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1 Basic

1.1 vimrc

```

set ts=4 sw=4 nu rnu et hls mouse=a
filetype indent on
sy on
inoremap jk <Esc>
inoremap {<CR> {<CR>}<C-o>0
nnoremap J 5j
nnoremap K 5k
nnoremap <F1> :w<bar>!g++ '%' -o run -std=c++20 -DLOCAL
-Wfatal-errors -fsanitize=address,undefined -g &&
echo done. && time ./run<CR>

```

1.2 default

```

#include <bits/stdc++.h>
using namespace std;
template<ranges::range T, class = enable_if_t<!
    is_convertible_v<T, string_view>>>
istream& operator>>(istream& s, T&&v) { for (auto& &&x
    : v) s >> x; return s; }
template<ranges::range T, class = enable_if_t<!
    is_convertible_v<T, string_view>>>
ostream& operator<<(ostream& s, T&&v) { for (auto& &&x
    : v) s << x << ' '; return s; }
#ifdef LOCAL
template<class... T> void dbg(T... x) { char e{}; ((
    cerr << e << x, e = ' '), ...); }
#define debug(x...) dbg(#x, '=', x, '\n')
#else
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#define debug(...) ((void)0)
#endif
#define all(v) (v).begin(), (v).end()
#define rall(v) (v).rbegin(), (v).rend()
#define ff first
#define ss second
using u32 = unsigned int;
using i64 = long long;
using u64 = unsigned long long;
using i128 = __int128;
using u128 = unsigned __int128;
template<class T> bool chmin(T& a, T b) { return (b < a
    and (a = b, true)); }
template<class T> bool chmax(T& a, T b) { return (a < b
    and (a = b, true)); }
template<class T> inline constexpr T inf =
    numeric_limits<T>::max() / 2;

```

1.3 judge

```

set -e
g++ -O3 a.cpp -o a
g++ -O3 ac.cpp -o c
g++ -O3 gen.cpp -o g

for ((i=0;;i++))
do
    echo "case $i"
    ./g > inp
    time ./a < inp > wa.out
    time ./c < inp > ac.out
    diff ac.out wa.out || break
done

```

1.4 Random

```

mt19937 rng(random_device{}());
i64 rand(i64 l = -lim, i64 r = lim) {
    return uniform_int_distribution<i64>(l, r)(rng);
}
double randr(double l, double r) {
    return uniform_real_distribution<double>(l, r)(rng);
}

```

1.5 Increase stack size

```
ulimit -s
```

2 Matching and Flow

2.1 Dinic

```
template<class Cap>
struct Dinic {
    struct Edge { int v; Cap w; int rev; };
    vector<vector<Edge>> G;
    int n, S, T;
    Dinic(int n, int S, int T) : n(n), S(S), T(T), G(n) {}
    void add_edge(int u, int v, Cap w) {
        G[u].push_back({v, w, (int)G[v].size()});
        G[v].push_back({u, 0, (int)G[u].size() - 1});
    }
    vector<int> dep;
    bool bfs() {
        dep.assign(n, 0);
        dep[S] = 1;
        queue<int> que;
        que.push(S);
        while (!que.empty()) {
            int u = que.front(); que.pop();
            for (auto [v, w, _] : G[u])
                if (!dep[v] and w) {
                    dep[v] = dep[u] + 1;
                    que.push(v);
                }
        }
        return dep[T] != 0;
    }
    Cap dfs(int u, Cap in) {
        if (u == T) return in;
        Cap out = 0;
        for (auto &[v, w, rev] : G[u]) {
            if (w and dep[v] == dep[u] + 1) {
                Cap f = dfs(v, min(w, in));
                w -= f, G[v][rev].w += f;
                in -= f, out += f;
                if (!in) break;
            }
        }
        if (in) dep[u] = 0;
        return out;
    }
    Cap maxflow() {
        Cap ret = 0;
        while (bfs()) {
            ret += dfs(S, inf<Cap>);
        }
        return ret;
    }
};
```

2.2 MCMF

```
template<class Cap>
struct MCMF {
    struct Edge { int v; Cap f, w; int rev; };
    vector<vector<Edge>> G;
    int n, S, T;
    MCMF(int n, int S, int T) : n(n), S(S), T(T), G(n) {}
    void add_edge(int u, int v, Cap cap, Cap cost) {
        G[u].push_back({v, cap, cost, (int)G[v].size()});
        G[v].push_back({u, 0, -cost, (int)G[u].size() - 1});
    }
    vector<Cap> dis;
    vector<bool> vis;
    bool spfa() {
        queue<int> que;
        dis.assign(n, inf<Cap>);
        vis.assign(n, false);
        que.push(S);
        vis[S] = 1;
        dis[S] = 0;
        while (!que.empty()) {
            int u = que.front(); que.pop();
            vis[u] = 0;
            for (auto [v, f, w, _] : G[u])
                if (f and chmin(dis[v], dis[u] + w))
                    if (!vis[v]) que.push(v), vis[v] = 1;
        }
        return dis[T] != inf<Cap>;
    }
};
```

```

    }
    Cap dfs(int u, Cap in) {
        if (u == T) return in;
        vis[u] = 1;
        Cap out = 0;
        for (auto &[v, f, w, rev] : G[u])
            if (f and !vis[v] and dis[v] == dis[u] + w) {
                Cap x = dfs(v, min(in, f));
                in -= x, out += x;
                f -= x, G[v][rev].f += x;
                if (!in) break;
            }
        if (in) dis[u] = inf<Cap>;
        vis[u] = 0;
        return out;
    }
    pair<Cap, Cap> maxflow() {
        Cap a = 0, b = 0;
        while (spfa()) {
            Cap x = dfs(S, inf<Cap>);
            a += x;
            b += x * dis[T];
        }
        return {a, b};
    }
};
```

2.3 HopcroftKarp

```
// l, r <= 1e5
struct HK {
    vector<int> g, l, r;
    int ans;
    HK(int n, int m, const vector<pair<int, int>> &e)
        : g(e.size()), l(n, -1), r(m, -1), ans{} {
        vector<int> deg(n + 1);
        for (auto [x, y] : e) deg[x]++;
        partial_sum(all(deg), deg.begin());
        for (auto [x, y] : e) g[--deg[x]] = y;
        vector<int> que(n);
        for (;;) {
            vector<int> a(n, -1), p(n, -1);
            int t = 0;
            for (int i = 0; i < n; i++) if (l[i] == -1)
                que[t++] = a[i] = p[i] = i;
            bool match = false;
            for (int i = 0; i < t; i++) {
                int x = que[i];
                if (~l[a[x]]) continue;
                for (int j = deg[x]; j < deg[x + 1]; j++) {
                    int y = g[j];
                    if (r[y] == -1) {
                        while (~y) r[y] = x, swap(l[x], y), x = p[x];
                        match = true, ans++;
                        break;
                    }
                    if (p[r[y]] == -1) {
                        que[t++] = y = r[y];
                        p[y] = x, a[y] = a[x];
                    }
                }
            }
            if (!match) break;
        }
    }
};
```

2.4 KM

```
i64 KM(vector<vector<int>> W) {
    const int n = W.size();
    vector<int> fl(n, -1), fr(n, -1), hr(n), hl(n);
    for (int i = 0; i < n; ++i) {
        hl[i] = *max_element(W[i].begin(), W[i].end());
    }
    auto Bfs = [&](int s) {
        vector<int> slk(n, INF), pre(n);
        vector<bool> vl(n, false), vr(n, false);
        queue<int> que;
        que.push(s);
        vr[s] = true;
        auto Check = [&](int x) -> bool {
            if (vl[x] == true, fl[x] != -1) {

```

```

    que.push(fl[x]);
    return vr[fl[x]] = true;
}
while (x != -1) swap(x, fr[fl[x] = pre[x]]);
return false;
};
while (true) {
    while (!que.empty()) {
        int y = que.front(); que.pop();
        for (int x = 0, d = 0; x < n; ++x) {
            if (!vl[x] and slk[x] >= (d = hl[x] + hr[y] -
W[x][y])) {
                if (pre[x] = y, d) slk[x] = d;
                else if (!Check(x)) return;
            }
        }
        int d = INF;
        for (int x = 0; x < n; ++x) {
            if (!vl[x] and d > slk[x]) d = slk[x];
        }
        for (int x = 0; x < n; ++x) {
            if (vl[x]) hl[x] += d;
            else slk[x] -= d;
            if (vr[x]) hr[x] -= d;
        }
        for (int x = 0; x < n; ++x) {
            if (!vl[x] and !slk[x] and !Check(x)) return;
        }
    }
};
for (int i = 0; i < n; ++i) Bfs(i);
i64 res = 0;
for (int i = 0; i < n; ++i) res += W[i][fl[i]];
return res;
}

```

2.5 SW

```

int w[kN][kN], g[kN], del[kN], v[kN];
void AddEdge(int x, int y, int c) {
    w[x][y] += c;
    w[y][x] += c;
}
pair<int, int> Phase(int n) {
    fill(v, v + n, 0), fill(g, g + n, 0);
    int s = -1, t = -1;
    while (true) {
        int c = -1;
        for (int i = 0; i < n; ++i) {
            if (del[i] || v[i]) continue;
            if (c == -1 || g[i] > g[c]) c = i;
        }
        if (c == -1) break;
        v[c] = 1, s = t, t = c;
        for (int i = 0; i < n; ++i) {
            if (del[i] || v[i]) continue;
            g[i] += w[c][i];
        }
    }
    return make_pair(s, t);
}
int GlobalMinCut(int n) {
    int cut = kInf;
    fill(del, 0, sizeof(del));
    for (int i = 0; i < n - 1; ++i) {
        int s, t; tie(s, t) = Phase(n);
        del[t] = 1, cut = min(cut, g[t]);
        for (int j = 0; j < n; ++j) {
            w[s][j] += w[t][j];
            w[j][s] += w[j][t];
        }
    }
    return cut;
}

```

2.6 GeneralMatching

```

struct GeneralMatching { // n <= 500
    const int BLOCK = 10;
    int n;
    vector<vector<int>> g;
    vector<int> hit, mat;

```

```

    std::priority_queue<pair<i64, int>, vector<pair<i64,
    int>>, greater<pair<i64, int>>> unmat;
    GeneralMatching(int _n) : n(_n), g(_n), mat(n, -1),
    hit(n) {}
    void add_edge(int a, int b) { // 0 <= a != b < n
        g[a].push_back(b);
        g[b].push_back(a);
    }
    int get_match() {
        for (int i = 0; i < n; i++) if (!g[i].empty()) {
            unmat.emplace(0, i);
        }
        // If WA, increase this
        // there are some cases that need >=1.3*n^2 steps
        for BLOCK=1
        // no idea what the actual bound needed here is.
        const int MAX_STEPS = 10 + 2 * n + n * n / BLOCK /
        2;
        mt19937 rng(random_device{}());
        for (int i = 0; i < MAX_STEPS; ++i) {
            if (unmat.empty()) break;
            int u = unmat.top().second;
            unmat.pop();
            if (mat[u] != -1) continue;
            for (int j = 0; j < BLOCK; j++) {
                ++hit[u];
                auto &e = g[u];
                const int v = e[rng() % e.size()];
                mat[u] = v;
                swap(u, mat[v]);
                if (u == -1) break;
            }
            if (u != -1) {
                mat[u] = -1;
                unmat.emplace(hit[u] * 100ULL / (g[u].size() +
1), u);
            }
        }
        int siz = 0;
        for (auto e : mat) siz += (e != -1);
        return siz / 2;
    }
};

```

3 Graph

3.1 Strongly Connected Component

```

struct SCC {
    int n;
    vector<vector<int>> G;
    vector<int> dfn, low, id;
    int scc{};
    SCC(int _n) : n{_n}, G{_n} {}
    void add_edge(int u, int v) {
        G[u].push_back(v);
    }
    void build() {
        dfn.assign(n, -1);
        low.assign(n, -1);
        id.assign(n, -1);
        vector<int> stk;
        int _t = 0;
        function<void(int)> dfs = [&](int u) {
            dfn[u] = low[u] = _t++;
            stk.push_back(u);
            for (int v : G[u]) {
                if (dfn[v] == -1) {
                    dfs(v);
                    chmin(low[u], low[v]);
                } else if (id[v] == -1) {
                    chmin(low[u], dfn[v]);
                }
            }
            if (dfn[u] == low[u]) {
                int t;
                do {
                    t = stk.back();
                    stk.pop_back();
                    id[t] = scc;
                } while (t != u);
                scc++;
            }
        };
    }
};

```

```
};
for (int i = 0; i < n; i++)
    if (dfn[i] == -1) dfs(i);
}
};
```

3.2 2-SAT

```
struct TwoSat {
    int n;
    vector<vector<int>> e;
    vector<bool> ans;
    TwoSat(int n) : n(n), e(2 * n), ans(n) {}
    void addClause(int u, bool f, int v, bool g) { // (u
        = f) or (v = g)
        e[2 * u + !f].push_back(2 * v + g);
        e[2 * v + !g].push_back(2 * u + f);
    }
    void addImply(int u, bool f, int v, bool g) { // (u =
        f) -> (v = g)
        e[2 * u + f].push_back(2 * v + g);
        e[2 * v + !g].push_back(2 * u + !f);
    }
    bool satisfiable() {
        vector<int> id(2 * n, -1), dfn(2 * n, -1), low(2 *
            n, -1);
        vector<int> stk;
        int now = 0, cnt = 0;
        function<void(int)> tarjan = [&](int u) {
            stk.push_back(u);
            dfn[u] = low[u] = now++;
            for (auto v : e[u]) {
                if (dfn[v] == -1) {
                    tarjan(v);
                    low[u] = min(low[u], low[v]);
                } else if (id[v] == -1) {
                    low[u] = min(low[u], dfn[v]);
                }
            }
            if (dfn[u] == low[u]) {
                int v;
                do {
                    v = stk.back();
                    stk.pop_back();
                    id[v] = cnt;
                } while (v != u);
                ++cnt;
            }
        };
        for (int i = 0; i < 2 * n; ++i) if (dfn[i] == -1)
            tarjan(i);
        for (int i = 0; i < n; ++i) {
            if (id[2 * i] == id[2 * i + 1]) return false;
            ans[i] = id[2 * i] > id[2 * i + 1];
        }
        return true;
    }
};
```

3.3 Manhattan MST

```
vector<tuple<int, int, int>> ManhattanMST(vector<Pt> P)
{
    vector<int> id(P.size());
    iota(all(id), 0);
    vector<tuple<int, int, int>> edges;
    for (int k = 0; k < 4; ++k) {
        sort(all(id), [&](int i, int j) -> bool {
            return (P[i] - P[j]).ff < (P[j] - P[i]).ss;
        });
        map<int, int> sweep;
        for (int i : id) {
            for (auto it = sweep.lower_bound(-P[i].ss); \
                it != sweep.end(); sweep.erase(it++)) {
                int j = it->ss;
                Pt d = P[i] - P[j];
                if (d.ss > d.ff) break;
                edges.emplace_back(d.ss + d.ff, i, j);
            }
            sweep[-P[i].ss] = i;
        }
        for (Pt &p : P) {
            if (k % 2) p.ff = -p.ff;
            else swap(p.ff, p.ss);
        }
    }
```

```
    }
}
return edges;
}
```

3.4 TreeHash

```
map<vector<int>, int> id;
vector<vector<int>> sub;
vector<int> siz;
int getid(const vector<int> &T) {
    if (id.count(T)) return id[T];
    int s = 1;
    for (int x : T) {
        s += siz[x];
    }
    sub.push_back(T);
    siz.push_back(s);
    return id[T] = id.size();
}
int dfs(int u, int f) {
    vector<int> S;
    for (int v : G[u]) if (v != f) {
        S.push_back(dfs(v, u));
    }
    sort(all(S));
    return getid(S);
}
```

3.5 Maximum IndependentSet

```
// n <= 40, (*500)
set<int> MI(const vector<vector<int>> &adj) {
    set<int> I, V;
    for (int i = 0; i < adj.size(); i++)
        V.insert(i);
    while (!V.empty()) {
        auto it = next(V.begin(), rng() % V.size());
        int cho = *it;
        I.insert(cho);
        V.extract(cho);
        for (int i : adj[cho]) {
            if (auto j = V.find(i); j != V.end())
                V.erase(j);
        }
    }
    return I;
}
```

3.6 Min Mean Weight Cycle

```
// d[i][j] == 0 if {i,j} !in E
long long d[1003][1003], dp[1003][1003];

pair<long long, long long> MMWC() {
    memset(dp, 0x3f, sizeof(dp));
    for (int i = 1; i <= n; ++i) dp[0][i] = 0;
    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= n; ++j) {
            for (int k = 1; k <= n; ++k) {
                dp[i][k] = min(dp[i - 1][j] + d[j][k], dp[i][k]);
            }
        }
    }
    long long au = 1ll << 31, ad = 1;
    for (int i = 1; i <= n; ++i) {
        if (dp[n][i] == 0x3f3f3f3f3f3f3f3f) continue;
        long long u = 0, d = 1;
        for (int j = n - 1; j >= 0; --j) {
            if ((dp[n][i] - dp[j][i]) * d > u * (n - j)) {
                u = dp[n][i] - dp[j][i];
                d = n - j;
            }
        }
        if (u * ad < au * d) au = u, ad = d;
    }
    long long g = __gcd(au, ad);
    return make_pair(au / g, ad / g);
}
```

3.7 Block Cut Tree

```
struct BlockCutTree {
    int n;
    vector<vector<int>> adj;
```

```

BlockCutTree(int _n) : n(_n), adj(_n) {}
void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}
pair<int, vector<pair<int, int>>> work() {
    vector<int> dfn(n, -1), low(n), stk;
    vector<pair<int, int>> edg;
    int cnt = 0, cur = 0;
    function<void(int)> dfs = [&](int x) {
        stk.push_back(x);
        dfn[x] = low[x] = cur++;
        for (auto y : adj[x]) {
            if (dfn[y] == -1) {
                dfs(y);
                low[x] = min(low[x], low[y]);
                if (low[y] == dfn[x]) {
                    int v;
                    do {
                        v = stk.back();
                        stk.pop_back();
                        edg.emplace_back(n + cnt, v);
                    } while (v != y);
                    edg.emplace_back(x, n + cnt);
                    cnt++;
                }
            } else {
                low[x] = min(low[x], dfn[y]);
            }
        }
    };
    for (int i = 0; i < n; i++) {
        if (dfn[i] == -1) {
            stk.clear();
            dfs(i);
        }
    }
    return {cnt, edg};
}
};

```

3.8 Heavy Light Decomposition

```

struct HLD {
    int n;
    vector<int> siz, top, dep, pa, in, out, seq;
    vector<vector<int>> G;
    HLD(int n) : n(n), G(n), siz(n), top(n),
        dep(n), pa(n), in(n), out(n), seq(n) {}
    int cur{};
    void addEdge(int u, int v) {
        G[u].push_back(v);
        G[v].push_back(u);
    }
    void work(int root = 0) {
        cur = 0;
        top[root] = root;
        dep[root] = 0;
        pa[root] = -1;
        dfs1(root);
        dfs2(root);
    }
    void dfs1(int u) {
        if (pa[u] != -1) {
            G[u].erase(find(all(G[u]), pa[u]));
        }
        siz[u] = 1;
        for (auto &v : G[u]) {
            pa[v] = u;
            dep[v] = dep[u] + 1;
            dfs1(v);
            siz[u] += siz[v];
            if (siz[v] > siz[G[u][0]]) {
                swap(v, G[u][0]);
            }
        }
    }
    void dfs2(int u) {
        in[u] = cur++;
        seq[in[u]] = u;
        for (int v : G[u]) {
            top[v] = (v == G[u][0] ? top[u] : v);
            dfs2(v);
        }
    }
}

```

```

    }
    out[u] = cur;
}
int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]]) swap(x, y);
        x = pa[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
}
int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}
int jump(int x, int k) {
    if (dep[x] < k) return -1;
    int d = dep[x] - k;
    while (dep[top[x]] > d) {
        x = pa[top[x]];
    }
    return seq[in[x] - dep[x] + d];
}
bool isAnc(int x, int y) {
    return in[x] <= in[y] and in[y] < out[x];
}
int rootPar(int r, int x) {
    if (r == x) return r;
    if (!isAnc(x, r)) return pa[x];
    auto it = upper_bound(all(G[x]), r, [&](int a, int b) -> bool {
        return in[a] < in[b];
    }) - 1;
    return *it;
}
int rootSiz(int r, int x) {
    if (r == x) return n;
    if (!isAnc(x, r)) return siz[x];
    return n - siz[rootPar(r, x)];
}
int rootLca(int a, int b, int c) {
    return lca(a, b) ^ lca(b, c) ^ lca(c, a);
}
};

```

3.9 Dominator Tree

```

struct Dominator {
    vector<vector<int>> g, r, rdom; int tk;
    vector<int> dfn, rev, fa, sdom, dom, val, rp;
    int n;
    Dominator(int n) : n(n), g(n), r(n), rdom(n), tk(0),
        dfn(n, -1), rev(n, -1), fa(n, -1), sdom(n, -1),
        dom(n, -1), val(n, -1), rp(n, -1) {}
    void addEdge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x] = tk] = x;
        fa[tk] = sdom[tk] = val[tk] = tk; tk++;
        for (int u : g[x]) {
            if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
            r[dfn[u]].push_back(dfn[x]);
        }
    }
    void merge(int x, int y) { fa[x] = y; }
    int find(int x, int c = 0) {
        if (fa[x] == x) return c ? -1 : x;
        if (int p = find(fa[x], 1); p != -1) {
            if (sdom[val[x]] > sdom[val[fa[x]]])
                val[x] = val[fa[x]];
            fa[x] = p;
            return c ? p : val[x];
        }
        return c ? fa[x] : val[x];
    }
    vector<int> build(int s) {
        // return the father of each node in dominator tree
        // p[i] = -2 if i is unreachable from s
        dfs(s);
        for (int i = tk - 1; i >= 0; --i) {
            for (int u : r[i])
                sdom[i] = min(sdom[i], sdom[find(u)]);
            if (i) rdom[sdom[i]].push_back(i);
            for (int u : rdom[i]) {
                int p = find(u);
                dom[u] = (sdom[p] == i ? i : p);
            }
        }
    }
}

```

```

    }
    if (i) merge(i, rp[i]);
}
vector<int> p(n, -2); p[s] = -1;
for (int i = 1; i < tk; ++i)
    if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
for (int i = 1; i < tk; ++i)
    p[rev[i]] = rev[dom[i]];
return p;
}
};

```

4 Data Structure

4.1 Lazy Segtree

```

template<class S, class T>
struct Seg {
    Seg<S, T> *ls{}, *rs{};
    int l, r;
    S d{};
    T f{};
    Seg(int _l, int _r) : l{_l}, r{_r} {
        if (r - l == 1) {
            return;
        }
        int mid = (l + r) / 2;
        ls = new Seg(l, mid);
        rs = new Seg(mid, r);
        pull();
    }
    void upd(const T &g) {
        g(d), g(f);
    }
    void pull() {
        d = ls->d + rs->d;
    }
    void push() {
        ls->upd(f);
        rs->upd(f);
        f = T{};
    }
    S query(int x, int y) {
        if (y <= l or r <= x) return S{};
        if (x <= l and r <= y) return d;
        push();
        return ls->query(x, y) + rs->query(x, y);
    }
    void apply(int x, int y, const T &g) {
        if (y <= l or r <= x) return;
        if (x <= l and r <= y) {
            upd(g);
            return;
        }
        push();
        ls->apply(x, y, g);
        rs->apply(x, y, g);
        pull();
    }
    void set(int p, const S &g) {
        if (p + 1 <= l or r <= p) return;
        if (r - l == 1) {
            d = g;
            return;
        }
        push();
        ls->set(p, g);
        rs->set(p, g);
        pull();
    }
    int findFirst(int x, int y, auto pred) {
        if (y <= l or r <= x or !pred(d)) return -1;
        if (r - l == 1) return l;
        push();
        int res = ls->findFirst(x, y, pred);
        return res == -1 ? rs->findFirst(x, y, pred) : res;
    }
    int findLast(int x, int y, auto pred) {
        if (y <= l or r <= x or !pred(d)) return -1;
        if (r - l == 1) return l;
        push();
        int res = rs->findLast(x, y, pred);
        return res == -1 ? ls->findLast(x, y, pred) : res;
    }
};

```

```

    }
};

```

4.2 Sparse Table

```

template<class T, auto F>
struct SparseTable {
    int n, lgN;
    vector<vector<T>> st;
    SparseTable(const vector<T> &V) {
        n = V.size();
        lgN = __lg(n);
        st.assign(lgN + 1, vector<T>(n));
        st[0] = V;
        for (int i = 0; (2 << i) <= n; i++)
            for (int j = 0; j + (2 << i) <= n; j++) {
                st[i + 1][j] = F(st[i][j], st[i][j] + (1 << i));
            }
    }
    T qry(int l, int r) { // [l, r)
        int h = __lg(r - l);
        return F(st[h][l], st[h][r - (1 << h)]);
    }
};

```

4.3 Binary Index Tree

```

template<class T>
struct BIT {
    int n;
    vector<T> a;
    BIT(int n) : n(n), a(n) {}
    int lowbit(int x) { return x & -x; }
    void add(int p, T x) {
        for (int i = p + 1; i <= n; i += lowbit(i))
            a[i - 1] += x;
    }
    T qry(int p) {
        T r{};
        for (int i = p + 1; i > 0; i -= lowbit(i))
            r += a[i - 1];
        return r;
    }
    T qry(int l, int r) { // [l, r)
        return qry(r - 1) - qry(l - 1);
    }
    int kth(T k) {
        int x = 0;
        for (int i = 1 << __lg(n); i; i >>= 1) {
            if (x + i <= n and k >= a[x + i - 1]) {
                x += i;
                k -= a[x - 1];
            }
        }
        return x;
    }
};

```

4.4 Special Segtree

```

struct Seg {
    Seg *ls, *rs;
    int l, r;
    vector<int> f, g;
    // f : intervals where covering [l, r]
    // g : intervals where interset with [l, r]
    Seg(int _l, int _r) : l{_l}, r{_r} {
        int mid = (l + r) >> 1;
        if (r - l == 1) return;
        ls = new Seg(l, mid);
        rs = new Seg(mid, r);
    }
    void insert(int x, int y, int id) {
        if (y <= l or r <= x) return;
        g.push_back(id);
        if (x <= l and r <= y) {
            f.push_back(id);
            return;
        }
        ls->insert(x, y, id);
        rs->insert(x, y, id);
    }
    void fix() {

```



```

    while (!f.empty() and use[f.back()]) f.pop_back();
    while (!g.empty() and use[g.back()]) g.pop_back();
}
int query(int x, int y) {
    if (y <= l or r <= x) return -1;
    fix();
    if (x <= l and r <= y) {
        return g.empty() ? -1 : g.back();
    }
    return max({f.empty() ? -1 : f.back(), ls->query(x, y), rs->query(x, y)});
}
};

```

4.5 Treap

```

mt19937 rng(random_device{}());
template<class S, class T>
struct Treap {
    struct Node {
        Node *ls{}, *rs{};
        int pos, siz;
        u32 pri;
        S d{}, e{};
        T f{};
        Node(int p, S x) : d{x}, e{x}, pos{p}, siz{1}, pri{rng()} {}
        void upd(T &g) {
            g(d), g(e), g(f);
        }
        void pull() {
            siz = Siz(ls) + Siz(rs);
            d = Get(ls) + e + Get(rs);
        }
        void push() {
            if (ls) ls->upd(f);
            if (rs) rs->upd(f);
            f = T{};
        }
    } *root{};
    static int Siz(Node *p) { return p ? p->siz : 0; }
    static S Get(Node *p) { return p ? p->d : S{}; }
    Treap() : root{} {}
    Node* Merge(Node *a, Node *b) {
        if (!a or !b) return a ? a : b;
        if (a->pri < b->pri) {
            a->push();
            a->rs = Merge(a->rs, b);
            a->pull();
            return a;
        } else {
            b->push();
            b->ls = Merge(a, b->ls);
            b->pull();
            return b;
        }
    }
    void Split(Node *p, Node *&a, Node *&b, int k) {
        if (!p) return void(a = b = nullptr);
        p->push();
        if (p->pos <= k) {
            a = p;
            Split(p->rs, a->rs, b, k);
            a->pull();
        } else {
            b = p;
            Split(p->ls, a, b->ls, k);
            b->pull();
        }
    }
    void insert(int p, S x) {
        Node *L, *R;
        Split(root, L, R, p);
        root = Merge(Merge(L, new Node(p, x)), R);
    }
    void erase(int x) {
        Node *L, *M, *R;
        Split(root, M, R, x);
        Split(M, L, M, x - 1);
        if (M) M = Merge(M->ls, M->rs);
        root = Merge(Merge(L, M), R);
    }
    S query() {

```

```

        return Get(root);
    }
};

```

4.6 LiChao Segtree

```

struct Line {
    i64 k, m; // y = k + mx;
    Line() : k{INF}, m{} {}
    Line(i64 _k, i64 _m) : k(_k), m(_m) {}
    i64 get(i64 x) {
        return k + m * x;
    }
};
struct Seg {
    Seg *ls{}, *rs{};
    int l, r, mid;
    Line line{};
    Seg(int _l, int _r) : l(_l), r(_r), mid((_l + _r) >> 1) {}
    if (r - l == 1) return;
    ls = new Seg(l, mid);
    rs = new Seg(mid, r);
}
void insert(Line L) {
    if (line.get(mid) > L.get(mid))
        swap(line, L);
    if (r - l == 1) return;
    if (L.m < line.m) {
        rs->insert(L);
    } else {
        ls->insert(L);
    }
}
i64 query(int p) {
    if (p < l or r <= p) return INF;
    if (r - l == 1) return line.get(p);
    return min({line.get(p), ls->query(p), rs->query(p)});
}
};

```

4.7 Persistent SegmentTree

```

template<class S>
struct Seg {
    Seg *ls{}, *rs{};
    int l, r;
    S d{};
    Seg(Seg* p) { (*this) = *p; }
    Seg(int l, int r) : l(l), r(r) {
        if (r - l == 1) {
            d = {};
            return;
        }
        int mid = (l + r) / 2;
        ls = new Seg(l, mid);
        rs = new Seg(mid, r);
        pull();
    }
    void pull() {
        d = ls->d + rs->d;
    }
    Seg* set(int p, const S &x) {
        Seg* n = new Seg(this);
        if (r - l == 1) {
            n->d = x;
            return n;
        }
        int mid = (l + r) / 2;
        if (p < mid) {
            n->ls = ls->set(p, x);
        } else {
            n->rs = rs->set(p, x);
        }
        n->pull();
        return n;
    }
    S query(int x, int y) {
        if (y <= l or r <= x) return {};
        if (x <= l and r <= y) return d;
        return ls->query(x, y) + rs->query(x, y);
    }
};

```

4.8 Blackmagic

```
#include <bits/extc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/hash_policy.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
template<class T>
using BST = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
__gnu_pbds::priority_queue<node, decltype(cmp),
    pairing_heap_tag> pq(cmp);
gp_hash_table<int, __gnu_pbds::priority_queue<node>::
    point_iterator> pqPos;
bst.insert((x << 20) + i);
bst.erase(bst.lower_bound(x << 20));
bst.order_of_key(x << 20) + 1;
*bst.find_by_order(x - 1) >> 20;
*--bst.lower_bound(x << 20) >> 20;
*bst.upper_bound((x + 1) << 20) >> 20;
```

4.9 Centroid Decomposition

```
struct CenDec {
    vector<vector<pair<int, i64>>> G;
    vector<vector<i64>> pdis;
    vector<int> pa, ord, siz;
    vector<bool> vis;
    int getsiz(int u, int f) {
        siz[u] = 1;
        for (auto [v, w] : G[u]) if (v != f and !vis[v])
            siz[u] += getsiz(v, u);
        return siz[u];
    }
    int find(int u, int f, int s) {
        for (auto [v, w] : G[u]) if (v != f and !vis[v])
            if (siz[v] * 2 >= s) return find(v, u, s);
        return u;
    };
    void caldis(int u, int f, i64 dis) {
        pdis[u].push_back(dis);
        for (auto [v, w] : G[u]) if (v != f and !vis[v]) {
            caldis(v, u, dis + w);
        }
    }
    int build(int u = 0) {
        u = find(u, u, getsiz(u, u));
        ord.push_back(u);
        vis[u] = 1;
        for (auto [v, w] : G[u]) if (!vis[v]) {
            pa[build(v)] = u;
        }
        caldis(u, -1, 0); // if need
        vis[u] = 0;
        return u;
    };
    CenDec(int n) : G(n), pa(n, -1), vis(n), siz(n), pdis
        (n) {}
};
```

4.10 2D BIT

```
template<class T>
struct BIT2D {
    vector<vector<T>> val;
    vector<vector<int>> Y;
    vector<int> X;
    int lowbit(int x) { return x & -x; }
    int getp(const vector<int> &v, int x) {
        return upper_bound(all(v), x) - v.begin();
    }
    BIT2D(vector<pair<int, int>> pos) {
        for (auto &[x, y] : pos) {
            X.push_back(x);
            swap(x, y);
        }
        sort(all(pos));
        sort(all(X));
        X.erase(unique(all(X)), X.end());
        Y.resize(X.size() + 1);
        val.resize(X.size() + 1);
        for (auto [y, x] : pos) {
            for (int i = getp(X, x); i <= X.size(); i +=
                lowbit(i))
```

```
                if (Y[i].empty() or Y[i].back() != y)
                    Y[i].push_back(y);
        }
        for (int i = 1; i <= X.size(); i++) {
            val[i].assign(Y[i].size() + 1, T{});
        }
    }
    void add(int x, int y, T v) {
        for (int i = getp(X, x); i <= X.size(); i += lowbit
            (i))
            for (int j = getp(Y[i], y); j <= Y[i].size(); j
                += lowbit(j))
                val[i][j] += v;
    }
    T qry(int x, int y) {
        T r{};
        for (int i = getp(X, x); i > 0; i -= lowbit(i))
            for (int j = getp(Y[i], y); j > 0; j -= lowbit(j)
                ) {
                r += val[i][j];
            }
        return r;
    }
};
```

5 Dynamic Programming

5.1 CDQ

```
auto cmp2 = [&](int a, int b) -> bool { return P[a][1]
    < P[b][1]; };
auto cdq = [&](auto self, auto l, auto r) {
    if (r - l == 1) return;
    auto mid = l + (r - l) / 2;
    self(self, l, mid);
    auto tmp = vector<int>(mid, r);
    sort(l, mid, cmp2);
    sort(mid, r, cmp2);
    for (auto i = l, j = mid; j < r; j++) {
        while (i != mid and P[*i][1] < P[*j][1]) {
            bit.add(P[*i][2], dp[*i]);
            i++;
        }
        dp[*j].upd(bit.qry(P[*j][2]));
    };
    for (auto i = l; i < mid; i++) bit.reset(P[*i][2]);
    copy(all(tmp), mid);
    self(self, mid, r);
}; cdq(cdq, all(ord));
```

6 Math

6.1 Theorem

- Pick's theorem

$$A = i + \frac{b}{2} - 1$$

- Laplacian matrix

$$L = D - A$$

- Extended Catalan number

$$\frac{1}{(k-1)n+1} \binom{kn}{n}$$

- Derangement $D_n = (n-1)(D_{n-1} + D_{n-2})$

- Möbius

$$\sum_{i|n} \mu(i) = [n=1] \sum_{i|n} \phi(i) = n$$

- Inversion formula

$$f(n) = \sum_{i=0}^n \binom{n}{i} g(i) \quad g(n) = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f(i)$$

$$f(n) = \sum_{d|n} g(d) \quad g(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) f(d)$$

- Sum of powers

$$\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j^- = 0$$

$$\text{note: } B_1^+ = -B_1^- \quad B_i^+ = B_i^-$$

- Cipolla's algorithm

$$\left(\frac{u}{p}\right) = u^{\frac{p-1}{2}}$$

$$1. \left(\frac{a^2 - n}{p} \right) = -1$$

$$2. x = (a + \sqrt{a^2 - n})^{\frac{p+1}{2}}$$

• Cayley's formula

number of trees on n labeled vertices: n^{n-2}

Let $T_{n,k}$ be the number of labelled forests on n vertices with k connected components, such that vertices $1, 2, \dots, k$ all belong to different connected components. Then $T_{n,k} = kn^{n-k-1}$.

• High order residue

$$[d^{\frac{p-1}{n \cdot p-1}} \equiv 1]$$

• Packing and Covering

$$|\text{Maximum Independent Set}| + |\text{Minimum Vertex Cover}| = |V|$$

• König's theorem

$$|\text{maximum matching}| = |\text{minimum vertex cover}|$$

• Dilworth's theorem

$$\text{width} = |\text{largest antichain}| = |\text{smallest chain decomposition}|$$

• Mirsky's theorem

$$\text{height} = |\text{longest chain}| = |\text{smallest antichain decomposition}| = |\text{minimum antichain partition}|$$

• Triangle center

- $G : (1,)$
- $O : (a^2(b^2 + c^2 - a^2),) = (\sin 2A,)$
- $I : (a,) = (\sin A)$
- $E : (-a, b, c) = (-\sin A, \sin B, \sin C)$
- $H : (\frac{1}{b^2 + c^2 - a^2},) = (\tan A,)$

• Lucas' Theorem :

For $n, m \in \mathbb{Z}^*$ and prime $P, C(m, n) \bmod P = \prod (C(m_i, n_i))$ where m_i is the i -th digit of m in base P .

• Stirling approximation :

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$

• Stirling Numbers(permutation $|P| = n$ with k cycles):

$$S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x + i)$$

• Stirling Numbers(Partition n elements into k non-empty set):

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

• Pick's Theorem : $A = i + b/2 - 1$

A : Area ; i : grid number in the inner ; b : grid number on the side

• Catalan number : $C_n = \binom{2n}{n} / (n+1)$

$$C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$

• Euler Characteristic:

$$\text{planar graph: } V - E + F - C = 1$$

$$\text{convex polyhedron: } V - E + F = 2$$

V, E, F, C : number of vertices, edges, faces(regions), and components

• Kirchhoff's theorem :

$A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$; Deleting any one row, one column, and cal the $\det(A)$

• Polyá' theorem (c is number of color , m is the number of cycle size):

$$(\sum_{i=1}^m c^{gcd(i,m)})/m$$

• Burnside lemma:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

• 錯排公式: (n 個人中, 每個人皆不再原來位置的組合數):

$$dp[0] = 1; dp[1] = 0;$$

$$dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$$

• Bell 數 (有 n 個人, 把他們拆組的方法總數):

$$B_0 = 1$$

$$B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$$

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$

• Wilson's theorem :

$$(p-1)! \equiv -1 \pmod{p}$$

• Fermat's little theorem :

$$a^p \equiv a \pmod{p}$$

• Euler's totient function:

$$A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$$

• 歐拉函數降冪公式:

$$A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$$

• 環相鄰塗異色:

$$(k-1)(-1)^n + (k-1)^n$$

• 6 的倍數:

$$(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$$

6.2 Linear Sieve

```
template<size_t N>
struct Sieve {
    array<bool, N + 1> isp{};
    array<int, N + 1> mu{}, phi{};
    vector<int> primes{};
    Sieve() {
        isp.fill(true);
        isp[0] = isp[1] = false;
        mu[1] = 1;
        phi[1] = 1;
        for (int i = 2; i <= N; i++) {
            if (isp[i]) {
                primes.push_back(i);
                mu[i] = -1;
                phi[i] = i - 1;
            }
            for (i64 p : primes) {
                if (p * i > N) break;
                isp[p * i] = false;
                if (i % p == 0) {
                    phi[p * i] = phi[i] * p;
                    break;
                }
                phi[p * i] = phi[i] * (p - 1);
                mu[p * i] = mu[p] * mu[i];
            }
        }
    }
};
```

6.3 Exgcd

```
pair<i64, i64> exgcd(i64 a, i64 b) { // ax + by = 1
    if (b == 0) return {1, 0};
    auto [x, y] = exgcd(b, a % b);
    return {y, x - a / b * y};
};
```

6.4 CRT

```
i64 CRT(vector<pair<i64, i64>> E) {
    i128 R = 0, M = 1;
    for (auto [r, m] : E) {
        i128 d = r - R, g = gcd<i64>(M, m);
        if (d % g != 0) return -1;
        i128 x = exgcd(M / g, m / g).ff * d / g;
        R += M * x;
        M = M * m / g;
        R = (R % M + M) % M;
    }
    return R;
}
```

6.5 Factorize

```
struct Factorize {
    i64 fmul(i64 a, i64 b, i64 p) {
        return (i128)a * b % p;
    }
    i64 fpow(i64 a, i64 b, i64 p) {
        i64 res = 1;
        for (; b; b >>= 1, a = fmul(a, a, p))
            if (b & 1) res = fmul(res, a, p);
        return res;
    }
    bool Check(i64 a, i64 u, i64 n, int t) {
        a = fpow(a, u, n);
        if (a == 0 or a == 1 or a == n - 1) return true;
        for (int i = 0; i < t; i++) {
            a = fmul(a, a, n);
            if (a == 1) return false;
            if (a == n - 1) return true;
        }
    }
};
```

```

    }
    return false;
};
bool IsPrime(i64 n) {
    constexpr array<i64, 7> kChk{2, 235, 9375, 28178,
    450775, 9780504, 1795265022};
    // for int: {2, 7, 61}
    if (n < 2) return false;
    if (n % 2 == 0) return n == 2;
    i64 u = n - 1;
    int t = 0;
    while (u % 2 == 0) u >>= 1, t++;
    for (auto v : kChk) if (!Check(v, u, n, t)) return
    false;
    return true;
}
i64 PollardRho(i64 n) {
    if (n % 2 == 0) return 2;
    i64 x = 2, y = 2, d = 1, p = 1;
    auto f = [](i64 x, i64 n, i64 p) -> i64 {
        return ((i128)x * x % n + p) % n;
    };
    while (true) {
        x = f(x, n, p);
        y = f(f(y, n, p), n, p);
        d = __gcd(abs(x - y), n);
        if (d != n and d != 1) return d;
        if (d == n) ++p;
    }
}
i64 PrimeFactor(i64 n) {
    return IsPrime(n) ? n : PrimeFactor(PollardRho(n));
}
};

```

6.6 NTT Prime List

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3

6.7 NTT

```

constexpr i64 cpow(i64 a, i64 b, i64 m) {
    i64 ret = 1;
    for (; b; b >>= 1, a = a * a % m)
        if (b & 1) ret = ret * a % m;
    return ret;
};
template<i64 M, i64 G>
struct NTT {
    static constexpr i64 iG = cpow(G, M - 2, M);
    void operator()(vector<i64> &v, bool inv) {
        int n = v.size();
        for (int i = 0, j = 0; i < n; i++) {
            if (i < j) swap(v[i], v[j]);
            for (int k = n / 2; (j ^= k) < k; k /= 2);
        }
        for (int mid = 1; mid < n; mid *= 2) {
            i64 w = cpow((inv ? iG : G), (M - 1) / (mid + mid), M);
            for (int i = 0; i < n; i += mid * 2) {
                i64 now = 1;
                for (int j = i; j < i + mid; j++, now = now * w % M) {
                    i64 x = v[j], y = v[j + mid];
                    v[j] = (x + y * now) % M;
                    v[j + mid] = (x - y * now) % M;
                }
            }
        }
        if (inv) {
            i64 in = cpow(n, M - 2, M);
            for (int i = 0; i < n; i++) v[i] = v[i] * in % M;
        }
    }
};
template<i64 M, i64 G>
vector<i64> convolution(vector<i64> f, vector<i64> g) {

```

```

    NTT<M, G> ntt;
    int sum = f.size() + g.size() - 1;
    int len = bit_ceil((u64)sum);
    f.resize(len); g.resize(len);
    ntt(f, 0), ntt(g, 0);
    for (int i = 0; i < len; i++) (f[i] *= g[i]) %= M;
    ntt(f, 1);
    f.resize(sum);
    for (int i = 0; i < sum; i++) if (f[i] < 0) f[i] += M;
    return f;
}
vector<i64> convolution_ll(const vector<i64> &f, const
    vector<i64> &g) {
    constexpr i64 M1 = 998244353, G1 = 3;
    constexpr i64 M2 = 985661441, G2 = 3;
    constexpr i64 M1M2 = M1 * M2;
    constexpr i64 M1m1 = M2 * cpow(M2, M1 - 2, M1);
    constexpr i64 M2m2 = M1 * cpow(M1, M2 - 2, M2);
    auto c1 = convolution<M1, G1>(f, g);
    auto c2 = convolution<M2, G2>(f, g);
    for (int i = 0; i < c1.size(); i++) {
        c1[i] = ((i128)c1[i] * M1m1 + (i128)c2[i] * M2m2) %
        M1M2;
    }
    return c1;
}

```

6.8 FWT

1. XOR Convolution

- $f(A) = (f(A_0) + f(A_1), f(A_0) - f(A_1))$
- $f^{-1}(A) = (f^{-1}(\frac{A_0+A_1}{2}), f^{-1}(\frac{A_0-A_1}{2}))$

2. OR Convolution

- $f(A) = (f(A_0), f(A_0) + f(A_1))$
- $f^{-1}(A) = (f^{-1}(A_0), f^{-1}(A_1) - f^{-1}(A_0))$

3. AND Convolution

- $f(A) = (f(A_0) + f(A_1), f(A_1))$
- $f^{-1}(A) = (f^{-1}(A_0) - f^{-1}(A_1), f^{-1}(A_1))$

6.9 FWT

```

void solve() {
    int n;
    cin >> n;

    vector<int> f(1 << n);
    vector<int> g(1 << n);
    for (int &x : f) cin >> x;
    for (int &x : g) cin >> x;

    auto OR = [&](auto f, auto g) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < (1 << n); j++)
                if (j >> i & 1) {
                    f[j] = add(f[j], f[j ^ (1 << i)]);
                    g[j] = add(g[j], g[j ^ (1 << i)]);
                }
        for (int i = 0; i < 1 << n; i++)
            f[i] = mul(f[i], g[i]);
        for (int i = 0; i < n; i++)
            for (int j = 0; j < (1 << n); j++)
                if (j >> i & 1) {
                    f[j] = add(f[j], mod - f[j ^ (1 << i)]);
                }
        for (int i = 0; i < 1 << n; i++)
            cout << f[i] << " \n"[i == (1 << n) - 1];
    };
    OR(f, g);

    auto AND = [&](auto f, auto g) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < (1 << n); j++)
                if (~j >> i & 1) {
                    f[j] = add(f[j], f[j ^ (1 << i)]);
                    g[j] = add(g[j], g[j ^ (1 << i)]);
                }
        for (int i = 0; i < 1 << n; i++)
            f[i] = mul(f[i], g[i]);
        for (int i = 0; i < n; i++)
            for (int j = 0; j < (1 << n); j++)
                if (~j >> i & 1) {

```

```

        f[j] = add(f[j], mod - f[j] ^ (1 << i));
    }
    for (int i = 0; i < 1 << n; i++)
        cout << f[i] << " \n"[i == (1 << n) - 1];
};
AND(f, g);

auto XOR = [&](auto f, auto g) {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < (1 << n); j++)
            if (j >> i & 1) {
                auto &a = f[j] ^ (1 << i);
                auto &b = f[j];
                tie(a, b) = pair(add(a, b), add(a, mod - b));
                auto &x = g[j] ^ (1 << i);
                auto &y = g[j];
                tie(x, y) = pair(add(x, y), add(x, mod - y));
            }
    for (int i = 0; i < (1 << n); i++)
        f[i] = mul(f[i], g[i]);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < (1 << n); j++)
            if (j >> i & 1) {
                auto &a = f[j] ^ (1 << i);
                auto &b = f[j];
                tie(a, b) = pair(mul(add(a, b), inv2), mul(
                    add(a, mod - b), inv2));
            }
    for (int i = 0; i < 1 << n; i++)
        cout << f[i] << " \n"[i == (1 << n) - 1];
};
XOR(f, g);
}

```

6.10 Lucas

```

// C(N, M) mod D
i64 Lucas(i64 N, i64 M, i64 D) {
    auto Factor = [&](i64 x) -> vector<pair<i64, i64>> {
        vector<pair<i64, i64>> r;
        for (i64 i = 2; x > 1; i++)
            if (x % i == 0) {
                i64 c = 0;
                while (x % i == 0) x /= i, c++;
                r.emplace_back(i, c);
            }
        return r;
    };
    auto Pow = [&](i64 a, i64 b, i64 m) -> i64 {
        i64 r = 1;
        for (; b >= 1; a = a * a % m)
            if (b & 1) r = r * a % m;
        return r;
    };
    vector<pair<i64, i64>> E;
    for (auto [p, q] : Factor(D)) {
        const i64 mod = Pow(p, q, 1 << 30);
        auto CountFact = [&](i64 x) -> i64 {
            i64 c = 0;
            while (x) c += (x /= p);
            return c;
        };
        auto CountBino = [&](i64 x, i64 y) { return
            CountFact(x) - CountFact(y) - CountFact(x - y); };
        auto Inv = [&](i64 x) -> i64 { return (exgcd(x, mod)
            ).ff % mod + mod % mod; };
        vector<i64> pre(mod + 1);
        pre[0] = pre[1] = 1;
        for (i64 i = 2; i <= mod; i++) pre[i] = (i % p == 0
            ? 1 : i) * pre[i - 1] % mod;
        function<i64(i64)> FactMod = [&](i64 n) -> i64 {
            if (n == 0) return 1;
            return FactMod(n / p) * Pow(pre[mod], n / mod,
                mod) % mod * pre[n % mod] % mod;
        };
        auto BinoMod = [&](i64 x, i64 y) -> i64 {
            return FactMod(x) * Inv(FactMod(y)) % mod * Inv(
                FactMod(x - y)) % mod;
        };
        i64 r = BinoMod(N, M) * Pow(p, CountBino(N, M), mod)
            % mod;
        E.emplace_back(r, mod);
    };
}

```

```

    return CRT(E);
}

```

6.11 Berlekamp Massey

```

template <int P>
vector<int> BerlekampMassey(vector<int> x) {
    vector<int> cur, ls;
    int lf = 0, ld = 0;
    for (int i = 0; i < (int)x.size(); ++i) {
        int t = 0;
        for (int j = 0; j < (int)cur.size(); ++j)
            (t += 1LL * cur[j] * x[i - j - 1] % P) %= P;
        if (t == x[i]) continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i, ld = (t + P - x[i]) % P;
            continue;
        }
        int k = 1LL * fpow(ld, P - 2, P) * (t + P - x[i]) % P;
        vector<int> c(i - lf - 1);
        c.push_back(k);
        for (int j = 0; j < (int)ls.size(); ++j)
            c.push_back(1LL * k * (P - ls[j]) % P);
        if (c.size() < cur.size()) c.resize(cur.size());
        for (int j = 0; j < (int)cur.size(); ++j)
            c[j] = (c[j] + cur[j]) % P;
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
            ls = cur, lf = i;
            ld = (t + P - x[i]) % P;
        }
        cur = c;
    }
    return cur;
}

```

6.12 Gauss Elimination

```

double Gauss(vector<vector<double>>& d) {
    int n = d.size(), m = d[0].size();
    double det = 1;
    for (int i = 0; i < m; ++i) {
        int p = -1;
        for (int j = i; j < n; ++j) {
            if (fabs(d[j][i]) < kEps) continue;
            if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p = j;
        }
        if (p == -1) continue;
        if (p != i) det *= -1;
        for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
        for (int j = 0; j < n; ++j) {
            if (i == j) continue;
            double z = d[j][i] / d[i][i];
            for (int k = 0; k < m; ++k) d[j][k] -= z * d[i][k];
        }
    }
    for (int i = 0; i < n; ++i) det *= d[i][i];
    return det;
}

```

6.13 Linear Equation

```

void linear_equation(vector<vector<double>>& d, vector<
    double> &aug, vector<double> &sol) {
    int n = d.size(), m = d[0].size();
    vector<int> r(n), c(m);
    iota(r.begin(), r.end(), 0);
    iota(c.begin(), c.end(), 0);
    for (int i = 0; i < m; ++i) {
        int p = -1, z = -1;
        for (int j = i; j < n; ++j) {
            for (int k = i; k < m; ++k) {
                if (fabs(d[r[j]][c[k]]) < eps) continue;
                if (p == -1 || fabs(d[r[j]][c[k]]) > fabs(d[r[p]
                    ][c[z]])) p = j, z = k;
            }
        }
        if (p == -1) continue;
        swap(r[p], r[i]), swap(c[z], c[i]);
        for (int j = 0; j < n; ++j) {
            if (i == j) continue;
            double z = d[r[j]][c[i]] / d[r[i]][c[i]];
            for (int k = 0; k < m; ++k) d[r[j]][c[k]] -= z *
                d[r[i]][c[k]];
        }
    }
}

```

```

    aug[r[j]] -= z * aug[r[i]];
}
}
vector<vector<double>> fd(n, vector<double>(m));
vector<double> faug(n), x(n);
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m; ++j) fd[i][j] = d[r[i]][c[j]];
    faug[i] = aug[r[i]];
}
d = fd, aug = faug;
for (int i = n - 1; i >= 0; --i) {
    double p = 0.0;
    for (int j = i + 1; j < n; ++j) p += d[i][j] * x[j];
    x[i] = (aug[i] - p) / d[i][i];
}
for (int i = 0; i < n; ++i) sol[c[i]] = x[i];
}

```

6.14 LinearRec

```

template <int P>
int LinearRec(const vector<int> &s, const vector<int> &
    coeff, int k) {
    int n = s.size();
    auto Combine = [&](const auto &a, const auto &b) {
        vector<int> res(n * 2 + 1);
        for (int i = 0; i <= n; ++i) {
            for (int j = 0; j <= n; ++j)
                (res[i + j] += 1LL * a[i] * b[j] % P) %= P;
        }
        for (int i = 2 * n; i > n; --i) {
            for (int j = 0; j < n; ++j)
                (res[i - 1 - j] += 1LL * res[i] * coeff[j] % P)
                    %= P;
        }
        res.resize(n + 1);
        return res;
    };
    vector<int> p(n + 1), e(n + 1);
    p[0] = e[1] = 1;
    for (; k > 0; k >= 1) {
        if (k & 1) p = Combine(p, e);
        e = Combine(e, e);
    }
    int res = 0;
    for (int i = 0; i < n; ++i) (res += 1LL * p[i + 1] *
        s[i] % P) %= P;
    return res;
}

```

6.15 SubsetConv

```

vector<int> SubsetConv(int n, const vector<int> &f,
    const vector<int> &g) {
    const int m = 1 << n;
    vector<vector<int>> a(n + 1, vector<int>(m)), b(n + 1,
        vector<int>(m));
    for (int i = 0; i < m; ++i) {
        a[__builtin_popcount(i)][i] = f[i];
        b[__builtin_popcount(i)][i] = g[i];
    }
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int s = 0; s < m; ++s) {
                if (s >> j & 1) {
                    a[i][s] += a[i][s ^ (1 << j)];
                    b[i][s] += b[i][s ^ (1 << j)];
                }
            }
        }
    }
    vector<vector<int>> c(n + 1, vector<int>(m));
    for (int s = 0; s < m; ++s) {
        for (int i = 0; i <= n; ++i) {
            for (int j = 0; j <= i; ++j) c[i][s] += a[j][s] * b[
                i - j][s];
        }
    }
    for (int i = 0; i <= n; ++i) {
        for (int j = 0; j < n; ++j) {
            for (int s = 0; s < m; ++s) {
                if (s >> j & 1) c[i][s] -= c[i][s ^ (1 << j)];
            }
        }
    }
}

```

```

}
}
}
vector<int> res(m);
for (int i = 0; i < m; ++i) res[i] = c[
    __builtin_popcount(i)][i];
return res;
}

```

6.16 SqrtMod

```

int SqrtMod(int n, int P) { // 0 <= x < P
    if (P == 2 or n == 0) return n;
    if (pow(n, (P - 1) / 2, P) != 1) return -1;
    mt19937 rng(12312);
    i64 z = 0, w;
    while (pow(w = (z * z - n + P) % P, (P - 1) / 2, P)
        != P - 1)
        z = rng() % P;
    const auto M = [P, w](auto &u, auto &v) {
        return make_pair(
            (u.ff * v.ff + u.ss * v.ss % P * w) % P,
            (u.ff * v.ss + u.ss * v.ff) % P
        );
    };
    pair<i64, i64> r(1, 0), e(z, 1);
    for (int w = (P + 1) / 2; w; w >= 1, e = M(e, e))
        if (w & 1) r = M(r, e);
    return r.ff; // sqrt(n) mod P where P is prime
}

```

6.17 FloorSum

```

// sigma 0 ~ n-1: (a * i + b) / m
i64 floor_sum(i64 n, i64 m, i64 a, i64 b) {
    u64 ans = 0;
    if (a < 0) {
        u64 a2 = (a % m + m) % m;
        ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
        a = a2;
    }
    if (b < 0) {
        u64 b2 = (b % m + m) % m;
        ans -= 1ULL * n * ((b2 - b) / m);
        b = b2;
    }
    while (true) {
        if (a >= m) {
            ans += n * (n - 1) / 2 * (a / m);
            a %= m;
        }
        if (b >= m) {
            ans += n * (b / m);
            b %= m;
        }
        u64 y_max = a * n + b;
        if (y_max < m) break;
        n = y_max / m;
        b = y_max % m;
        swap(m, a);
    }
    return ans;
}

```

6.18 LinearProgrammingSimplex

```

// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// x = simplex(A, b, c); (A <= 100 x 100)
vector<double> simplex(
    const vector<vector<double>> &a,
    const vector<double> &b,
    const vector<double> &c) {
    int n = (int)a.size(), m = (int)a[0].size() + 1;
    vector val(n + 2, vector<double>(m + 1));
    vector<int> idx(n + m);
    iota(all(idx), 0);
    int r = n, s = m - 1;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j)
            val[i][j] = -a[i][j];
    }
}

```

```

    val[i][m - 1] = 1;
    val[i][m] = b[i];
    if (val[r][m] > val[i][m])
        r = i;
}
copy(all(c), val[n].begin());
val[n + 1][m - 1] = -1;
for (double num; ; ) {
    if (r < n) {
        swap(idx[s], idx[r + m]);
        val[r][s] = 1 / val[r][s];
        for (int j = 0; j <= m; ++j) if (j != s)
            val[r][j] *= -val[r][s];
        for (int i = 0; i <= n + 1; ++i) if (i != r) {
            for (int j = 0; j <= m; ++j) if (j != s)
                val[i][j] += val[r][j] * val[i][s];
            val[i][s] *= val[r][s];
        }
    }
    r = s = -1;
    for (int j = 0; j < m; ++j)
        if (s < 0 || idx[s] > idx[j])
            if (val[n + 1][j] > eps || val[n + 1][j] > -eps
                && val[n][j] > eps)
                s = j;
    if (s < 0) break;
    for (int i = 0; i < n; ++i) if (val[i][s] < -eps) {
        if (r < 0
            || (num = val[r][m] / val[r][s] - val[i][m] /
                val[i][s]) < -eps
            || num < eps && idx[r + m] > idx[i + m])
            r = i;
    }
    if (r < 0) {
        // Solution is unbounded.
        return vector<double>{};
    }
}
if (val[n + 1][m] < -eps) {
    // No solution.
    return vector<double>{};
}
vector<double> x(m - 1);
for (int i = m; i < n + m; ++i)
    if (idx[i] < m - 1)
        x[idx[i]] = val[i - m][m];
return x;
}

```

7 Geometry

7.1 2D Point

```

using Pt = pair<double, double>;
using numbers::pi;
constexpr double eps = 1e-9;
Pt operator+(Pt a, Pt b) { return {a.ff + b.ff, a.ss + b.ss}; }
Pt operator-(Pt a, Pt b) { return {a.ff - b.ff, a.ss - b.ss}; }
Pt operator*(Pt a, double b) { return {a.ff * b, a.ss * b}; }
Pt operator/(Pt a, double b) { return {a.ff / b, a.ss / b}; }
double operator*(Pt a, Pt b) { return a.ff * b.ff + a.ss * b.ss; }
double operator^(Pt a, Pt b) { return a.ff * b.ss - a.ss * b.ff; }
double abs(Pt a) { return sqrt(a * a); }
double cro(Pt a, Pt b, Pt c) { return (b - a) ^ (c - a); }
int sig(double x) { return (x > -eps) - (x < eps); }
Pt rot(Pt u, double a) {
    Pt v{sin(a), cos(a)};
    return {u ^ v, u * v};
}
bool inedge(Pt a, Pt b, Pt c) {
    return ((a - b) ^ (c - b)) == 0 and (a - b) * (c - b)
        <= 0;
}
bool banana(Pt a, Pt b, Pt c, Pt d) {
    if (inedge(a, c, b) or inedge(a, d, b) or \
        inedge(c, a, d) or inedge(c, b, d))

```

```

        return true;
    return sig(cro(a, b, c)) * sig(cro(a, b, d)) < 0 and \
        sig(cro(c, d, a)) * sig(cro(c, d, b)) < 0;
}
Pt Inter(Pt a, Pt b, Pt c, Pt d) {
    double s = cro(c, d, a), t = -cro(c, d, b);
    return (a * t + b * s) / (s + t);
}
struct Line {
    Pt a{}, b{};
    Line() {}
    Line(Pt _a, Pt _b) : a{_a}, b{_b} {}
};
Pt Inter(Line L, Line R) {
    return Inter(L.a, L.b, R.a, R.b);
}

```

7.2 Convex Hull

```

vector<Pt> Hull(vector<Pt> P) {
    sort(all(P));
    P.erase(unique(all(P)), P.end());
    P.insert(P.end(), rall(P));
    vector<Pt> stk;
    for (auto p : P) {
        while (stk.size() >= 2 and \
            cro(*++stk.rbegin(), stk.back(), p) <= 0 and \
            (*++stk.rbegin() < stk.back()) == (stk.back() <
                p)) {
            stk.pop_back();
        }
        stk.push_back(p);
    }
    stk.pop_back();
    return stk;
}

```

7.3 Convex Hull trick

```

template<class T>
struct Convex {
    int n;
    vector<T> A, V, L, U;
    Convex(const vector<T> &_A) : A(_A), n(_A.size()) {
        // n >= 3
        auto it = max_element(all(A));
        L.assign(A.begin(), it + 1);
        U.assign(it, A.end());
        U.push_back(A[0]);
        for (int i = 0; i < n; i++) {
            V.push_back(A[(i + 1) % n] - A[i]);
        }
    }
    int inside(T p, const vector<T> &h, auto f) { // 0:
        out, 1: on, 2: in
        auto it = lower_bound(all(h), p, f);
        if (it == h.end()) return 0;
        if (it == h.begin()) return p == *it;
        return 1 - sig(cro(*prev(it), p, *it));
    }
    int inside(T p) {
        return min(inside(p, L, less{}), inside(p, U,
            greater{}));
    }
    static bool cmp(T a, T b) { return sig(a ^ b) > 0; }
    int tangent(T v, bool close = true) {
        assert(v != T{});
        auto l = V.begin(), r = V.end() - 1;
        if (v < T{}) l = r, r = V.begin();
        if (close) return (lower_bound(l, r, v, cmp) - V.
            begin()) % n;
        return (upper_bound(l, r, v, cmp) - V.begin()) % n;
    }
    array<int, 2> tangent2(T p) {
        array<int, 2> t{-1, -1};
        if (inside(p) == 2) return t;
        if (auto it = lower_bound(all(L), p); it != L.end()
            and p == *it) {
            int s = it - L.begin();
            return {(s + 1) % n, (s - 1 + n) % n};
        }
        if (auto it = lower_bound(all(U), p, greater{}); it
            != U.end() and p == *it) {
            int s = it - U.begin() + L.size() - 1;

```



```

    return {(s + 1) % n, (s - 1 + n) % n};
}
for (int i = 0; i != t[0]; i = tangent((A[t[0] = i]
- p), 0));
for (int i = 0; i != t[1]; i = tangent((p - A[t[1]
= i]), 1));
return t;
}
int Find(int l, int r, T a, T b) {
    if (r < l) r += n;
    int s = sig(cro(a, b, A[l % n]));
    while (r - l > 1) {
        (sig(cro(a, b, A[(l + r) / 2 % n])) == s ? l : r)
        = (l + r) / 2;
    }
    return l % n;
};
vector<int> LineIntersect(T a, T b) { // A_x A_x+1
    interset with ab
    assert(a != b);
    int l = tangent(a - b), r = tangent(b - a);
    if (sig(cro(a, b, A[l])) * sig(cro(a, b, A[r])) >=
    0) return {};
    return {Find(l, r, a, b), Find(r, l, a, b)};
}
};

```

7.4 Dynamic Convex Hull

```

template<class T, class Comp = less<T>>
struct DynamicHull {
    set<T, Comp> H;
    DynamicHull() {}
    void insert(T p) {
        if (inside(p)) return;
        auto it = H.insert(p).ff;
        while (it != H.begin() and prev(it) != H.begin() \
        and cross(*prev(it), 2), *prev(it), *it) <= 0) {
            it = H.erase(--it);
        }
        while (it != --H.end() and next(it) != --H.end() \
        and cross(*it, *next(it), *next(it), 2) <= 0) {
            it = --H.erase(++it);
        }
    }
    int inside(T p) { // 0: out, 1: on, 2: in
        auto it = H.lower_bound(p);
        if (it == H.end()) return 0;
        if (it == H.begin()) return p == *it;
        return 1 - sig(cross(*prev(it), p, *it));
    }
};

```

7.5 Half Plane Intersection

```

vector<Pt> HPI(vector<Line> P) {
    const int n = P.size();
    sort(all(P), [&](Line L, Line R) -> bool {
        Pt u = L.b - L.a, v = R.b - R.a;
        bool f = Pt(sig(u.ff), sig(u.ss)) < Pt{};
        bool g = Pt(sig(v.ff), sig(v.ss)) < Pt{};
        if (f != g) return f < g;
        return (sig(u ^ v) ? sig(u ^ v) : sig(cro(L.a, R.a,
        R.b))) > 0;
    });
    auto Same = [&](Line L, Line R) {
        Pt u = L.b - L.a, v = R.b - R.a;
        return sig(u ^ v) == 0 and sig(u * v) == 1;
    };
    deque<Pt> inter;
    deque<Line> seg;
    for (int i = 0; i < n; i++) if (i == 0 or !Same(P[i -
    1], P[i])) {
        while (seg.size() >= 2 and sig(cro(inter.back(), P[
        i].b, P[i].a)) == 1) {
            seg.pop_back(), inter.pop_back();
        }
        while (seg.size() >= 2 and sig(cro(inter[0], P[i].b
        , P[i].a)) == 1) {
            seg.pop_front(), inter.pop_front();
        }
        if (!seg.empty()) inter.push_back(Inter(seg.back(),
        P[i]));
        seg.push_back(P[i]);
    }
}

```

```

}
while (seg.size() >= 2 and sig(cro(inter.back(), seg
[0].b, seg[0].a)) == 1) {
    seg.pop_back(), inter.pop_back();
}
inter.push_back(Inter(seg[0], seg.back()));
return vector<Pt>(all(inter));
}

```

7.6 Minimal Enclosing Circle

```

using circle = pair<Pt, double>;
struct MES {
    MES() {}
    bool inside(const circle &c, Pt p) {
        return abs(p - c.ff) <= c.ss + eps;
    };
    circle get_cir(Pt a, Pt b) {
        return circle((a + b) / 2., abs(a - b) / 2.);
    }
    circle get_cir(Pt a, Pt b, Pt c) {
        Pt p = (b - a) / 2.;
        p = Pt(-p.ss, p.ff);
        double t = ((c - a) * (c - b)) / (2 * (p * (c - a))
        );
        p = ((a + b) / 2.) + (p * t);
        return circle(p, abs(p - a));
    }
    circle get_mes(vector<Pt> P) {
        if (P.empty()) return circle{Pt(0, 0), 0};
        mt19937 rng(random_device{}());
        shuffle(all(P), rng);
        circle C{P[0], 0};
        for (int i = 1; i < P.size(); i++) {
            if (inside(C, P[i])) continue;
            C = get_cir(P[i], P[0]);
            for (int j = 1; j < i; j++) {
                if (inside(C, P[j])) continue;
                C = get_cir(P[i], P[j]);
                for (int k = 0; k < j; k++) {
                    if (inside(C, P[k])) continue;
                    C = get_cir(P[i], P[j], P[k]);
                }
            }
        }
        return C;
    }
};

```

7.7 Minkowski

```

vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) { // P
, Q need sort
    const int n = P.size(), m = Q.size();
    P.push_back(P[0]), P.push_back(P[1]);
    Q.push_back(Q[0]), Q.push_back(Q[1]);
    vector<Pt> R;
    for (int i = 0, j = 0; i < n or j < m; ) {
        R.push_back(P[i] + Q[j]);
        auto v = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
        if (v >= 0) i++;
        if (v <= 0) j++;
    }
    return R;
}

```

7.8 TriangleCenter

```

Pt TriangleCircumCenter(Pt a, Pt b, Pt c) {
    Pt res;
    double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
    double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
    double ax = (a.x + b.x) / 2;
    double ay = (a.y + b.y) / 2;
    double bx = (c.x + b.x) / 2;
    double by = (c.y + b.y) / 2;
    double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay)
    ) / (sin(a1) * cos(a2) - sin(a2) * cos(a1));
    return Pt(ax + r1 * cos(a1), ay + r1 * sin(a1));
}
Pt TriangleMassCenter(Pt a, Pt b, Pt c) {
    return (a + b + c) / 3.0;
}

```



```
Pt TriangleOrthoCenter(Pt a, Pt b, Pt c) {
    return TriangleMassCenter(a, b, c) * 3.0 -
        TriangleCircumCenter(a, b, c) * 2.0;
}

Pt TriangleInnerCenter(Pt a, Pt b, Pt c) {
    Pt res;
    double la = abs(b - c);
    double lb = abs(a - c);
    double lc = abs(a - b);
    res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb + lc);
    res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb + lc);
    return res;
}
```

8 Stringology

8.1 KMP

```
vector<int> build_fail(string s) {
    const int len = s.size();
    vector<int> f(len, -1);
    for (int i = 1, p = -1; i < len; i++) {
        while (~p and s[p + 1] != s[i]) p = f[p];
        if (s[p + 1] == s[i]) p++;
        f[i] = p;
    }
    return f;
}
```

8.2 Z-algorithm

```
vector<int> zalgo(string s) {
    if (s.empty()) return {};
    int len = s.size();
    vector<int> z(len);
    z[0] = len;
    for (int i = 1, l = 1, r = 1; i < len; i++) {
        z[i] = i < r ? min(z[i - l], r - i) : 0;
        while (i + z[i] < len and s[i + z[i]] == s[z[i]]) z[i]++;
        if (i + z[i] > r) l = i, r = i + z[i];
    }
    return z;
}
```

8.3 Manacher

```
vector<int> manacher(const string &s) {
    string p = "@#";
    for (char c : s) p += c + '#';
    p += '$';
    vector<int> dp(p.size());
    int mid = 0, r = 1;
    for (int i = 1; i < p.size() - 1; i++) {
        auto &k = dp[i];
        k = i < mid + r ? min(dp[mid * 2 - i], mid + r - i) : 0;
        while (p[i + k + 1] == p[i - k - 1]) k++;
        if (i + k > mid + r) mid = i, r = k;
    }
    return vector<int>(dp.begin() + 2, dp.end() - 2);
}
```

8.4 SuffixArray

```
namespace sfx {
#define fup(a, b) for (int i = a; i < b; i++)
#define fdn(a, b) for (int i = b - 1; i >= a; i--)
constexpr int N = 5e5 + 5;
bool _t[N * 2];
int H[N], RA[N], x[N], _p[N];
int SA[N * 2], _s[N * 2], _c[N * 2], _q[N * 2];
void pre(int *sa, int *c, int n, int z) {
    fill_n(sa, n, 0), copy_n(c, z, x);
}
void induce(int *sa, int *c, int *s, bool *t, int n, int z) {
    copy_n(c, z - 1, x + 1);
    fup(0, n) if (sa[i] and !t[sa[i] - 1])
        sa[x[sa[i] - 1]++] = sa[i] - 1;
    copy_n(c, z, x);
}
```

```
fdn(0, n) if (sa[i] and t[sa[i] - 1])
    sa[--x[sa[i] - 1]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
    fill_n(c, z, 0);
    fup(0, n) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) { fup(0, n) sa[--c[s[i]]] = i; return; }
    fdn(0, n - 1)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    pre(sa, c, n, z);
    fup(1, n) if (t[i] and !t[i - 1])
        sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    fup(0, n) if (sa[i] and t[sa[i]] and !t[sa[i] - 1]) {
        bool neq = last < 0 or !equal(s + sa[i], s + p[q[sa[i]] + 1], s + last);
        ns[q[last = sa[i]]] = nmzx += neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    pre(sa, c, n, z);
    fdn(0, nn) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
vector<int> build(vector<int> s, int n) {
    copy_n(begin(s), n, _s);
    _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    vector<int> sa(n);
    fup(0, n) sa[i] = SA[i + 1];
    return sa;
}
vector<int> lcp_array(vector<int> &s, vector<int> &sa) {
    int n = s.size();
    vector<int> rnk(n);
    fup(0, n) rnk[sa[i]] = i;
    vector<int> lcp(n - 1);
    int h = 0;
    fup(0, n) {
        if (h > 0) h--;
        if (rnk[i] == 0) continue;
        int j = sa[rnk[i] - 1];
        for (; j + h < n and i + h < n; h++)
            if (s[j + h] != s[i + h]) break;
        lcp[rnk[i] - 1] = h;
    }
    return lcp;
}
```

8.5 SimpleSuffixArray

```
struct SuffixArray {
    int n;
    vector<int> suf, rk, S;
    SuffixArray(vector<int> _S) : S(_S) {
        n = S.size();
        suf.assign(n, 0);
        rk.assign(n * 2, -1);
        iota(all(suf), 0);
        for (int i = 0; i < n; i++) rk[i] = S[i];
        for (int k = 2; k < n + n; k *= 2) {
            auto cmp = [&](int a, int b) -> bool {
                return rk[a] == rk[b] ? (rk[a + k / 2] < rk[b + k / 2]) : (rk[a] < rk[b]);
            };
            sort(all(suf), cmp);
            auto tmp = rk;
            tmp[suf[0]] = 0;
            for (int i = 1; i < n; i++) {
                tmp[suf[i]] = tmp[suf[i - 1]] + cmp(suf[i - 1], suf[i]);
            }
            rk.swap(tmp);
        }
    }
}
```

```
};
```

8.6 PalindromicTree

```
struct PAM {
    struct Node {
        int fail, len, dep;
        array<int, 26> ch;
        Node(int _len) : len{_len}, fail{}, ch{}, dep{} {};
    };
    vector<Node> g;
    vector<int> id;
    int odd, even, lst;
    string S;
    int new_node(int len) {
        g.emplace_back(len);
        return g.size() - 1;
    }
    PAM() : odd(new_node(-1)), even(new_node(0)) {
        lst = g[even].fail = odd;
    }
    int up(int p) {
        while (S.rbegin()[g[p].len + 1] != S.back())
            p = g[p].fail;
        return p;
    }
    int add(char c) {
        S += c;
        lst = up(lst);
        c -= 'a';
        if (!g[lst].ch[c]) g[lst].ch[c] = new_node(g[lst].len + 2);
        int p = g[lst].ch[c];
        g[p].fail = (lst == odd ? even : g[up(g[lst].fail)].ch[c]);
        lst = p;
        g[lst].dep = g[g[lst].fail].dep + 1;
        id.push_back(lst);
        return lst;
    }
    void del() {
        S.pop_back();
        id.pop_back();
        lst = id.empty() ? odd : id.back();
    }
};
```

8.7 SmallestRotation

```
string Rotate(const string &s) {
    int n = s.length();
    string t = s + s;
    int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}
```

8.8 Aho-Corasick

```
struct ACAuto {
    static const int sigma = 26;
    struct Node {
        array<Node*, sigma> ch{};
        Node *fail = nullptr;
        int cnt = 0;
        vector<int> id;
    };
    *root;
    ACAuto() : root(new Node()) {}
    void insert(const string &s, int id) {
        auto p = root;
        for (char c : s) {
            int d = c - 'a';
            if (!p->ch[d]) p->ch[d] = new Node();
            p = p->ch[d];
        }
        p->id.emplace_back(id);
    }
};
```

```
vector<Node*> ord;
void build() {
    root->fail = root;
    queue<Node*> que;
    for (int i = 0; i < sigma; i++) {
        if (root->ch[i]) {
            root->ch[i]->fail = root;
            que.emplace(root->ch[i]);
        }
        else {
            root->ch[i] = root;
        }
    }
    while (!que.empty()) {
        auto p = que.front(); que.pop();
        ord.emplace_back(p);
        for (int i = 0; i < sigma; i++) {
            if (p->ch[i]) {
                p->ch[i]->fail = p->fail->ch[i];
                que.emplace(p->ch[i]);
            }
            else {
                p->ch[i] = p->fail->ch[i];
            }
        }
    }
}
void walk(const string &s) {
    auto p = root;
    for (const char &c : s) {
        int d = c - 'a';
        (p = p->ch[d])->cnt++;
    }
}
void count(vector<int> &cnt) {
    reverse(all(ord));
    for (auto p : ord) {
        p->fail->cnt += p->cnt;
        for (int id : p->id)
            cnt[id] = p->cnt;
    }
}
```

9 Misc

9.1 Fraction Binary Search

```
// Binary search on Stern-Brocot Tree
// Parameters:
// n: Q_n is the set of all rational numbers whose
// denominator does not exceed n
// pred: pair<i64, i64> -> bool
// Return value:
// x: {p, q} p/q is smaller value in Q_n that satisfy
// pred()
// Complexity: O(log^2 n)
using Pt = pair<i64, i64>;
Pt operator+(Pt a, Pt b) { return {a.ff + b.ff, a.ss + b.ss}; }
Pt operator*(i64 a, Pt b) { return {a * b.ff, a * b.ss}; }
```

```
pair<i64, i64> FractionSearch(i64 n, const auto &pred) {
    pair<i64, i64> low{0, 1}, hei{1, 0};
    while (true) {
        bool cur = pred(low + hei);
        auto &fr{cur ? low : hei}, &to{cur ? hei : low};
        u64 L = 1, R = 2;
        while (pred(fr + R * to) == cur) {
            debug(L, R);
            L *= 2;
            R *= 2;
            if ((fr + L * to).ss > n) return to;
        }
        while (L + 1 < R) {
            u64 M = (L + R) / 2;
            (pred(fr + M * to) == cur ? L : R) = M;
        }
        fr = fr + L * to;
    }
}
```

9.2 de Bruijn sequence

```
constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L;
    int buf[MAXC * MAXN];
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) { //
        alphabet, len, k
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;
```

9.3 HilbertCurve

```
long long hilbert(int n, int x, int y) {
    long long res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 111 * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
}
```

9.4 DLX

```
namespace dlx {
    int lt[maxn], rg[maxn], up[maxn], dn[maxn], cl[maxn],
        rw[maxn], bt[maxn], s[maxn], head, sz, ans;
    void init(int c) {
        for (int i = 0; i < c; ++i) {
            up[i] = dn[i] = bt[i] = i;
            lt[i] = i == 0 ? c : i - 1;
            rg[i] = i == c - 1 ? c : i + 1;
            s[i] = 0;
        }
        rg[c] = 0, lt[c] = c - 1;
        up[c] = dn[c] = -1;
        head = c, sz = c + 1;
    }
    void insert(int r, const vector<int> &col) {
        if (col.empty()) return;
        int f = sz;
        for (int i = 0; i < (int)col.size(); ++i) {
            int c = col[i], v = sz++;
            dn[bt[c]] = v;
            up[v] = bt[c], bt[c] = v;
            rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
            rw[v] = r, cl[v] = c;
            ++s[c];
            if (i > 0) lt[v] = v - 1;
        }
        lt[f] = sz - 1;
    }
    void remove(int c) {
        lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        for (int i = dn[c]; i != c; i = dn[i]) {
            for (int j = rg[i]; j != i; j = rg[j])
                up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[j]];
        }
    }
    void restore(int c) {
        for (int i = up[c]; i != c; i = up[i]) {
            for (int j = lt[i]; j != i; j = lt[j])
                ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
        }
    }
}
```

```
}
lt[rg[c]] = c, rg[lt[c]] = c;
}
// Call dlx::make after inserting all rows.
void make(int c) {
    for (int i = 0; i < c; ++i)
        dn[bt[i]] = i, up[i] = bt[i];
}
void dfs(int dep) {
    if (dep >= ans) return;
    if (rg[head] == head) return ans = dep, void();
    if (dn[rg[head]] == rg[head]) return;
    int c = rg[head];
    int w = c;
    for (int x = c; x != head; x = rg[x]) if (s[x] < s[w])
        w = x;
    remove(w);
    for (int i = dn[w]; i != w; i = dn[i]) {
        for (int j = rg[i]; j != i; j = rg[j]) remove(cl[j]);
        dfs(dep + 1);
        for (int j = lt[i]; j != i; j = lt[j]) restore(cl[j]);
    }
    restore(w);
}
int solve() {
    ans = 1e9, dfs(0);
    return ans;
}
```

9.5 NextPerm

```
i64 next_perm(i64 x) {
    i64 y = x | (x - 1);
    return (y + 1) | (((~y & ~y) - 1) >> (__builtin_ctz(
        x) + 1));
}
```

9.6 FastIO

```
struct FastIO {
    const static int ibufsiz = 4<<20, obufsiz = 18<<20;
    char ibuf[ibufsiz], *ipos = ibuf, obuf[obufsiz], *
        opos = obuf;
    FastIO() { fread(ibuf, 1, ibufsiz, stdin); }
    ~FastIO() { fwrite(obuf, 1, opos - obuf, stdout); }
    template<class T> FastIO& operator>>(T &x) {
        bool sign = 0; while (!isdigit(*ipos)) { if (*ipos
            == '-') sign = 1; ++ipos; }
        x = *ipos++ & 15;
        while (isdigit(*ipos)) x = x * 10 + (*ipos++ & 15);
        if (sign) x = -x;
        return *this;
    }
    template<class T> FastIO& operator<<(T n) {
        static char _buf[18];
        char* _pos = _buf;
        if (n < 0) *opos++ = '-', n = -n;
        do *_pos++ = '0' + n % 10; while (n /= 10);
        while (_pos != _buf) *opos++ = *--_pos;
        return *this;
    }
    FastIO& operator<<(char ch) { *opos++ = ch; return *
        this; }
} FIO;
#define cin FIO
#define cout FIO
```

9.7 Python FastIO

```
import sys
sys.stdin.readline()
sys.stdout.write()
```

9.8 Trick

```
dp[61][0][0][0][7] = 1;
for (int h = 60; h >= 0; h--) {
    int s = (n >> h & 1) * 7;
    for (int x = 0; x < 8; x++) if (__builtin_parity(x)
        == 0) {
        for (int y = 0; y < 8; y++)
            if (((y & ~s) & x) == 0) {
                for (int a = 0; a < A[0]; a++)
                    for (int b = 0; b < A[1]; b++)
```

```

        for (int c = 0; c < A[2]; c++) {
            if (dp[h + 1][a][b][c][y] == 0) continue;
            i64 i = ((x >> 2 & 1LL) << h) % A[0];
            i64 j = ((x >> 1 & 1LL) << h) % A[1];
            i64 k = ((x >> 0 & 1LL) << h) % A[2];
            auto &val =
                dp[h][[(i + a) % A[0]][(j + b) % A[1]][(k
+ c) % A[2]][y & ~(s ^ x)]];
            val = add(val, dp[h + 1][a][b][c][y]);
        }
    }
}
pair<i64, i64> Split(i64 x) {
    if (x == 1) return {0, 0};
    i64 h = __lg(x);
    i64 fill = (1LL << (h + 1)) - 1;
    i64 l = (1LL << h) - 1 - max(0LL, fill - x - (1LL <<
(h - 1)));
    i64 r = x - 1 - l;
    return {l, r};
};
~
auto [ls, l] = DP(lo);
auto [rs, r] = DP(hi);
if (r < K) {
    cout << "Impossible\n";
    return;
}
if (l == K) cout << ls << '\n';
else if (r == K) cout << rs << '\n';
else {
    cout << (ls * (r - K) + rs * (K - l)) / (r - l) <<
'\n';
}
}
~
auto F = [&](int L, int R) -> i64 {
    static vector<int> cnt(n);
    static int l = 0, r = -1;
    static i64 ans = 0;

    auto Add = [&](int x) {
        ans += cnt[A[x]]++;
    };
    auto Del = [&](int x) {
        ans -= --cnt[A[x]];
    };

    while (r < R) Add(++r);
    while (L < l) Add(--l);
    while (R < r) Del(r--);
    while (l < L) Del(l++);

    return ans;
};

vector<i64> dp(n), tmp(n);
function<void(int, int, int, int)> sol = [&](int l,
int r, int x, int y) {
    if (l > r) return;
    int mid = (l + r) / 2;
    int z = mid;
    for (int i = min(y, mid - 1); i >= x; i--)
        if (chmin(tmp[mid], dp[i] + F(i + 1, mid))) {
            z = i;
        }
    if (l == r) return;
    sol(l, mid - 1, x, z);
    sol(mid + 1, r, z, y);
};

for (int i = 0; i < n; i++)
    dp[i] = F(0, i);

for (int i = 2; i <= m; i++) {
    tmp.assign(n, inf<i64>);
    sol(0, n - 1, 0, n - 1);
    dp = tmp;
}

cout << dp[n - 1] << '\n';

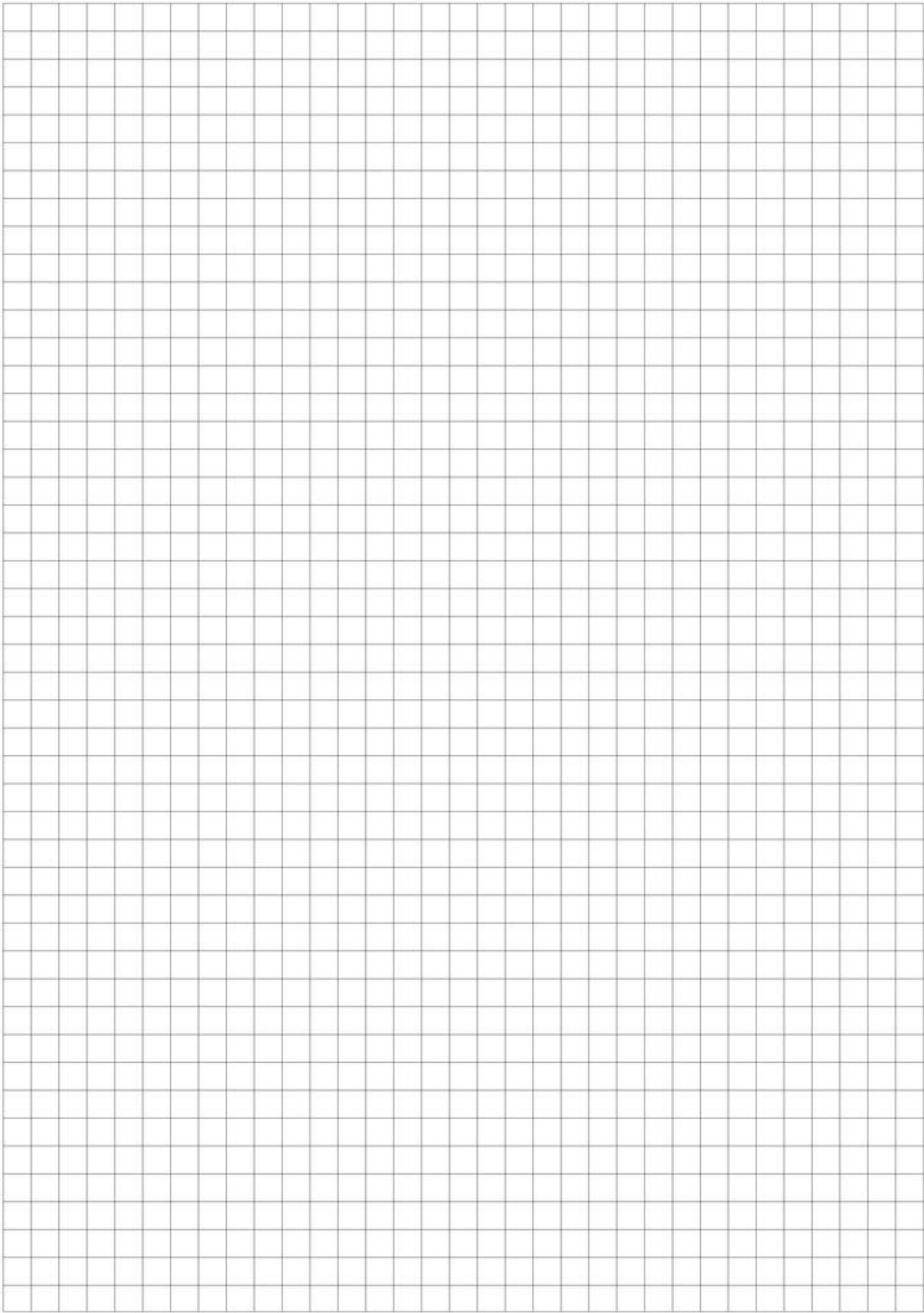
```

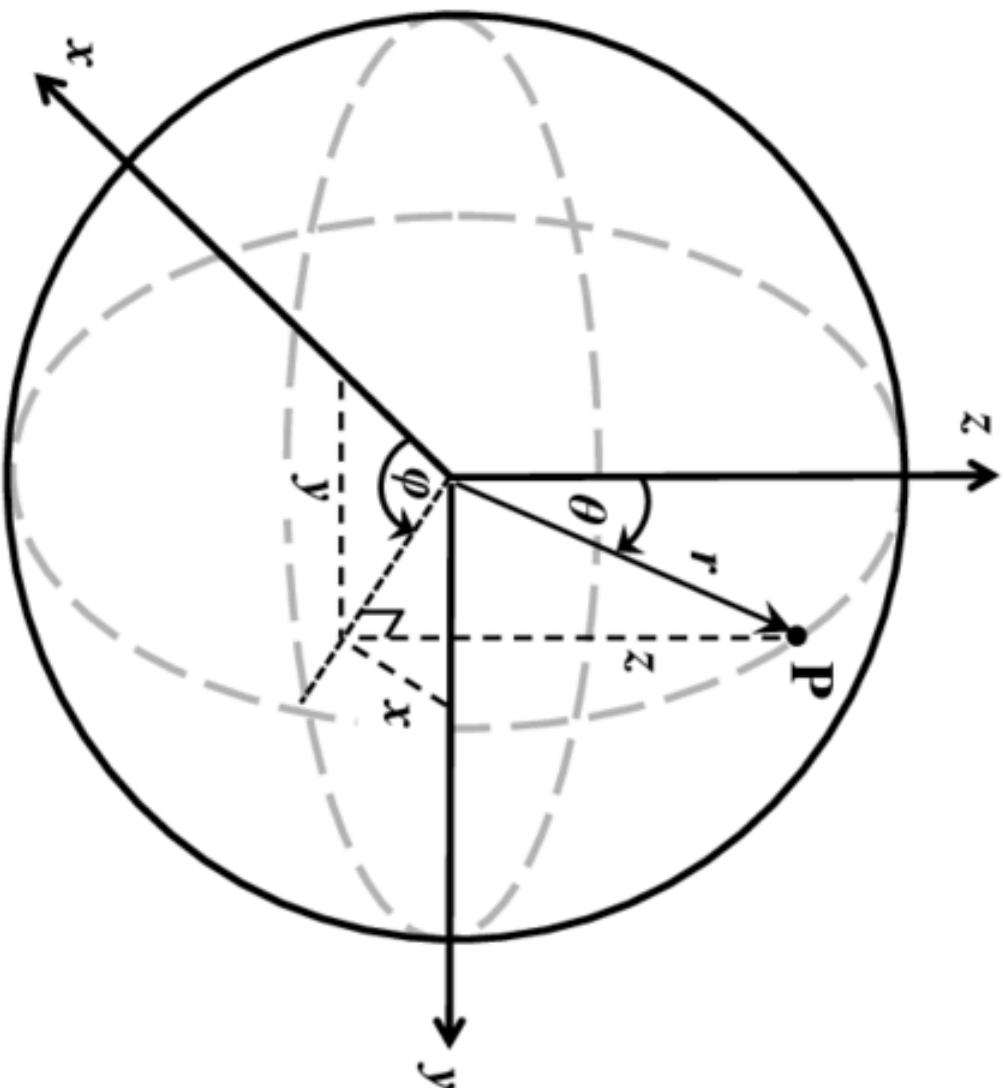
```

}
}

9.9 PyTrick
from itertools import permutations
op = ['+', '-', '*', '/']
a, b, c, d = input().split()
ans = set()
for (x,y,z,w) in permutations([a, b, c, d]):
    for op1 in op:
        for op2 in op:
            for op3 in op:
                val = eval(f"{x}{op1}{y}{op2}{z}{op3}{w}")
                if (op1 == '/' and op2 == '/' and op3 == '/') or
                    val < 0:
                    continue
                ans.add(val)
print(len(ans))
#
from decimal import *
from fractions import *
s = input()
n = int(input())
f = Fraction(s)
g = Fraction(s).limit_denominator(n)
h = f * 2 - g
if h.numerator <= n and h.denominator <= n and h < g:
    g = h
print(g.numerator, g.denominator)

```





$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}(z/r)$$

$$\varphi = \tan^{-1}(y/x)$$