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```

1 Basic

1.1 default

```
#include <bits/stdc++.h>
using namespace std:
#ifdef LOCAL
template < class ...T> void dbg(T ...x) { char e{}; ((
    cerr << e << x, e = ' '), ...); }

template < class T> void org(T l, T r) { while (l != r)
    cerr << ' ' << *l++; cerr << '\n'; }

#define debug(x...) dbg("(", #x, ") =", x, '\n')
#define orange(x...) dbg("[", #x, ") ="), org(x)
#*log</pre>
#else
#pragma GCC optimize("03,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#define debug(...) ((void)0)
#define orange(...) ((void)0)
#endif
#define ff first
#define ss second
#define all(v) (v).begin(), (v).end()
#define rall(v) (v).rbegin(), (v).rend()
template<class T> bool chmin(T &a, T b) { return b < a</pre>
      and (a = b, true); }
template<class T> bool chmax(T &a, T b) { return a < b
      and (a = b, true); }
```

1.2 vimrc

2 Matching and Flow

2.1 Dinic

```
template<class Cap>
struct Dinic {
  struct Edge { int v; Cap w; int rev; };
  vector<vector<Edge>> G;
  int n, S, T;
  Dinic(int _n, int _S, int _T) : n(_n), S(_S), T(_T),
     G(_n) \{ \}
  void add_edge(int u, int v, Cap w) {
    G[u].push_back({v, w, (int)G[v].size()});
    G[v].push_back({u, 0, (int)G[u].size() - 1});
  vector<int> dep;
  bool bfs() +
    dep.assign(n, 0);
    dep[S] = 1;
    queue<int> que;
    que.push(S);
    while (!que.empty()) {
      int u = que.front(); que.pop();
for (auto [v, w, _] : G[u])
  if (!dep[v] and w) {
           dep[v] = dep[u] + 1;
           que.push(v);
     return dep[T] != 0;
  Cap dfs(int u, Cap in) {
    if (u == T) return in;
    Cap out = 0;
    for (auto &[v, w, rev] : G[u]) {
      if (w \text{ and } dep[v] == dep[u] + 1) {
         Cap f = dfs(v, min(w, in));
         w -= f, G[v][rev].w += f;
         in -= f, out += f;
         if (!in) break;
    if (in) dep[u] = 0;
    return out;
  Cap maxflow() {
    Cap ret = 0;
    while (bfs()) {
      ret += dfs(S, INF);
     return ret;
};
2.2 zkwDinic
template<class Cap>
struct zkwDinic {
  struct Edge { int v; Cap w, f; int rev; };
  vector<vector<Edge>> G;
  int n, S, T;
  zkwDinic(int _n, int _S, int _T) : n(_n), S(_S), T(_T)
     ), G(_n) {}
  void add_edge(int u, int v, Cap w, Cap f) {
    G[u].push_back({v, w, f, (int)G[v].size()});
G[v].push_back({u, -w, 0, (int)G[u].size() - 1});
  vector<Cap> dis;
  vector<bool> vis;
  bool spfa() {
    queue<int> que:
    dis.assign(n, INF);
    vis.assign(n, false);
    que.push(S);
    vis[S] = 1;
    dis[S] = 0;
    while (!que.empty()) {
      int u = que.front(); que.pop();
       vis[u] = 0:
       for (auto [v, w, f, _] : G[u])
         if (f and chmin(dis[v], dis[u] + w))
           if (!vis[v]) que.push(v), vis[v] = 1;
     return dis[T] != INF;
```

```
Cap dfs(int u, Cap in) {
                                                                    while (x != -1) swap(x, fr[fl[x] = pre[x]]);
    if (u == T) return in;
                                                                    return false;
    vis[u] = 1;
    Cap out = 0;
                                                                  while (true) {
    for (auto &[v, w, f, rev] : G[u])
                                                                    while (!que.empty()) {
      if (f and !vis[v] and dis[v] == dis[u] + w) {
                                                                      int y = que.front(); que.pop();
                                                                      for (int x = 0, d = 0; x < n; ++x) {
        Cap x = dfs(v, min(in, f));
        in -= x, out += x;
f -= x, G[v][rev].f += x;
                                                                        if (!vl[x] \text{ and } slk[x] >= (d = hl[x] + hr[y] -
                                                                   W[x][y]) {
        if (!in) break;
                                                                          if (pre[x] = y, d) slk[x] = d;
                                                                          else if (!Check(x)) return;
    if (in) dis[u] = INF;
    vis[u] = 0;
                                                                      }
    return out;
                                                                    int d = INF;
  pair<Cap, Cap> maxflow() {
                                                                    for (int x = 0; x < n; ++x) {
                                                                      if (!vl[x] \text{ and } d > slk[x]) d = slk[x];
    Cap a = 0, b = 0;
    while (spfa()) {
      Cap x = dfs(S, INF);
                                                                    for (int x = 0; x < n; ++x) {
                                                                      if (v1[x]) h1[x] += d;
      a += x;
      b += x * dis[T];
                                                                      else slk[x] -= d;
                                                                      if (vr[x]) hr[x] -= d;
    return {a, b};
                                                                    for (int x = 0; x < n; ++x) {
};
                                                                      if (!vl[x] and !slk[x] and !Check(x)) return;
2.3 HopcroftKarp
struct HopcroftKarp {
                                                                };
                                                                for (int i = 0; i < n; ++i) Bfs(i);</pre>
  std::vector<int> g, 1, r;
  int ans;
                                                                i64 res = 0;
                                                                for (int i = 0; i < n; ++i) res += W[i][fl[i]];</pre>
  HopcroftKarp(int n, int m, const std::vector<pair<int</pre>
    . int>> &e`
                                                                return res;
    : g(e.size()), l(n, -1), r(m, -1), ans(0) {
    vector<int> deg(n + 1);
                                                              2.5 GeneralMatching
    for (auto [x, y] : e) deg[x]++;
    partial_sum(all(deg), deg.begin());
                                                             struct GeneralMatching {
    for (auto [x, y] : e) g[--deg[x]] = y;
                                                                const int BLOCK = 10;
    vector<int> que(n);
                                                                int n:
    for (;;) {
                                                                vector<vector<int> > g;
      vector<int> a(n, -1), p(n, -1);
                                                                vector<int> hit, mat;
                                                                std::priority_queue<pair<i64, int>, vector<pair<i64,
      int t = 0:
      for (int i = 0; i < n; i++) if (1[i] == -1)
                                                                  int>>, greater<pair<i64, int>>> unmat;
                                                                GeneralMatching(int _n) : n(_n), g(_n), mat(n, -1),
        que[t++] = a[i] = p[i] = i;
      bool match = false;
                                                                  hit(n) {}
      for (int i = 0; i < t; i++) {
                                                                void add_edge(int a, int b) { // 0 <= a != b < n</pre>
        int x = que[i];
                                                                  g[a].push_back(b);
        if (~l[a[x]]) continue;
                                                                  g[b].push_back(a);
        for (int j = deg[x]; j < deg[x + 1]; j++) {
          int y = g[j];
                                                                int get_match() {
          if(r[y] == -1)
                                                                  for (int i = 0; i < n; i++) if (!g[i].empty()) {
             while (\sim y) r[y] = x, swap(1[x], y), x = p[x
                                                                    unmat.emplace(0, i);
    ];
             match = true, ans++;
                                                                  // If WA, increase this
            break;
                                                                  // there are some cases that need >=1.3*n^2 steps
                                                                  for BLOCK=1
          if(p[r[y]] == -1)
                                                                  // no idea what the actual bound needed here is.
             que[t++] = y = r[y], p[y] = x, a[y] = a[x];
                                                                  const int MAX_STEPS = 10 + 2 * n + n * n / BLOCK /
        }
                                                                  mt19937 rng(random_device{}());
      if (!match) break;
                                                                  for (int i = 0; i < MAX_STEPS; ++i) {</pre>
                                                                    if (unmat.empty()) break;
                                                                    int u = unmat.top().second;
};
                                                                    unmat.pop()
                                                                    if (mat[u] != -1) continue;
2.4 KM
                                                                    for (int j = 0; j < BLOCK; j++) {</pre>
i64 KM(vector<vector<int>> W) {
                                                                      ++hit[u];
  const int n = W.size();
                                                                      auto &e = g[u];
  vector<int> fl(n, -1), fr(n, -1), hr(n), hl(n);
for (int i = 0; i < n; ++i) {</pre>
                                                                      const int v = e[rng() % e.size()];
                                                                      mat[u] = v
   hl[i] = *max_element(W[i].begin(), W[i].end());
                                                                      swap(u, mat[v]);
                                                                      if (u == -1) break;
  auto Bfs = [&](int s) {
    vector<int> slk(n, INF), pre(n);
vector<bool> vl(n, false), vr(n, false);
                                                                    if (u != -1) {
                                                                      mat[u] = -1
                                                                      unmat.emplace(hit[u] * 100ULL / (g[u].size() +
    queue<int> que;
    que.push(s);
                                                                  1), u);
    vr[s] = true;
    auto Check = [&](int x) -> bool {
      if (vl[x] = true, fl[x] != -1) {
                                                                  int siz = 0;
        que.push(fl[x]);
                                                                  for (auto e : mat) siz += (e != -1);
        return vr[fl[x]] = true;
                                                                  return siz / 2;
```

```
};
```

3 Graph

3.1 2-SAT

```
struct TwoSAT {
  vector<vector<int>> G;
   int n;
  TwoSAT({\color{red} int \ \_n}) \ : \ n(\_n), \ G(\_n \ * \ 2) \ \{\}
   int ne(int x) { return x < n ? x + n : x - n; }</pre>
  void add_edge(int u, int v) { // u or v
G[ne(u)].push_back(v);
     G[ne(v)].push_back(u);
  vector<int> solve() {
     vector<int> ans(n * 2, -1), id(n * 2), stk, \
low(n * 2), dfn(n * 2), vis(n * 2);
     int _t = 0, scc_cnt = 0;
function<void(int)> dfs = [&](int u) {
       dfn[u] = low[u] = _t++;
        stk.push_back(u);
       vis[u] = 1;
        for (int v : G[u]) {
          if (!vis[v])
          dfs(v), chmin(low[u], low[v]);
else if (vis[v] == 1)
             chmin(low[u], dfn[v]);
       if (dfn[u] == low[u]) {
          for (int x = -1; x != u; ) {
  x = stk.back(); stk.pop_back();
             vis[x] = 2, id[x] = scc\_cnt;
             if (ans[x] == -1) {
               ans[x] = 1;
               ans[ne(x)] = 0;
            }
          scc_cnt++;
       }
     for (int i = 0; i < n + n; i++)</pre>
       if (!vis[i]) dfs(i);
     for (int i = 0; i < n; i++)
       if (id[i] == id[ne(i)])
          return {};
     ans.resize(n);
     return ans;
};
```

3.2 ManhattanMST

```
vector<tuple<int, int, int>> ManhattanMST(vector<Pt> P)
  vector<int> id(P.size());
  iota(all(id), 0);
  vector<tuple<int, int, int>> edges;
for (int k = 0; k < 4; ++k) {</pre>
    sort(all(id), [&](int i, int j) -> bool {
  return (P[i] - P[j]).ff < (P[j] - P[i]).ss;</pre>
    });
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-P[i].ss); \
           it != sweep.end(); sweep.erase(it++)) {
         int j = it->ss;
         Pt d = P[i] - P[j];
         if (d.ss > d.ff) break;
         edges.emplace_back(d.ss + d.ff, i, j);
      sweep[-P[i].ss] = i;
    for (Pt &p : P) {
      if (k % 2) p.ff = -p.ff;
       else swap(p.ff, p.ss);
  return edges;
```

4 Data Structure

```
4.1 Blackmagic
#include <bits/extc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/hash_policy.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
template<class T>
using BST = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>
gnu_pbds::priority_queue<node, decltype(cmp),</pre>
    pairing_heap_tag> pq(cmp);
gp_hash_table<int, gnu_pbds::priority_queue<node>::
    point_iterator> pqPos;
bst.insert((x << 20) + i);
bst.erase(bst.lower_bound(x << 20));</pre>
bst.order_of_key(x << 20) + 1;
*bst.find_by_order(x - 1) >> 20;
*--bst.lower_bound(x << 20) >> 20;
*bst.upper_bound((x + 1) << 20) >> 20;
4.2 Centroid Decomposition
struct CenDec {
  vector<vector<pair<int, int>>> anc;
  vector<int> Mdis:
  CenDec(const vector<vector<int>> &G) : anc(G.size()),
     Mdis(G.size(), INF) {
    const int n = G.size();
    vector<int> siz(n);
    vector<bool> vis(n);
    function<int(int, int)> getsiz = [&](int u, int f)
      siz[u] = 1;
      for (int v : G[u]) if (v != f and !vis[v])
        siz[u] += getsiz(v, u);
      return siz[u];
    function<int(int, int, int)> find = [&](int u, int
    f, int s) {
      for (int v : G[u]) if (v != f and !vis[v])
        if (siz[v] * 2 >= s) return find(v, u, s);
      return u;
    function<void(int, int, int, int)> caldis = [&](int
     u, int f, int a, int d) {
      anc[u].emplace_back(a, d);
for (int v : G[u]) if (v != f and !vis[v])
        caldis(v, u, a, d + 1);
    function<void(int)> build = [&](int u) {
      u = find(u, u, getsiz(u, u));
      vis[u] = 1;
      for (int v : G[u]) if (!vis[v]) {
        caldis(v, u, u, 1);
        build(v);
      vis[u] = 0;
    build(0);
  void add(int p) {
    Mdis[p] = 0;
for (auto [v, d] : anc[p])
      chmin(Mdis[v], d);
  int que(int p)
    int r = Mdis[p];
```

5 Math

}

};

return r:

5.1 Exgcd

```
pair<i64, i64> exgcd(i64 a, i64 b) { // ax + by = 1
  if (b == 0) return {1, 0};
  auto [x, y] = exgcd(b, a % b);
```

for (auto [v, d] : anc[p])

chmin(r, Mdis[v] + d);

```
return {y, x - a / b * y};
                                                                 static constexpr i64 iG = cpow(G, M - 2, M);
                                                                 void operator()(vector<i64> &v, bool inv) {
                                                                   int n = v.size();
5.2 CRT
                                                                    for (int i = 0, j = 0; i < n; i++) {
                                                                     if (i < j) swap(v[i], v[j]);</pre>
i64 CRT(vector<pair<i64, i64>> E) {
                                                                      for (int k = n / 2; (j ^{-} k) < k; k /= 2);
  i128 R = 0, M = 1;
  for (auto [r, m] : E) {
                                                                   for (int mid = 1; mid < n; mid *= 2) {</pre>
    i128 d = r - R, g = gcd < i64 > (M, m);
                                                                     i64 \text{ w} = \text{cpow}((inv ? iG : G), (M - 1) / (mid + mid))
    if (d % g != 0) return -1;
                                                                    ), M);
    i128 x = exgcd(M / g, m / g).ff * d / g;
                                                                      for (int i = 0; i < n; i += mid * 2) {
    R += M * x;
                                                                        i64 \text{ now} = 1;
    M = M * m / g;
                                                                        for (int j = i; j < i + mid; j++, now = now * w</pre>
    R = (R \% M + M) \% M;
                                                                     % M) {
                                                                          i64 \times v[j], y = v[j + mid];
  return R;
                                                                          v[j] = (x + y * now) % M;
                                                                          v[j + mid] = (x - y * now) % M;
5.3 Factorize
                                                                     }
struct Factorize {
  i64 fmul(i64 a, i64 b, i64 p) {
                                                                   if (inv) {
    return (i128)a * b % p;
                                                                      i64 in = cpow(n, M - 2, M);
                                                                      for (int i = 0; i < n; i++) v[i] = v[i] * in % M;
  i64 fpow(i64 a, i64 b, i64 p) {
    i64 res = 1;
                                                                 }
    for (; b; b >>= 1, a = fmul(a, a, p))
                                                               };
      if (b & 1) res = fmul(res, a, p);
                                                               template<i64 M, i64 G>
                                                               vector<i64> convolution(vector<i64> f, vector<i64> g) {
                                                                 NTT<M. G> ntt:
  bool Check(i64 a, i64 u, i64 n, int t) {
                                                                 int sum = f.size() + g.size() - 1;
    a = fpow(a, u, n);
                                                                 int len = bit_ceil((u64)sum);
    if (a == 0 or a == 1 or a == n - 1) return true;
                                                                 f.resize(len); g.resize(len);
    for (int i = 0; i < t; i++) {
                                                                 ntt(f, 0), ntt(g, 0);
      a = fmul(a, a, n);
if (a == 1) return false;
                                                                 for (int i = 0; i < len; i++) (f[i] *= g[i]) %= M;
                                                                 ntt(f, 1);
      if (a == n - 1) return true;
                                                                 f.resize(sum);
    }
                                                                 for (int i = 0; i < sum; i++) if (f[i] < 0) f[i] += M
    return false;
                                                                 return f:
  bool IsPrime(i64 n) {
    constexpr array<i64, 7> kChk{2, 235, 9375, 28178,
                                                               vector<i64> convolution_ll(const vector<i64> &f, const
    450775, 9780504, 1795265022};
// for int: {2, 7, 61}
                                                                    vector<i64> &g) {
                                                                 constexpr i64 M1 = 998244353, G1 = 3;
    if (n < 2) return false;
                                                                 constexpr i64 M2 = 985661441, G2 = 3;
    if (n % 2 == 0) return n == 2;
                                                                 constexpr i64 M1M2 = M1 * M2;
    i64 u = n - 1;
                                                                 constexpr i64 M1m1 = M2 * cpow(M2, M1 - 2, M1);
    int t = 0:
                                                                 constexpr i64 M2m2 = M1 * cpow(M1, M2 - 2, M2);
    while (u % 2 == 0) u >>= 1, t++;
                                                                 auto c1 = convolution<M1, G1>(f, g);
auto c2 = convolution<M2, G2>(f, g);
    for (auto v : kChk) if (!Check(v, u, n, t)) return
    false:
                                                                 for (int i = 0; i < c1.size(); i++) {</pre>
    return true:
                                                                   c1[i] = ((i128)c1[i] * M1m1 + (i128)c2[i] * M2m2) %
                                                                     M1M2;
  i64 PollardRho(i64 n) {
                                                                 }
    if (n % 2 == 0) return 2;
                                                                 return c1;
    i64 x = 2, y = 2, d = 1, p = 1;
                                                               }
    auto f = [](i64 x, i64 n, i64 p) -> i64 {
  return ((i128)x * x % n + p) % n;
                                                               5.5 FWT
                                                               void xorfwt(int v[], int 1, int r) {
    while (true) {
                                                                 if (r - 1 == 1) return;
      x = f(x, n, p);
                                                                 int m = 1 + r >> 1;
      y = f(f(y, n, p), n, p);
                                                                 xorfwt(v, 1, m), xorfwt(v, m, r);
for (int i = 1, j = m; i < m; ++i, ++j) {</pre>
      d = \_\_gcd(abs(x - y), n);
      if (d != n and d != 1) return d;
                                                                   int x = v[i] + v[j];
      if (d == n) ++p;
                                                                   v[j] = v[i] - v[j], v[i] = x;
    }
  }
                                                               }
};
                                                               void xorifwt(int v[], int l, int r) {
5.4 NTT
                                                                 if (r - 1 == 1) return;
// 17 -> 3
                                                                 int m = 1 + r >> 1;
                                                                 for (int i = 1, j = m; i < m; ++i, ++j) {
  int x = (v[i] + v[j]) / 2;</pre>
// 97 -> 5
// 193 -> 5
// 998244353 -> 3
                                                                    v[j] = (v[i] - v[j]) / 2, v[i] = x;
// 985661441 -> 3
constexpr i64 cpow(i64 a, i64 b, i64 m) {
                                                                 xorifwt(v, l, m), xorifwt(v, m, r);
  i64 ret = 1;
                                                               }
  for (; b; b >>= 1, a = a * a % m)
                                                               void andfwt(int v[], int l, int r) {
    if (b & 1) ret = ret * a % m;
  return ret;
                                                                 if (r - l == 1) return;
                                                                 int m = 1 + r >> 1;
template<i64 M, i64 G>
                                                                 and fwt(v, 1, m), and fwt(v, m, r);
struct NTT {
                                                                 for (int i = 1, j = m; i < m; ++i, ++j) v[i] += v[j];
```

```
}
                                                                 return CRT(E);
void andifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int^m = 1 + r >> 1;
                                                               5.8 FloorSum
  andifwt(v, 1, m), andifwt(v, m, r);
                                                               // sigma 0 ~ n-1: (a * i + b) / m
  for (int i = 1, j = m; i < m; ++i, ++j) v[i] -= v[j];
                                                              i64 floor_sum(i64 n, i64 m, i64 a, i64 b) {
                                                                 u64 \ ans = 0:
                                                                 if (a < 0) +
void orfwt(int v[], int l, int r) {
                                                                   u64 a2 = (a \% m + m) \% m;
  if (r - 1 == 1) return;
                                                                   ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
  int m = 1 + r >> 1;
                                                                   a = a2:
  orfwt(v, 1, m), orfwt(v, m, r); for (int i = 1, j = m; i < m; ++i, ++j) v[j] += v[i];
                                                                 if (b < 0)
                                                                   u64 b2 = (b \% m + m) \% m;
                                                                   ans -= 1ULL * n * ((b2 - b) / m);
void orifwt(int v[], int l, int r) {
                                                                   b = b2:
  if (r - l == 1) return;
  int m = 1 + r >> 1;
                                                                 while (true) {
  if (a >= m) {
                                                                     ans += n * (n - 1) / 2 * (a / m);
                                                                     a %= m:
5.6 FWT
                                                                   if (b >= m) {
  1. XOR Convolution
                                                                     ans += n * (b / m);
       • f(A) = (f(A_0) + f(A_1), f(A_0) - f(A_1))
• f^{-1}(A) = (f^{-1}(\frac{A_0 + A_1}{2}), f^{-1}(\frac{A_0 - A_1}{2}))
                                                                     b %= m:
                                                                   u64 y_max = a * n + b;
  2. OR Convolution
                                                                   if (y_max < m) break;</pre>
        • f(A) = (f(A_0), f(A_0) + f(A_1))
                                                                   n = y_max / m;
       • f^{-1}(A) = (f^{-1}(A_0), f^{-1}(A_1) - f^{-1}(A_0))
                                                                   b = y_max % m;
                                                                   swap(m, a);
  3. AND Convolution
       • f(A) = (f(A_0) + f(A_1), f(A_1))
       • f^{-1}(A) = (f^{-1}(A_0) - f^{-1}(A_1), f^{-1}(A_1))
                                                                 return ans;
      Lucas
                                                               6
                                                                    Geometry
i64 Lucas(i64 N, i64 M, i64 D) { // C(N, M) mod D
  auto Factor = [&](i64 x) -> vector<pair<i64, i64>> {
                                                                   2D Point
    vector<pair<i64, i64>> r;
                                                               using Pt = pair<i64, i64>;
    for (i64 i = 2; x > 1; i++)
                                                               constexpr double eps = 1e-9;
      if(x \% i == 0) {
                                                              Pt operator+(Pt a, Pt b) { return {a.ff + b.ff, a.ss +
        i64 c = 0
                                                                   b.ss}; }
        while (x \% i == 0) x /= i, c++;
                                                              Pt operator-(Pt a, Pt b) { return {a.ff - b.ff, a.ss -
        r.emplace_back(i, c);
                                                                   b.ss}; }
                                                               Pt operator*(Pt a, double b) { return {a.ff * b, a.ss *
    return r;
                                                                    b}; }
                                                               Pt operator/(Pt a, double b) { return {a.ff / b, a.ss /
  auto Pow = [&](i64 a, i64 b, i64 m) -> i64 {
                                                                    b}; }
    i64 r = 1;
                                                               double operator*(Pt a, Pt b) { return a.ff * b.ff + a.
    for (; b; b >>= 1, a = a * a % m)
                                                                   ss * b.ss; }
      if (b & 1) r = r * a % m;
                                                               double operator*(Pt a, Pt b) { return a.ff * b.ss - a.
    return r;
                                                                   ss * b.ff; }
  };
                                                               double abs(Pt a) { return sqrt(a * a); }
  vector<pair<i64, i64>> E;
for (auto [p, q] : Factor(D)) {
                                                               double cro(Pt a, Pt b, Pt c) { return (b - a) ^ (c - a)
    const i64 mod = Pow(p, q, 1 << 30);
                                                               int sig(double x) { return (x > -eps) - (x < eps); }</pre>
    auto CountFact = [&](i64 x) -> i64 {
                                                              Pt Inter(Pt a, Pt b, Pt c, Pt d) {
      i64 c = 0;
                                                                 double s = cro(c, d, a), t = -cro(c, d, b);
return (a * t + b * s) / (s + t);
      while (x) c += (x /= p);
      return c;
    };
                                                               struct Line ·
    auto CountBino = [&](i64 x, i64 y) { return
                                                                 Pt a{}, b{};
    CountFact(x) - CountFact(y) - CountFact(x - y); \ \};
                                                                 Line() {}
    auto Inv = [\&](i64 x) \rightarrow i64 \{ return (exgcd(x, mod)) \}
                                                                 Line(Pt _a, Pt _b) : a\{_a\}, b\{_b\} {}
    ).ff % mod + mod) % mod; };
    vector<i64> pre(mod + 1);
                                                              Pt Inter(Line L, Line R) {
    pre[0] = pre[1] = 1;
                                                                 return Inter(L.a, L.b, R.a, R.b);
    for (i64 i = 2; i <= mod; i++) pre[i] = (i % p == 0
     ? 1 : i) * pre[i - 1] % mod;
    function<i64(i64)> FactMod = [&](i64 n) -> i64 {
                                                               6.2 Convex Hull
      if (n == 0) return 1;
      return FactMod(n / p) * Pow(pre[mod], n / mod,
                                                              vector<Pt> Hull(vector<Pt> P) {
    mod) % mod * pre[n % mod] % mod;
                                                                 sort(all(P));
                                                                 P.erase(unique(all(P)), P.end());
    auto BinoMod = [&](i64 x, i64 y) -> i64 {
                                                                 P.insert(P.end(), rall(P));
                                                                 vector<Pt> stk;
      return FactMod(x) * Inv(FactMod(y)) % mod * Inv(
                                                                 for (auto p : P)
    FactMod(x - y)) % mod;
                                                                   while (stk.size() >= 2 and \
    i64 r = BinoMod(N, M) * Pow(p, CountBino(N, M), mod
                                                                       cro(*++stk.rbegin(), stk.back(), p) <= 0 and \</pre>
                                                                       (*++stk.rbegin() < stk.back()) == (stk.back() <
    ) % mod;
    E.emplace_back(r, mod);
                                                                    p)) {
```

```
stk.pop_back();
                                                                seg.push_back(P[i]);
    stk.push_back(p);
                                                              while (seg.size() >= 2 and sig(cro(inter.back(), seg
                                                                [0].b, seg[0].a) == 1) {
                                                                seg.pop_back(), inter.pop_back();
 stk.pop_back();
  return stk;
                                                              inter.push_back(Inter(seg[0], seg.back()));
                                                              return vector<Pt>(all(inter));
6.3 Convex Hull trick
vector<Pt> Hull(vector<Pt> P) {
                                                                 Minimal Enclosing Circle
                                                            6.6
  sort(all(P));
                                                           using circle = pair<Pt, double>;
  P.erase(unique(all(P)), P.end());
                                                            struct MES {
 P.insert(P.end(), rall(P));
                                                              MES() {}
  vector<Pt> stk;
                                                              bool inside(const circle &c, Pt p) {
  for (auto p : P) {
                                                                return abs(p - c.ff) <= c.ss + eps;</pre>
    while (stk.size() >= 2 and \
        cro(*++stk.rbegin(), stk.back(), p) <= 0 and \</pre>
                                                              circle get_cir(Pt a, Pt b) {
        (*++stk.rbegin() < stk.back()) == (stk.back() <
                                                                return circle((a + b) / 2., abs(a - b) / 2.);
     p)) {
      stk.pop_back();
                                                              circle get_cir(Pt a, Pt b, Pt c) {
                                                                Pt p = (b - a) / 2.;
    stk.push_back(p);
                                                                p = Pt(-p.ss, p.ff);
                                                                double t = ((c - a) * (c - b)) / (2 * (p * (c - a)))
 stk.pop_back();
  return stk;
                                                                p = ((a + b) / 2.) + (p * t);
}
                                                                return circle(p, abs(p - a));
      Dynamic Convex Hull
                                                              circle get_mes(vector<Pt> P) {
template<class T, class Comp = less<T>>
                                                                if (P.empty()) return circle{Pt(0, 0), 0};
struct DynamicHull {
                                                                mt19937 rng(random_device{}());
                                                                shuffle(all(P), rng);
 set<T. Comp> H:
 DynamicHull() {}
                                                                circle C{P[0], 0};
  void insert(T p) {
                                                                for (int i = 1; i < P.size(); i++) {</pre>
    if (inside(p)) return;
                                                                  if (inside(C, P[i])) continue;
    auto it = H.insert(p).ff;
                                                                  C = get_cir(P[i], P[0]);
    while (it != H.begin() and prev(it) != H.begin() \
                                                                  for (int j = 1; j < i; j++) {
  if (inside(C, P[j])) continue;</pre>
        and cross(*prev(it, 2), *prev(it), *it) <= 0) {
      it = H.erase(--it);
                                                                    C = get_cir(P[i], P[j]);
                                                                    for (int k = 0; k < j; k++) {
    while (it != --H.end() and next(it) != --H.end() \
                                                                      if (inside(C, P[k])) continue;
        and cross(*it, *next(it), *next(it, 2)) <= 0) {</pre>
                                                                      C = get_cir(P[i], P[j], P[k]);
      it = --H.erase(++it);
                                                                 }
 bool inside(T p) {
                                                                return C:
    auto it = H.lower_bound(p);
    if (it == H.end()) return false;
                                                           };
    if (it == H.begin()) return p == *it;
    return cross(*prev(it), p, *it) <= 0;
                                                                 Minkowski
                                                           vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
};
                                                              auto reorder = [&](auto &R) -> void {
                                                                auto cmp = [&](Pt a, Pt b) -> bool {
6.5 Half Plane Intersection
                                                                  return Pt(a.ss, a.ff) < Pt(b.ss, b.ff);</pre>
vector<Pt> HPI(vector<Line> P) {
                                                                rotate(R.begin(), min_element(all(R), cmp), R.end()
  const int n = P.size();
  sort(all(P), [&](Line L, Line R) -> bool {
                                                                R.push_back(R[0]), R.push_back(R[1]);
    Pt u = L.b - L.a, v = R.b - R.a;
    bool f = Pt(sig(u.ff), sig(u.ss)) < Pt{};</pre>
                                                              const int n = P.size(), m = Q.size();
    bool g = Pt(sig(v.ff), sig(v.ss)) < Pt{};</pre>
                                                              reorder(P), reorder(Q);
    if (f != g) return f < g;</pre>
    return (sig(u ^ v) ? sig(u ^ v) : sig(cro(L.a, R.a,
                                                              vector<Pt> R:
                                                              for (int i = 0,
                                                                              j = 0, s; i < n or j < m; ) {
     R.b))) > 0;
                                                                R.push_back(P[i] + Q[j]);
  });
                                                                s = sig((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
  auto Same = [&](Line L, Line R) {
   Pt u = L.b - L.a, v = R.b - R.a;
                                                                i += (s >= 0), j += (s <= 0);
                                                              }
    return sig(u \wedge v) == 0 and sig(u * v) == 1;
                                                              return R;
                                                           }
  deque <Pt> inter:
  deque <Line> seg;
                                                                 Stringology
  for (int i = 0; i < n; i++) if (i == 0 or !Same(P[i -
     1], P[i])) {
                                                            7.1 Z-algorithm
    while (seg.size() >= 2 and sig(cro(inter.back(), P[
    i].b, P[i].a)) == 1) {
                                                            vector<int> zalgo(string s) {
      seg.pop_back(), inter.pop_back();
                                                              if (s.empty()) return {};
                                                              int len = s.size();
                                                              vector<int> z(len);
    while (seg.size() >= 2 and sig(cro(inter[0], P[i].b
    , P[i].a)) == 1) {
                                                              z[0] = len;
      seg.pop_front(), inter.pop_front();
                                                              for (int i = 1, l = 1, r = 1; i < len; i++) {
                                                                z[i] = i < r ? min(z[i-1], r-i) : 0;
    if (!seg.empty()) inter.push_back(Inter(seg.back(),
                                                                while (i + z[i] < len and s[i + z[i]] == s[z[i]]) z
     P[i]));
                                                                [i]++:
```

```
if (i + z[i] > r) l = i, r = i + z[i];
                                                                     vector<int> lcp(n - 1);
                                                                     int h = 0;
  return z;
                                                                     fup(0, n) {
}
                                                                       if (h > 0) h--;
                                                                       if (rnk[i] == 0) continue;
7.2 Manacher
                                                                       int j = sa[rnk[i] - 1];
                                                                       for (; j + h < n \text{ and } i + h < n; h++)
vector<int> manacher(const string &s) {
  string p = "@#"
                                                                         if (s[j + h] != s[i + h]) break;
  for (char c : s) p += c + '#';
                                                                       lcp[rnk[i] - 1] = h;
 p += '$'
  vector<int> dp(p.size());
                                                                     return lcp;
  int mid = 0, r = 1;
                                                                }
  for (int i = 1; i < p.size() - 1; i++) {</pre>
    auto &k = dp[i];
                                                                7.4 PalindromicTree
    k = i < mid + r ? min(dp[mid * 2 - i], mid + r - i)
     : 0;
                                                                struct PAM {
    while (p[i + k + 1] == p[i - k - 1]) k++;
                                                                  struct Node -
    if (i + k > mid + r) mid = i, r = k;
                                                                     int fail, len, dep;
                                                                     array<int, 26> ch;
 return vector<int>(dp.begin() + 2, dp.end() - 2);
                                                                     Node(int _len) : len{_len}, fail{}, ch{}, dep{} {};
                                                                   vector<Node> g;
7.3 SuffixArrau
                                                                   vector<int> id;
namespace sfx {
                                                                   int odd, even, lst;
#define fup(a, b) for (int i = a; i < b; i++)
#define fdn(a, b) for (int i = b - 1; i >= a; i--)
                                                                   string S;
                                                                   int new_node(int len) {
  constexpr int N = 5e5 + 5;
                                                                     g.emplace_back(len);
 bool _t[N * 2];
                                                                     return g.size() - 1;
 int H[N], RA[N], x[N], _p[N]; int SA[N * 2], _s[N * 2], _c[N * 2], _q[N * 2];
                                                                   PAM() : odd(new_node(-1)), even(new_node(0)) {
 void pre(int *sa, int *c, int n, int z) {
                                                                     lst = g[even].fail = odd;
    fill_n(sa, n, 0), copy_n(c, z, x);
                                                                   int up(int p) {
  void induce(int *sa, int *c, int *s, bool *t, int n,
                                                                     while (S.rbegin()[g[p].len + 1] != S.back())
    int z) {
                                                                       p = g[p].fail;
    copy_n(c, z - 1, x + 1);

fup(0, n) if (sa[i] and !t[sa[i] - 1])
                                                                     return p;
      sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
                                                                   int add(char c) {
    copy_n(c, z, x);

fdn(0, n) if (sa[i] and t[sa[i] - 1])
                                                                     S += c;
                                                                     lst = up(lst);
      sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
                                                                     c -= 'a'
                                                                     if (!g[lst].ch[c]) g[lst].ch[c] = new_node(g[lst].
  void sais(int *s, int *sa, int *p, int *q, bool *t,
                                                                     len + 2);
    int *c, int n, int z) {
                                                                     int p = g[lst].ch[c];
    bool uniq = t[n - 1] = true;
                                                                     g[p].fail = (lst == odd ? even : g[up(g[lst].fail)]
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
                                                                     ].ch[c]);
    last = -1;
                                                                     lst = p
    fill_n(c, z, 0);
                                                                     g[lst].dep = g[g[lst].fail].dep + 1;
    fup(0, n) uniq &= ++c[s[i]] < 2;
                                                                     id.push_back(lst);
    partial_sum(c, c + z, c);
                                                                     return 1st;
    if (uniq) { fup(0, n) sa[--c[s[i]]] = i; return; }
    fdn(0, n-1)
                                                                   void del() {
      t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i]
                                                                     S.pop_back()
    + 1]);
                                                                     id.pop_back();
    pre(sa, c, n, z);
fup(1, n) if (t[i] and !t[i - 1])
  sa[--x[s[i]]] = p[q[i] = nn++] = i;
                                                                     lst = id.empty() ? odd : id.back();
                                                                }:
    \begin{array}{l} induce(sa,\ c,\ s,\ t,\ n,\ z);\\ fup(0,\ n)\ if\ (sa[i]\ and\ t[sa[i]]\ and\ !t[sa[i]\ -\ 1]) \end{array}
                                                                8
                                                                      Misc
    bool neq = last < 0 or !equal(s + sa[i], s + p[q[
sa[i]] + 1], s + last);</pre>
      ns[q[last = sa[i]]] = nmxz += neq;
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz)
     + 1);
    pre(sa, c, n, z);
    fdn(0, nn) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
 vector<int> build(vector<int> s, int n) {
    copy_n(begin(s), n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    vector<int> sa(n);
    fup(0, n) sa[i] = SA[i + 1];
    return sa;
  vector<int> lcp_array(vector<int> &s, vector<int> &sa
    ) {
    int n = int(s.size());
    vector<int> rnk(n)
    fup(0, n) rnk[sa[i]] = i;
```