*

Proof. Since $\mathbf{f} \in C^1(E)$, it follows from Lemma 2 that there is an ϵ-neighborhood $N_\epsilon(\hat{\mathbf{x}}) \subset E$ and a constant $K > 0$ such that $\forall \mathbf{x}, \mathbf{y} \in N_\epsilon(\mathbf{x}_0)$,

$$|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y})| \leq K|\mathbf{x} - \mathbf{y}|.$$

Let $b = \epsilon/2$. Then the continuous function $\mathbf{f}(\mathbf{x})$ is bounded on the compact set

$$N_0 = \{\mathbf{x} \in \mathbb{R}^n || \mathbf{x} - \hat{\mathbf{x}}| \leq b\}$$

Let

$$M = \max_{\mathbf{x} \in N_0} |\mathbf{f}(\mathbf{x})|.$$

Let the successive approximations $\mathbf{u}_k(t)$ be defined by

$$\mathbf{u}_0(t) = \mathbf{x}_0$$
$$\mathbf{u}_{k+1}(t) = \mathbf{x}_0 + \int_0^t \mathbf{f}(\mathbf{u}_k(s))ds$$

.

Assuming that $\exists$ an $m > 0$ such that $\mathbf{u}_k(t)$ is defined and continuous on $[-m, m]$ and satisfies

$$\max_{t \in [-m,m]} |\mathbf{u}_k(t) - \mathbf{x}_0| \leq b.$$

Therefore

$$||\mathbf{u}_{k+1}(t) - \mathbf{x}_0|| \leq \int_0^t ||\mathbf{f}(\mathbf{u}_k(s))||ds \leq Ma \tag{3.3}$$

for all $t \in [-m, m]$. Thus, specifying $0 < m \leq b/M$, it follows that $\mathbf{u}_k(t)$ is defined, continuous, and satisfies equation (3.11) for all $t \in [-m, m], k = 1, 2, 3...$

It follows from the Lipshitz condition from Lemma 1 for $\mathbf{f}$ that for all $t \in [-m, m]$,

$$\begin{aligned}
||\mathbf{u}_2(t) - \mathbf{u}_1(t)|| &\leq \int_0^t ||\mathbf{f}(\mathbf{u}_1(s)) - \mathbf{f}(\mathbf{u}_0(s))||ds \\
&\leq K \int_0^t ||\mathbf{u}_1(s) - \mathbf{u}_0(s)||ds \\
&\leq Km \max_{t\in[-m,m]} |\mathbf{u}_1(t) - \mathbf{x}_0| \\
&\leq Kmb.
\end{aligned}$$

Now assume

$$\max_{t\in[-m,m]} |\mathbf{u}_j(t) - \mathbf{u}_{j-1}(t)| \leq (Ka)^{j-1}b \tag{3.4}$$

for some integer $j \geq 2$, then for any $t \in [-m, m]$,

$$\begin{aligned}
||\mathbf{u}_{j+1}(t) - \mathbf{u}_j(t)|| &\leq \int_0^t ||\mathbf{f}(\mathbf{u}_j(s)) - \mathbf{f}(\mathbf{u}_{j-1}(s))||ds \\
&\leq K \int_0^t ||\mathbf{u}_j(s) - \mathbf{u}_{j-1}(s)||ds \\
&\leq Km \max_{t\in[-m,m]} |\mathbf{u}_j(t) - \mathbf{u}_{j-1}(t)| \\
&\leq (Km)^j b.
\end{aligned}$$

Let $\alpha = Km$ and choose $0 < m < 1/K$. Then for any $p > k \geq N$ and $t \in [-m, m]$,

$$\begin{aligned}
||\mathbf{u}_m(t) - \mathbf{u}_k(t)|| &\leq \sum_{j=k}^{p-1} ||\mathbf{u}_{j+1}(t) - \mathbf{u}_j(t)|| \\
&\leq \sum_{j=N}^{\infty} ||\mathbf{u}_{j+1}(t) - \mathbf{u}_j(t)|| \\
&\leq \sum_{j=N}^{\infty} \alpha^j b \\
&= \frac{\alpha^N}{1-\alpha} b.
\end{aligned}$$

As $N \to \infty$, $\dfrac{\alpha^N}{1-\alpha} b \to 0$.
Thus, for all $\epsilon > 0$, $\exists N$ such that $p, k \geq N$ implies

$$||\mathbf{u}_p - \mathbf{u}_k|| = \max_{t \in [-m,m]} |\mathbf{u}_p(t) - \mathbf{u}_k(t)| < \epsilon. \tag{3.5}$$

By definition, $\{\mathbf{u}_k\}$ is now a Cauchy sequence in $C[-m, m]$. $\mathbf{u}_k(t)$ converges to the continuous function $\mathbf{u}(t)$ uniformly as $k \to \infty$.

When

$$\mathbf{u}(t) = \lim_{k \to \infty} \mathbf{u}_k(t), \tag{3.6}$$

then the limit of the successive approximations satisfies

$$\mathbf{u}(t) = \hat{\mathbf{x}} + \int_0^t \mathbf{f}(\mathbf{u}(s)) ds \tag{3.7}$$

and $\mathbf{u}'(t) = \mathbf{f}(\mathbf{u}(t))$ with $\mathbf{u}(0) = \hat{\mathbf{x}}$. Thus $\mathbf{u}(t)$ is a solution of the system.

In order to prove uniqueness, suppose $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are two distinct solutions of the system on $[-m, m]$. By the Maximum Principle, the continuous function $||\mathbf{u}(t) - \mathbf{v}(t)||$ finds its maximum on $t_1 \in [-m, m]$.

$$\begin{aligned}
||\mathbf{u} - \mathbf{v}|| &= \max_{t \in [-m,m]} |\mathbf{u}(t) - \mathbf{v}(t)| \\
&= |\int_0^{t_1} \mathbf{f}(\mathbf{u}(s)) - \mathbf{f}(\mathbf{v}(s)) ds| \\
&\leq \int_0^{|t_1|} |\mathbf{f}(\mathbf{u}(s)) - \mathbf{f}(\mathbf{v}(s))| ds \\
&\leq K \int_0^{|t_1|} |\mathbf{u}(s) - \mathbf{v}(s)| ds \\
&\leq Km \max_{t \in [-m,m]} |\mathbf{u}(t) - \mathbf{v}(t)| \\
&= Km ||\mathbf{u} - \mathbf{v}||.
\end{aligned}$$

When $Km < 1$ by construction of the existence proof. Thus $||\mathbf{u} - \mathbf{v}|| = 0$, so $\mathbf{u}(t) = \mathbf{v}(t)$ on $[-m, m]$.

$\square$

Theorem 2 (Dependence on Initial Conditions)**.** *Let E be an open subset of $\mathbb{R}^{n+m}$ containing $\mathbf{x}_0$ and assume $\mathbf{f} \in C^1(E)$. It follows that $\exists$ an $m > 0$ and $\gamma > 0$ such that for all $\mathbf{y} \in N_\gamma(\hat{\mathbf{x}})$, the initial value problem with*

$$\mathbf{x}' = \mathbf{f}(\mathbf{x})$$

$$\mathbf{x}(0) = \mathbf{y}$$

has a unique solution $\mathbf{u}(t, \mathbf{y})$ with $\mathbf{u} \in C^1(G)$ where $G = [-m, m] \times N_\gamma(\hat{\mathbf{x}}) \subset \mathbb{R}^n$. Furthermore, for each $\mathbf{y} \in N_\gamma(\hat{\mathbf{x}})$, $\mathbf{u}(t, \mathbf{y})$ is a twice continuously differentiable function of t for $t \in [-m, m]$.

Theorem 3 (Dependence on Parameters)**.** *Let E be an open subset of $\mathbb{R}^{n+r}$ containing the point $(\hat{\mathbf{x}}, \hat{\mathbf{q}})$ where $\hat{\mathbf{x}} \in \mathbb{R}^n$ and $\hat{\mathbf{q}} \in \mathbb{R}^r$ and assume $\mathbf{f} \in C^1(E)$. It follows that $\exists$ an $m > 0$ and $\gamma > 0$ such that for all $\mathbf{y}, \mathbf{q} \in N_\gamma(\hat{\mathbf{x}})$, the initial value problem with*

$$\mathbf{x}' = \mathbf{f}(\mathbf{x}, \mathbf{q})$$

$$\mathbf{x}(0) = \mathbf{y}$$

has a unique solution $\mathbf{u}(t, \mathbf{y}, \mathbf{q})$ with $\mathbf{u} \in C^1(G)$ where $G = [-m, m] \times N_\gamma(\hat{\mathbf{x}}) \times N_\gamma(\hat{\mathbf{q}})$.

3.3 Analysis on the IDM

Claim 1. *The function defined for the IDM has continuous partial derivatives.*

Proof. The partial derivatives can be found directly.

$$\frac{\partial f_1}{\partial x_1} = 0.$$
$$\frac{\partial f_2}{\partial x_1} = \hat{\alpha_0} x_2^\delta + \hat{\alpha_1} x_2^4 + \hat{\alpha_2} x_2^3 + \hat{\alpha_3} x_2^{5/2} + \hat{\alpha_4} x_2^2 + \hat{\alpha_5} x_2^{3/2} + \hat{\alpha_6} x_2 + \hat{\alpha_7} x_2^{1/2} + \hat{\alpha_8}.$$

where $\hat{\alpha_0} = 0$, $\hat{\alpha_i} = \alpha_i \cdot \dfrac{2}{(x_{\alpha-1} - l - x_\alpha)}$, and $i \in \{1, .., 8\}$.

The only point at which this is not continuous is the same point at which the original function is not defined. That is, when $(x_{\alpha-1} - l - x_\alpha) = 0$, which means that the gap between the cars is zero. The function is constructed with this problematic area in mind, since this would indicate a collision. Keeping the data collision free is of utmost importance.

$$\frac{\partial f_1}{x_2} = 1.$$
$$\frac{\partial f_2}{x_2} = \delta\alpha_0 x_2^{\delta-1} + 4\alpha_1 x_2^3 + 3\alpha_2 x_2^2 + (5/2)\alpha_3 x_2^{3/2} + 2\alpha_4 x_2 + (3/2)\alpha_5 x_2^{1/2} + \alpha_6 + (1/2)\alpha_7 x_2^{-1/2}.$$

There is one discontinuity at $x_2 = 0$, which is due to the last term(and the first one, if $\delta \leq 1$). When $x_2 = 0$, the car is stopped, so by heuristic reasoning, this should result in divergent behavior. Additionally, this behavior should not occur on the freeway except in extreme conditions. When $s_1 = 0$ in the simplified version of the model, $\alpha_7 = 0$, then this term is nonexistent.

Therefore, on the domain $E \subset \mathbb{R}$ where $x_1 \in \{\mathbb{R}_{\geq 0} \backslash (x_{\alpha-1} - l - x_\alpha)\}$ and $x_2 \in \{\mathbb{R}_+\}$, then $\mathbf{f}$ has continuous partial derivatives. $\square$

Chapter 4

Simulation Results

The position and velocity in these simulations were found using ode45 in Matlab, which uses Runge-Kutta methods to solve systems of ordinary differential equations. The parameters that were used to attain the results will be in the form $q = (T, a, b, v_0, \delta, l, x_{\alpha-1}, v_{\alpha-1}, s_0, s_1)$

4.1 Two-Car

Before simulating a general number of cars, we set up a two-car simulation(see listing process.m). The parameters used were $(1.6, .73, 1.67, 3.33, 4, l, x_1, v_1, 2, 3)$ and each simulation watched the first two minute interval.

The basic assumption here is that the velocity of the first car v_1 would be a sine function of time. In order to make this a reasonable parameter, I let value of this function be centered at the desired speed v_0 and vary by 1.2 m/s(2.68 miles/hr, 4.2 km/hr) . This sine function goes through one cycle in a minute.

The position x_1 will be found by integrating over the velocity on the time period and adding the position that the first car starts at.

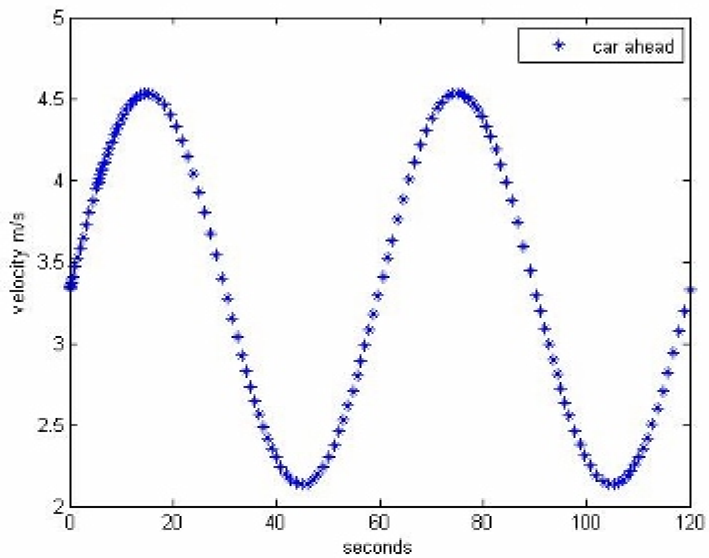

Figure 4.1: This is v_1, assuming we know it follows the sine function described above.

4.1.1 Example One

The first car started 100 m ahead of the second car and followed a sinusoidal velocity as described above.. The second car started at position 0 at a speed of .055 m/s. Since the first car was far away and traveling near desired speed, the second car accelerated to desired speed and remained there with no significant disruption.

4.1.2 Example Two

In order to examine how the cars affect each other, they needed to be closer together. Therefore, the first car was started 20 m away. The second car also started at the desired speed. However, as the second car approached the first one, it was forced to decrease speed and wait for the first car to increase the distance between them again. Although the velocity of the second car is still around the desired speed, there is more variation since it is dependent on the behavior of car one.

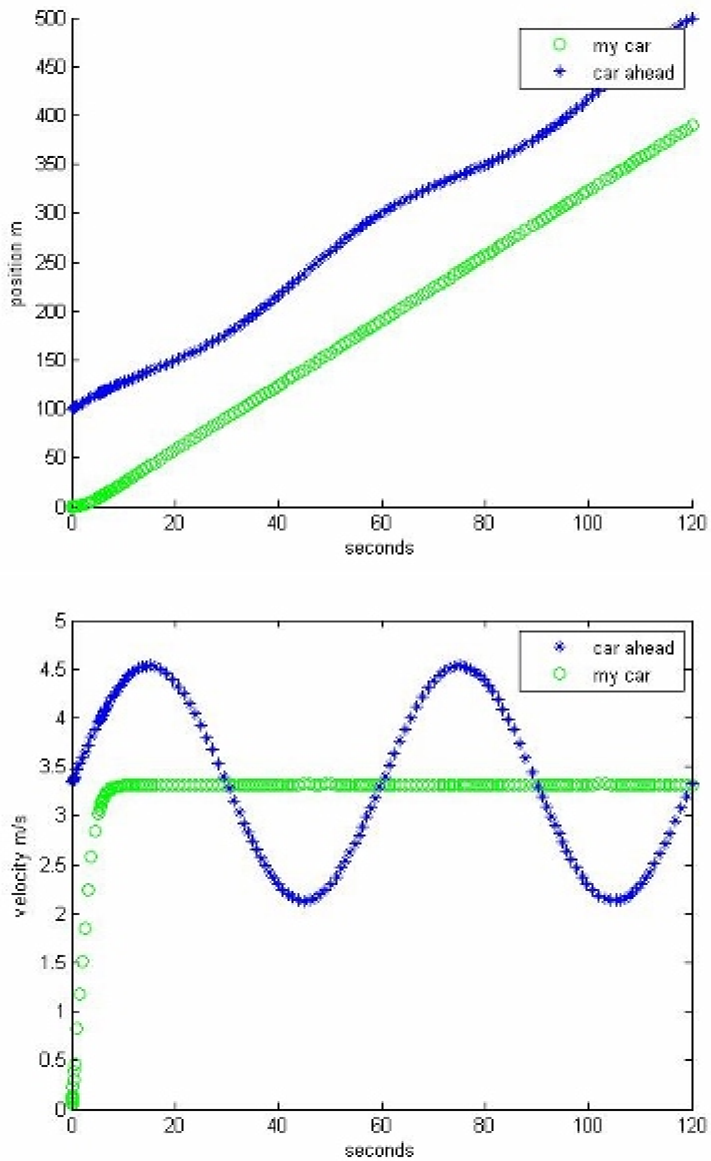

Figure 4.2: Example One: These graphs simulate the movement of each car as they move independently of each other

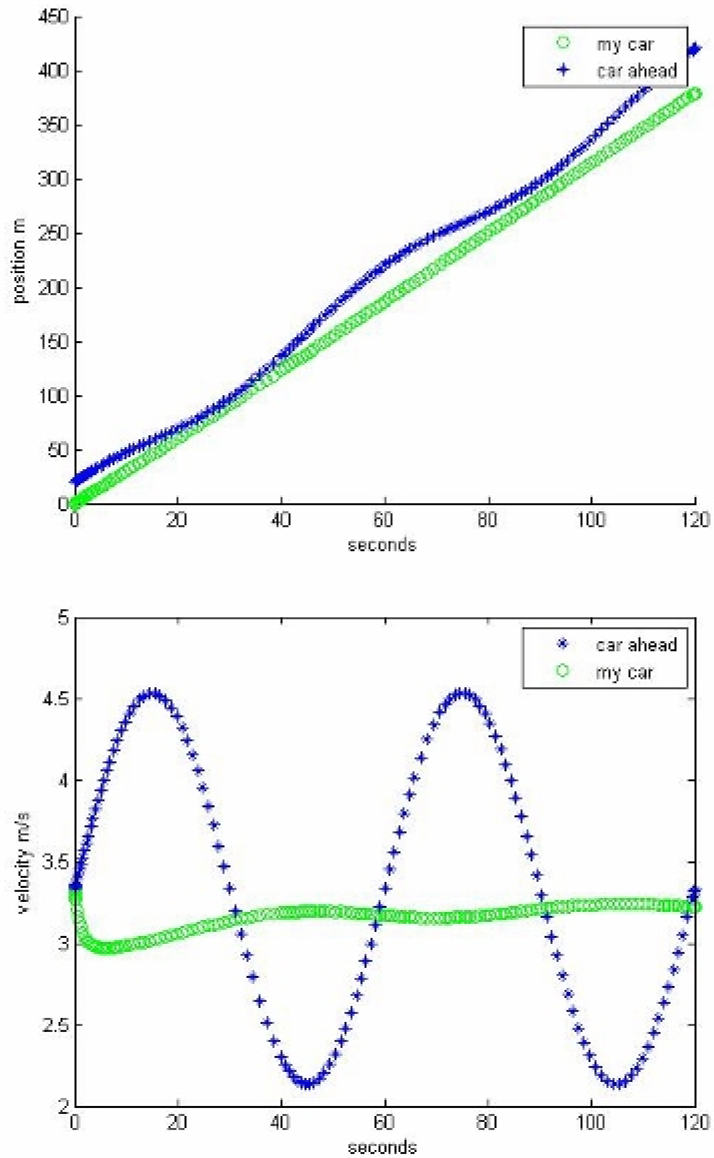

Figure 4.3: Example Two: These graphs simulate the movement of each car as they interact with each other

4.2 N-Car Simulation

Given a general number of cars, the IDM can still be solved with the ode solver. Consider a vector $\mathbf{x}$ with N car positions, followed by N car velocities.
Now, applying the function $\mathbf{f}$ results in a new system:

$$\begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \\ v_1' \\ v_2' \\ \vdots \\ v_n' \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \\ v_1' \\ v_2' \\ \vdots \\ v_n' \end{pmatrix} = \mathbf{f} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} x_{n+1} \\ x_{n+2} \\ \vdots \\ x_{2n} \\ g(x_1, x_{n+1}, x_n - x_{n+1}) \\ g(x_2, x_{n+2}, x_{n+1} - x_{n+2}) \\ \vdots \\ g(x_n, x_{2*n}, x_{2*n-1} - x_{2*n}) \end{pmatrix} \tag{4.1}$$

where

$$g(x(i), x(n+i), x(n+i-1) - x(n+i)) = \alpha_0 x(n+i)^\delta + \alpha_1 x(n+i)^4 + \alpha_2 x(n+i)^3 + \alpha_3 x(n+i)^{5/2} + ...$$
$$\alpha_4 x(n+i)^2 + \alpha_5 x(n+i)^{3/2} + \alpha_6 x(n+i) + \alpha_7 x(n+i)^{1/2} + \alpha_8.$$

In the simulations below, N is set to 20 cars. Of course, in order to record the behavior of the first car, we need information about the car in front of it. In the free traffic scenario, this imaginary further car is set at some distance that will not be reached by any car in the first two minutes at the speed (800 m) and is assumed to be traveling at the desired speed. In the congested traffic scenario, it is set both closer (300 m) and slower (about 1/4th of the desired speed) in order to establish the beginning of the next jam. Equivalently, this could be considered the beginning of the a bottlenecked area, since the specific problematic loacation is known.

Every input parameter except those that are related to the previous car's behavior is set to the same values as the simple example.

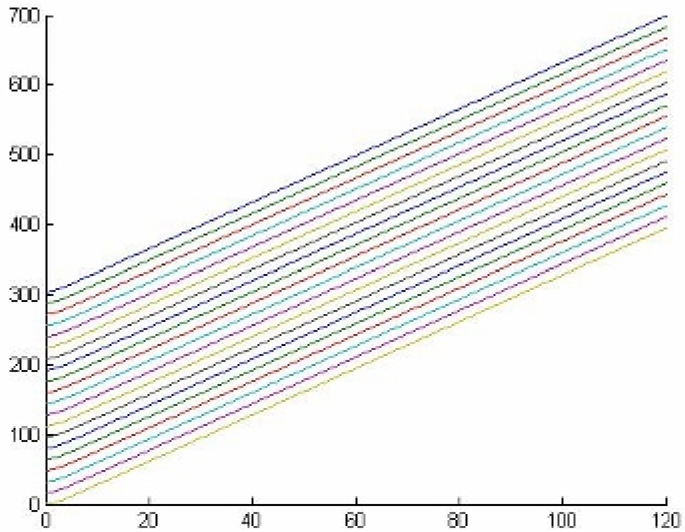

Figure 4.4: This graph simulates the movement of each car as they behave independently

4.2.1 Free Traffic

In the free traffic state, all of the cars are set to be a set distance apart to begin with. They are all traveling at the same speed, and there is no need for them to interact. Unsurprisingly, the pattern is uniform and linear.

4.2.2 Changing Traffic States

In order to simulate mutiple stages of traffic, the cars were separated into two groups. (see listing multiprocess.m) The front group's speed was lowered and the cars were pushed closer together to begin with. The back group has a fast speed to begin with, so they caught up with the front group quickly, making the first distinct state a traffic jam. However, as there was no limit on the first car, it was able to speed up to the desired speed, followed by the others. Eventually, there is another slow car on the road that the first car cannot pass. At that point it slows down, again followed by both groups.

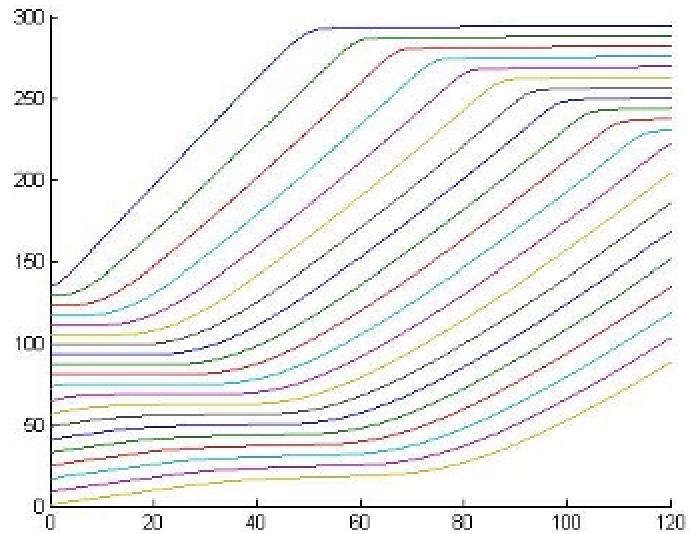

Figure 4.5: The cars begin in a jam situation, move into a downstream, then free state, before entering into an upstream

Chapter 5

Conclusions and Discussion

5.1 Limitations

One important limitation to note is that the Intelligent Driver Model (IDM) does not include any provisions for multi-lane traffic. The simulations performed were demonstrated on one lane traffic in order to examine the behavior of this model alone. However, appending a lane-changing model like MOBIL can explore the effects of multi-lane car following without developing a more complex model [8].

There are a few significant differences between behavior that human drivers demonstrate and those represented by the Individual Driver Model. Two realistic disadvantages of the IDM are that drivers can use spatial and temporal anticipation. Although the IDM only uses the information from the car directly in front, it is certainly reasonable to expect that a driver can see several cars ahead, as well as identify developing traffic situations that will demand future action. However, it is well known that it can take upwards of two seconds for the human brain to react to an ongoing traffic situation, which is beyond the time represented by T, which is the time given to put actions into practice [4]. It is also evident that many drivers suffer from limited or divisible attention spans, as well as perception errors.

Although the first two differences are a stabilizing influence, as they help drivers react appropriately to situations, any perception errors or high reaction time are destabilizing influences that can negatively affect the flow of traffic. The question, then, is whether the effects of these changes are dominated by the stabilizing or destabilizing influences, or whether they effectively nullify each other.

One method of examining these influences is to institute a meta-model onto established models. The Human Driver Model (HDM) is a meta-model that establishes rules based on physical systems onto time-continuous microscopic models [15]. In particular, the HDM can be applied to models that take the general form

$$\frac{\partial v_\alpha}{\partial t} = af(\Delta x_\alpha, v_\alpha, \Delta v_\alpha) \tag{5.1}$$

which certainly matches the form of the IDM.

5.1.1 Finite Reaction Time and Imperfect Estimations

A reaction time T' is instituted. Although T' and T may be on the same order, they are conceptually disparate parameters. Whereas T is a parameter that represents the time it takes to change the car's behavior, T' is a physiological parameter that represents the time it takes for a person to realize they need to make the change. Equation (5.1) can be evaluated at the time $t - T'$.

Estimation errors must, by their nature, include measures of stochasticity, as the severity of the error will depend highly on the person and situation. Since velocity is reported reasonably accurately by the speedometer, a driver may be expected to make their biggest errors in terms of measuring the distance and speed of the car ahead. The HDM uses a Weiner Process to model this estimation error [15] .

5.1.2 Spatial and Temporal Anticipation

The spatial anticipation correction first delineates the acceleration calculation of equation (5.1) into a "free" road behavior and any interaction terms. Then the interaction terms can be summed up to reflect the number of cars ahead that are expected to affect the car.
Assuming that drivers are aware of the need for a reaction time, they will attempt to improve their

driving ability by trying to anticipate the changing traffic status. In order to estimate the future velocity, a constant-acceleration heuristic rule can be applied, and in order to estimate the future distance and velocity difference, a constant-velocity heuristic will be used. In order to use these heuristics, it is assumed that this process only compensates for unsurprising traffic situations. For example, if a light turns yellow, the driver will expect the car in front of them to slow down at a constant rate. However, this will not be able to compensate for sudden braking behavior. The equations containing these heuristics will also maintain the reaction time and estimation errors.

5.2 Conclusion

There are many advantages to having the IDM as a tool. One that was not previously discussed in this paper is that there is an equivalent macroscopic model. The choice of which to use is not just a scientific question, but also a philosophical one: does traffic flow as one being, or does it act as individual particles? Looking at traffic from a microscopic view makes a lot of sense, since each driver makes their own decisions. However, it is especially clear with car-following models that unless one is completely alone on the road, the driver alone is not responsible for their behavior. In this case, the microscopic method was clearly a superior choice, because the main application is to use the Adaptive Cruise Control that only has an affect on the car it is equipped upon. Adaptive Cruise Control also does not have a large effect while in stop-and-go traffic, mostly because the driver themselves has greatly limited options.

Despite the limitations seen above, the IDM (without corrections) is still a complex equation, weaving together a wide base of factors. Much like many other models, was written in a format that was useful to directly implement it, but not for observing the underlying mathematical composition. By transforming it into a familiar presentation, the dependence upon different components becomes available to explore in a methodological fashion. By looking at the function as a direct ODE, general analytic theory enabled conclusions to be made about existence, uniqueness, and stability. One extension of this paper would be to determine parameter sensitivity. In the simulations performed, the initial placement and velocity of the cars were the most drastic factors to witness different behavior. Deviating from the reasonable parameters given did not affect the situation greatly unless they were changed to reflect very unlikely scenarios. However, one reason for this seeming stability is that every driver was assumed to have the same abilities and desires, which is not an entirely realistic scenario. Another extension is the calibration of parameters, namely, by the least squares optimization method shown in chapter 2. This can also demonstrate whether the nominal guess q_0 was a good choice in the first place and can provide added legitimacy to previously performed simulations in diverse situations.

Finally, returning to the idea of the next generation of the ACC, a few conjectures may be made. Certainly, adding conditions to avoid rear-end collisions will widen the parameter pool further, but it should not drastically change the careful construction of the IDM. As sensor technology increases in scope and power, it is likely that prediction of heavy traffic areas independent of expected bottlenecks will be relayed to the ACC, opening the door to a more complicated strategy layer that could very well make even more specialized use of the IDM. In particular, the HDM's advantage of spatial anticipation may disappear as the sensors become more sensitive to more information.

Appendix

```
% % % % % % % % %idm.m % % % % % % % % % % % % % % %
% % % % % % % % % % % % % % % % % % % % % % % % % % %

function f=idm(t,x,a,b,T,v0,l,s0,s1,del,A,om,t0,xup)

% %This should output the function values of the IDM for one car % %
% %x(1)  position, x(2) velocity % %

% % %calculate position and velocity for the car ahead % % %
va=v0+A*sin(om*t);
xa=xup+v0*(t-t0)+((A/om)*cos(om*(t0))-cos(om*t));

% % %gap to car ahead% % %
s=xa-l-x(1);
ss=s^2;

% % %coefficients for the RHS % % %
alpha0=-a/(v0^del);
alpha1=-1/(4*b*ss);
alpha2=(va/(2*b*ss)-(sqrt(a)*T)/(sqrt(b)*ss));
alpha3=(sqrt(a)*s1)/(sqrt(b*v0)*ss);
alpha4=((sqrt(a)*s0)/((sqrt(b)*ss)))-((a*T^2)/ss)...
    -((va*va)/(4*b*ss))-((sqrt(a)*va*T)/((sqrt(b)*ss)));
alpha5=((-2*sqrt(a)*s0*s1*va)/(sqrt(b*v0)*ss))...
    -((2*a*s1*T)/(sqrt(v0)*ss));
alpha6=((-2*a*s1^2)/(v0*ss))-...
    ((sqrt(a)*s0*va)/(sqrt(b)*ss))-((2*a*s0*T)/ss);
alpha7=((-2*a*s0*s1)/(sqrt(v0)*ss));
alpha8=((-a*s0^2)/ss)+a;


% % %Set function equations% % %
f=zeros(2,1);
f(1)=x(2);
f(2)=alpha0*x(2)^del+alpha1*x(2)^4+alpha2*x(2)^3+alpha3*x(2)^(5/2)...
    +alpha4*x(2)^2+alpha5*x(2)^(3/2)+alpha6*x(2)+alpha7*x(2)^(1/2)+alpha8;
end

% % % % % % % % % % % % % % % % % % % % % % % % % % % % %
```

```
% % % % % % % % % % % % %process.m % % %  %  %  % % % % % %
%  % % % % % % % % % % % % % % % % % % % % % % % % % % %

%Simulation of two cars using the IDM %
%Set Parameters
a=.73;      %acceleration
b=1.67;     %comfortable deceleration
T=1.6;      %safe time gap
v0=120/36;  %desired speed
l=4;        %average length of car
s0=2;       %jam distance
s1=3;       %jam distance nonlinear
del=4;      %acceleration exponent


xup=100;    %how far ahead the other car is
om=2*pi/60; %rate of oscillation of ahead car
A=1.2;      %amplitude of oscillation of ahead car

y0=[0 2/36];       %initial position, velocity
tspan=[0 120];     %two minute time window


%Solve the system using ode45
t0=tspan(1);
[tout,yout]=ode45(@(t,y) idm(t,y,a,b,T,v0,l,s0,s1,del,A,om,t0,xup),tspan,y0);
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

%plot the results

hold all;
figure(1);
plot(tout,yout(:,1),'og');
xa=xup+v0*(tout-t0)+(A/om)*(cos(om*tout)-cos(om*t0));
plot(tout,xa,'*b');
legend('my car','car ahead');

hold all;
figure(2);
plot(tout,yout(:,2),'og');
va=v0+A*sin(om*tout);
plot(tout,va,'*b');
legend('car ahead', 'my car');
```

```
% % % % % % % % % % %multiprocess.m % % % % % % %
% % % % % % % % % % % % % % % % % % % % % % % %
% % %%Simulation of N cars using the IDM % % % %
%Set Parameters

a=.73;      %acceleration
b=1.67;     %comfortable deceleration
T=1.6;      %safe time gap
v0=120/36;  %desired speed
l=4;        %average length of car
s0=2;       %jam distance
s1=3;       %jam distance nonlinear
del=4;      %acceleration exponent

n=20;        %number of cars

```

```
y0 = zeros(2*n,1);

for i=1:n/2
    y0(i)= 21+(s0+l)*(n-i)
end
for i=(n/2)+1:n
    y0(i)=(s0+l+2)*(n-i)
end

for i=n+1:n+(n/2)
    y0(i)=30/36
end

for i=n+(n/2)+1:2*n
    y0(i)=120/36
end

%initial position, velocity
tspan=linspace(0, 120, 600);      %two minute time window

%Solve the system using ode45
[tout,yout]=ode45(@(t,y) multidm(t,y,n,a,b,T,v0,l,s0,s1,del),tspan,y0);

    hold all;
    figure(1);
    for i=1:n
        plot(tout,yout(:,i));
    end
```

```
% % % % % % % % % %multidm.m % % % % % % % % % %
% % % % % % % % % % % % % % % % % % % % % % % %
function f=multidm(t,x,n,a,b,T,v0,l,s0,s1,del)
%This should output the function values of the IDM for N cars%
%x(1)..x(n)  position, x(n+1)..x(2n) velocity%
%x(1) is the front car, x(n) is the last car%
%initialize vector%

f = zeros(2*n,1);
s=zeros(n,1);

alpha0=0.0;
alpha1=0.0;
    alpha2=0.0;
    alpha3=0.0;
    alpha4=0.0;
    alpha5=0.0;
    alpha6=0.0;
    alpha7=0.0;
    alpha8=0.0;

  %calculate the behavior of the 1st car%
va = v0/4;
xinf = 300;
s(1)= xinf-l-x(1);
ss=s(1)^2;
alpha0=-a/(v0^del);
alpha1=-1/(4*b*(ss));
alpha2=(va/(2*b*ss))-(sqrt(a)*T)/(sqrt(b)*ss);
alpha3=(sqrt(a)*s1)/(sqrt(b*v0)*ss);
alpha4=((sqrt(a)*s0)/(sqrt(b)*ss))-((a*T^2)/ss)-((va*va)/(4*b*ss))-...
```

```
    ((sqrt(a)*va*T)/(sqrt(b)*ss));
alpha5=((-2*sqrt(a)*s0*s1*va)/(sqrt(b*v0)*ss))-((2*a*s1*T)/(sqrt(v0)*ss));
alpha6=((-2*a*s1^2)/(v0*ss))-((sqrt(a)*s0*va)/(sqrt(b)*ss))-((2*a*s0*T)/ss);
alpha7=((-2*a*s0*s1)/(sqrt(v0)*ss));
alpha8=((-a*s0^2)/ss)+a;

f(1)=x(n+1);
f(n+1) = alpha0*x(n+1)^del+alpha1*x(n+1)^4+alpha2*x(n+1)^3+alpha3*x(n+1)^(5/2)...
    +alpha4*x(n+1)^2+alpha5*x(n+1)^(3/2)+alpha6*x(n+1)+alpha7*x(n+1)^(1/2)+alpha8;

%calculate the behavior for the 2 to the nth car%

for i=2:n;

       alpha0=0.0;
    alpha1= 0.0;
    alpha2=0.0;
    alpha3= 0.0;
    alpha4=0.0;
    alpha5= 0.0;
    alpha6= 0.0;
    alpha7= 0.0;
    alpha8= 0.0;

    va = x(n+i-1);
    s  = x(i-1)-l-x(i);
    ss=s^2;


    alpha0=-a/(v0^del);
    alpha1=-1/(4*b*(ss));
    alpha2=(va/(2*b*ss))-(sqrt(a)*T)/(sqrt(b)*ss);
    alpha3=(sqrt(a)*s1)/(sqrt(b*v0)*ss);
    alpha4=((sqrt(a)*s0)/(sqrt(b)*ss))-((a*T^2)/ss)-((va*va)/(4*b*ss))-...
    ((sqrt(a)*va*T)/(sqrt(b)*ss));
    alpha5=((-2*sqrt(a)*s0*s1*va)/(sqrt(b*v0)*ss))-((2*a*s1*T)/(sqrt(v0)*ss));
    alpha6=((-2*a*s1^2)/(v0*ss))-...
    ((sqrt(a)*s0*va)/(sqrt(b)*ss))-((2*a*s0*T)/ss);
    alpha7=((-2*a*s0*s1)/(sqrt(v0)*ss));
    alpha8=((-a*s0^2)/ss)+a;

    temp=0;
    f(i)=x(n+i);
    temp=x(n+i);

    f(n+i) = alpha0*temp^del+alpha1*temp^4+alpha2*temp^3 ...
        +alpha3*temp^(5/2)+alpha4*temp^2+alpha5*temp^(3/2)...
        +alpha6*temp+alpha7*temp^(1/2)+alpha8;
 end;



end
```

Bibliography

[1] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama. Dynamical model of traffic congestion and numerical simulation. Physical Review E, 51(2):1035–1042, 1995.

[2] M. Brackstone and M. McDonald. Car-following:a historical review. Transportation Research Part F: Traffic Psychology and Behavior, 2(4):181–196, 1999.

[3] D.C. Gazis, R. Herman, and R.W. Rothery. Nonlinear follow-the-leader models of traffic flow. Operations Research, 9(4):545–567, 1961.

[4] Y. He, M. Chowdhury, T. Khan, M. Rahman, and Y. Zhou. Analysis of car following models. Journal of Intelligent Transport Systems, Submitted, 2011.

[5] D. Helbing. Gas-kinetic derivation of navier-stokes-like traffic equations. Physical Review E, 53:2366–2381, 1996.

[6] D. Helbing, A. Hennecke, V. Shvetsov, and M. Treiber. Micro- and macrosimulation of freeway traffic. Mathematical and Computer Modelling, 35(5):517–547, 2002.

[7] J. Hu and W. Li. Theory of ordinary differential equations. University Lecture, 2005.

[8] A. Kesting, M. Treiber, and D. Helbing. General lane-changing model mobil for car-following models. Transportation Research Record 1999, 2007.

[9] A. Kesting, M. Treiber, M. Schönhof, and D. Helbing. Adaptive cruise control design for active congestion avoidance. Transportation Research Part C, 16:668–683, 2008.

[10] R.D. Kühne. Macroscopic freeway model for dense traffic-stop-start waves and incident detection. In I. Volmuller and R. Hamerslag, editors, Proceedings of the 9th International Symposium on Transportation and Traffic Theory, pages 21–42. VNU Science Press, July 1984.

[11] A. Larsson. Driver usage and understanding of adaptive cruise control. Applied Ergonomics, 43:501–506, 2011.

[12] G.F. Newell. Nonlinear effects in the dynamics of car following. Operations Research, 9(2):209–229, 1961.

[13] L. Perko. Differential Equations and Dynamical Systems. Springer, 2nd edition, 1996.

[14] M. Treiber, A. Hennecke, and D. Helbing. Congested traffic states in empirical observations and microscopic simulations. Physical Review E, 62(2):1805–1823, 2000.

[15] M. Treiber, A. Kesting, and D. Helbing. Delays, inaccuracies and anticipation in microscopic traffic models. Physica A Statistical Mechanics and its Applications, 360(1):71–88, 2006.