Contents

1 Basic

1.1 Random

```
mt19937 rng(random_device{}());
i64 rand(i64 l = -lim, i64 r = lim) {
  return uniform_int_distribution<i64>(l, r)(rng); }
double randr(double l, double r) {
  return uniform_real_distribution<double>(l, r)(rng);
}
```

1.2 PBDS

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
template<class T>
using BST = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
// __gnu_pbds::priority_queue<node, decltype(cmp), pairing_heap_tag> pq(cmp);
// gp_hash_table<int, gnu_pbds::priority_queue<node>::point_iterator> pqPos;
// bst.insert((x << 20) + i);
// bst.erase(bst.lower_bound(x << 20));
// bst.order_of_key(x << 20) + 1;
// *bst.find_by_order(x - 1) >> 20;
// *--bst.lower_bound(x << 20) >> 20;
// *bst.upper_bound((x + 1) << 20) >> 20;
```

1.3 vimrc

```
se sts=4 sw=4 so=5 ls=2 nu rnu et hls mouse=a is ic scs
filetype indent on | imap jk <Esc>
nmap <F9> :w<bar>!g++ '%' -o run -std=c++20 -DLOCAL -Wfatal-errors -fsanitize=address,undefined -g && echo done. && time ./run<CR>
nmap <F8> :!time ./run<CR> | map ; : | map <C-l> :nohl<CR>
ca hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \| md5sum \| cut -c-6
" setxkbmap -option caps:ctrl_modifier
```

1.4 optimize

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```

2 Matching and Flow

2.1 KM [e65495] (9e13bc|493813)

```
// max weight, for min negate the weights
template<class T> // O(N^3), N <= 800
T KM(const vector<vector<T>> &w) {
  const int n = w.size();
  vector<T> lx(n), ly(n);
  vector<int> mx(n, -1), my(n, -1), pa(n);
  auto aug = [&](int y) {
    for (int x, z; y != -1; y = z) {
      x = pa[y]; z = mx[x];
      my[y] = x; mx[x] = y;
    }
  };
  auto bfs = [&](int s) {
    vector<T> sy(n, inf<T>);
    vector<bool> vx(n), vy(n);
    queue<int> q;
    q.push(s);
    while (true) {
      while (q.size()) {
        int x = q.front(); q.pop();
        vx[x] = 1;
        for (int y = 0; y < n; y++) {
          if (vy[y]) continue;
          T d = lx[x] + ly[y] - w[x][y];
          if (d == 0) {
            pa[y] = x;
            if (my[y] == -1)
              return aug(y);
            vy[y] = 1;
            q.push(my[y]);
          } else if (chmin(sy[y], d)) {
            pa[y] = x;
          }
        }
      } /* SPLIT-HASH */
      T cut = inf<T>;
      for (int y = 0; y < n; y++)
        if (!vy[y])
          chmin(cut, sy[y]);
      for (int j = 0; j < n; j++) {
        if (vx[j]) lx[j] -= cut;
        if (vy[j]) ly[j] += cut;
        else     sy[j] -= cut;
      }
      for (int y = 0; y < n; y++)
        if (!vy[y] and sy[y] == 0) {
          if (my[y] == -1)
            return aug(y);
          vy[y] = 1;
          q.push(my[y]);
        }
    }
  };
  for (int x = 0; x < n; x++)
    lx[x] = ranges::max(w[x]);
  for (int x = 0; x < n; x++)
    bfs(x);
  T ans = 0;
  for (int x = 0; x < n; x++)
    ans += w[x][mx[x]];
  return ans;
}
```

2.2 Model

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer. Also, f is a mincost valid flow.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited; $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - \left(\sum_{e \in E(v)} w(e)\right) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection cheat sheet: S, T 分別代表 $0, 1$ 側，最小化總花費。

i 為 0 時花費 c	(i, T, c)
i 為 1 時花費 c	(S, i, c)
$i \in I$ 有任何一個為 0 時花費 c	$(i, w, \infty), (w, T, c)$
$i \in I$ 有任何一個為 1 時花費 c	$(S, w, c), (w, i, \infty)$
i 為 0 時得到 c	直接得到 c；(S, i, c)
i 為 1 時得到 c	直接得到 c；(i, T, c)
i 為 0，j 為 1 時花費 c	(i, j, c)
i, j 不同時花費 c	$(i, j, c), (j, i, c)$
i, j 同時是 0 時得到 c	直接得到 c；$(S, w, c), (w, i, \infty), (w, j, \infty)$
i, j 同時是 1 時得到 c	直接得到 c；$(i, w, \infty), (j, w, \infty), (w, T, c)$

- Submodular functions minimization
  - For a function $f : 2^V \to \mathbb{R}$, f is a submodular function iff
    * $\forall S, T \subseteq V$, $f(S) + f(T) \geq f(S \cup T) + f(S \cap T)$, or
    * $\forall X \subseteq Y \subseteq V$, $x \notin Y$, $f(X \cup \{x\}) - f(X) \geq f(Y \cup \{x\}) - f(Y)$.
  - To minimize $\sum_i \theta_i(x_i) + \sum_{i<j} \phi_{ij}(x_i, x_j) + \sum_{i<j<k} \psi_{ijk}(x_i, x_j, x_k)$
  - If $\theta_i(1) \geq \theta_i(0)$, add edge $(S, i, \theta_i(1) - \theta_i(0))$ and $\theta_i(0)$ to answer; otherwise, $(i, T, \theta_i(0) - \theta_i(1))$ and $\theta_i(1)$.
  - Add edges $(i, j, \phi_{ij}(0, 1) + \phi_{ij}(1, 0) - \phi_{ij}(0, 0) - \phi_{ij}(1, 1))$.
  - Denote x_{ijk} as helper nodes. Let $P = \psi_{ijk}(0,0,0) + \psi_{ijk}(0,1,1) + \psi_{ijk}(1,0,1) + \psi_{ijk}(1,1,0) - \psi_{ijk}(0,0,1) - \psi_{ijk}(0,1,0) - \psi_{ijk}(1,0,0) - \psi_{ijk}(1,1,1)$. Add $-P$ to answer. If $P \geq 0$, add edges (i, x_{ijk}, P), (j, x_{ijk}, P), (k, x_{ijk}, P), (x_{ijk}, T, P); otherwise $(x_{ijk}, i, -P)$, $(x_{ijk}, j, -P)$, $(x_{ijk}, k, -P)$, $(S, x_{ijk}, -P)$.
  - The minimum cut of this graph will be the the minimum value of the function above.
- Dual of minimum cost maximum flow
  1. Capacity c_{uv}, Flow f_{uv}, Cost w_{uv}, Required Flow difference for vertex b_u.
  2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\begin{aligned} \min \sum_{uv} w_{uv} f_{uv} \\ -f_{uv} \geq -c_{uv} \\ \sum_{v} f_{vu} - \sum_{v} f_{uv} = -b_u \end{aligned} \Leftrightarrow \begin{aligned} \min \sum_{u} b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv}) \\ p_u \geq 0 \end{aligned}$$

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     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
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  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- 0/1 quadratic programming

$$\sum_{x} c_x x + \sum_{y} c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x\bar{y} + x'\bar{y'})$$

can be minimized by the mincut of the following graph:

  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

2.3 Dinic [9c3711] (d625f1|05394d)

```
template<class Cap>
struct Flow {
  struct Edge { int v; Cap w; int rev; };
  vector<vector<Edge>> G;
  int n;
  Flow(int n) : n(n), G(n) {}
  void addEdge(int u, int v, Cap w) {
    G[u].push_back({v, w, (int)G[v].size()});
    G[v].push_back({u, 0, (int)G[u].size() - 1});
  }
  vector<int> dep;
  bool bfs(int s, int t) {
    dep.assign(n, 0);
    dep[s] = 1;
    queue<int> que;
    que.push(s);
    while (!que.empty()) {
      int u = que.front(); que.pop();
      for (auto [v, w, rev] : G[u])
        if (!dep[v] and w) {
          dep[v] = dep[u] + 1;
          que.push(v);
        }
    }
    return dep[t] != 0;
  } /* SPLIT-HASH */
  Cap dfs(int u, Cap in, int t) {
    if (u == t) return in;
    Cap out = 0;
    for (auto &[v, w, rev] : G[u]) {
      if (w and dep[v] == dep[u] + 1) {
        Cap f = dfs(v, min(w, in), t);
        w -= f;
        G[v][rev].w += f;
        in -= f;
        out += f;
        if (!in) break;
      }
    }
    if (in) dep[u] = 0;
    return out;
  }
  Cap maxFlow(int s, int t) {
    Cap ret = 0;
    while (bfs(s, t)) ret += dfs(s, inf<Cap>, t);
    return ret;
  }
};
```

2.4 MCMF [e6e7cb] (c451e4|2bdcbc)

```
template<class T>
struct MCMF {
  struct Edge { int v; T f, w; int rev; };
  vector<vector<Edge>> G;
  const int n;
  MCMF(int n) : n(n), G(n) {}
  void addEdge(int u, int v, T f, T c) {
    G[u].push_back({v, f, c, ssize(G[v])});
    G[v].push_back({u, 0, -c, ssize(G[u]) - 1});
  }
  vector<T> dis;
  vector<bool> vis;
  bool spfa(int s, int t) {
    queue<int> que;
    dis.assign(n, inf<T>);
    vis.assign(n, false);
    que.push(s);
    vis[s] = 1;
    dis[s] = 0;
    while (!que.empty()) {
      int u = que.front(); que.pop();
      vis[u] = 0;
      for (auto [v, f, w, _] : G[u])
        if (f and chmin(dis[v], dis[u] + w))
          if (!vis[v]) {
            que.push(v);
            vis[v] = 1;
          }
    }
    return dis[t] != inf<T>;
  } /* SPLIT-HASH */
  T dfs(int u, T in, int t) {
    if (u == t) return in;
    vis[u] = 1;
    T out = 0;
    for (auto &[v, f, w, rev] : G[u])
      if (f and !vis[v] and dis[v] == dis[u] + w) {
        T x = dfs(v, min(in, f), t);
        in -= x;
        out += x;
        f -= x;
        G[v][rev].f += x;
        if (!in) break;
      }
    if (in) dis[u] = inf<T>;
    vis[u] = 0;
    return out;
  }
  pair<T, T> maxFlow(int s, int t) {
    T a = 0, b = 0;
    while (spfa(s, t)) {
      T x = dfs(s, inf<T>, t);
      a += x;
      b += x * dis[t];
    }
    return {a, b};
  }
};
```

2.5 HopcroftKarp [a760ee]

```
// Complexity: O(m sqrt(n))
// edge (u \in A) -> (v \in B) : G[u].push_back(v);
struct HK {
  const int n, m;
  vector<int> l, r, a, p;
  int ans;
  HK(int n, int m) : n(n), m(m), l(n, -1), r(m, -1),
    ans{} {}
  void work(const auto &G) {
    for (bool match = true; match; ) {
      match = false;
      queue<int> q;
      a.assign(n, -1), p.assign(n, -1);
      for (int i = 0; i < n; i++)
        if (l[i] == -1) q.push(a[i] = p[i] = i);
      while (!q.empty()) {
        int z, x = q.front(); q.pop();
        if (l[a[x]] != -1) continue;
        for (int y : G[x]) {
          if (r[y] == -1) {
            for (z = y; z != -1; ) {
              r[z] = x;
              swap(l[x], z);
              x = p[x];
            }
            match = true;
            ans++;
            break;
          } else if (p[r[y]] == -1) {
            q.push(z = r[y]);
            p[z] = x;
            a[z] = a[x];
          }
        }
      }
    }
  }
};
```

2.6 SW [f57872] (8dccd8|4f2b3c)

```
int w[kN][kN], g[kN], del[kN], v[kN];
void AddEdge(int x, int y, int c) {
  w[x][y] += c;
  w[y][x] += c;
}
pair<int, int> Phase(int n) {
  fill(v, v + n, 0), fill(g, g + n, 0);
  int s = -1, t = -1;
  while (true) {
    int c = -1;
    for (int i = 0; i < n; ++i) {
      if (del[i] || v[i]) continue;
      if (c == -1 || g[i] > g[c]) c = i;
    }
    if (c == -1) break;
    v[c] = 1, s = t, t = c;
```

```
    for (int i = 0; i < n; ++i) {
      if (del[i] || v[i]) continue;
      g[i] += w[c][i];
    }
  }
  return make_pair(s, t);
} /* SPLIT-HASH */
int GlobalMinCut(int n) {
  int cut = kInf;
  fill(del, 0, sizeof(del));
  for (int i = 0; i < n - 1; ++i) {
    int s, t; tie(s, t) = Phase(n);
    del[t] = 1, cut = min(cut, g[t]);
    for (int j = 0; j < n; ++j) {
      w[s][j] += w[t][j];
      w[j][s] += w[j][t];
    }
  }
  return cut;
} // O(V^3), can be O(VE + V^2 log V)?
```

2.7 GeneralMatching [79808a]

```
struct GeneralMatching { // n <= 500
  const int BLOCK = 10;
  int n;
  vector<vector<int> > g;
  vector<int> hit, mat;
  std::priority_queue<pair<i64, int>, vector<pair<i64,
    int>>, greater<pair<i64, int>>> unmat;
  GeneralMatching(int _n) : n(_n), g(_n), mat(n, -1),
    hit(n) {}
  void add_edge(int a, int b) { // 0 <= a != b < n
    g[a].push_back(b);
    g[b].push_back(a);
  }
  int get_match() {
    for (int i = 0; i < n; i++) if (!g[i].empty()) {
      unmat.emplace(0, i);
    }
    // If WA, increase this
    // there are some cases that need >=1.3*n^2 steps
    for BLOCK=1
    // no idea what the actual bound needed here is.
    const int MAX_STEPS = 10 + 2 * n + n * n / BLOCK /
    2;
    mt19937 rng(random_device{}());
    for (int i = 0; i < MAX_STEPS; ++i) {
      if (unmat.empty()) break;
      int u = unmat.top().second;
      unmat.pop();
      if (mat[u] != -1) continue;
      for (int j = 0; j < BLOCK; j++) {
        ++hit[u];
        auto &e = g[u];
        const int v = e[rng() % e.size()];
        mat[u] = v;
        swap(u, mat[v]);
        if (u == -1) break;
      }
      if (u != -1) {
        mat[u] = -1;
        unmat.emplace(hit[u] * 100ULL / (g[u].size() +
    1), u);
      }
    }
    int siz = 0;
    for (auto e : mat) siz += (e != -1);
    return siz / 2;
  }
};
```

3 Graph

3.1 2-SAT [4f92ea]

```
struct TwoSat {
  int n;
  vector<vector<int>> G;
  vector<bool> ans;
  vector<int> id, dfn, low, stk;
  TwoSat(int n) : n(n), G(2 * n), ans(n),
    id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1) {}
  void addClause(int u, bool f, int v, bool g) { // (u
    = f) or (v = g)
    G[2 * u + !f].push_back(2 * v + g);
    G[2 * v + !g].push_back(2 * u + f);
  }
  void addImply(int u, bool f, int v, bool g) { // (u =
     f) -> (v = g)
    G[2 * u + f].push_back(2 * v + g);
    G[2 * v + !g].push_back(2 * u + !f);
  }
  int cur = 0, scc = 0;
  void dfs(int u) {
    stk.push_back(u);
    dfn[u] = low[u] = cur++;
    for (int v : G[u]) {
      if (dfn[v] == -1) {
        dfs(v);
        chmin(low[u], low[v]);
      } else if (id[v] == -1) {
        chmin(low[u], dfn[v]);
      }
    }
    if (dfn[u] == low[u]) {
      int x;
      do {
        x = stk.back();
        stk.pop_back();
        id[x] = scc;
      } while (x != u);
      scc++;
    }
  }
  bool satisfiable() {
    for (int i = 0; i < n * 2; i++)
      if (dfn[i] == -1) {
        dfs(i);
      }
    for (int i = 0; i < n; ++i) {
      if (id[2 * i] == id[2 * i + 1]) {
        return false;
      }
      ans[i] = id[2 * i] > id[2 * i + 1];
    }
    return true;
  }
};
```

3.2 Tree [0ff741] (480942|27e618)

```
struct Tree {
  int n, lgN;
  vector<vector<int>> G;
  vector<vector<int>> st;
  vector<int> in, out, dep, pa, seq;
  Tree(int n) : n(n), G(n), in(n), out(n), dep(n), pa(n
    , -1) {}
  int cmp(int a, int b) {
    return dep[a] < dep[b] ? a : b;
  }
  void dfs(int u) {
    erase(G[u], pa[u]);
    in[u] = seq.size();
    seq.push_back(u);
    for (int v : G[u]) {
      dep[v] = dep[u] + 1;
      pa[v] = u;
      dfs(v);
    }
    out[u] = seq.size();
  }
  void build() {
    seq.reserve(n);
    dfs(0);
    lgN = __lg(n);
    st.assign(lgN + 1, vector<int>(n));
    st[0] = seq;
    for (int i = 0; i < lgN; i++)
      for (int j = 0; j + (2 << i) <= n; j++)
        st[i + 1][j] = cmp(st[i][j], st[i][j + (1 << i)
    ]);
  }
  int inside(int x, int y) {
    return in[x] <= in[y] and in[y] < out[x];
  }
  int lca(int x, int y) {
    if (x == y) return x;
    if ((x = in[x] + 1) > (y = in[y] + 1))
```

```
      swap(x, y);
    int h = __lg(y - x);
    return pa[cmp(st[h][x], st[h][y - (1 << h)])];
  }
  int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
  }
  int rootPar(int r, int x) {
    if (r == x) return -1;
    if (!inside(x, r)) return pa[x];
    return *--upper_bound(all(G[x]), r,
      [&](int a, int b) -> bool {
        return in[a] < in[b];
      });
  }
  int size(int x) { return out[x] - in[x]; }
  int rootSiz(int r, int x) {
    if (r == x) return n;
    if (!inside(x, r)) return size(x);
    return n - size(rootPar(r, x));
  }
  int rootLca(int a, int b, int c) {
    return lca(a, b) ^ lca(b, c) ^ lca(c, a);
  } /* SPLIT-HASH */
  vector<int> virTree(vector<int> ver) {
    sort(all(ver), [&](int a, int b) {
      return in[a] < in[b];
    });
    for (int i = ver.size() - 1; i > 0; i--)
      ver.push_back(lca(ver[i], ver[i - 1]));
    sort(all(ver), [&](int a, int b) {
      return in[a] < in[b];
    });
    ver.erase(unique(all(ver)), ver.end());
    return ver;
  }
  void inplace_virTree(vector<int> &ver) {
    vector<int> ex; // O(n), need sort before
    for (int i = 0; i + 1 < ver.size(); i++)
      if (!inside(ver[i], ver[i + 1]))
        ex.push_back(lca(ver[i], ver[i + 1]));
    vector<int> stk, pa(ex.size(), -1);
    for (int i = 0; i < ex.size(); i++) {
      int lst = -1;
      while (stk.size() and in[ex[stk.back()]] >= in[ex
    [i]]) {
        lst = stk.back();
        stk.pop_back();
      }
      if (lst != -1) pa[lst] = i;
      if (stk.size()) pa[i] = stk.back();
      stk.push_back(i);
    }
    vector<bool> vis(ex.size());
    auto dfs = [&](auto self, int u) -> void {
      vis[u] = 1;
      if (pa[u] != -1 and !vis[pa[u]])
        self(self, pa[u]);
      if (ex[u] != ver.back())
        ver.push_back(ex[u]);
    };
    const int s = ver.size();
    for (int i = 0; i < ex.size(); i++)
      if (!vis[i]) dfs(dfs, i);
    inplace_merge(ver.begin(), ver.begin() + s, ver.end
    (),
        [&](int a, int b) { return in[a] < in[b]; });
    ver.erase(unique(all(ver)), ver.end());
  }
};
```

3.3 Functional Graph [b2e271]

```
// bel[x]: x is belong bel[x]-th jellyfish
// len[x]: cycle length of x-th jellyfish
// ord[x]: order of x in cycle (x == root[x])
struct FunctionalGraph {
  int n, _t = 0;
  vector<vector<int>> G;
  vector<int> f, bel, dep, ord, root, in, out, len;
  FunctionalGraph(int n) : n(n), G(n), root(n),
     bel(n, -1), dep(n), ord(n), in(n), out(n) {}
  void dfs(int u) {
    in[u] = _t++;
    for (int v : G[u]) if (bel[v] == -1) {
      dep[v] = dep[u] + 1;
      root[v] = root[u];
      bel[v] = bel[u];
      dfs(v);
    }
    out[u] = _t;
  };
  void build(const auto &_f) {
    f = _f;
    for (int i = 0; i < n; i++)
      G[f[i]].push_back(i);
    vector<int> vis(n, -1);
    for (int i = 0; i < n; i++) if (vis[i] == -1) {
      int x = i;
      while (vis[x] == -1) {
        vis[x] = i;
        x = f[x];
      }
      if (vis[x] != i) continue;
      int s = x, l = 0;
      do {
        bel[x] = len.size();
        ord[x] = l++;
        root[x] = x;
        x = f[x];
      } while (x != s);
      len.push_back(l);
    }
    for (int i = 0; i < n; i++)
      if (root[i] == i)
        dfs(i);
  }
  int dist(int x, int y) { // x -> y
    if (bel[x] != bel[y])
      return -1;
    if (dep[x] < dep[y])
      return -1;
    if (dep[y] != 0) {
      if (in[y] <= in[x] and in[x] < out[y])
        return dep[x] - dep[y];
      return -1;
    }
    return dep[x] + (ord[y] - ord[root[x]] + len[bel[x
    ]]) % len[bel[x]];
  }
};
```

3.4 Manhattan MST [2bf037]

```
// {w, u, v}
vector<tuple<int, int, int>> ManhattanMST(vector<Pt> P)
    {
  vector<int> id(P.size());
  iota(all(id), 0);
  vector<tuple<int, int, int>> edg;
  for (int k = 0; k < 4; k++) {
    sort(all(id), [&](int i, int j) {
        return (P[i] - P[j]).ff < (P[j] - P[i]).ss;
      });
    map<int, int> sweep;
    for (int i : id) {
      auto it = sweep.lower_bound(-P[i].ss);
      while (it != sweep.end()) {
        int j = it->ss;
        Pt d = P[i] - P[j];
        if (d.ss > d.ff)
          break;
        edg.emplace_back(d.ff + d.ss, i, j);
        it = sweep.erase(it);
      }
      sweep[-P[i].ss] = i;
    }
    for (Pt &p : P)
      if (k % 2) p.ff = -p.ff;
      else swap(p.ff, p.ss);
  }
  return edg;
}
```

3.5 Count Cycles [c7e8f2]

```
// ord = sort by deg decreasing, rk[ord[i]] = i
// D[i] = edge point from rk small to rk big
for (int x : ord) { // c3
 for (int y : D[x]) vis[y] = 1;
```

```
 for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
 for (int y : D[x]) vis[y] = 0;
}
for (int x : ord) { // c4
 for (int y : D[x]) for (int z : adj[y])
  if (rk[z] > rk[x]) c4 += vis[z]++;
 for (int y : D[x]) for (int z : adj[y])
  if (rk[z] > rk[x]) --vis[z];
} // both are O(M*sqrt(M)), test @ 2022 CCPC guangzhou
```

3.6 Maximum Clique [3ca044] (00fbbb|298686)

```
constexpr size_t kN = 150;
using bits = bitset<kN>;
struct MaxClique {
  bits G[kN], cs[kN];
  int ans, sol[kN], q, cur[kN], d[kN], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].reset();
  }
  void addEdge(int u, int v) {
    G[u][v] = G[v][u] = 1;
  }
  void preDfs(vector<int> &v, int i, bits mask) {
    if (i < 4) {
      for (int x : v) d[x] = (G[x] & mask).count();
      sort(all(v), [&](int x, int y) {
        return d[x] > d[y];
      });
    }
    vector<int> c(v.size());
    cs[1].reset(), cs[2].reset();
    int l = max(ans - q + 1, 1), r = 2, tp = 0, k;
    for (int p : v) {
      for (k = 1;
        (cs[k] & G[p]).any(); ++k);
      if (k >= r) cs[++r].reset();
      cs[k][p] = 1;
      if (k < l) v[tp++] = p;
    }
    for (k = l; k < r; ++k)
      for (auto p = cs[k]._Find_first(); p < kN; p = cs
    [k]._Find_next(p))
        v[tp] = p, c[tp] = k, ++tp;
    dfs(v, c, i + 1, mask);
  } /* SPLIT-HASH */
  void dfs(vector<int> &v, vector<int> &c, int i, bits
    mask) {
    while (!v.empty()) {
      int p = v.back();
      v.pop_back();
      mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr;
      for (int x : v)
        if (G[p][x]) nr.push_back(x);
      if (!nr.empty()) preDfs(nr, i, mask & G[p]);
      else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back();
      --q;
    }
  }
  int solve() {
    vector<int> v(n);
    iota(all(v), 0);
    ans = q = 0;
    preDfs(v, 0, bits(string(n, '1')));
    return ans;
  }
} cliq;
```

3.7 Min Mean Weight Cycle [cdb7d3]

```
// d[i][j] == 0 if {i,j} !in E
long long d[1003][1003], dp[1003][1003];

pair<long long, long long> MMWC() {
 memset(dp, 0x3f, sizeof(dp));
 for (int i = 1; i <= n; ++i) dp[0][i] = 0;
 for (int i = 1; i <= n; ++i) {
  for (int j = 1; j <= n; ++j) {
   for (int k = 1; k <= n; ++k) {
    dp[i][k] = min(dp[i - 1][j] + d[j][k], dp[i][k]);
   }
  }
 }
 long long au = 1ll << 31, ad = 1;
 for (int i = 1; i <= n; ++i) {
  if (dp[n][i] == 0x3f3f3f3f3f3f3f3f) continue;
  long long u = 0, d = 1;
  for (int j = n - 1; j >= 0; --j) {
   if ((dp[n][i] - dp[j][i]) * d > u * (n - j)) {
    u = dp[n][i] - dp[j][i];
    d = n - j;
   }
  }
  if (u * ad < au * d) au = u, ad = d;
 }
 long long g = __gcd(au, ad);
 return make_pair(au / g, ad / g);
}
```

3.8 Block Cut Tree [c8aef1]

```
struct BlockCutTree {
  int n;
  vector<vector<int>> adj;
  BlockCutTree(int _n) : n(_n), adj(_n) {}
  void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  pair<int, vector<pair<int, int>>> work() {
    vector<int> dfn(n, -1), low(n), stk;
    vector<pair<int, int>> edg;
    int cnt = 0, cur = 0;
    function<void(int)> dfs = [&](int x) {
      stk.push_back(x);
      dfn[x] = low[x] = cur++;
      for (auto y : adj[x]) {
        if (dfn[y] == -1) {
          dfs(y);
          low[x] = min(low[x], low[y]);
          if (low[y] == dfn[x]) {
            int v;
            do {
              v = stk.back();
              stk.pop_back();
              edg.emplace_back(n + cnt, v);
            } while (v != y);
            edg.emplace_back(x, n + cnt);
            cnt++;
          }
        } else {
          low[x] = min(low[x], dfn[y]);
        }
      }
    };
    for (int i = 0; i < n; i++) {
      if (dfn[i] == -1) {
        stk.clear();
        dfs(i);
      }
    }
    return {cnt, edg};
  }
};
```

3.9 Heavy Light D [b3b663] (323e32|20e9cc)

```
struct HLD {
  int n;
  vector<int> siz, dep, pa, in, out, seq, top, tail;
  vector<vector<int>> G;
  HLD(int n) : n(n), G(n), siz(n), dep(n), pa(n),
    in(n), out(n), top(n), tail(n) {}
  void build(int root = 0) {
    top[root] = root;
    dep[root] = 0;
    pa[root] = -1;
    dfs1(root);
    dfs2(root);
  }
  void dfs1(int u) {
    erase(G[u], pa[u]);
    siz[u] = 1;
    for (auto &v : G[u]) {
      pa[v] = u;
      dep[v] = dep[u] + 1;
      dfs1(v);
```

```
      siz[u] += siz[v];
      if (siz[v] > siz[G[u][0]]) {
        swap(v, G[u][0]);
      }
    }
  }
  void dfs2(int u) {
    in[u] = seq.size();
    seq.push_back(u);
    tail[u] = u;
    for (int v : G[u]) {
      top[v] = (v == G[u][0] ? top[u] : v);
      dfs2(v);
      if (v == G[u][0]) {
        tail[u] = tail[v];
      }
    }
    out[u] = seq.size();
  } /* SPLIT-HASH */
  int lca(int x, int y) {
    while (top[x] != top[y]) {
      if (dep[top[x]] < dep[top[y]]) swap(x, y);
      x = pa[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
  }
  int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
  }
  int jump(int x, int k) {
    if (dep[x] < k) return -1;
    int d = dep[x] - k;
    while (dep[top[x]] > d) {
      x = pa[top[x]];
    }
    return seq[in[x] - dep[x] + d];
  }
  bool isAnc(int x, int y) {
    return in[x] <= in[y] and in[y] < out[x];
  }
  int rootPar(int r, int x) {
    if (r == x) return r;
    if (!isAnc(x, r)) return pa[x];
    auto it = upper_bound(all(G[x]), r, [&](int a, int
    b) -> bool {
      return in[a] < in[b];
    }) - 1;
    return *it;
  }
  int rootSiz(int r, int x) {
    if (r == x) return n;
    if (!isAnc(x, r)) return siz[x];
    return n - siz[rootPar(r, x)];
  }
  int rootLca(int a, int b, int c) {
    return lca(a, b) ^ lca(b, c) ^ lca(c, a);
  }
};
```

3.10 Dominator Tree [e09ba5]

```
struct Dominator {
  vector<vector<int>> g, r, rdom; int tk;
  vector<int> dfn, rev, fa, sdom, dom, val, rp;
  int n;
  Dominator(int n) : n(n), g(n), r(n), rdom(n), tk(0),
    dfn(n, -1), rev(n, -1), fa(n, -1), sdom(n, -1),
    dom(n, -1), val(n, -1), rp(n, -1) {}
  void add_edge(int x, int y) { g[x].push_back(y); }
  void dfs(int x) {
    rev[dfn[x] = tk] = x;
    fa[tk] = sdom[tk] = val[tk] = tk; tk++;
    for (int u : g[x]) {
      if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
      r[dfn[u]].push_back(dfn[x]);
    }
  }
  void merge(int x, int y) { fa[x] = y; }
  int find(int x, int c = 0) {
    if (fa[x] == x) return c ? -1 : x;
    if (int p = find(fa[x], 1); p != -1) {
      if (sdom[val[x]] > sdom[val[fa[x]]])
        val[x] = val[fa[x]];
      fa[x] = p;
      return c ? p : val[x];
    }
    return c ? fa[x] : val[x];
  }
  vector<int> build(int s) {
    // return the father of each node in dominator tree
    // p[i] = -2 if i is unreachable from s
    dfs(s);
    for (int i = tk - 1; i >= 0; --i) {
      for (int u : r[i])
        sdom[i] = min(sdom[i], sdom[find(u)]);
      if (i) rdom[sdom[i]].push_back(i);
      for (int u : rdom[i]) {
        int p = find(u);
        dom[u] = (sdom[p] == i ? i : p);
      }
      if (i) merge(i, rp[i]);
    }
    vector<int> p(n, -2); p[s] = -1;
    for (int i = 1; i < tk; ++i)
      if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
    for (int i = 1; i < tk; ++i)
      p[rev[i]] = rev[dom[i]];
    return p;
  }
};
```

3.11 Matroid Intersection [0ef8fe]

```
template<class Matroid1, class Matroid2>
vector<bool> MatroidIntersection(Matroid1 &m1, Matroid2
     &m2) {
  const int N = m1.size();
  vector<bool> I(N);
  while (true) {
    m1.set(I);
    m2.set(I);
    vector<vector<int>> E(N + 2);
    const int s = N, t = N + 1;
    for (int i = 0; i < N; i++) {
      if (I[i]) { continue; }
      auto c1 = m1.circuit(i);
      auto c2 = m2.circuit(i);
      if (c1.empty()) {
        E[s].push_back(i);
      } else {
        for (int y : c1) if (y != i) {
          E[y].push_back(i);
        }
      }
      if (c2.empty()) {
        E[i].push_back(t);
      } else {
        for (int y : c2) if (y != i) {
          E[i].push_back(y);
        }
      }
    }
    vector<int> pre(N + 2, -1);
    queue<int> que;
    que.push(s);
    while (que.size() and pre[t] == -1) {
      int u = que.front();
      que.pop();
      for (int v : E[u]) {
        if (pre[v] == -1) {
          pre[v] = u;
          que.push(v);
        }
      }
    }
    if (pre[t] == -1) { break; }
    for (int p = pre[t]; p != s; p = pre[p]) {
      I[p] = !I[p];
    }
  }
  return I;
}
```

3.12 G Series-Parallel Graph [55713a] (08bf82|757cf1)

```
/* Vertex:  {u, -1}
 * Edge:    {u, v};        u < v
 * Series:  (e1, v1, e2) => e3; e1 < e2
```

```
 * Parallel: (e1, e2)   => e3; e1 = e2
 * Dangling: (v1, e1, v2) => v3; e1 = {v1, v2}
*/
struct GSPGraph {
  int N;
  vector<pair<int, int>> S;
  vector<vector<int>> tree;
  vector<bool> isrt;
  int getv(int e, int u) { return S[e].ff ^ S[e].ss ^ u
    ; }
  int newNode(pair<int, int> s, vector<int> sub) {
    S[N] = s, tree[N] = sub;
    for (int x : sub) isrt[x] = false;
    return N++;
  }
  GSPGraph(int n, const vector<pair<int, int>> &edge) {
    N = edge.size();
    S = edge;
    S.resize(N * 2 + n, {-1, -1});
    tree.resize(N * 2 + n);
    isrt.assign(N * 2 + n, true);
    vector<vector<int>> G(n);
    vector<int> vid(n), deg(n);
    unordered_map<pair<int, int>, int> eid;
    queue<int> que;
    auto add = [&](int e) {
      auto [u, v] = S[e];
      if (auto it = eid.find(S[e]); it != eid.end()) {
        it->ss = e = newNode(S[e], {e, it->ss});
        if (--deg[u] == 2) que.push(u);
        if (--deg[v] == 2) que.push(v);
      } else eid[S[e]] = e;
      G[u].push_back(e);
      G[v].push_back(e);
    }; /* SPLIT-HASH */
    for (int i = N - 1; i >= 0; i--) {
      S[i] = minmax({S[i].ff, S[i].ss});
      add(i);
    }
    for (int i = 0; i < n; i++) {
      S[vid[i] = N++] = {i, -1};
      deg[i] += ssize(G[i]);
      if (deg[i] <= 2) que.push(i);
    }
    auto pop = [&](int x) {
      while (!isrt[G[x].back()]) G[x].pop_back();
      int e = G[x].back();
      isrt[e] = false;
      return e;
    };
    while (que.size()) {
      int u = que.front(); que.pop();
      if (deg[u] == 1) {
        int e = pop(u), v = getv(e, u);
        vid[v] = newNode(
          {v, -1}, {vid[S[e].ff], e, vid[S[e].ss]}
        );
        if (--deg[v] == 2) que.push(v);
      } else if (deg[u] == 2) {
        int e1 = pop(u), e2 = pop(u);
        if (S[e1] > S[e2]) swap(e1, e2);
        add(newNode(
          minmax(getv(e1, u), getv(e2, u)),
          {e1, vid[u], e2}
        ));
      }
    }
    S.resize(N);
    tree.resize(N);
    isrt.resize(N);
  }
};
```

4 Data Structure

4.1 Lazy Segtree [514506] (8da578|bee1cb)

```
template<class S, class T>
struct Seg {
  Seg *ls{}, *rs{}; S sum{}; T tag{}; int l, r;
  Seg(int _l, int _r) : l(_l), r(_r) {
    if (r - l == 1)
      return;
    int m = (l + r) / 2;
    ls = new Seg(l, m);
    rs = new Seg(m, r);
    pull();
  }
  void pull() { sum = ls->sum + rs->sum; }
  void push() {
    ls->apply(tag);
    rs->apply(tag);
    tag = T{};
  }
  void apply(const T &f) { f(tag); f(sum); }
  S query(int x, int y) {
    if (y <= l or r <= x)
      return {};
    if (x <= l and r <= y)
      return sum;
    push();
    return ls->query(x, y) + rs->query(x, y);
  }
  void apply(int x, int y, const T &f) {
    if (y <= l or r <= x)
      return;
    if (x <= l and r <= y)
      return apply(f);
    push();
    ls->apply(x, y, f);
    rs->apply(x, y, f);
    pull();
  } /* SPLIT-HASH */
  void set(int p, const S &e) {
    if (p < l or p >= r)
      return;
    if (r - l == 1)
      return sum = e, void();
    push();
    ls->set(p, e);
    rs->set(p, e);
    pull();
  }
  pair<int, S> findFirst(int x, int y, auto &&pred, S
    cur = {}) {
    if (y <= l or r <= x)
      return { -1, cur };
    if (x <= l and r <= y and !pred(cur + sum))
      return { -1, cur + sum };
    if (r - l == 1)
      return { l, cur + sum };
    push();
    auto L = ls->findFirst(x, y, pred, cur);
    if (L.ff != -1) return L;
    return rs->findFirst(x, y, pred, L.ss);
  }
  pair<int, S> findLast(int x, int y, auto &&pred, S
    cur = {}) {
    if (y <= l or r <= x)
      return { -1, cur };
    if (x <= l and r <= y and !pred(sum + cur))
      return { -1, sum + cur };
    if (r - l == 1)
      return { l, sum + cur };
    push();
    auto R = rs->findLast(x, y, pred, cur);
    if (R.ff != -1) return R;
    return ls->findLast(x, y, pred, R.ss);
  }
};
```

4.2 Fenwick Tree [197d13]

```
template<class T>
struct Fenwick {
  int n;
  vector<T> a;
  Fenwick(int _n) : n(_n), a(_n) {}
  int lob(int x) { return x & -x; }
  void add(int p, T x) {
    assert(p < n);
    for (int i = p + 1; i <= n; i += lob(i)) {
      a[i - 1] = a[i - 1] + x;
    }
  }
  T sum(int p) { // sum [0, p]
    T s{};
    for (int i = min(p, n) + 1; i > 0; i -= lob(i)) {
```

```
      s = s + a[i - 1];
    }
    return s;
  }
  int findFirst(auto &&pred) { // min{ k | pred(k) }
    T s{};
    int p = 0;
    for (int i = 1 << __lg(n); i; i >>= 1) {
      if (p + i <= n and !pred(s + a[p + i - 1])) {
        p += i;
        s = s + a[p - 1];
      }
    }
    return p == n ? -1 : p;
  }
};
```

4.3 Interval Segtree [360bb9]

```
struct Seg {
  Seg *ls, *rs;
  int l, r;
  vector<int> f, g;
  // f : intervals where covering [l, r]
  // g : intervals where interset with [l, r]
  Seg(int _l, int _r) : l{_l}, r{_r} {
    int mid = (l + r) >> 1;
    if (r - l == 1) return;
    ls = new Seg(l, mid);
    rs = new Seg(mid, r);
  }
  void insert(int x, int y, int id) {
    if (y <= l or r <= x) return;
    g.push_back(id);
    if (x <= l and r <= y) {
      f.push_back(id);
      return;
    }
    ls->insert(x, y, id);
    rs->insert(x, y, id);
  }
  void fix() {
    while (!f.empty() and use[f.back()]) f.pop_back();
    while (!g.empty() and use[g.back()]) g.pop_back();
  }
  int query(int x, int y) {
    if (y <= l or r <= x) return -1;
    fix();
    if (x <= l and r <= y) {
      return g.empty() ? -1 : g.back();
    }
    return max({f.empty() ? -1 : f.back(), ls->query(x,
     y), rs->query(x, y)});
  }
};
```

4.4 PrefixMax Sum Segtree [3a9bce]

```
// O(Nlog^2N)!
const int kC = 1E6;
struct Seg {
  static Seg pool[kC], *top;
  Seg *ls{}, *rs{};
  int l, r;
  i64 sum = 0, rsum = 0, mx = 0;
  Seg() {}
  Seg(int _l, int _r, const vector<i64> &v) : l(_l), r(
    _r) {
    if (r - l == 1) {
      sum = mx = v[l];
      return;
    }
    int m = (l + r) / 2;
    ls = new (top++) Seg(l, m, v);
    rs = new (top++) Seg(m, r, v);
    pull();
  }
  i64 cal(i64 h) { // sigma i in [l, r) max(h, v[i])
    if (r - l == 1) {
      return max(mx, h);
    }
    if (mx <= h) {
      return h * (r - l);
    }
    if (ls->mx >= h) {
      return ls->cal(h) + rsum;
    }
    return h * (ls->r - ls->l) + rs->cal(h);
  }
  void pull() {
    rsum = rs->cal(ls->mx);
    sum = ls->sum + rsum;
    mx = max(ls->mx, rs->mx);
  }
  void set(int p, i64 h) {
    if (r - l == 1) {
      sum = mx = h;
      return;
    }
    int m = (l + r) / 2;
    if (p < m) {
      ls->set(p, h);
    } else {
      rs->set(p, h);
    }
    pull();
  }
  i64 query(int p, i64 h) { // sigma i in [0, p) max(h,
     v[i])
    if (p <= l) {
      return 0;
    }
    if (p >= r) {
      return cal(h);
    }
    return ls->query(p, h) + rs->query(p, max(h, ls->mx
    ));
  }
} Seg::pool[kC], *Seg::top = Seg::pool;
```

4.5 Disjoint Set Union-undo [17d60d]

```
template<class T>
struct DSU {
  vector<T> tag;
  vector<int> f, siz, stk;
  int cc;
  DSU(int n) : f(n, -1), siz(n, 1), tag(n), cc(n) {}
  int find(int x) { return f[x] < 0 ? x : find(f[x]); }
  bool merge(int x, int y) {
    x = find(x);
    y = find(y);
    if (x == y) return false;
    if (siz[x] > siz[y]) swap(x, y);
    f[x] = y;
    siz[y] += siz[x];
    tag[x] = tag[x] - tag[y];
    stk.push_back(x);
    cc--;
    return true;
  }
  void apply(int x, T s) {
    x = find(x);
    tag[x] = tag[x] + s;
  }
  void undo() {
    int x = stk.back();
    int y = f[x];
    stk.pop_back();
    tag[x] = tag[x] + tag[y];
    siz[y] -= siz[x];
    f[x] = -1;
    cc++;
  }
  bool same(int x, int y) { return find(x) == find(y);
    }
  int size(int x) { return siz[find(x)]; }
};
```

4.6 Centroid Decomposition [438db7]

```
struct CenDec {
  vector<vector<pair<int, i64>>> G;
  vector<vector<i64>> pdis;
  vector<int> pa, ord, siz;
  vector<bool> vis;
  int getsiz(int u, int f) {
    siz[u] = 1;
    for (auto [v, w] : G[u]) if (v != f and !vis[v])
      siz[u] += getsiz(v, u);
    return siz[u];
  }
```

```
  int find(int u, int f, int s) {
    for (auto [v, w] : G[u]) if (v != f and !vis[v])
      if (siz[v] * 2 >= s) return find(v, u, s);
    return u;
  };
  void caldis(int u, int f, i64 dis) {
    pdis[u].push_back(dis);
    for (auto [v, w] : G[u]) if (v != f and !vis[v]) {
      caldis(v, u, dis + w);
    }
  }
  int build(int u = 0) {
    u = find(u, u, getsiz(u, u));
    ord.push_back(u);
    vis[u] = 1;
    for (auto [v, w] : G[u]) if (!vis[v]) {
      pa[build(v)] = u;
    }
    caldis(u, -1, 0); // if need
    vis[u] = 0;
    return u;
  };
  CenDec(int n) : G(n), pa(n, -1), vis(n), siz(n), pdis
    (n) {}
};
```

4.7 2D BIT [805424]

```
template<class T>
struct BIT2D {
  vector<vector<T>> val;
  vector<vector<int>> Y;
  vector<int> X;
  int lowbit(int x) { return x & -x; }
  int getp(const vector<int> &v, int x) {
    return upper_bound(all(v), x) - v.begin();
  }
  BIT2D(vector<pair<int, int>> pos) {
    for (auto &[x, y] : pos) {
      X.push_back(x);
      swap(x, y);
    }
    sort(all(pos));
    sort(all(X));
    X.erase(unique(all(X)), X.end());
    Y.resize(X.size() + 1);
    val.resize(X.size() + 1);
    for (auto [y, x] : pos) {
      for (int i = getp(X, x); i <= X.size(); i +=
    lowbit(i))
        if (Y[i].empty() or Y[i].back() != y)
          Y[i].push_back(y);
    }
    for (int i = 1; i <= X.size(); i++) {
      val[i].assign(Y[i].size() + 1, T{});
    }
  }
  void add(int x, int y, T v) {
    for (int i = getp(X, x); i <= X.size(); i += lowbit
    (i))
      for (int j = getp(Y[i], y); j <= Y[i].size(); j
    += lowbit(j))
        val[i][j] += v;
  }
  T qry(int x, int y) {
    T r{};
    for (int i = getp(X, x); i > 0; i -= lowbit(i))
      for (int j = getp(Y[i], y); j > 0; j -= lowbit(j)
    ) {
        r += val[i][j];
      }
    return r;
  }
};
```

4.8 Big Binary [dbe18b]

```
struct BigBinary : map<int, int> {
  void split(int x) {
    auto it = lower_bound(x);
    if (it != begin()) {
      it--;
      if (it->ss > x) {
        (*this)[x] = it->ss;
        it->ss = x;
      }
    }
  }
  void add(int x) {
    split(x);
    auto it = find(x);
    while (it != end() and it->ff == x) {
      x = it->ss;
      it = erase(it);
    }
    (*this)[x] = x + 1;
  }
  void sub(int x) {
    split(x);
    auto it = lower_bound(x);
    // assert(it != end());
    auto [l, r] = *it;
    erase(it);
    if (l + 1 < r) (*this)[l + 1] = r;
    if (x < l)   (*this)[x] = l;
  }
};
```

4.9 Splay Tree [650110] (4b1e58|2c0471|13d3ca)

```
struct Node {
  Node *ch[2]{}, *p{};
  Info info{}, sum{};
  Tag tag{};
  int size{};
  bool rev{};
} pool[int(1E5 + 10)], *top = pool;
Node *newNode(Info a) {
  Node *t = top++;
  t->info = t->sum = a;
  t->size = 1;
  return t;
}
int size(const Node *x) { return x ? x->size : 0; }
Info get(const Node *x) { return x ? x->sum : Info{}; }
int dir(const Node *x) { return x->p->ch[1] == x; }
bool nroot(const Node *x) { return x->p and x->p->ch[
    dir(x)] == x; }
void reverse(Node *x) { if (x) x->rev = !x->rev; }
void update(Node *x, const Tag &f) {
  if (!x) return;
  f(x->tag);
  f(x->info);
  f(x->sum);
}
void push(Node *x) {
  if (x->rev) {
    swap(x->ch[0], x->ch[1]);
    reverse(x->ch[0]);
    reverse(x->ch[1]);
    x->rev = false;
  }
  update(x->ch[0], x->tag);
  update(x->ch[1], x->tag);
  x->tag = Tag{};
}
void pull(Node *x) {
  x->size = size(x->ch[0]) + 1 + size(x->ch[1]);
  x->sum = get(x->ch[0]) + x->info + get(x->ch[1]);
} /* SPLIT-HASH */
void rotate(Node *x) {
  Node *y = x->p, *z = y->p;
  push(y);
  int d = dir(x);
  push(x);
  Node *w = x->ch[d ^ 1];
  if (nroot(y)) {
    z->ch[dir(y)] = x;
  }
  if (w) {
    w->p = y;
  }
  (x->ch[d ^ 1] = y)->ch[d] = w;
  (y->p = x)->p = z;
  pull(y);
  pull(x);
}
void splay(Node *x) {
  while (nroot(x)) {
    Node *y = x->p;
```

```
    if (nroot(y)) {
      rotate(dir(x) == dir(y) ? y : x);
    }
    rotate(x);
  }
}
Node *nth(Node *x, int k) {
  assert(size(x) > k);
  while (true) {
    push(x);
    int left = size(x->ch[0]);
    if (left > k) {
      x = x->ch[0];
    } else if (left < k) {
      k -= left + 1;
      x = x->ch[1];
    } else {
      break;
    }
  }
  splay(x);
  return x;
} /* SPLIT-HASH */
Node *split(Node *x) {
  assert(x);
  push(x);
  Node *l = x->ch[0];
  if (l) l->p = x->ch[0] = nullptr;
  pull(x);
  return l;
}
Node *join(Node *x, Node *y) {
  if (!x or !y) return x ? x : y;
  y = nth(y, 0);
  push(y);
  y->ch[0] = x;
  if (x) x->p = y;
  pull(y);
  return y;
}
Node *find_first(Node *x, auto &&pred) {
  Info pre{};
  while (true) {
    push(x);
    if (pred(pre + get(x->ch[0]))) {
      x = x->ch[0];
    } else if (pred(pre + get(x->ch[0]) + x->info) or !
    x->ch[1]) {
      break;
    } else {
      pre = pre + get(x->ch[0]) + x->info;
      x = x->ch[1];
    }
  }
  splay(x);
  return x;
}
```

4.10 Link Cut Tree [7ef9ee] (ebadb5|d1bbee)

```
namespace lct {
Node *access(Node *x) {
  Node *last = {};
  while (x) {
    splay(x);
    push(x);
    x->ch[0] = last;
    pull(x);
    last = x;
    x = x->p;
  }
  return last;
}
void make_root(Node *x) {
  access(x);
  splay(x);
  reverse(x);
}
Node *find_root(Node *x) {
  push(x = access(x));
  while (x->ch[1]) {
    push(x = x->ch[1]);
  }
  splay(x);
  return x;
} /* SPLIT-HASH */
bool link(Node *x, Node *y) {
  if (find_root(x) == find_root(y)) {
    return false;
  }
  make_root(x);
  x->p = y;
  return true;
}
bool cut(Node *a, Node *b) {
  make_root(a);
  access(b);
  splay(a);
  if (a->ch[0] == b) {
    split(a);
    return true;
  }
  return false;
}
Info query(Node *a, Node *b) {
  make_root(b);
  return get(access(a));
}
void set(Node *x, Info v) {
  splay(x);
  push(x);
  x->info = v;
  pull(x);
} }
```

4.11 Static Top Tree [eda4f7] (56a00a|20d546|98bd4b)

```
template<class Vertex, class Path>
struct StaticTopTree {
  enum Type { Rake, Compress, Combine, Convert };
  int stt_root;
  vector<vector<int>> &G;
  vector<int> P, L, R, S;
  vector<Type> T;
  vector<Vertex> f;
  vector<Path> g;
  int buf;
  int dfs(int u) {
    int s = 1, big = 0;
    for (int &v : G[u]) {
      erase(G[v], u);
      int t = dfs(v);
      s += t;
      if (chmax(big, t)) swap(G[u][0], v);
    }
    return s;
  }
  int add(int l, int r, Type t) {
    int x = buf++;
    P[x] = -1, L[x] = l, R[x] = r, T[x] = t;
    if (l != -1) P[l] = x, S[x] += S[l];
    if (r != -1) P[r] = x, S[x] += S[r];
    return x;
  }
  int merge(auto l, auto r, Type t) {
    if (r - l == 1) return *l;
    int s = 0;
    for (auto i = l; i != r; i++) s += S[*i];
    auto m = l;
    while (s > S[*m]) s -= 2 * S[*m++];
    return add(merge(l, m, t), merge(m, r, t), t);
  } /* SPLIT-HASH */
  int pathCluster(int u) {
    vector<int> chs{pointCluster(u)};
    while (!G[u].empty()) chs.push_back(pointCluster(u
    = G[u][0]));
    return merge(all(chs), Type::Compress);
  }
  int pointCluster(int u) {
    vector<int> chs;
    for (int v : G[u] | views::drop(1))
      chs.push_back(add(pathCluster(v), -1, Type::
      Convert));
    if (chs.empty()) return add(u, -1, Type::Convert);
    return add(u, merge(all(chs), Type::Rake), Type::
    Combine);
```

```
  }
  StaticTopTree(vector<vector<int>> &_G, int root = 0)
    : G(_G) {
    const int n = G.size();
    P.assign(4 * n, -1);
    L.assign(4 * n, -1);
    R.assign(4 * n, -1);
    S.assign(4 * n, 1);
    T.assign(4 * n, Type::Rake);
    buf = n;
    dfs(root);
    stt_root = pathCluster(root);
    f.resize(buf);
    g.resize(buf);
  }
  void update(int x) {
    if (T[x] == Rake) f[x] = f[L[x]] * f[R[x]];
    else if (T[x] == Compress) g[x] = g[L[x]] + g[R[x]];
    else if (T[x] == Combine) g[x] = f[L[x]] + f[R[x]];
    else if (T[L[x]] == Rake) g[x] = Path(f[L[x]]);
    else f[x] = Vertex(g[L[x]]);
  } /* SPLIT-HASH */
  void set(int x, const Vertex &v) {
    f[x] = v;
    for (x = P[x]; x != -1; x = P[x])
      update(x);
  }
  Vertex get() { return g[stt_root]; }
};
struct Path;
struct Vertex {
  Vertex() {}
  Vertex(const Path&);
};
struct Path {
  Path() {};
  Path(const Vertex&);
};
Vertex operator*(const Vertex &a, const Vertex &b) {
  return {};
}
Path operator+(const Vertex &a, const Vertex &b) {
  return {};
}
Path operator+(const Path &a, const Path &b) {
  return {};
}
Vertex::Vertex(const Path &x) {}
Path::Path(const Vertex &x) {}
/*
 * (root) 1 - 2 (heavy)
 *    / \   \
 *   3 4   5
 * type V: subtree DP info (Commutative Semigroup)
 * type P: path DP info (Semigroup)
 * V(2) + V(5) -> P(2)
 * V(1) + (V(3) * V(4)) -> P(1)
 * ans: V(P(1) + P(2))
*/
```

5 Math

5.1 Linear Sieve [86c066]

```
vector<int> primes, minp;
vector<int> mu, phi;
vector<bool> isp;
void Sieve(int n) {
  minp.assign(n + 1, 0);
  primes.clear();
  isp.assign(n + 1, 0);
  mu.resize(n + 1);
  phi.resize(n + 1);
  mu[1] = phi[1] = 1;
  for (int i = 2; i <= n; i++) {
    if (minp[i] == 0) {
      minp[i] = i;
      isp[i] = 1;
      primes.push_back(i);
      mu[i] = -1;
      phi[i] = i - 1;
    }
    for (i64 p : primes) {
      if (p * i > n)
        break;
      minp[i * p] = p;
      if (p == minp[i]) {
        phi[p * i] = phi[i] * p;
        break;
      }
      phi[p * i] = phi[i] * (p - 1);
      mu[p * i] = mu[p] * mu[i];
    }
  }
}
```

5.2 Exgcd [280acd]

```
// ax + by = gcd(a, b)
i64 exgcd(i64 a, i64 b, i64 &x, i64 &y) {
  if (b == 0) return x = 1, y = 0, a;
  i64 g = exgcd(b, a % b, y, x);
  y -= a / b * x;
  return g; }
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */
```

5.3 Chinese Remainder Theorem [9a0377]

```
// O(NlogC)
// E = {(m, r), ...}: x mod m_i = r_i
// return {M, R} x mod M = R
// return {-1, -1} if no solution
pair<i64, i64> CRT(vector<pair<i64, i64>> E) {
  i128 R = 0, M = 1;
  for (auto [m, r] : E) {
    i64 g, x, y, d;
    g = exgcd(M, m, x, y);
    d = r - R;
    if (d % g != 0) {
      return {-1, -1};
    }
    R += d / g * M * x;
    M = M * m / g;
    R = (R % M + M) % M;
  }
  return {M, R};
}
```

5.4 Factorize [eece29] (2a59db|ba92f0)

```
u64 mul(u64 a, u64 b, u64 M) {
  i64 r = a * b - M * u64(1.L / M * a * b);
  return r + M * ((r < 0) - (r >= (i64)M));
}
u64 power(u64 a, u64 b, u64 M) {
  u64 r = 1;
  for (; b; b /= 2, a = mul(a, a, M))
    if (b & 1) r = mul(r, a, M);
  return r;
}
bool isPrime(u64 n) {
  if (n < 2 or n % 6 % 4 != 1) return (n | 1) == 3;
  auto magic = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
  u64 s = __builtin_ctzll(n - 1), d = n >> s;
  for (u64 x : magic) {
    u64 p = power(x % n, d, n), i = s;
    while (p != 1 and p != n - 1 and x % n && i--)
      p = mul(p, p, n);
    if (p != n - 1 and i != s) return 0;
  }
  return 1;
} /* SPLIT-HASH */
u64 pollard(u64 n) {
  u64 c = 1;
  auto f = [&](u64 x) { return mul(x, x, n) + c; };
  u64 x = 0, y = 0, p = 2, q, t = 0;
  while (t++ % 128 or gcd(p, n) == 1) {
    if (x == y) c++, y = f(x = 2);
    if (q = mul(p, x > y ? x - y : y - x, n)) p = q;
    x = f(x); y = f(f(y));
  }
  return gcd(p, n);
}
u64 primeFactor(u64 n) {
  return isPrime(n) ? n : primeFactor(pollard(n));
}
```

5.5 Theorem

- Pick's Theorem
  $A = i + \frac{b}{2} - 1$
  A: Area、i: grid number in the inner、b: grid number on the side

- Matrix-Tree theorem
  undirected graph
  $D_{ii}(G) = \text{deg}(i), D_{ij} = 0, i \neq j$
  $A_{ij}(G) = A_{ji}(G) = \#e(i,j), i \neq j$
  $L(G) = D(G) - A(G)$
  $t(G) = \det L(G)\binom{1,2,\cdots,i-1,i+1,\cdots,n}{1,2,\cdots,i-1,i+1,\cdots,n}$
  leaf to root
  $D_{ii}^{out}(G) = \text{deg}^{\text{out}}(i), D_{ij}^{out} = 0, i \neq j$
  $A_{ij}(G) = \#e(i,j), i \neq j$
  $L^{out}(G) = D^{out}(G) - A(G)$
  $t^{root}(G,k) = \det L^{out}(G)\binom{1,2,\cdots,k-1,k+1,\cdots,n}{1,2,\cdots,k-1,k+1,\cdots,n}$
  root to leaf
  $L^{in}(G) = D^{in}(G) - A(G)$
  $t^{leaf}(G,k) = \det L^{in}(G)\binom{1,2,\cdots,k-1,k+1,\cdots,n}{1,2,\cdots,k-1,k+1,\cdots,n}$

- Derangement
  $D_n = (n-1)(D_{n-1} + D_{n-2}) = nD(n-1) + (-1)^n$

- Möbius Inversion
  $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(\frac{n}{d}) f(d)$

- Euler Inversion
  $\sum_{i|n} \varphi(i) = n$

- Binomial Inversion
  $f(n) = \sum_{i=0}^{n} \binom{n}{i} g(i) \Leftrightarrow \ g(n) = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} f(i)$

- Subset Inversion
  $f(S) = \sum_{T \subseteq S} g(T) \Leftrightarrow g(S) = \sum_{T \subseteq S} (-1)^{|S|-|T|} f(T)$

- Min-Max Inversion
  $\text{max}_{i \in S}\, x_i = \sum_{T \subseteq S} (-1)^{|T|-1} \text{min}_{j \in T}\, x_j$

- Ex Min-Max Inversion
  $\underset{i \in S}{\text{kthmax}}\, x_i = \sum_{T \subseteq S} (-1)^{|T|-k} \binom{|T|-1}{k-1} \text{min}_{j \in T}\, x_j$

- Lcm-Gcd Inversion
  $\underset{i \in S}{\text{lcm}} x_i = \prod_{T \subseteq S} \left(\text{gcd}_{j \in T}\, x_j\right)^{(-1)^{|T|-1}}$

- Sum of powers
  $\sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} \ B_k^+ \ n^{m+1-k}$
  $\sum_{j=0}^{m} \binom{m+1}{j} B_j^- = 0$
  note: $B_1^+ = -B_1^-, B_i^+ = B_i^-$

- Cayley's formula
  number of trees on n labeled vertices: n^{n-2}
  Let $T_{n,k}$ be the number of labelled forests on n vertices with k connected components, such that vertices 1, 2, ..., k all belong to different connected components. Then $T_{n,k} = kn^{n-k-1}$.

- High order residue
  $[d^{\frac{p-1}{(n,p-1)}} \equiv 1]$

- Packing and Covering
  $|$maximum independent set$| + |$minimum vertex cover$| = |V|$

- Kőnig's theorem
  $|$maximum matching$| = |$minimum vertex cover$|$

- Dilworth's theorem
  width $= |$largest antichain$| = |$smallest chain decomposition$|$

- Mirsky's theorem
  height $= |$longest chain$| = |$smallest antichain decomposition$| = |$minimum anticlique partition$|$

- Lucas'Theorem
  For $n, m \in \mathbb{Z}^*$ and prime P, $\binom{m}{n}$ mod $P = \Pi\binom{m_i}{n_i}$ where m_i is the i-th digit of m in base P.

- Stirling approximation
  $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n e^{\frac{1}{12n}}$

- 1st Stirling Numbers(permutation $|P| = n$ with k cycles)
  $S(n,k) =$ coefficient of x^k in $\Pi_{i=0}^{n-1}(x+i)$
  $S(n+1,k) = nS(n,k) + S(n,k-1)$

- 2nd Stirling Numbers(Partition n elements into k non-empty set)
  $S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$
  $S(n+1,k) = kS(n,k) + S(n,k-1)$

- Catalan number
  $C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n-1}$
  $\binom{n+m}{n} - \binom{n+m}{n+1} = (m+n)! \frac{n-m+1}{n+1}$ for $n \geq m$
  $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
  $C_0 = 1$ and $C_{n+1} = 2(\frac{2n+1}{n+2})C_n$
  $C_0 = 1$ and $C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$ for $n \geq 0$

- Extended Catalan number
  $\frac{1}{(k-1)n+1}\binom{kn}{n}$

- Calculate $c[i-j] += a[i] \times b[j]$ for $a[n], b[m]$
  1. a=reverse(a); c=mul(a,b); c=reverse(c[:n]);
  2. b=reverse(b); c=mul(a,b); c=rshift(c,m-1);

- Eulerian number (permutation $1 \sim n$ with m $a[i] > a[i-1]$)
  $A(n,m) = \sum_{i=0}^{m} (-1)^i \binom{n+1}{i} (m+1-i)^n$
  $A(n,m) = (n-m)A(n-1,m-1) + (m+1)A(n-1,m)$

- Hall's theorem
  Let $G = (X+Y, E)$ be a bipartite graph. For $W \subseteq X$, let $N(W) \subseteq Y$ denotes the adjacent vertices set of W. Then, G has a X'-perfect matching (matching contains $X' \subseteq X$) iff $\forall W \subseteq X', |W| \leq |N(W)|$.

- Tutte Matrix:
  For a graph $G = (V,E)$, its maximum matching $= \frac{rank(A)}{2}$ where $A_{ij} = ((i,j) \in E ? (i < j ? x_{ij} : -x_{ji}) : 0)$ and x_{ij} are random numbers.

- Erdős–Gallai theorem
  There exists a simple graph with degree sequence $d_1 \geq \cdots \geq d_n$ iff $\sum_{i=1}^{n} d_i$ is even and $\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \text{min}(d_i, k), \forall 1 \leq k \leq n$

- Euler Characteristic
  planar graph: $V - E + F - C = 1$
  convex polyhedron: $V - E + F = 2$
  V, E, F, C: number of vertices, edges, faces(regions), and components

- Burnside Lemma
  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

- Polya theorem
  $|Y^x/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$
  $m = |Y|$: num of colors, c(g) : num of cycle

- Cayley's Formula
  Given a degree sequence $d_1, \ldots, d_n$ of a labeled tree, there are $\frac{(n-2)!}{(d_1-1)!\cdots(d_n-1)!}$ spanning trees.

- Find a Primitive Root of n:
  n has primitive roots iff $n = 2, 4, p^k, 2p^k$ where p is an odd prime.
  1. Find $\phi(n)$ and all prime factors of $\phi(n)$, says $P = \{p_1, ..., p_m\}$
  2. $\forall g \in [2,n)$, if $g^{\frac{\phi(n)}{p_i}} \neq 1, \forall p_i \in P$, then g is a primitive root.
  3. Since the smallest one isn't too big, the algorithm runs fast.
  4. n has exactly $\phi(\phi(n))$ primitive roots.

- Taylor series
  $f(x) = f(c) + f'(c)(x-c) + \frac{f^{(2)}(c)}{2!}(x-c)^2 + \frac{f^{(3)}(c)}{3!}(x-c)^3 + \cdots$

- Lagrange Multiplier
  $\text{min}\, f(x,y)$, subject to $g(x,y) = 0$
  $\frac{\partial f}{\partial x} + \lambda \frac{\partial g}{\partial x} = 0$
  $\frac{\partial f}{\partial y} + \lambda \frac{\partial g}{\partial y} = 0$
  $g(x,y) = 0$

- Calculate $f(x+n)$ where $f(x) = \sum_{i=0}^{n-1} a_i x^i$
  $f(x+n) = \sum_{i=0}^{n-1} a_i (x+n)^i = \sum_{i=0}^{n-1} x^i \cdot \frac{1}{i!} \sum_{j=i}^{n-1} \frac{a_j}{j!} \cdot \frac{n^{j-i}}{(j-i)!}$

- Bell 數（有 n 個人，把他們拆組的方法總數）
  $B_0 = 1$
  $B_n = \sum_{k=0}^{n} s(n,k) \quad (second - stirling)$
  $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$

- Wilson's theorem
  $(p-1)! \equiv -1(\mod p)$
  $(p^q!)_p \equiv \begin{cases} 1, & (p=2) \wedge (q \geq 3), \\ -1, & \text{otherwise.} \end{cases} \pmod{p}^q$

- Fermat's little theorem
  $a^p \equiv a \pmod{p}$

- Euler's theorem
  $a^b \equiv \begin{cases} a^{b \bmod \varphi(m)}, & \gcd(a,m) = 1, \\ a^b, & \gcd(a,m) \neq 1, b < \varphi(m), \\ a^{(b \bmod \varphi(m)) + \varphi(m)}, & \gcd(a,m) \neq 1, b \geq \varphi(m). \end{cases} \pmod{m}$

- 環狀著色（相鄰塗異色）
  $(k-1)(-1)^n + (k-1)^n$

5.6 FloorBlock [ee541d]

```
vector<i64> floorBlock(i64 x) { // x >= 0
  vector<i64> itv;
  for (i64 l = 1, r; l <= x; l = r) {
    r = x / (x / l) + 1;
    itv.push_back(l);
  }
  itv.push_back(x + 1);
  return itv;
}
```

5.7 FloorCeil [9a0a64]

```
i64 ifloor(i64 a, i64 b) {
  if (b < 0) a = -a, b = -b;
  if (a < 0) return (a - b + 1) / b;
  return a / b;
}

i64 iceil(i64 a, i64 b) {
  if (b < 0) a = -a, b = -b;
  if (a > 0) return (a + b - 1) / b;
  return a / b;
}
```

5.8 NTT Prime List

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2748779069441	3	6597069766657	5
39582418599937	5	79164837199873	5
1231453023109121	3	1337006139375617	3
4179340454199820289	3	1945555039024054273	3
9223372036737335297	3		

5.9 NTT [b808df] (316f19|12c7ec|ade009|7d0f40)

```
template<i64 M, i64 root>
struct NTT {
  static const int Log = 21;
  array<i64, Log + 1> e{}, ie{};
  NTT() {
    static_assert(__builtin_ctz(M - 1) >= Log);
    e[Log] = power(root, (M - 1) >> Log, M);
    ie[Log] = power(e[Log], M - 2, M);
    for (int i = Log - 1; i >= 0; i--) {
      e[i] = e[i + 1] * e[i + 1] % M;
      ie[i] = ie[i + 1] * ie[i + 1] % M;
    }
  }
  void operator()(vector<i64> &v, bool inv) {
    int n = v.size();
    for (int i = 0, j = 0; i < n; i++) {
      if (i < j) swap(v[i], v[j]);
      for (int k = n / 2; (j ^= k) < k; k /= 2);
    }
    for (int m = 1; m < n; m *= 2) {
      i64 w = (inv ? ie : e)[__lg(m) + 1];
      for (int i = 0; i < n; i += m * 2) {
        i64 cur = 1;
        for (int j = i; j < i + m; j++) {
          i64 g = v[j], t = cur * v[j + m] % M;
          v[j] = (g + t) % M;
          v[j + m] = (g - t + M) % M;
          cur = cur * w % M;
        }
      }
    }
    if (inv) {
      i64 in = power(n, M - 2, M);
      for (int i = 0; i < n; i++) v[i] = v[i] * in % M;
    }
  }
}; /* SPLIT-HASH */
template<int M, int G>
vector<i64> convolution(vector<i64> f, vector<i64> g) {
  static NTT<M, G> ntt;
  int n = ssize(f) + ssize(g) - 1;
  int len = bit_ceil(1ull * n);
  f.resize(len);
  g.resize(len);
  ntt(f, 0), ntt(g, 0);
  for (int i = 0; i < len; i++) {
    (f[i] *= g[i]) %= M;
  }
  ntt(f, 1);
  f.resize(n);
  return f;
} /* SPLIT-HASH */
vector<i64> inv(vector<i64> f) {
  const int n = f.size();
  int k = 1;
  vector<i64> g{inv(f[0])}, t;
  for (i64 &x : f) {
    x = (mod - x) % mod;
  }
  t.reserve(n);
  while (k < n) {
    k = min(k * 2, n);
    g.resize(k);
    t.assign(f.begin(), f.begin() + k);
    auto h = g * t;
    h.resize(k);
    (h[0] += 2) %= mod;
    g = g * h;
    g.resize(k);
  }
  g.resize(n);
  return g;
} /* SPLIT-HASH */
// CRT
vector<i64> convolution_ll(const vector<i64> &f, const
    vector<i64> &g) {
  constexpr i64 M1 = 998244353, G1 = 3;
  constexpr i64 M2 = 985661441, G2 = 3;
  constexpr i64 M1M2 = M1 * M2;
  constexpr i64 M1m1 = M2 * power(M2, M1 - 2, M1);
  constexpr i64 M2m2 = M1 * power(M1, M2 - 2, M2);
  auto c1 = convolution<M1, G1>(f, g);
  auto c2 = convolution<M2, G2>(f, g);
  for (int i = 0; i < c1.size(); i++) {
    c1[i] = ((i128)c1[i] * M1m1 + (i128)c2[i] * M2m2) %
      M1M2;
  }
  return c1;
}
// 2D convolution
vector<vector<i64>> operator*(vector<vector<i64>> f,
    vector<vector<i64>> g) {
  const int n = f.size() + g.size() - 1;
  const int m = f[0].size() + g[0].size() - 1;
  int len = bit_ceil(1ull * max(n, m));
  f.resize(len);
  g.resize(len);
  for (auto &v : f) {
    v.resize(len);
    ntt(v, 0);
  }
  for (auto &v : g) {
    v.resize(len);
    ntt(v, 0);
  }
  for (int i = 0; i < len; i++)
    for (int j = 0; j < i; j++) {
      swap(f[i][j], f[j][i]);
      swap(g[i][j], g[j][i]);
    }
  for (int i = 0; i < len; i++) {
    ntt(f[i], 0);
    ntt(g[i], 0);
  }
  for (int i = 0; i < len; i++)
    for (int j = 0; j < len; j++) {
      f[i][j] = mul(f[i][j], g[i][j]);
    }
  for (int i = 0; i < len; i++) {
    ntt(f[i], 1);
  }
  for (int i = 0; i < len; i++)
    for (int j = 0; j < i; j++) {
      swap(f[i][j], f[j][i]);
    }
  for (auto &v : f) {
    ntt(v, 1);
    v.resize(m);
  }
  f.resize(n);
  return f;
}
```

5.10 FWT

1. XOR Convolution
   - $f(A) = (f(A_0) + f(A_1), f(A_0) - f(A_1))$
   - $f^{-1}(A) = (f^{-1}(\frac{A_0+A_1}{2}), f^{-1}(\frac{A_0-A_1}{2}))$

2. OR Convolution
   - $f(A) = (f(A_0), f(A_0) + f(A_1))$
   - $f^{-1}(A) = (f^{-1}(A_0), f^{-1}(A_1) - f^{-1}(A_0))$

3. AND Convolution
   - $f(A) = (f(A_0) + f(A_1), f(A_1))$
   - $f^{-1}(A) = (f^{-1}(A_0) - f^{-1}(A_1), f^{-1}(A_1))$

5.11 FWT [948582]

```
void ORop(i64 &x, i64 &y) { y = (y + x) % mod; }
void ORinv(i64 &x, i64 &y) { y = (y - x + mod) % mod; }

void ANDop(i64 &x, i64 &y) { x = (x + y) % mod; }
void ANDinv(i64 &x, i64 &y) { x = (x - y + mod) % mod;
    }

void XORop(i64 &x, i64 &y) { tie(x, y) = pair{(x + y) %
     mod, (x - y + mod) % mod}; }
void XORinv(i64 &x, i64 &y) { tie(x, y) = pair{(x + y)
    * inv2 % mod, (x - y + mod) * inv2 % mod}; }

void FWT(vector<i64> &f, auto &op) {
  const int s = f.size();
  for (int i = 1; i < s; i *= 2)
    for (int j = 0; j < s; j += i * 2)
      for (int k = 0; k < i; k++)
        op(f[j + k], f[i + j + k]);
}
// FWT(f, XORop), FWT(g, XORop)
// f[i] *= g[i]
// FWT(f, XORinv)
```

5.12 Xor Basis [0a6958]

```
struct Basis {
  array<int, kD> bas{}, tim{};
  void insert(int x, int t) {
    for (int i = kD - 1; i >= 0; i--)
      if (x >> i & 1) {
        if (!bas[i]) {
          bas[i] = x;
          tim[i] = t;
          return;
        }
        if (t > tim[i]) {
          swap(x, bas[i]);
          swap(t, tim[i]);
        }
        x ^= bas[i];
      }
  }
  bool query(int x) {
    for (int i = kD - 1; i >= 0; i--)
      chmin(x, x ^ bas[i]);
    return x == 0;
  }
};
```

5.13 Lucas [d777ff]

```
// comb(n, m) % M, M = p^k
// O(M)-O(log(n))
struct Lucas {
  const i64 p, M;
  vector<i64> f;
  Lucas(int p, int M) : p(p), M(M), f(M + 1) {
    f[0] = 1;
    for (int i = 1; i <= M; i++) {
      f[i] = f[i - 1] * (i % p == 0 ? 1 : i) % M;
    }
  }
  i64 CountFact(i64 n) {
    i64 c = 0;
    while (n) c += (n /= p);
    return c;
  }
  // (n! without factor p) % p^k
  i64 ModFact(i64 n) {
    i64 r = 1;
    while (n) {
      r = r * power(f[M], n / M % 2, M) % M * f[n % M]
    % M;
      n /= p;
    }
    return r;
  }
  i64 ModComb(i64 n, i64 m) {
    if (m < 0 or n < m) return 0;
    i64 c = CountFact(n) - CountFact(m) - CountFact(n -
     m);
    i64 r = ModFact(n) * power(ModFact(m), M / p * (p -
     1) - 1, M) % M
              * power(ModFact(n - m), M / p * (p - 1) -
     1, M) % M;
    return r * power(p, c, M) % M;
  }
};
```

5.14 Min25 Sieve [15b088] (a9e103|6ea3f1)

```
// Prefix Sums of Multiplicative Functions
// O(N^0.75 / logN)
// calc f(1) + ... + f(N)
// where f is multiplicative function
// construct completely multiplicative functions
// g_i s.t. for all prime x, f(x) = sigma c_i*g_i(x)
// def gsum(x) = g(1) + ... + g(x)
// call apply(g_i, gsum_i, c_i) and call work(f)
struct Min25 {
  const i64 N, sqrtN;
  vector<i64> Q;
  vector<i64> Fp, S;
  int id(i64 x) { return x <= sqrtN ? Q.size() - x : N
    / x - 1; }
  Min25(i64 N) : N(N), sqrtN(isqrt(N)) {
    // sieve(sqrtN);
    for (i64 l = 1, r; l <= N; l = r + 1) {
      Q.push_back(N / l);
      r = N / (N / l);
    }
    Fp.assign(Q.size(), 0);
    S.assign(Q.size(), 0);
  }
  void apply(const auto &f, const auto &fsum, i64 coef)
     {
    vector<i64> F(Q.size());
    for (int i = 0; i < Q.size(); i++) {
      F[i] = fsum(Q[i]) - 1;
    }
    for (i64 p : primes) {
      auto t = F[id(p - 1)];
      for (int i = 0; i < Q.size(); i++) {
        if (Q[i] < p * p) {
          break;
        }
        F[i] -= (F[id(Q[i] / p)] - t) * f(p);
      }
    }
    for (int i = 0; i < Q.size(); i++) {
      Fp[i] += F[i] * coef;
    }
  } /* SPLIT-HASH */
  i64 work(const auto &f) {
    S = Fp;
    for (i64 p : primes | views::reverse) {
      i64 t = Fp[id(p)];
      for (int i = 0; i < Q.size(); i++) {
        if (Q[i] < p * p) {
          break;
        }
        for (i64 pw = p; pw * p <= Q[i]; pw *= p) {
          S[i] += (S[id(Q[i] / pw)] - t) * f(p, pw);
          S[i] += f(p, pw * p);
        }
      }
    }
    for (int i = 0; i < Q.size(); i++) {
      S[i]++;
    }
    return S[0];
  }
};
```

5.15 Berlekamp Massey [485387]

```
template<int P>
vector<int> BerlekampMassey(vector<int> x) {
 vector<int> cur, ls;
 int lf = 0, ld = 0;
 for (int i = 0; i < (int)x.size(); ++i) {
  int t = 0;
  for (int j = 0; j < (int)cur.size(); ++j)
```

```
    (t += 1LL * cur[j] * x[i - j - 1] % P) %= P;
  if (t == x[i]) continue;
  if (cur.empty()) {
   cur.resize(i + 1);
   lf = i, ld = (t + P - x[i]) % P;
   continue;
  }
  int k = 1LL * fpow(ld, P - 2, P) * (t + P - x[i]) % P
    ;
  vector<int> c(i - lf - 1);
  c.push_back(k);
  for (int j = 0; j < (int)ls.size(); ++j)
   c.push_back(1LL * k * (P - ls[j]) % P);
  if (c.size() < cur.size()) c.resize(cur.size());
  for (int j = 0; j < (int)cur.size(); ++j)
   c[j] = (c[j] + cur[j]) % P;
  if (i - lf + (int)ls.size() >= (int)cur.size()) {
   ls = cur, lf = i;
   ld = (t + P - x[i]) % P;
  }
  cur = c;
 }
 return cur;
}
```

5.16 LinearRec [b8082e]

```
template <int P>
int LinearRec(const vector<int> &s, const vector<int> &
    coeff, int k) {
  int n = s.size();
  auto Combine = [&](const auto &a, const auto &b) {
    vector<int> res(n * 2 + 1);
    for (int i = 0; i <= n; ++i) {
      for (int j = 0; j <= n; ++j)
        (res[i + j] += 1LL * a[i] * b[j] % P) %= P;
    }
    for (int i = 2 * n; i > n; --i) {
      for (int j = 0; j < n; ++j)
        (res[i - 1 - j] += 1LL * res[i] * coeff[j] % P)
     %= P;
    }
    res.resize(n + 1);
    return res;
  };
  vector<int> p(n + 1), e(n + 1);
  p[0] = e[1] = 1;
  for (; k > 0; k >>= 1) {
    if (k & 1) p = Combine(p, e);
    e = Combine(e, e);
  }
  int res = 0;
  for (int i = 0; i < n; ++i) (res += 1LL * p[i + 1] *
    s[i] % P) %= P;
  return res;
}
```

5.17 SubsetConv [85faed]

```
vector<i64> SubsetConv(vector<i64> f, vector<i64> g) {
  const int n = f.size();
  const int U = __lg(n) + 1;
  vector F(U, vector<i64>(n));
  auto G = F, H = F;
  for (int i = 0; i < n; i++) {
    F[popcount<u64>(i)][i] = f[i];
    G[popcount<u64>(i)][i] = g[i];
  }
  for (int i = 0; i < U; i++) {
    FWT(F[i], ORop);
    FWT(G[i], ORop);
  }
  for (int i = 0; i < U; i++)
    for (int j = 0; j <= i; j++)
      for (int k = 0; k < n; k++)
        H[i][k] = (H[i][k] + F[i - j][k] * G[j][k]) %
     mod;
  for (int i = 0; i < U; i++) FWT(H[i], ORinv);
  for (int i = 0; i < n; i++) f[i] = H[popcount<u64>(i)
    ][i];
  return f;
}
```

5.18 SqrtMod [1f43aa]

```
// 0 <= x < p, s.t. x^2 mod p = n
int SqrtMod(int n, int P) {
  if (P == 2 or n == 0) return n;
  if (power(n, (P - 1) / 2, P) != 1) return -1;
  mt19937 rng(12312);
  i64 z = 0, w;
  while (power(w = (z * z - n + P) % P, (P - 1) / 2, P)
      != P - 1)
    z = rng() % P;
  const auto M = [P, w](auto &u, auto &v) {
    return pair{
      (u.ff * v.ff + u.ss * v.ss % P * w) % P,
      (u.ff * v.ss + u.ss * v.ff) % P
    };
  };
  pair<i64, i64> r{1, 0}, e{z, 1};
  for (int w = (P + 1) / 2; w; w >>= 1, e = M(e, e))
    if (w & 1) r = M(r, e);
  return r.ff;
}
```

5.19 DiscreteLog [505d09]

```
template<class T>
T BSGS(T x, T y, T M) {
 // x^? \equiv y (mod M)
 T t = 1, c = 0, g = 1;
 for (T M_ = M; M_ > 0; M_ >>= 1) g = g * x % M;
 for (g = gcd(g, M); t % g != 0; ++c) {
  if (t == y) return c;
  t = t * x % M;
 }
 if (y % g != 0) return -1;
 t /= g, y /= g, M /= g;
 T h = 0, gs = 1;
 for (; h * h < M; ++h) gs = gs * x % M;
 unordered_map<T, T> bs;
 for (T s = 0; s < h; bs[y] = ++s) y = y * x % M;
 for (T s = 0; s < M; s += h) {
  t = t * gs % M;
  if (bs.count(t)) return c + s + h - bs[t];
 }
 return -1;
}
```

5.20 FloorSum [d06654]

```
// sigma 0 ~ n-1: (a * i + b) / m
i64 floorSum(i64 n, i64 m, i64 a, i64 b) {
  u64 ans = 0;
  if (a < 0) {
    u64 a2 = (a % m + m) % m;
    ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
    a = a2;
  }
  if (b < 0) {
    u64 b2 = (b % m + m) % m;
    ans -= 1ULL * n * ((b2 - b) / m);
    b = b2;
  }
  while (true) {
    if (a >= m) {
      ans += n * (n - 1) / 2 * (a / m);
      a %= m;
    }
    if (b >= m) {
      ans += n * (b / m);
      b %= m;
    }
    u64 y_max = a * n + b;
    if (y_max < m) break;
    n = y_max / m;
    b = y_max % m;
    swap(m, a);
  }
  return ans;
}
```

5.21 LP Simplex [2e718d] (ab8fd0|6eb679)

```
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// x = simplex(A, b, c); (A <= 100 x 100)
vector<double> simplex(
    const vector<vector<double>> &a,
    const vector<double> &b,
    const vector<double> &c) {
```

```
  int n = (int)a.size(), m = (int)a[0].size() + 1;
  vector val(n + 2, vector<double>(m + 1));
  vector<int> idx(n + m);
  iota(all(idx), 0);
  int r = n, s = m - 1;
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j)
      val[i][j] = -a[i][j];
    val[i][m - 1] = 1;
    val[i][m] = b[i];
    if (val[r][m] > val[i][m])
      r = i;
  }
  copy(all(c), val[n].begin());
  val[n + 1][m - 1] = -1;
  for (double num; ; ) {
    if (r < n) {
      swap(idx[s], idx[r + m]);
      val[r][s] = 1 / val[r][s];
      for (int j = 0; j <= m; ++j) if (j != s)
        val[r][j] *= -val[r][s];
      for (int i = 0; i <= n + 1; ++i) if (i != r) {
        for (int j = 0; j <= m; ++j) if (j != s)
          val[i][j] += val[r][j] * val[i][s];
        val[i][s] *= val[r][s];
      }
    } /* SPLIT-HASH */
    r = s = -1;
    for (int j = 0; j < m; ++j)
      if (s < 0 || idx[s] > idx[j])
        if (val[n + 1][j] > eps || val[n + 1][j] > -eps
 && val[n][j] > eps)
          s = j;
    if (s < 0) break;
    for (int i = 0; i < n; ++i) if (val[i][s] < -eps) {
      if (r < 0
        || (num = val[r][m] / val[r][s] - val[i][m] /
 val[i][s]) < -eps
        || num < eps && idx[r + m] > idx[i + m])
        r = i;
    }
    if (r < 0) {
      //  Solution is unbounded.
      return vector<double>{};
    }
  }
  if (val[n + 1][m] < -eps) {
    //  No solution.
    return vector<double>{};
  }
  vector<double> x(m - 1);
  for (int i = m; i < n + m; ++i)
    if (idx[i] < m - 1)
      x[idx[i]] = val[i - m][m];
  return x;
}
```

5.22 Lagrange Interpolation [6e0daa]

```
struct Lagrange {
  int deg{};
  vector<i64> C;
  Lagrange(const vector<i64> &P) {
    deg = P.size() - 1;
    C.assign(deg + 1, 0);
    for (int i = 0; i <= deg; i++) {
      i64 q = comb(-i) * comb(i - deg) % mod;
      if ((deg - i) % 2 == 1) {
        q = mod - q;
      }
      C[i] = P[i] * q % mod;
    }
  }
  i64 operator()(i64 x) { // 0 <= x < mod
    if (0 <= x and x <= deg) {
      i64 ans = comb(x) * comb(deg - x) % mod;
      if ((deg - x) % 2 == 1) {
        ans = (mod - ans);
      }
      return ans * C[x] % mod;
    }
    vector<i64> pre(deg + 1), suf(deg + 1);
    for (int i = 0; i <= deg; i++) {
      pre[i] = (x - i);
      if (i) {
        pre[i] = pre[i] * pre[i - 1] % mod;
      }
    }
    for (int i = deg; i >= 0; i--) {
      suf[i] = (x - i);
      if (i < deg) {
        suf[i] = suf[i] * suf[i + 1] % mod;
      }
    }
    i64 ans = 0;
    for (int i = 0; i <= deg; i++) {
      ans += (i == 0 ? 1 : pre[i - 1]) * (i == deg ? 1
 : suf[i + 1]) % mod * C[i];
      ans %= mod;
    }
    if (ans < 0) ans += mod;
    return ans;
  }
};
```

5.23 polyop-luogu [2dbb45] (43ad78|db2503|07e6ef)

```
constexpr int mod = 998'244'353;
// fpow / modinv / mul / add / sub
int get_root(int n, int P = mod){ // ensure 0 <= n < p
 if (P == 2 or n == 0)
  return n;
 auto check = [&](ll x)
 { return fpow(x, (P - 1) / 2); };
 if (check(n) != 1)
  return -1;
 mt19937 rnd(7122);
 ll z = 1, w;
 while (check(w = (z * z - n + P) % P) != P - 1)
  z = rnd() % P;
 const auto M = [P, w](auto &u, auto &v)
 {
  auto [a, b] = u;
  auto [c, d] = v;
  return make_pair((a * c + b * d % P * w) % P,
       (a * d + b * c) % P);
 };
 pair<ll, ll> r(1, 0), e(z, 1);
 for (int q = (P + 1) / 2; q; q >>= 1, e = M(e, e))
  if (q & 1)
   r = M(r, e);
 return int(r.first);//sqrt(n) mod P where P is prime
} /* SPLIT-HASH */
template <int MOD, int G, int MAXN>
struct NTT {
 static_assert(MAXN == (MAXN & -MAXN));
 int roots[MAXN];
 NTT() {
  int r = fpow(G, (MOD - 1) / MAXN);
  for (int i = MAXN >> 1; i; i >>= 1) {
   roots[i] = 1;
   for (int j = 1; j < i; j++)
    roots[i + j] = mul(roots[i + j - 1], r);
   r = mul(r, r);
   // for (int j = 0; j < i; j ++) // FFT (tested)
   // roots [i+j] = polar <llf >(1 , PI * j / i);
  }
 }
 // n must be 2^k, and 0 <= F[i] < MOD
 void operator()(int F[], int n, bool inv = false) {
  for (int i = 0, j = 0; i < n; i++) {
   if (i < j)
    swap(F[i], F[j]);
   for (int k = n >> 1; (j ^= k) < k; k >>= 1) ;
  }
  for (int s = 1; s < n; s *= 2)
   for (int i = 0; i < n; i += s * 2)
    for (int j = 0; j < s; j++) {
     int a = F[i+j], b=mul(F[i+j+s], roots[s+j]);
     F[i + j] = add(a, b);
     F[i + j + s] = sub(a, b);
    }
  if (!inv)
   return;
  const int invn = modinv(n);
  for (int i = 0; i < n; i++)
   F[i] = mul(F[i], invn);
```

```
  reverse(F + 1, F + n);
 }
}; /* SPLIT-HASH */
NTT<mod, 3, 1 << 23> ntt;

#define fi(l, r) for (size_t i = (l); i < (r); i++)
using S = vector<int>;
auto Mul(auto a, auto b, size_t sz) {
 a.resize(sz), b.resize(sz);
 ntt(a.data(), sz);
 ntt(b.data(), sz);
 fi(0, sz) a[i] = mul(a[i], b[i]);
 return ntt(a.data(), sz, true), a;
}
S Newton(const S &v, int init, auto &&iter) {
 S Q = {init};
 for (int sz = 2; Q.size() < v.size(); sz *= 2) {
  S A{begin(v), begin(v) + min(sz, int(v.size()))};
  A.resize(sz * 2), Q.resize(sz * 2);
  iter(Q, A, sz * 2);
  Q.resize(sz);
 }
 return Q.resize(v.size()), Q;
}
S Inv(const S &v) { // v[0] != 0
 return Newton(v, modinv(v[0]),
     [](S &X, S &A, int sz) {
  ntt(X.data(), sz), ntt(A.data(), sz);
  for (int i = 0; i < sz; i++)
  X[i] = mul(X[i], sub(2, mul(X[i], A[i])));
  ntt(X.data(), sz, true); });
}
S Dx(S A) {
 fi(1, A.size()) A[i - 1] = mul(i, A[i]);
 return A.empty() ? A : (A.pb(), A);
}
S Sx(S A) {
 A.insert(A.begin(), 0);
 fi(1, A.size()) A[i] = mul(modinv(int(i)), A[i]);
 return A;
}
S Ln(const S &A) { // coef[0] == 1; res[0] == 0
 auto B = Sx(Mul(Dx(A),Inv(A),bit_ceil(A.size()*2)));
 return B.resize(A.size()), B;
}
S Exp(const S &v) { // coef[0] == 0; res[0] == 1
 return Newton(v, 1,
     [](S &X, S &A, int sz) {
  auto Y = X; Y.resize(sz / 2); Y = Ln(Y);
  fi(0, Y.size()) Y[i] = sub(A[i], Y[i]);
  Y[0] = add(Y[0], 1); X = Mul(X, Y, sz); });
}
S Pow(S a, ll M) { // period mod*(mod-1)
 assert(!a.empty() && a[0] != 0);
 const auto imul = [&a](int s) {
 for (int &x: a) x = mul(x, s); };
 int c = a[0];
 imul(modinv(c));
 a = Ln(a);
 imul(int(M % mod));
 a = Exp(a);
 imul(fpow(c, t(M % (mod - 1))));
 return a; // mod x^N where N=a.size()
}
S Sqrt(const S &v) { // need: QuadraticResidue
 assert(!v.empty() && v[0] != 0);
 const int r = get_root(v[0]);
 assert(r != -1);
 return Newton(v, r,
     [](S &X, S &A, int sz) {
  auto Y = X; Y.resize(sz / 2);
  auto B = Mul(A, Inv(Y), sz);
  for (int i = 0, inv2 = mod / 2 + 1; i < sz; i++)
  X[i] = mul(inv2, add(X[i], B[i])); });
}
S Mul(auto &&a, auto &&b) {
 const auto n = a.size() + b.size() - 1;
 auto R = Mul(a, b, bit_ceil(n));
 return R.resize(n), R;
}
S MulT(S a, S b, size_t k) {
 assert(b.size());
 reverse(ALL(b));
 auto R = Mul(a, b);
 R = vector(R.begin() + b.size() - 1, R.end());
 return R.resize(k), R;
}
S Eval(const S &f, const S &x) {
 if (f.empty())
  return vector(x.size(), 0);
 const int n = int(max(x.size(), f.size()));
 auto q = vector(n * 2, S(2, 1));
 S ans(n);
 fi(0, x.size()) q[i + n][1] = sub(0, x[i]);
 for (int i = n - 1; i > 0; i--)
  q[i] = Mul(q[i << 1], q[i << 1 | 1]);
 q[1] = MulT(f, Inv(q[1]), n);
 for (int i = 1; i < n; i++) {
  auto L = q[i << 1], R = q[i << 1 | 1];
  q[i << 1 | 0] = MulT(q[i], R, L.size());
  q[i << 1 | 1] = MulT(q[i], L, R.size());
 }
 for (int i = 0; i < n; i++)
  ans[i] = q[i + n][0];
 return ans.resize(x.size()), ans;
}
pair<S, S> DivMod(const S &A, const S &B) {
 assert(!B.empty() && B.back() != 0);
 if (A.size() < B.size())
  return {{}, A};
 const auto sz = A.size() - B.size() + 1;
 S X = B;
 reverse(ALL(X));
 X.resize(sz);
 S Y = A;
 reverse(ALL(Y));
 Y.resize(sz);
 S Q = Mul(Inv(X), Y);
 Q.resize(sz);
 reverse(ALL(Q));
 X = Mul(Q, B);
 Y = A;
 fi(0, Y.size()) Y[i] = sub(Y[i], X[i]);
 while (Y.size() && Y.back() == 0)
  Y.pb();
 while (Q.size() && Q.back() == 0)
  Q.pb();
 return {Q, Y};
} // empty means zero polynomial
int LinearRecursionKth(S a, S c, int64_t k)
{
 const auto d = a.size();
 assert(c.size() == d + 1);
 const auto sz = bit_ceil(2 * d + 1), o = sz / 2;
 S q = c;
 for (int &x : q)
  x = sub(0, x);
 q[0] = 1;
 S p = Mul(a, q);
 p.resize(sz);
 q.resize(sz);
 for (int r; r = (k & 1), k; k >>= 1)
 {
  fill(d + ALL(p), 0);
  fill(d + 1 + ALL(q), 0);
  ntt(p.data(), sz);
  ntt(q.data(), sz);
  for (size_t i = 0; i < sz; i++)
   p[i] = mul(p[i], q[(i + o) & (sz - 1)]);
  for (size_t i = 0, j = o; j < sz; i++, j++)
   q[i] = q[j] = mul(q[i], q[j]);
  ntt(p.data(), sz, true);
  ntt(q.data(), sz, true);
  for (size_t i = 0; i < d; i++)
   p[i] = p[i << 1 | r];
  for (size_t i = 0; i <= d; i++)
   q[i] = q[i << 1];
 } // Bostan-Mori
 return mul(p[0], modinv(q[0]));
} // a_n = \sum c_j a_(n-j), c_0 is not used

int n; S arr(n); arr = Ln(arr);
```

6 Geometry

6.1 Basic [2c8e70]

```
using numbers::pi;
template<class T> inline constexpr T eps =
    numeric_limits<T>::epsilon() * 1E6;
using Real = long double;
struct Pt {
  Real x{}, y{};
  Pt operator+(Pt a) const { return {x + a.x, y + a.y};
    }
  Pt operator-(Pt a) const { return {x - a.x, y - a.y};
    }
  Pt operator*(Real k) const { return {x * k, y * k}; }
  Pt operator/(Real k) const { return {x / k, y / k}; }
  Real operator*(Pt a) const { return x * a.x + y * a.y
    ; }
  Real operator^(Pt a) const { return x * a.y - y * a.x
    ; }
  auto operator<=>(const Pt&) const = default;
  bool operator==(const Pt&) const = default;
};
int sgn(Real x) { return (x > -eps<Real>) - (x < eps<
    Real>); }
Real ori(Pt a, Pt b, Pt c) { return (b - a) ^ (c - a);
    }
Pt norm(Pt u) { return {-u.y, u.x}; }
bool argcmp(const Pt &a, const Pt &b) { // arg(a) < arg
    (b)
  int f = (-norm(a) > Pt{} ? 1 : -1) * (a != Pt{});
  int g = (-norm(b) > Pt{} ? 1 : -1) * (b != Pt{});
  return f == g ? (a ^ b) > 0 : f < g;
}
Real abs2(Pt a) { return a * a; }
// floating point only
Pt rotate(Pt u, Real a) {
  Pt v{sinl(a), cosl(a)};
  return {u ^ v, u * v};
}
Real abs(Pt a) { return sqrtl(a * a); }
Real arg(Pt x) { return atan2l(x.y, x.x); }
Pt unit(Pt x) { return x / abs(x); }
struct Line {
  Pt a, b;
  Pt dir() const { return b - a; }
};
int PtSide(Pt p, Line L) {
  return sgn(ori(L.a, L.b, p)); // for int
  return sgn(ori(L.a, L.b, p) / abs(L.a - L.b));
}
bool PtOnSeg(Pt p, Line L) {
  return PtSide(p, L) == 0 and sgn((p - L.a) * (p - L.b
    )) <= 0;
}
Pt proj(Pt p, Line l) {
  Pt dir = unit(l.b - l.a);
  return l.a + dir * (dir * (p - l.a));
}
struct Cir {
  Pt o; double r;
};
bool disjunct(const Cir &a, const Cir &b) {
  return sgn(abs(a.o - b.o) - a.r - b.r) >= 0;
}
bool contain(const Cir &a, const Cir &b) {
  return sgn(a.r - b.r - abs(a.o - b.o)) >= 0;
}
```

6.2 Point to Segment Distance [0c07fc]

```
double PtSegDist(Pt p, Line l) {
  double ans = min(abs(p - l.a), abs(p - l.b));
  if (sgn(abs(l.a - l.b)) == 0) return ans;
  if (sgn((l.a - l.b) * (p - l.b)) < 0) return ans;
  if (sgn((l.b - l.a) * (p - l.a)) < 0) return ans;
  return min(ans, abs(ori(p, l.a, l.b)) / abs(l.a - l.b
    ));
}
double SegDist(Line l, Line m) {
  return PtSegDist({0, 0}, {l.a - m.a, l.b - m.b});
}
```

6.3 Point in Polygon [ae764a]

```
int inPoly(Pt p, const vector<Pt> &P) {
  const int n = P.size();
  int cnt = 0;
  for (int i = 0; i < n; i++) {
    Pt a = P[i], b = P[(i + 1) % n];
    if (PtOnSeg(p, {a, b})) return 1; // on edge
    if ((sgn(a.y - p.y) == 1) ^ (sgn(b.y - p.y) == 1))
      cnt += sgn(ori(a, b, p));
  }
  return cnt == 0 ? 0 : 2; // out, in
}
```

6.4 Intersection of Lines [31415c]

```
bool isInter(Line l, Line m) {
  if (PtOnSeg(m.a, l) or PtOnSeg(m.b, l) or
    PtOnSeg(l.a, m) or PtOnSeg(l.b, m))
    return true;
  return PtSide(m.a, l) * PtSide(m.b, l) < 0 and
      PtSide(l.a, m) * PtSide(l.b, m) < 0;
}
Pt LineInter(Line l, Line m) {
  double s = ori(m.a, m.b, l.a), t = ori(m.a, m.b, l.b)
    ;
  return (l.b * s - l.a * t) / (s - t);
}
bool strictInter(Line l, Line m) {
  int la = PtSide(m.a, l);
  int lb = PtSide(m.b, l);
  int ma = PtSide(l.a, m);
  int mb = PtSide(l.b, m);
  if (la == 0 and lb == 0) return false;
  return la * lb < 0 and ma * mb < 0;
}
```

6.5 X of Circle and Line [a53f3c]

```
vector<Pt> CircleLineInter(Cir c, Line l) {
  Pt H = proj(c.o, l);
  Pt dir = unit(l.b - l.a);
  double h = abs(H - c.o);
  if (sgn(h - c.r) > 0) return {};
  double d = sqrt(max((double)0., c.r * c.r - h * h));
  if (sgn(d) == 0) return {H};
  return {H - dir *d, H + dir * d};
  // Counterclockwise
}
```

6.6 Intersection of Circles [3c00f3]

```
vector<Pt> CircleInter(Cir a, Cir b) {
  double d2 = abs2(a.o - b.o), d = sqrt(d2);
  if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r + b.
    r) return {};
  Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r -
    a.r * a.r) / (2 * d2));
  double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) * (
    a.r + b.r - d) * (-a.r + b.r + d));
  Pt v = rotate(b.o - a.o) * A / (2 * d2);
  if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
  return {u - v, u + v}; // counter clockwise of a
}
```

6.7 Area of Circle and Polygon [6783c6]

```
double CirclePoly(Cir C, const vector<Pt> &P) {
  auto arg = [&](Pt p, Pt q) { return atan2(p ^ q, p *
    q); };
  double r2 = C.r * C.r / 2;
  auto tri = [&](Pt p, Pt q) {
    Pt d = q - p;
    auto a = (d * p) / abs2(d), b = (abs2(p) - C.r * C.
    r)/ abs2(d);
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a - sqrt(det)), t = min(1., -a +
    sqrt(det));
    if (t < 0 or 1 <= s) return arg(p, q) * r2;
    Pt u = p + d * s, v = p + d * t;
    return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) *
    r2;
  };
  double sum = 0.0;
  for (int i = 0; i < P.size(); i++)
    sum += tri(P[i] - C.o, P[(i + 1) % P.size()] - C.o);
  return sum;
}
```

6.8 Area of Sector [58d858]

```
//  AOB * r^2 / 2
double Sector(Pt a, Pt b, double r) {
  double theta = atan2(a.y, a.x) - atan2(b.y, b.x);
```

```
  while (theta <= 0) theta += 2 * pi;
  while (theta >= 2 * pi) theta -= 2 * pi;
  theta = min(theta, 2 * pi - theta);
  return r * r * theta / 2;
}
```

6.9 Union of Polygons [0cc68d]

```
// Area[i] : area covered by at least i polygon
vector<double> PolyUnion(const vector<vector<Pt>> &P) {
  const int n = P.size();
  vector<double> Area(n + 1);
  vector<Line> Ls;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < P[i].size(); j++)
      Ls.push_back({P[i][j], P[i][(j + 1) % P[i].size()]});
  auto cmp = [&](Line &l, Line &r) {
    Pt u = l.b - l.a, v = r.b - r.a;
    if (argcmp(u, v)) return true;
    if (argcmp(v, u)) return false;
    return PtSide(l.a, r) < 0;
  };
  sort(all(Ls), cmp);
  for (int l = 0, r = 0; l < Ls.size(); l = r) {
    while (r < Ls.size() and !cmp(Ls[l], Ls[r])) r++;
    Line L = Ls[l];
    vector<pair<Pt, int>> event;
    for (auto [c, d] : Ls) {
      if (sgn((L.a - L.b) ^ (c - d)) != 0) {
        int s1 = PtSide(c, L) == 1;
        int s2 = PtSide(d, L) == 1;
        if (s1 ^ s2) event.emplace_back(LineInter(L, {c, d}), s1 ? 1 : -1);
      } else if (PtSide(c, L) == 0 and sgn((L.a - L.b) * (c - d)) > 0) {
        event.emplace_back(c, 2);
        event.emplace_back(d, -2);
      }
    }
    sort(all(event), [&](auto i, auto j) {
      return (L.a - i.ff) * (L.a - L.b) < (L.a - j.ff) * (L.a - L.b);
    });
    int cov = 0, tag = 0;
    Pt lst{0, 0};
    for (auto [p, s] : event) {
      if (cov >= tag) {
        Area[cov] += lst ^ p;
        Area[cov - tag] -= lst ^ p;
      }
      if (abs(s) == 1) cov += s;
      else tag += s / 2;
      lst = p;
    }
  }
  for (int i = n - 1; i >= 0; i--) Area[i] += Area[i + 1];
  for (int i = 1; i <= n; i++) Area[i] /= 2;
  return Area;
};
```

6.10 Union of Circles [f29049]

```
// Area[i] : area covered by at least i circle
vector<double> CircleUnion(const vector<Cir> &C) {
  const int n = C.size();
  vector<double> Area(n + 1);
  auto check = [&](int i, int j) {
    if (!contain(C[i], C[j]))
      return false;
    return sgn(C[i].r - C[j].r) > 0 or (sgn(C[i].r - C[j].r) == 0 and i < j);
  };
  struct Teve {
    double ang; int add; Pt p;
    bool operator<(const Teve &b) { return ang < b.ang; }
  };
  auto ang = [&](Pt p) { return atan2(p.y, p.x); };
  for (int i = 0; i < n; i++) {
    int cov = 1;
    vector<Teve> event;
    for (int j = 0; j < n; j++) if (i != j) {
      if (check(j, i)) cov++;
      else if (!check(i, j) and !disjunct(C[i], C[j]))
    {
        auto I = CircleInter(C[i], C[j]);
        assert(I.size() == 2);
        double a1 = ang(I[0] - C[i].o), a2 = ang(I[1] - C[i].o);
        event.push_back({a1, 1, I[0]});
        event.push_back({a2, -1, I[1]});
        if (a1 > a2) cov++;
      }
    }
    if (event.empty()) {
      Area[cov] += pi * C[i].r * C[i].r;
      continue;
    }
    sort(all(event));
    event.push_back(event[0]);
    for (int j = 0; j + 1 < event.size(); j++) {
      cov += event[j].add;
      Area[cov] += (event[j].p ^ event[j + 1].p) / 2.;
      double theta = event[j + 1].ang - event[j].ang;
      if (theta < 0) theta += 2 * pi;
      Area[cov] += (theta - sin(theta)) * C[i].r * C[i].r / 2.;
    }
  }
  return Area;
}
```

6.11 TanLs of Circle and Point [bebedd]

```
vector<Line> CircleTangent(Cir c, Pt p) {
  vector<Line> z;
  double d = abs(p - c.o);
  if (sgn(d - c.r) == 0) {
    Pt i = rotate(p - c.o);
    z.push_back({p, p + i});
  } else if (d > c.r) {
    double o = acos(c.r / d);
    Pt i = unit(p - c.o);
    Pt j = rotate(i, o) * c.r;
    Pt k = rotate(i, -o) * c.r;
    z.push_back({c.o + j, p});
    z.push_back({c.o + k, p});
  }
  return z;
}
```

6.12 TangentLines of Circles [fd34e8]

```
vector<Line> CircleTangent(Cir c1, Cir c2, int sign1) {
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = abs2(c1.o - c2.o);
  if (sgn(d_sq) == 0) return ret;
  double d = sqrt(d_sq);
  Pt v = (c2.o - c1.o) / d;
  double c = (c1.r - sign1 * c2.r) / d;
  if (c * c > 1) return ret;
  double h = sqrt(max(0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c + sign2 * h * v.x);
    Pt p1 = c1.o + n * c1.r;
    Pt p2 = c2.o + n * (c2.r * sign1);
    if (sgn(p1.x - p2.x) == 0 && sgn(p1.y - p2.y) == 0)
      p2 = p1 + rotate(c2.o - c1.o);
    ret.push_back({p1, p2});
  }
 return ret;
}
```

6.13 Convex Hull [c856e7]

```
vector<Pt> Hull(vector<Pt> P) {
  sort(all(P));
  P.erase(unique(all(P)), P.end());
  if (P.size() <= 1) return P;
  P.insert(P.end(), P.rbegin() + 1, P.rend());
  vector<Pt> stk;
  for (auto p : P) {
    auto it = stk.rbegin();
    while (stk.rend() - it >= 2 and \
      ori(*next(it), *it, p) <= 0 and \
      (*next(it) < *it) == (*it < p)) {
      it++;
    }
```

```
    stk.resize(stk.rend() - it);
    stk.push_back(p);
  }
  stk.pop_back();
  return stk;
}
```

6.14 Convex Hull trick [70dc20] (3b219c|603737)

```
struct Convex {
  int n;
  vector<Pt> A, V, L, U;
  Convex(const vector<Pt> &_A) : A(_A), n(_A.size()) {
    // n >= 3
    auto it = max_element(all(A));
    L.assign(A.begin(), it + 1);
    U.assign(it, A.end()), U.push_back(A[0]);
    for (int i = 0; i < n; i++) {
      V.push_back(A[(i + 1) % n] - A[i]);
    }
  }
  int inside(Pt p, const vector<Pt> &h, auto f) {
    auto it = lower_bound(all(h), p, f);
    if (it == h.end()) return 0;
    if (it == h.begin()) return p == *it;
    return 1 - sgn(ori(*prev(it), p, *it));
  }
  // 0: out, 1: on, 2: in
  int inside(Pt p) {
    return min(inside(p, L, less{}), inside(p, U,
    greater{}));
  }
  static bool cmp(Pt a, Pt b) { return sgn(a ^ b) > 0;
    }
  // A[i] is a far/closer tangent point
  int tangent(Pt v, bool close = true) {
    assert(v != Pt{});
    auto l = V.begin(), r = V.begin() + L.size() - 1;
    if (v < Pt{}) l = r, r = V.end();
    if (close) return (lower_bound(l, r, v, cmp) - V.
    begin()) % n;
    return (upper_bound(l, r, v, cmp) - V.begin()) % n;
  } /* SPLIT-HASH */
  // closer tangent point
  array<int, 2> tangent2(Pt p) {
    array<int, 2> t{-1, -1};
    if (inside(p) == 2) return t;
    if (auto it = lower_bound(all(L), p); it != L.end()
     and p == *it) {
      int s = it - L.begin();
      return {(s + 1) % n, (s - 1 + n) % n};
    }
    if (auto it = lower_bound(all(U), p, greater{}); it
     != U.end() and p == *it) {
      int s = it - U.begin() + L.size() - 1;
      return {(s + 1) % n, (s - 1 + n) % n};
    }
    for (int i = 0; i != t[0]; i = tangent((A[t[0] = i]
     - p), 0));
    for (int i = 0; i != t[1]; i = tangent((p - A[t[1]
    = i]), 1));
    return t;
  }
  int find(int l, int r, Line L) {
    if (r < l) r += n;
    int s = PtSide(A[l % n], L);
    return *ranges::partition_point(views::iota(l, r),
      [&](int m) {
        return PtSide(A[m % n], L) == s;
      }) - 1;
  };
  // Line A_x A_x+1 interset with L
  vector<int> intersect(Line L) {
    int l = tangent(L.a - L.b), r = tangent(L.b - L.a);
    if (PtSide(A[l], L) * PtSide(A[r], L) >= 0) return
    {};
    return {find(l, r, L) % n, find(r, l, L) % n};
  }
};
```

6.15 Dynamic Convex Hull [b6e83b]

```
template<class T, class Comp = less<T>>
struct DynamicHull {
  set<T, Comp> H;
  void insert(T p) {
    if (inside(p)) return;
    auto it = H.insert(p).ff;
    while (it != H.begin() and prev(it) != H.begin() \
        and ori(*prev(it, 2), *prev(it), *it) <= 0) {
      it = H.erase(--it);
    }
    while (it != --H.end() and next(it) != --H.end() \
        and ori(*it, *next(it), *next(it, 2)) <= 0) {
      it = --H.erase(++it);
    }
  }
  int inside(T p) { // 0: out, 1: on, 2: in
    auto it = H.lower_bound(p);
    if (it == H.end()) return 0;
    if (it == H.begin()) return p == *it;
    return 1 - sgn(ori(*prev(it), p, *it));
  }
};
// DynamicHull<Pt> D;
// DynamicHull<Pt, greater<>> U;
// D.inside(p) and U.inside(p)
```

6.16 Half Plane Intersection [b913b6]

```
bool cover(Line L, Line P, Line Q) {
  // return PtSide(LineInter(P, Q), L) <= 0; for double
  i128 u = (Q.a - P.a) ^ Q.dir();
  i128 v = P.dir() ^ Q.dir();
  i128 x = P.dir().x * u + (P.a - L.a).x * v;
  i128 y = P.dir().y * u + (P.a - L.a).y * v;
  return sgn(x * L.dir().y - y * L.dir().x) * sgn(v) >=
     0;
}
vector<Line> HPI(vector<Line> P) {
  sort(all(P), [&](Line l, Line m) {
    if (argcmp(l.dir(), m.dir())) return true;
    if (argcmp(m.dir(), l.dir())) return false;
    return ori(m.a, m.b, l.a) > 0;
  });
  int n = P.size(), l = 0, r = -1;
  for (int i = 0; i < n; i++) {
    if (i and !argcmp(P[i - 1].dir(), P[i].dir()))
    continue;
    while (l < r and cover(P[i], P[r - 1], P[r])) r--;
    while (l < r and cover(P[i], P[l], P[l + 1])) l++;
    P[++r] = P[i];
  }
  while (l < r and cover(P[l], P[r - 1], P[r])) r--;
  while (l < r and cover(P[r], P[l], P[l + 1])) l++;
  if (r - l <= 1 or !argcmp(P[l].dir(), P[r].dir()))
    return {}; // empty
  if (cover(P[l + 1], P[l], P[r]))
    return {}; // infinity
  return vector(P.begin() + l, P.begin() + r + 1);
}
```

6.17 Minkowski [27b78f]

```
// P, Q, R(return) are counterclockwise order convex
    polygon
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
  assert(P.size() >= 2 and Q.size() >= 2);
  auto cmp = [&](Pt a, Pt b) {
    return Pt{a.y, a.x} < Pt{b.y, b.x};
  };
  auto reorder = [&](auto &R) {
    rotate(R.begin(), min_element(all(R), cmp), R.end()
    );
    R.push_back(R[0]), R.push_back(R[1]);
  };
  const int n = P.size(), m = Q.size();
  reorder(P), reorder(Q);
  vector<Pt> R;
  for (int i = 0, j = 0, s; i < n or j < m; ) {
    R.push_back(P[i] + Q[j]);
    s = sgn((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
    if (s >= 0) i++;
    if (s <= 0) j++;
  }
  return R;
}
```

6.18 Minimal Enclosing Circle [a05bc4]

```
Pt Center(Pt a, Pt b, Pt c) {
  Pt x = (a + b) / 2;
  Pt y = (b + c) / 2;
```

```
  return LineInter({x, x + rotate(b - a)}, {y, y +
    rotate(c - b)});
}
Cir MEC(vector<Pt> P) {
  mt19937 rng(time(0));
  shuffle(all(P), rng);
  Cir C{};
  for (int i = 0; i < P.size(); i++) {
    if (C.inside(P[i])) continue;
    C = {P[i], 0};
    for (int j = 0; j < i; j++) {
      if (C.inside(P[j])) continue;
      C = {(P[i] + P[j]) / 2, abs(P[i] - P[j]) / 2};
      for (int k = 0; k < j; k++) {
        if (C.inside(P[k])) continue;
        C.o = Center(P[i], P[j], P[k]);
        C.r = abs(C.o - P[i]);
      }
    }
  }
  return C;
}
```

6.19 Point In Circumcircle [f499c7]

```
// p[0], p[1], p[2] should be counterclockwise order
int inCC(const array<Pt, 3> &p, Pt a) {
  i128 det = 0;
  for (int i = 0; i < 3; i++)
    det += i128(abs2(p[i]) - abs2(a)) * ori(a, p[(i +
    1) % 3], p[(i + 2) % 3]);
  return (det > 0) - (det < 0); // in:1, on:0, out:-1
}
```

6.20 Delaunay Triangulation [5aab34] (5c4d17|b007ea)

```
bool inCC(const array<Pt, 3> &p, Pt a) {
  i128 det = 0;
  for (int i = 0; i < 3; i++)
    det += i128(abs2(p[i]) - abs2(a)) * ori(a, p[(i +
    1) % 3], p[(i + 2) % 3]);
  return det > 0;
}
struct Edge {
  int id;
  list<Edge>::iterator rit;
};
vector<list<Edge>> Delaunay(const vector<Pt> &P) {
  assert(is_sorted(all(P))); // need sorted before!
  const int n = P.size();
  vector<list<Edge>> E(n);
  auto addEdge = [&](int u, int v, auto a, auto b) {
    a = E[u].insert(a, {v});
    b = E[v].insert(b, {u});
    return array{b->rit = a, a->rit = b};
  };
  auto divide = [&](auto &&self, int l, int r) -> int {
    if (r - l <= 1) return l;
    int m = (l + r) / 2;
    array<int, 2> t{self(self, l, m), self(self, m, r)
    };
    int w = t[P[t[1]].y < P[t[0]].y];
    auto low = [&](int s) {
      for (Edge e : E[t[s]]) {
        if (ori(P[t[1]], P[t[0]], P[e.id]) > 0 or
          PtOnSeg(P[e.id], {P[t[0]], P[t[1]]})) {
          t[s] = e.id;
          return true;
        }
      }
      return false;
    }; /* SPLIT-HASH */
    while (low(0) or low(1));
    array its = addEdge(t[0], t[1], E[t[0]].begin(), E[
    t[1]].end());
    while (true) {
      Line L{P[t[0]], P[t[1]]};
      auto cand = [&](int s) -> optional<list<Edge>::
    iterator> {
        auto nxt = [&](auto it) {
          if (s == 0) return (++it == E[t[0]].end() ? E
    [t[0]].begin() : it);
          return --(it == E[t[1]].begin() ? E[t[1]].end
    () : it);
        };
        if (E[t[s]].empty()) return {};
        auto lst = nxt(its[s]), it = nxt(lst);
        while (PtSide(P[it->id], L) > 0 and inCC({L.a,
      L.b, P[lst->id]}, P[it->id])) {
          E[t[s ^ 1]].erase(lst->rit);
          E[t[s]].erase(lst);
          it = nxt(lst = it);
        }
        return PtSide(P[lst->id], L) > 0 ? optional{lst
      } : nullopt;
      };
      auto lc = cand(0), rc = cand(1);
      if (!lc and !rc) break;
      int sd = !lc or (rc and inCC({L.a, L.b, P[(*lc)->
      id]}, P[(*rc)->id]));
      auto lst = *(sd ? rc : lc);
      t[sd] = lst->id;
      its[sd] = lst->rit;
      its = addEdge(t[0], t[1], ++its[0], its[1]);
    }
    return w;
  };
  divide(divide, 0, n);
  return E;
};
```

6.21 Triangle Center [085b8e]

```
Pt TriangleCircumCenter(Pt a, Pt b, Pt c) {
 Pt res;
 double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
 double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
 double ax = (a.x + b.x) / 2;
 double ay = (a.y + b.y) / 2;
 double bx = (c.x + b.x) / 2;
 double by = (c.y + b.y) / 2;
 double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay)
    ) / (sin(a1) * cos(a2) - sin(a2) * cos(a1));
 return Pt(ax + r1 * cos(a1), ay + r1 * sin(a1));
}
Pt TriangleMassCenter(Pt a, Pt b, Pt c) {
 return (a + b + c) / 3.0;
}
Pt TriangleOrthoCenter(Pt a, Pt b, Pt c) {
 return TriangleMassCenter(a, b, c) * 3.0 -
    TriangleCircumCenter(a, b, c) * 2.0;
}
Pt TriangleInnerCenter(Pt a, Pt b, Pt c) {
 Pt res;
 double la = abs(b - c);
 double lb = abs(a - c);
 double lc = abs(a - b);
 res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb +
    lc);
 res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb +
    lc);
 return res;
}
```

7 Stringology

7.1 KMP [d5eccd]

```
vector<int> buildFail(string s) {
  const int len = s.size();
  vector<int> f(len, -1);
  for (int i = 1, p = -1; i < len; i++) {
    while (~p and s[p + 1] != s[i]) p = f[p];
    if (s[p + 1] == s[i]) p++;
    f[i] = p;
  }
  return f;
}
```

7.2 Z-algorithm [a70d58]

```
vector<int> zalgo(string s) {
  if (s.empty()) return {};
  int len = s.size();
  vector<int> z(len);
  z[0] = len;
  for (int i = 1, l = 1, r = 1; i < len; i++) {
    z[i] = i < r ? min(z[i - l], r - i) : 0;
    while (i + z[i] < len and s[i + z[i]] == s[z[i]]) z
    [i]++;
    if (i + z[i] > r) l = i, r = i + z[i];
  }
```

```cpp
  return z;
}
```

7.3 Manacher [77c4a7]

```cpp
vector<int> manacher(string_view s) {
  string p = "@#";
  for (char c : s) {
    p += c;
    p += '#';
  }
  p += '$';
  vector<int> dp(p.size());
  int mid = 0, r = 1;
  for (int i = 1; i < p.size() - 1; i++) {
    auto &k = dp[i];
    k = i < mid + r ? min(dp[mid * 2 - i], mid + r - i)
      : 0;
    while (p[i + k + 1] == p[i - k - 1]) k++;
    if (i + k > mid + r) mid = i, r = k;
  }
  return vector<int>(dp.begin() + 2, dp.end() - 2);
}
```

7.4 SAIS C++20 [06a2fa] (2a7f73|e7bb63)

```cpp
auto sais(const auto &s) {
  const int n = (int)s.size(), z = ranges::max(s) + 1;
  if (n == 1) return vector{0};
  vector<int> c(z); for (int x : s) ++c[x];
  partial_sum(all(c), begin(c));
  vector<int> sa(n); auto I = views::iota(0, n);
  vector<bool> t(n); t[n - 1] = true;
  for (int i = n - 2; i >= 0; i--)
    t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i +
    1]);
  auto is_lms = views::filter([&t](int x) {
    return x && t[x] & !t[x - 1];
  });
  auto induce = [&] {
    for (auto x = c; int y : sa)
      if (y-- and !t[y]) sa[x[s[y] - 1]++] = y;
    for (auto x = c; int y : sa | views::reverse)
      if (y-- and t[y]) sa[--x[s[y]]] = y;
  };
  vector<int> lms, q(n); lms.reserve(n);
  for (auto x = c; int i : I | is_lms) {
    q[i] = int(lms.size());
    lms.push_back(sa[--x[s[i]]] = i);
  }
  induce(); vector<int> ns(lms.size());
  for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
      int len = min({n - i, n - j, lms[q[i] + 1] - i});
      ns[q[i]] = nz += lexicographical_compare(
        s.begin() + j, s.begin() + j + len,
        s.begin() + i, s.begin() + i + len
      );
    }
    j = i;
  }
  ranges::fill(sa, 0); auto nsa = sais(ns);
  for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
  return induce(), sa;
} /* SPLIT-HASH */
// sa[i]: sa[i]-th suffix is the
// i-th lexicographically smallest suffix.
// lcp[i]: LCP of suffix sa[i] and suffix sa[i + 1].
struct Suffix {
  int n;
  vector<int> sa, rk, lcp;
  Suffix(const auto &s) : n(s.size()),
    lcp(n - 1), rk(n) {
    vector<int> t(n + 1); // t[n] = 0
    copy(all(s), t.begin()); // s shouldn't contain 0
    sa = sais(t); sa.erase(sa.begin());
    for (int i = 0; i < n; i++) rk[sa[i]] = i;
    for (int i = 0, h = 0; i < n; i++) {
      if (!rk[i]) { h = 0; continue; }
      for (int j = sa[rk[i] - 1];
          i + h < n and j + h < n
          and s[i + h] == s[j + h];) ++h;
      lcp[rk[i] - 1] = h ? h-- : 0;
    }
  }
};
```

7.5 Aho-Corasick [95f63b] (491636|781e36)

```cpp
const int sigma = ;

struct Node {
  Node *ch[sigma]{};
  Node *fail{}, *next{};
  bool end{};
} pool[i64(1E6)]{};

struct ACauto {
  int top;
  Node *root;
  ACauto() {
    top = 0;
    root = new (pool + top++) Node();
  }
  int add(string_view s) {
    auto p = root;
    for (char c : s) {
      c -= ;
      if (!p->ch[c]) {
        p->ch[c] = new (pool + top++) Node();
      }
      p = p->ch[c];
    }
    p->end = true;
    return p - pool;
  } /* SPLIT-HASH */
  vector<Node*> ord;
  void build() {
    queue<Node*> que;
    root->fail = root;
    for (auto &p : root->ch) {
      if (p) {
        p->fail = root;
        que.push(p);
      } else {
        p = root;
      }
    }
    while (!que.empty()) {
      auto p = que.front();
      que.pop();
      ord.push_back(p);
      p->next = (p->fail->end ? p->fail : p->fail->next
    );
      for (int i = 0; i < sigma; i++) {
        if (p->ch[i]) {
          p->ch[i]->fail = p->fail->ch[i];
          que.push(p->ch[i]);
        } else {
          p->ch[i] = p->fail->ch[i];
        }
      }
    }
  }
};
```

7.6 Palindromic Tree [bee744] (448867|810e73)

```cpp
// 迴文樹的每個節點代表一個迴文串
// len[i] 表示第 i 個節點的長度
// fail[i] 表示第 i 個節點的失配指針
// fail[i] 是 i 的次長迴文後綴
// dep[i] 表示第 i 個節點有幾個迴文後綴
// nxt[i][c] 表示在節點 i 兩邊加上字元 c 得到的點
// nxt 邊構成了兩顆分別以 odd 和 even 為根的向下的樹
// len[odd] = -1, len[even] = 0
// fail 邊構成了一顆以 odd 為根的向上的樹
// fail[even] = odd
// 0 ~ node size 是一個好的 dp 順序
// walk 是構建迴文樹時 lst 經過的節點
struct PAM {
  vector<array<int, 26>> nxt;
  vector<int> fail, len, dep, walk;
  int odd, even, lst;
  string S;
  int newNode(int l) {
    fail.push_back(0);
    nxt.push_back({});
    len.push_back(l);
    dep.push_back(0);
    return fail.size() - 1;
```

```
  }
  PAM() : odd(newNode(-1)), even(newNode(0)) {
    lst = fail[even] = odd;
  }
  void reserve(int l) {
    fail.reserve(l + 2);
    len.reserve(l + 2);
    nxt.reserve(l + 2);
    dep.reserve(l + 2);
    walk.reserve(l);
  }
  void build(string_view s) {
    reserve(s.size());
    for (char c : s) {
      walk.push_back(add(c));
    }
  } /* SPLIT-HASH */
  int up(int p) {
    while (S.rbegin()[len[p] + 1] != S.back()) {
      p = fail[p];
    }
    return p;
  }
  int add(char c) {
    S += c;
    lst = up(lst);
    c -= 'a';
    if (!nxt[lst][c]) {
      nxt[lst][c] = newNode(len[lst] + 2);
    }
    int p = nxt[lst][c];
    fail[p] = (lst == odd ? even : nxt[up(fail[lst])][c
    ]);
    lst = p;
    dep[lst] = dep[fail[lst]] + 1;
    return lst;
  }
};
```

7.7 Suffix Automaton [105a6e] (425e0d|efeb0a)

```
struct SAM {
  vector<array<int, 26>> nxt;
  vector<int> fail, len;
  int lst = 0;
  int newNode() {
    fail.push_back(0);
    len.push_back(0);
    nxt.push_back({});
    return fail.size() - 1;
  }
  SAM() : lst(newNode()) {}
  void reset() {
    lst = 0;
  }
  int add(int c) {
    if (nxt[lst][c] and len[nxt[lst][c]] == len[lst] +
    1) { // 廣義
      return lst = nxt[lst][c];
    }
    int cur = newNode();
    len[cur] = len[lst] + 1;
    while (lst and nxt[lst][c] == 0) {
      nxt[lst][c] = cur;
      lst = fail[lst];
    } /* SPLIT-HASH */
    int p = nxt[lst][c];
    if (p == 0) {
      fail[cur] = 0;
      nxt[0][c] = cur;
    } else if (len[p] == len[lst] + 1) {
      fail[cur] = p;
    } else {
      int t = newNode();
      nxt[t] = nxt[p];
      fail[t] = fail[p];
      len[t] = len[lst] + 1;
      while (nxt[lst][c] == p) {
        nxt[lst][c] = t;
        lst = fail[lst];
      }
      fail[p] = fail[cur] = t;
    }
    return lst = cur;
```

```
  }
  vector<int> order() { // 長度遞減
    vector<int> cnt(len.size());
    for (int i = 0; i < len.size(); i++)
      cnt[len[i]]++;
    partial_sum(rall(cnt), cnt.rbegin());
    vector<int> ord(cnt[0]);
    for (int i = len.size() - 1; i >= 0; i--)
      ord[--cnt[len[i]]] = i;
    return ord;
  }
};
```

7.8 Lyndon Factorization [822807]

```
// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
// min rotate: last < n of duval_min(s + s)
// max rotate: last < n of duval_max(s + s)
// min suffix: last of duval_min(s)
// max suffix: last of duval_max(s + -1)
vector<int> duval(const auto &s) {
  int n = s.size(), i = 0;
  vector<int> pos;
  while (i < n) {
    int j = i + 1, k = i;
    while (j < n and s[k] <= s[j]) { // >=
      if (s[k] < s[j]) k = i; // >
      else k++;
      j++;
    }
    while (i <= k) {
      pos.push_back(i);
      i += j - k;
    }
  }
  pos.push_back(n);
  return pos;
}
```

7.9 SmallestRotation [b6ba3b]

```
string Rotate(const string &s) {
 int n = s.length();
 string t = s + s;
 int i = 0, j = 1;
 while (i < n && j < n) {
  int k = 0;
  while (k < n && t[i + k] == t[j + k]) ++k;
  if (t[i + k] <= t[j + k]) j += k + 1;
  else i += k + 1;
  if (i == j) ++j;
 }
 int pos = (i < n ? i : j);
 return t.substr(pos, n);
}
```

8 Misc

8.1 Fraction Binary Search [be56a1]

```
// Binary search on Stern-Brocot Tree
// Parameters: n, pred
// n: Q_n is the set of all rational numbers whose
    denominator does not exceed n
// pred: pair<i64, i64> -> bool, pred({0, 1}) must be
    true
// Return value: {{a, b}, {x, y}}
// a/b is bigger value in Q_n that satisfy pred()
// x/y is smaller value in Q_n that not satisfy pred()
// Complexity: O(log^2 n)
using Pt = pair<i64, i64>;
Pt operator+(Pt a, Pt b) { return {a.ff + b.ff, a.ss +
    b.ss}; }
Pt operator*(i64 a, Pt b) { return {a * b.ff, a * b.ss
    }; }
pair<pair<i64, i64>, pair<i64, i64>> FractionSearch(i64
     n, const auto &pred) {
  pair<i64, i64> low{0, 1}, hei{1, 0};
  while (low.ss + hei.ss <= n) {
    bool cur = pred(low + hei);
    auto &fr{cur ? low : hei}, &to{cur ? hei : low};
    u64 L = 1, R = 2;
    while ((fr + R * to).ss <= n and pred(fr + R * to)
    == cur) {
      L *= 2;
      R *= 2;
```

```
    }
    while (L + 1 < R) {
      u64 M = (L + R) / 2;
      ((fr + M * to).ss <= n and pred(fr + M * to) ==
    cur ? L : R) = M;
    }
    fr = fr + L * to;
  }
  return {low, hei};
}
```

8.2 de Bruijn sequence [d87b1e]

```
constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
  int C, N, K, L;
  int buf[MAXC * MAXN];
  void dfs(int *out, int t, int p, int &ptr) {
    if (ptr >= L) return;
    if (t > N) {
      if (N % p) return;
      for (int i = 1; i <= p && ptr < L; ++i)
        out[ptr++] = buf[i];
    } else {
      buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
      for (int j = buf[t - p] + 1; j < C; ++j)
        buf[t] = j, dfs(out, t + 1, t, ptr);
    }
  }
  void solve(int _c, int _n, int _k, int *out) { //
    alphabet, len, k
    int p = 0;
    C = _c, N = _n, K = _k, L = N + K - 1;
    dfs(out, 1, 1, p);
    if (p < L) fill(out + p, out + L, 0);
  }
} dbs;
```

8.3 HilbertCurve [eccbe9]

```
i64 hilbert(int n, int x, int y) {
  i64 pos = 0;
  for (int s = (1 << n) / 2; s; s /= 2) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    pos += 1LL * s * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return pos;
}
```

8.4 Grid Intersection [dad212]

```
int det(Pt a, Pt b) { return a.ff * b.ss - a.ss * b.ff;
    }
// find p s.t (d1 * p, d2 * p) = x
Pt gridInter(Pt d1, Pt d2, Pt x) {
  swap(d1.ss, d2.ff);
  int s = det(d1, d2);
  int a = det(x, d2);
  int b = det(d1, x);
  assert(s != 0);
  if (a % s != 0 or b % s != 0) {
    return //{-1, -1};
  }
  return {a / s, b / s};
}
```

8.5 NextPerm [b6145d]

```
i64 next_perm(i64 x) {
  i64 y = x | (x - 1);
  return (y + 1) | (((~y & -~y) - 1) >> (__builtin_ctz(
    x) + 1));
}
```

8.6 HeapSize [5ce699]

```
pair<i64, i64> Split(i64 x) {
  if (x == 1) return {0, 0};
  i64 h = __lg(x);
  i64 fill = (1LL << (h + 1)) - 1;
  i64 l = (1LL << h) - 1 - max(0LL, fill - x - (1LL <<
    (h - 1)));
  i64 r = x - 1 - l;
  return {l, r};
}
```

8.7 Python

```
import sys
sys.stdin.readline()
sys.stdout.write(...)
from decimal import *
setcontext(Context(prec=MAX_PREC, Emax=MAX_EMAX,
    rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction('3.14159').limit_denominator(10).numerator #22
```

8.8 Kotlin

```
import java.util.*
import java.math.BigInteger;
import kotlin.math.*
private class Scanner {
  val lines = java.io.InputStreamReader(System.`in`).
    readLines()
  var curLine = 0
  var st = StringTokenizer(lines[0])
  fun next(): String {
    while(!st.hasMoreTokens())
      st = StringTokenizer(lines[++curLine])
    return st.nextToken()
  }
  fun nextInt() = next().toInt()
  fun nextLong() = next().toLong()
}
fun Long.toBigInteger() = BigInteger.valueOf(this)
fun Int.toBigInteger() = BigInteger.valueOf(toLong())
fun main() {
  val sc = Scanner()
  val buf = StringBuilder()

  val mp = Array(5) { Array(5) { -1 } }
  val dx = intArrayOf( 1, 0 )
  val dy = intArrayOf( 0, 1 )
  val v = ArrayList<Int>()

  fun dfs(x: Int, y: Int, s: Int = 0) {
    for((dx,dy) in dx zip dy) dfs(x+dx, y+dy, s)
  }
  dfs(0,0)

  val st = v.toSet().toIntArray().sorted()
  println("${st.joinToString()}\n") // st.sort()

  for(i in 1..sc.nextInt()) {
    val x = st.binarySearch(sc.nextInt())
    buf.append("$x\n")
  }

  val a = BigInteger(sc.next())
  val b = sc.nextLong().toBigInteger()
  println(a * b)
  print(buf)
}
```