Contents

1 Basic

1.1 vimrc

```
set ts=4 sw=4 nu rnu et cin hls mouse=a
color default
sy on
inoremap {<CR> {<CR>}<C-o>O
inoremap jk <Esc>
nnoremap J 5j
nnoremap K 5k
nnoremap <F8> :w<bar>!g++ -std=c++20 -DLOCAL -Wfatal-
    errors -Wshadow -O2 -g -fsanitize=address,undefined
     -o run "%" && echo "done." && time ./run<CR>
```

1.2 default

```
#include <bits/stdc++.h>
using namespace std;
#ifdef LOCAL
template<class ...T> void dbg(T ...x) { char e{}; ((
    cerr << e << x, e = ' '), ...); }
template<class T> void org(T l, T r) { while (l != r)
    cerr << ' ' << *l++; cerr << '\n'; }
#define debug(x...) dbg("(", #x, ") =", x, '\n')
#define orange(x...) dbg("[", #x, ") ="), org(x)
#else
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#define debug(...) ((void)0)
#define orange(...) ((void)0)
#endif
#define ff first
#define ss second
#define all(v) (v).begin(), (v).end()
#define rall(v) (v).rbegin(), (v).rend()
template<class T> bool chmin(T &a, T b) { return b < a
    and (a = b, true); }
template<class T> bool chmax(T &a, T b) { return a < b
    and (a = b, true); }
template<class ...T> int add(T ...x) { int t{}; return
    (((t += x) %= mod), ...), t; }
template<class ...T> int mul(T ...x) { i64 t{1}; return
     (((t *= x) %= mod), ...), t; }
```

1.3 judge

```
set -e
g++ -O3 a.cpp -o a
g++ -O3 ac.cpp -o c
g++ -O3 gen.cpp -o g

for ((i=0;;i++))
do
  echo "case $i"
  ./g > inp
  time ./a < inp > wa.out
  time ./c < inp > ac.out
  diff ac.out wa.out || break
done
```

1.4 Random

```
mt19937 rng(random_device{}());
i64 rand(i64 l = -lim, i64 r = lim) {
  return uniform_int_distribution<i64>(l, r)(rng);
}
double randr(double l, double r) {
  return uniform_real_distribution<double>(l, r)(rng);
}
```

2 Matching and Flow

2.1 Dinic

```
template<class Cap>
struct Dinic {
  struct Edge { int v; Cap w; int rev; };
  vector<vector<Edge>> G;
  int n, S, T;
  Dinic(int _n, int _S, int _T) : n(_n), S(_S), T(_T),
    G(_n) {}
  void add_edge(int u, int v, Cap w) {
    G[u].push_back({v, w, (int)G[v].size()});
    G[v].push_back({u, 0, (int)G[u].size() - 1});
  }
  vector<int> dep;
  bool bfs() {
    dep.assign(n, 0);
    dep[S] = 1;
    queue<int> que;
    que.push(S);
    while (!que.empty()) {
      int u = que.front(); que.pop();
      for (auto [v, w, _] : G[u])
        if (!dep[v] and w) {
          dep[v] = dep[u] + 1;
          que.push(v);
        }
    }
```

```
    return dep[T] != 0;
  }
  Cap dfs(int u, Cap in) {
    if (u == T) return in;
    Cap out = 0;
    for (auto &[v, w, rev] : G[u]) {
      if (w and dep[v] == dep[u] + 1) {
        Cap f = dfs(v, min(w, in));
        w -= f, G[v][rev].w += f;
        in -= f, out += f;
        if (!in) break;
      }
    }
    if (in) dep[u] = 0;
    return out;
  }
  Cap maxflow() {
    Cap ret = 0;
    while (bfs()) {
      ret += dfs(S, INF);
    }
    return ret;
  }
};
```

2.2 zkwDinic

```
template<class Cap>
struct zkwDinic {
  struct Edge { int v; Cap w, f; int rev; };
  vector<vector<Edge>> G;
  int n, S, T;
  zkwDinic(int _n, int _S, int _T) : n(_n), S(_S), T(_T
    ), G(_n) {}
  void add_edge(int u, int v, Cap w, Cap f) {
    G[u].push_back({v, w, f, (int)G[v].size()});
    G[v].push_back({u, -w, 0, (int)G[u].size() - 1});
  }
  vector<Cap> dis;
  vector<bool> vis;
  bool spfa() {
    queue<int> que;
    dis.assign(n, INF);
    vis.assign(n, false);
    que.push(S);
    vis[S] = 1;
    dis[S] = 0;
    while (!que.empty()) {
      int u = que.front(); que.pop();
      vis[u] = 0;
      for (auto [v, w, f, _] : G[u])
        if (f and chmin(dis[v], dis[u] + w))
          if (!vis[v]) que.push(v), vis[v] = 1;
    }
    return dis[T] != INF;
  }
  Cap dfs(int u, Cap in) {
    if (u == T) return in;
    vis[u] = 1;
    Cap out = 0;
    for (auto &[v, w, f, rev] : G[u])
      if (f and !vis[v] and dis[v] == dis[u] + w) {
        Cap x = dfs(v, min(in, f));
        in -= x, out += x;
        f -= x, G[v][rev].f += x;
        if (!in) break;
      }
    if (in) dis[u] = INF;
    vis[u] = 0;
    return out;
  }
  pair<Cap, Cap> maxflow() {
    Cap a = 0, b = 0;
    while (spfa()) {
      Cap x = dfs(S, INF);
      a += x;
      b += x * dis[T];
    }
    return {a, b};
  }
};
```

2.3 HopcroftKarp

```
struct HopcroftKarp {
  std::vector<int> g, l, r;
  int ans;
  HopcroftKarp(int n, int m, const std::vector<pair<int
    , int>> &e)
    : g(e.size()), l(n, -1), r(m, -1), ans(0) {
    vector<int> deg(n + 1);
    for (auto [x, y] : e) deg[x]++;
    partial_sum(all(deg), deg.begin());
    for (auto [x, y] : e) g[--deg[x]] = y;
    vector<int> que(n);
    for (;;) {
      vector<int> a(n, -1), p(n, -1);
      int t = 0;
      for (int i = 0; i < n; i++) if (l[i] == -1)
        que[t++] = a[i] = p[i] = i;
      bool match = false;
      for (int i = 0; i < t; i++) {
        int x = que[i];
        if (~l[a[x]]) continue;
        for (int j = deg[x]; j < deg[x + 1]; j++) {
          int y = g[j];
          if (r[y] == -1) {
            while (~y) r[y] = x, swap(l[x], y), x = p[x
    ];
            match = true, ans++;
            break;
          }
          if (p[r[y]] == -1)
            que[t++] = y = r[y], p[y] = x, a[y] = a[x];
        }
      }
      if (!match) break;
    }
  }
};
```

2.4 KM

```
i64 KM(vector<vector<int>> W) {
  const int n = W.size();
  vector<int> fl(n, -1), fr(n, -1), hr(n), hl(n);
  for (int i = 0; i < n; ++i) {
    hl[i] = *max_element(W[i].begin(), W[i].end());
  }
  auto Bfs = [&](int s) {
    vector<int> slk(n, INF), pre(n);
    vector<bool> vl(n, false), vr(n, false);
    queue<int> que;
    que.push(s);
    vr[s] = true;
    auto Check = [&](int x) -> bool {
      if (vl[x] = true, fl[x] != -1) {
        que.push(fl[x]);
        return vr[fl[x]] = true;
      }
      while (x != -1) swap(x, fr[fl[x] = pre[x]]);
      return false;
    };
    while (true) {
      while (!que.empty()) {
        int y = que.front(); que.pop();
        for (int x = 0, d = 0; x < n; ++x) {
          if (!vl[x] and slk[x] >= (d = hl[x] + hr[y] -
     W[x][y])) {
            if (pre[x] = y, d) slk[x] = d;
            else if (!Check(x)) return;
          }
        }
      }
      int d = INF;
      for (int x = 0; x < n; ++x) {
        if (!vl[x] and d > slk[x]) d = slk[x];
      }
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x) {
        if (!vl[x] and !slk[x] and !Check(x)) return;
      }
    }
```

```
  };
  for (int i = 0; i < n; ++i) Bfs(i);
  i64 res = 0;
  for (int i = 0; i < n; ++i) res += W[i][fl[i]];
  return res;
}
```

2.5 SW

```
int w[kN][kN], g[kN], del[kN], v[kN];
void AddEdge(int x, int y, int c) {
  w[x][y] += c;
  w[y][x] += c;
}
pair<int, int> Phase(int n) {
  fill(v, v + n, 0), fill(g, g + n, 0);
  int s = -1, t = -1;
  while (true) {
    int c = -1;
    for (int i = 0; i < n; ++i) {
      if (del[i] || v[i]) continue;
      if (c == -1 || g[i] > g[c]) c = i;
    }
    if (c == -1) break;
    v[c] = 1, s = t, t = c;
    for (int i = 0; i < n; ++i) {
      if (del[i] || v[i]) continue;
      g[i] += w[c][i];
    }
  }
  return make_pair(s, t);
}
int GlobalMinCut(int n) {
  int cut = kInf;
  fill(del, 0, sizeof(del));
  for (int i = 0; i < n - 1; ++i) {
    int s, t; tie(s, t) = Phase(n);
    del[t] = 1, cut = min(cut, g[t]);
    for (int j = 0; j < n; ++j) {
      w[s][j] += w[t][j];
      w[j][s] += w[j][t];
    }
  }
  return cut;
}
```

2.6 GeneralMatching

```
struct GeneralMatching {
  const int BLOCK = 10;
  int n;
  vector<vector<int> > g;
  vector<int> hit, mat;
  std::priority_queue<pair<i64, int>, vector<pair<i64,
    int>>, greater<pair<i64, int>>> unmat;
  GeneralMatching(int _n) : n(_n), g(_n), mat(n, -1),
    hit(n) {}
  void add_edge(int a, int b) { // 0 <= a != b < n
    g[a].push_back(b);
    g[b].push_back(a);
  }
  int get_match() {
    for (int i = 0; i < n; i++) if (!g[i].empty()) {
      unmat.emplace(0, i);
    }
    // If WA, increase this
    // there are some cases that need >=1.3*n^2 steps
    for BLOCK=1
    // no idea what the actual bound needed here is.
    const int MAX_STEPS = 10 + 2 * n + n * n / BLOCK /
    2;
    mt19937 rng(random_device{}());
    for (int i = 0; i < MAX_STEPS; ++i) {
      if (unmat.empty()) break;
      int u = unmat.top().second;
      unmat.pop();
      if (mat[u] != -1) continue;
      for (int j = 0; j < BLOCK; j++) {
        ++hit[u];
        auto &e = g[u];
        const int v = e[rng() % e.size()];
        mat[u] = v;
        swap(u, mat[v]);
        if (u == -1) break;
      }
      if (u != -1) {
        mat[u] = -1;
        unmat.emplace(hit[u] * 100ULL / (g[u].size() +
    1), u);
      }
    }
    int siz = 0;
    for (auto e : mat) siz += (e != -1);
    return siz / 2;
  }
};
```

3 Graph

3.1 2-SAT

```
struct TwoSAT {
  vector<vector<int>> G;
  int n;
  TwoSAT(int _n) : n(_n), G(_n * 2) {}
  int ne(int x) { return x < n ? x + n : x - n; }
  void add_edge(int u, int v) { // u or v
    G[ne(u)].push_back(v);
    G[ne(v)].push_back(u);
  }
  vector<int> solve() {
    vector<int> ans(n * 2, -1), id(n * 2), stk, \
      low(n * 2), dfn(n * 2), vis(n * 2);
    int _t = 0, scc_cnt = 0;
    function<void(int)> dfs = [&](int u) {
      dfn[u] = low[u] = _t++;
      stk.push_back(u);
      vis[u] = 1;
      for (int v : G[u]) {
        if (!vis[v])
          dfs(v), chmin(low[u], low[v]);
        else if (vis[v] == 1)
          chmin(low[u], dfn[v]);
      }
      if (dfn[u] == low[u]) {
        for (int x = -1; x != u; ) {
          x = stk.back(); stk.pop_back();
          vis[x] = 2, id[x] = scc_cnt;
          if (ans[x] == -1) {
            ans[x] = 1;
            ans[ne(x)] = 0;
          }
        }
        scc_cnt++;
      }
    };
    for (int i = 0; i < n + n; i++)
      if (!vis[i]) dfs(i);
    for (int i = 0; i < n; i++)
      if (id[i] == id[ne(i)])
        return {};
    ans.resize(n);
    return ans;
  }
};
```

3.2 ManhattanMST

```
vector<tuple<int, int, int>> ManhattanMST(vector<Pt> P)
    {
  vector<int> id(P.size());
  iota(all(id), 0);
  vector<tuple<int, int, int>> edges;
  for (int k = 0; k < 4; ++k) {
    sort(all(id), [&](int i, int j) -> bool {
      return (P[i] - P[j]).ff < (P[j] - P[i]).ss;
    });
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-P[i].ss); \
          it != sweep.end(); sweep.erase(it++)) {
        int j = it->ss;
        Pt d = P[i] - P[j];
        if (d.ss > d.ff) break;
        edges.emplace_back(d.ss + d.ff, i, j);
      }
      sweep[-P[i].ss] = i;
```

```
    }
    for (Pt &p : P) {
      if (k % 2) p.ff = -p.ff;
      else swap(p.ff, p.ss);
    }
  }
  return edges;
}
```

3.3 TreeHash

```
u64 TreeHash(const vector<vector<int>> &G) {
  const int n = G.size();
  vector<int> cen;
  vector<u64> pw(n, 1);
  for (int i = 1; i < n; i++) pw[i] = pw[i - 1] * u64(1
    e9 + 123);
  auto dfs = [&](auto self, int u, int fa) -> int {
    int siz = 1;
    bool f = true;
    for (int v : G[u]) if (v != fa) {
      int s = self(self, v, u);
      f &= (s * 2 <= n);
      siz += s;
    }
    f &= ((n - siz) * 2 <= n);
    if (f) cen.push_back(u);
    return siz;
  }; dfs(dfs, 0, -1);
  auto cal = [&](auto self, int u, int fa) -> pair<u64,
     int> {
    vector<pair<u64, int>> U;
    int siz = 1;
    u64 h = G[u].size();
    for (int v : G[u]) if (v != fa) {
      U.push_back(self(self, v, u));
    }
    sort(all(U));
    for (auto [v, s] : U) {
      h = h * pw[s] + v;
      siz += s;
    }
    return {h, siz};
  };
  vector<u64> H;
  for (int c : cen) H.push_back(cal(cal, c, -1).ff);
  return ranges::min(H);
};
```

3.4 MaximumIndependentSet

3.5 MinMeanWeightCycle

```
// d[i][j] == 0 if {i,j} !in E
long long d[1003][1003], dp[1003][1003];

pair<long long, long long> MMWC() {
 memset(dp, 0x3f, sizeof(dp));
 for (int i = 1; i <= n; ++i) dp[0][i] = 0;
 for (int i = 1; i <= n; ++i) {
  for (int j = 1; j <= n; ++j) {
   for (int k = 1; k <= n; ++k) {
    dp[i][k] = min(dp[i - 1][j] + d[j][k], dp[i][k]);
   }
  }
 }
 long long au = 1ll << 31, ad = 1;
 for (int i = 1; i <= n; ++i) {
  if (dp[n][i] == 0x3f3f3f3f3f3f3f3f) continue;
  long long u = 0, d = 1;
  for (int j = n - 1; j >= 0; --j) {
   if ((dp[n][i] - dp[j][i]) * d > u * (n - j)) {
    u = dp[n][i] - dp[j][i];
    d = n - j;
   }
  }
  if (u * ad < au * d) au = u, ad = d;
 }
 long long g = __gcd(au, ad);
 return make_pair(au / g, ad / g);
}
```

4 Data Structure

4.1 Lazy Segtree

```
template<class S, class T>
struct Seg {
  Seg<S, T> *ls{}, *rs{};
  int l, r;
  S d{};
  T f{};
  Seg(int _l, int _r, const vector<Info> &v) : l{_l}, r
    {_r} {
    if (r - l == 1) {
      d = v[l];
      return;
    }
    int mid = l + r >> 1;
    ls = new Seg(l, mid, v);
    rs = new Seg(mid, r, v);
    pull();
  }
  void upd(const T &g) {
    g(d), g(f);
  }
  void pull() {
    d = ls->d + rs->d;
  }
  void push() {
    ls->upd(f);
    rs->upd(f);
    f = T{};
  }
  S prod(int x, int y) {
    if (y <= l or r <= x) return S{};
    if (x <= l and r <= y) return d;
    push();
    return ls->prod(x, y) + rs->prod(x, y);
  }
  void apply(int x, int y, const T &g) {
    if (y <= l or r <= x) return;
    if (x <= l and r <= y) {
      upd(g);
      return;
    }
    push();
    ls->apply(x, y, g);
    rs->apply(x, y, g);
    pull();
  }
};
```

4.2 Treap

```
mt19937 rng(random_device{}());
template<class S, class T>
struct Treap {
  struct Node {
    Node *ls{}, *rs{};
    int pos, siz;
    u32 pri;
    S d{}, e{};
    T f{};
    Node(int p, S x) : d{x}, e{x}, pos{p}, siz{1}, pri{
    rng()} {}
    void upd(T &g) {
      g(d), g(e), g(f);
    }
    void pull() {
      siz = Siz(ls) + Siz(rs);
      d = Get(ls) + e + Get(rs);
    }
    void push() {
      if (ls) ls->upd(f);
      if (rs) rs->upd(f);
      f = T{};
    }
  } *root{};
  static int Siz(Node *p) { return p ? p->siz : 0; }
  static S Get(Node *p) { return p ? p->d : S{}; }
  Treap() : root{} {}
  Node* Merge(Node *a, Node *b) {
    if (!a or !b) return a ? a : b;
    if (a->pri < b->pri) {
      a->push();
```

```
      a->rs = Merge(a->rs, b);
      a->pull();
      return a;
    } else {
      b->push();
      b->ls = Merge(a, b->ls);
      b->pull();
      return b;
    }
  }
  void Split(Node *p, Node *&a, Node *&b, int k) {
    if (!p) return void(a = b = nullptr);
    p->push();
    if (p->pos <= k) {
      a = p;
      Split(p->rs, a->rs, b, k);
      a->pull();
    } else {
      b = p;
      Split(p->ls, a, b->ls, k);
      b->pull();
    }
  }
  void insert(int p, S x) {
    Node *L, *R;
    Split(root, L, R, p);
    root = Merge(Merge(L, new Node(p, x)), R);
  }
  void erase(int x) {
    Node *L, *M, *R;
    Split(root, M, R, x);
    Split(M, L, M, x - 1);
    if (M) M = Merge(M->ls, M->rs);
    root = Merge(Merge(L, M), R);
  }
  S query() {
    return Get(root);
  }
};
```

4.3 LiChao Segtree

```
struct Line {
  i64 k, m; // y = k + mx;
  Line() : k{INF}, m{} {}
  Line(i64 _k, i64 _m) : k(_k), m(_m) {}
  i64 get(i64 x) {
    return k + m * x;
  }
};
struct Seg {
  Seg *ls{}, *rs{};
  int l, r, mid;
  Line line{};
  Seg(int _l, int _r) : l(_l), r(_r), mid(_l + _r >> 1)
      {
    if (r - l == 1) return;
    ls = new Seg(l, mid);
    rs = new Seg(mid, r);
  }
  void insert(Line L) {
    if (line.get(mid) > L.get(mid))
      swap(line, L);
    if (r - l == 1) return;
    if (L.m < line.m) {
      rs->insert(L);
    } else {
      ls->insert(L);
    }
  }
  i64 query(int p) {
    if (p < l or r <= p) return INF;
    if (r - l == 1) return line.get(p);
    return min({line.get(p), ls->query(p), rs->query(p)
    });
  }
};
```

4.4 Persistent SegmentTree

```
struct Seg {
  Seg *ls{}, *rs{};
  int l, r;
  i64 sum{};
  Seg(Seg* p) { (*this) = *p; }
  Seg(int _l, int _r, const vector<int> &v) : l{_l}, r{
    _r} {
    if (r - l == 1) {
      sum = v[l];
      return;
    }
    int mid = l + r >> 1;
    ls = new Seg(l, mid, v);
    rs = new Seg(mid, r, v);
    pull();
  }
  void pull() {
    sum = ls->sum + rs->sum;
  }
  Seg* modify(int p, int v) {
    Seg* ret = new Seg(this);
    if (r - l == 1) {
      ret->sum = v;
      return ret;
    }
    if (p < (l + r >> 1)) ret->ls = ret->ls->modify(p,
    v);
    else ret->rs = ret->rs->modify(p, v);
    ret->pull();
    return ret;
  }
  i64 query(int x, int y) {
    if (y <= l or r <= x) return 0;
    if (x <= l and r <= y) return sum;
    return ls->query(x, y) + rs->query(x, y);
  }
};
```

4.5 Blackmagic

```
#include <bits/extc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/hash_policy.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
template<class T>
using BST = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
gnu_pbds::priority_queue<node, decltype(cmp),
    pairing_heap_tag> pq(cmp);
gp_hash_table<int, gnu_pbds::priority_queue<node>::
    point_iterator> pqPos;
bst.insert((x << 20) + i);
bst.erase(bst.lower_bound(x << 20));
bst.order_of_key(x << 20) + 1;
*bst.find_by_order(x - 1) >> 20;
*--bst.lower_bound(x << 20) >> 20;
*bst.upper_bound((x + 1) << 20) >> 20;
```

4.6 Centroid Decomposition

```
struct CenDec {
  vector<vector<pair<int, int>>> anc;
  vector<int> Mdis;
  CenDec(const vector<vector<int>> &G) : anc(G.size()),
     Mdis(G.size(), INF) {
    const int n = G.size();
    vector<int> siz(n);
    vector<bool> vis(n);
    function<int(int, int)> getsiz = [&](int u, int f)
    {
      siz[u] = 1;
      for (int v : G[u]) if (v != f and !vis[v])
        siz[u] += getsiz(v, u);
      return siz[u];
    };
    function<int(int, int, int)> find = [&](int u, int
    f, int s) {
      for (int v : G[u]) if (v != f and !vis[v])
        if (siz[v] * 2 >= s) return find(v, u, s);
      return u;
    };
    function<void(int, int, int, int)> caldis = [&](int
     u, int f, int a, int d) {
      anc[u].emplace_back(a, d);
      for (int v : G[u]) if (v != f and !vis[v])
        caldis(v, u, a, d + 1);
```

```
    };
    function<void(int)> build = [&](int u) {
      u = find(u, u, getsiz(u, u));
      vis[u] = 1;
      for (int v : G[u]) if (!vis[v]) {
        caldis(v, u, u, 1);
        build(v);
      }
      vis[u] = 0;
    };
    build(0);
  }
  void add(int p) {
    Mdis[p] = 0;
    for (auto [v, d] : anc[p])
      chmin(Mdis[v], d);
  }
  int que(int p) {
    int r = Mdis[p];
    for (auto [v, d] : anc[p])
      chmin(r, Mdis[v] + d);
    return r;
  }
};
```

5 Dynamic Programming

6 CDQ

../code/DynamicProgramming/CDQ.cpp

7 Math

7.1 Theorem

- Pick's theorem

  $A = i + \frac{b}{2} - 1$

- Laplacian matrix

  $L = D - A$

- Extended Catalan number

  $\frac{1}{(k-1)n+1}\binom{kn}{n}$

- Derangement $D_n = (n-1)(D_{n-1} + D_{n-2})$

- Möbius

  $\sum_{i|n} \mu(i) = [n = 1] \sum_{i|n} \phi(i) = n$

- Inversion formula

  $f(n) = \sum_{i=0}^{n} \binom{n}{i} g(i)\ g(n) = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} f(i)$

  $f(n) = \sum_{d|n} g(d)\ g(n) = \sum_{d|n} \mu(\frac{n}{d}) f(d)$

- Sum of powers

  $\sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k}\ B_k^+\ n^{m+1-k}$

  $\sum_{j=0}^{m} \binom{m+1}{j} B_j^- = 0$

  note : $B_1^+ = -B_1^-\ B_i^+ = B_i^-$

- Cipolla's algorithm

  $\left(\frac{u}{p}\right) = u^{\frac{p-1}{2}}$

  1. $\left(\frac{a^2 - n}{p}\right) = -1$
  2. $x = (a + \sqrt{a^2 - n})^{\frac{p+1}{2}}$

- High order residue

  $[d^{\frac{p-1}{(n,p-1)}} \equiv 1]$

- Packing and Covering

  $|MaximumIndependentSet| + |MinimumVertexCover| = |V|$

- Kőnig's theorem

  $|maximummatching| = |minimumvertexcover|$

- Dilworth's theorem

  $width = |largestantichain| = |smallestchaindecomposition|$

- Mirsky's theorem

  $height = |longestchain| = |smallestantichaindecomposition| = |minimumanticliquepartition|$

- Triangle center

  - $G : (1,)$
  - $O : (a^2(b^2 + c^2 - a^2),) = (sin2A,)$
  - $I : (a,) = (sinA)$
  - $E : (-a, b, c) = (-sinA, sinB, sinC)$
  - $H : (\frac{1}{b^2+c^2-a^2},) = (tanA,)$

- Lucas'Theorem :
  For $n, m \in \mathbb{Z}^*$ and prime P, $C(m, n) \mod P = \Pi(C(m_i, n_i))$ where m_i is the i-th digit of m in base P.

- Stirling approximation :
  $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n e^{\frac{1}{12n}}$

- Stirling Numbers(permutation $|P| = n$ with k cycles):
  $S(n, k) =$ coefficient of x^k in $\Pi_{i=0}^{n-1}(x + i)$

- Stirling Numbers(Partition n elements into k non-empty set):
  $S(n, k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$

- Pick's Theorem : $A = i + b/2 - 1$
  A: Area 、i: grid number in the inner 、b: grid number on the side

- Catalan number : $C_n = \binom{2n}{n}/(n+1)$
  $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)!\frac{n-m+1}{n+1} \quad for \quad n \geq m$
  $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
  $C_0 = 1 \quad and \quad C_{n+1} = 2(\frac{2n+1}{n+2})C_n$
  $C_0 = 1 \quad and \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} \quad for \quad n \geq 0$

- Euler Characteristic:
  planar graph: $V - E + F - C = 1$
  convex polyhedron: $V - E + F = 2$
  V, E, F, C: number of vertices, edges, faces(regions), and components

- Kirchhoff's theorem :
  $A_{ii} = deg(i), A_{ij} = (i, j) \in E\ ?\ -1 : 0$, Deleting any one row, one column, and cal the det(A)

- Polya' theorem (c is number of color，m is the number of cycle size):
  $(\sum_{i=1}^{m} c^{gcd(i,m)})/m$

- Burnside lemma:
  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

- 錯排公式: (n 個人中，每個人皆不再原來位置的組合數):
  $dp[0] = 1; dp[1] = 0;$
  $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$

- Bell 數 (有 n 個人, 把他們拆組的方法總數) :
  $B_0 = 1$
  $B_n = \sum_{k=0}^{n} s(n, k) \quad (second - stirling)$
  $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$

- Wilson's theorem :
  $(p-1)! \equiv -1 (mod\ p)$

- Fermat's little theorem :
  $a^p \equiv a (mod\ p)$

- Euler's totient function:
  $A^{B^C}\ mod\ p = pow(A, pow(B, C, p-1)) mod\ p$

- 歐拉函數降冪公式:
  $A^B \mod C = A^{B \mod \phi(c) + \phi(c)} \mod C$

- 6 的倍數:
  $(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$

7.2 Exgcd

```
pair<i64, i64> exgcd(i64 a, i64 b) { // ax + by = 1
  if (b == 0) return {1, 0};
  auto [x, y] = exgcd(b, a % b);
  return {y, x - a / b * y};
};
```

7.3 CRT

```
i64 CRT(vector<pair<i64, i64>> E) {
  i128 R = 0, M = 1;
  for (auto [r, m] : E) {
    i128 d = r - R, g = gcd<i64>(M, m);
    if (d % g != 0) return -1;
    i128 x = exgcd(M / g, m / g).ff * d / g;
    R += M * x;
    M = M * m / g;
    R = (R % M + M) % M;
  }
  return R;
}
```

7.4 Factorize

```
struct Factorize {
  i64 fmul(i64 a, i64 b, i64 p) {
    return (i128)a * b % p;
  }
  i64 fpow(i64 a, i64 b, i64 p) {
    i64 res = 1;
    for (; b; b >>= 1, a = fmul(a, a, p))
      if (b & 1) res = fmul(res, a, p);
    return res;
  }
  bool Check(i64 a, i64 u, i64 n, int t) {
    a = fpow(a, u, n);
    if (a == 0 or a == 1 or a == n - 1) return true;
    for (int i = 0; i < t; i++) {
      a = fmul(a, a, n);
      if (a == 1) return false;
      if (a == n - 1) return true;
    }
    return false;
  };
  bool IsPrime(i64 n) {
    constexpr array<i64, 7> kChk{2, 235, 9375, 28178,
    450775, 9780504, 1795265022};
    // for int: {2, 7, 61}
    if (n < 2) return false;
    if (n % 2 == 0) return n == 2;
    i64 u = n - 1;
    int t = 0;
    while (u % 2 == 0) u >>= 1, t++;
    for (auto v : kChk) if (!Check(v, u, n, t)) return
    false;
    return true;
  }
  i64 PollardRho(i64 n) {
    if (n % 2 == 0) return 2;
    i64 x = 2, y = 2, d = 1, p = 1;
    auto f = [](i64 x, i64 n, i64 p) -> i64 {
      return ((i128)x * x % n + p) % n;
    };
    while (true) {
      x = f(x, n, p);
      y = f(f(y, n, p), n, p);
      d = __gcd(abs(x - y), n);
      if (d != n and d != 1) return d;
      if (d == n) ++p;
    }
  }
};
```

7.5 NTT

```
// 17 -> 3
// 97 -> 5
// 193 -> 5
// 998244353 -> 3
// 985661441 -> 3
constexpr i64 cpow(i64 a, i64 b, i64 m) {
  i64 ret = 1;
  for (; b; b >>= 1, a = a * a % m)
    if (b & 1) ret = ret * a % m;
  return ret;
};
template<i64 M, i64 G>
struct NTT {
  static constexpr i64 iG = cpow(G, M - 2, M);
  void operator()(vector<i64> &v, bool inv) {
    int n = v.size();
    for (int i = 0, j = 0; i < n; i++) {
      if (i < j) swap(v[i], v[j]);
      for (int k = n / 2; (j ^= k) < k; k /= 2);
    }
    for (int mid = 1; mid < n; mid *= 2) {
      i64 w = cpow((inv ? iG : G), (M - 1) / (mid + mid
    ), M);
      for (int i = 0; i < n; i += mid * 2) {
        i64 now = 1;
        for (int j = i; j < i + mid; j++, now = now * w
     % M) {
          i64 x = v[j], y = v[j + mid];
          v[j] = (x + y * now) % M;
          v[j + mid] = (x - y * now) % M;
        }
      }
    }
    if (inv) {
      i64 in = cpow(n, M - 2, M);
      for (int i = 0; i < n; i++) v[i] = v[i] * in % M;
    }
  }
};
template<i64 M, i64 G>
vector<i64> convolution(vector<i64> f, vector<i64> g) {
  NTT<M, G> ntt;
  int sum = f.size() + g.size() - 1;
  int len = bit_ceil((u64)sum);
  f.resize(len); g.resize(len);
  ntt(f, 0), ntt(g, 0);
  for (int i = 0; i < len; i++) (f[i] *= g[i]) %= M;
  ntt(f, 1);
  f.resize(sum);
  for (int i = 0; i < sum; i++) if (f[i] < 0) f[i] += M
    ;
  return f;
}
vector<i64> convolution_ll(const vector<i64> &f, const
    vector<i64> &g) {
  constexpr i64 M1 = 998244353, G1 = 3;
  constexpr i64 M2 = 985661441, G2 = 3;
  constexpr i64 M1M2 = M1 * M2;
  constexpr i64 M1m1 = M2 * cpow(M2, M1 - 2, M1);
  constexpr i64 M2m2 = M1 * cpow(M1, M2 - 2, M2);
  auto c1 = convolution<M1, G1>(f, g);
  auto c2 = convolution<M2, G2>(f, g);
  for (int i = 0; i < c1.size(); i++) {
    c1[i] = ((i128)c1[i] * M1m1 + (i128)c2[i] * M2m2) %
     M1M2;
  }
  return c1;
}
```

7.6 FWT

1. XOR Convolution
   - $f(A) = (f(A_0) + f(A_1), f(A_0) - f(A_1))$
   - $f^{-1}(A) = (f^{-1}(\frac{A_0+A_1}{2}), f^{-1}(\frac{A_0-A_1}{2}))$
2. OR Convolution
   - $f(A) = (f(A_0), f(A_0) + f(A_1))$
   - $f^{-1}(A) = (f^{-1}(A_0), f^{-1}(A_1) - f^{-1}(A_0))$
3. AND Convolution
   - $f(A) = (f(A_0) + f(A_1), f(A_1))$
   - $f^{-1}(A) = (f^{-1}(A_0) - f^{-1}(A_1), f^{-1}(A_1))$

7.7 FWT

```
void xorfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  xorfwt(v, l, m), xorfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) {
    int x = v[i] + v[j];
    v[j] = v[i] - v[j], v[i] = x;
  }
}

void xorifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  for (int i = l, j = m; i < m; ++i, ++j) {
    int x = (v[i] + v[j]) / 2;
```

```
    v[j] = (v[i] - v[j]) / 2, v[i] = x;
  }
  xorifwt(v, l, m), xorifwt(v, m, r);
}

void andfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  andfwt(v, l, m), andfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[i] += v[j];
}

void andifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  andifwt(v, l, m), andifwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[i] -= v[j];
}

void orfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  orfwt(v, l, m), orfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[j] += v[i];
}

void orifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  orifwt(v, l, m), orifwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[j] -= v[i];
}
```

7.8 Lucas

```
i64 Lucas(i64 N, i64 M, i64 D) { // C(N, M) mod D
  auto Factor = [&](i64 x) -> vector<pair<i64, i64>> {
    vector<pair<i64, i64>> r;
    for (i64 i = 2; x > 1; i++)
      if (x % i == 0) {
        i64 c = 0;
        while (x % i == 0) x /= i, c++;
        r.emplace_back(i, c);
      }
    return r;
  };
  auto Pow = [&](i64 a, i64 b, i64 m) -> i64 {
    i64 r = 1;
    for (; b; b >>= 1, a = a * a % m)
      if (b & 1) r = r * a % m;
    return r;
  };
  vector<pair<i64, i64>> E;
  for (auto [p, q] : Factor(D)) {
    const i64 mod = Pow(p, q, 1 << 30);
    auto CountFact = [&](i64 x) -> i64 {
      i64 c = 0;
      while (x) c += (x /= p);
      return c;
    };
    auto CountBino = [&](i64 x, i64 y) { return
    CountFact(x) - CountFact(y) - CountFact(x - y); };
    auto Inv = [&](i64 x) -> i64 { return (exgcd(x, mod
    ).ff % mod + mod) % mod; };
    vector<i64> pre(mod + 1);
    pre[0] = pre[1] = 1;
    for (i64 i = 2; i <= mod; i++) pre[i] = (i % p == 0
     ? 1 : i) * pre[i - 1] % mod;
    function<i64(i64)> FactMod = [&](i64 n) -> i64 {
      if (n == 0) return 1;
      return FactMod(n / p) * Pow(pre[mod], n / mod,
    mod) % mod * pre[n % mod] % mod;
    };
    auto BinoMod = [&](i64 x, i64 y) -> i64 {
      return FactMod(x) * Inv(FactMod(y)) % mod * Inv(
    FactMod(x - y)) % mod;
    };
    i64 r = BinoMod(N, M) * Pow(p, CountBino(N, M), mod
    ) % mod;
    E.emplace_back(r, mod);
  };
  return CRT(E);
}
```

7.9 Berlekamp Massey

```
template <int P>
vector<int> BerlekampMassey(vector<int> x) {
 vector<int> cur, ls;
 int lf = 0, ld = 0;
 for (int i = 0; i < (int)x.size(); ++i) {
  int t = 0;
  for (int j = 0; j < (int)cur.size(); ++j)
   (t += 1LL * cur[j] * x[i - j - 1] % P) %= P;
  if (t == x[i]) continue;
  if (cur.empty()) {
   cur.resize(i + 1);
   lf = i, ld = (t + P - x[i]) % P;
   continue;
  }
  int k = 1LL * fpow(ld, P - 2, P) * (t + P - x[i]) % P
    ;
  vector<int> c(i - lf - 1);
  c.push_back(k);
  for (int j = 0; j < (int)ls.size(); ++j)
   c.push_back(1LL * k * (P - ls[j]) % P);
  if (c.size() < cur.size()) c.resize(cur.size());
  for (int j = 0; j < (int)cur.size(); ++j)
   c[j] = (c[j] + cur[j]) % P;
  if (i - lf + (int)ls.size() >= (int)cur.size()) {
   ls = cur, lf = i;
   ld = (t + P - x[i]) % P;
  }
  cur = c;
 }
 return cur;
}
```

7.10 Gauss Elimination

```
double Gauss(vector<vector<double>> &d) {
 int n = d.size(), m = d[0].size();
 double det = 1;
 for (int i = 0; i < m; ++i) {
  int p = -1;
  for (int j = i; j < n; ++j) {
   if (fabs(d[j][i]) < kEps) continue;
   if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p = j;
  }
  if (p == -1) continue;
  if (p != i) det *= -1;
  for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
  for (int j = 0; j < n; ++j) {
   if (i == j) continue;
   double z = d[j][i] / d[i][i];
   for (int k = 0; k < m; ++k) d[j][k] -= z * d[i][k];
  }
 }
 for (int i = 0; i < n; ++i) det *= d[i][i];
 return det;
}
```

7.11 Linear Equation

```
void linear_equation(vector<vector<double>> &d, vector<
    double> &aug, vector<double> &sol) {
  int n = d.size(), m = d[0].size();
  vector<int> r(n), c(m);
  iota(r.begin(), r.end(), 0);
  iota(c.begin(), c.end(), 0);
  for (int i = 0; i < m; ++i) {
    int p = -1, z = -1;
    for (int j = i; j < n; ++j) {
      for (int k = i; k < m; ++k) {
        if (fabs(d[r[j]][c[k]]) < eps) continue;
        if (p == -1 || fabs(d[r[j]][c[k]]) > fabs(d[r[p
    ]][c[z]])) p = j, z = k;
      }
    }
    if (p == -1) continue;
    swap(r[p], r[i]), swap(c[z], c[i]);
    for (int j = 0; j < n; ++j) {
      if (i == j) continue;
      double z = d[r[j]][c[i]] / d[r[i]][c[i]];
      for (int k = 0; k < m; ++k) d[r[j]][c[k]] -= z *
    d[r[i]][c[k]];
      aug[r[j]] -= z * aug[r[i]];
    }
```

```
  }
  vector<vector<double>> fd(n, vector<double>(m));
  vector<double> faug(n), x(n);
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m; ++j) fd[i][j] = d[r[i]][c[j
    ]];
    faug[i] = aug[r[i]];
  }
  d = fd, aug = faug;
  for (int i = n - 1; i >= 0; --i) {
    double p = 0.0;
    for (int j = i + 1; j < n; ++j) p += d[i][j] * x[j
    ];
    x[i] = (aug[i] - p) / d[i][i];
  }
  for (int i = 0; i < n; ++i) sol[c[i]] = x[i];
}
```

7.12 LinearRec

```
template <int P>
int LinearRec(const vector<int> &s, const vector<int> &
    coeff, int k) {
  int n = s.size();
  auto Combine = [&](const auto &a, const auto &b) {
    vector<int> res(n * 2 + 1);
    for (int i = 0; i <= n; ++i) {
      for (int j = 0; j <= n; ++j)
        (res[i + j] += 1LL * a[i] * b[j] % P) %= P;
    }
    for (int i = 2 * n; i > n; --i) {
      for (int j = 0; j < n; ++j)
        (res[i - 1 - j] += 1LL * res[i] * coeff[j] % P)
     %= P;
    }
    res.resize(n + 1);
    return res;
  };
  vector<int> p(n + 1), e(n + 1);
  p[0] = e[1] = 1;
  for (; k > 0; k >>= 1) {
    if (k & 1) p = Combine(p, e);
    e = Combine(e, e);
  }
  int res = 0;
  for (int i = 0; i < n; ++i) (res += 1LL * p[i + 1] *
    s[i] % P) %= P;
  return res;
}
```

7.13 SubsetConv

```
vector<int> SubsetConv(int n, const vector<int> &f,
    const vector<int> &g) {
 const int m = 1 << n;
 vector<vector<int>> a(n + 1, vector<int>(m)), b(n + 1,
     vector<int>(m));
 for (int i = 0; i < m; ++i) {
  a[__builtin_popcount(i)][i] = f[i];
  b[__builtin_popcount(i)][i] = g[i];
 }
 for (int i = 0; i <= n; ++i) {
  for (int j = 0; j < n; ++j) {
   for (int s = 0; s < m; ++s) {
    if (s >> j & 1) {
     a[i][s] += a[i][s ^ (1 << j)];
     b[i][s] += b[i][s ^ (1 << j)];
    }
   }
  }
 }
 vector<vector<int>> c(n + 1, vector<int>(m));
 for (int s = 0; s < m; ++s) {
  for (int i = 0; i <= n; ++i) {
   for (int j = 0; j <= i; ++j) c[i][s] += a[j][s] * b[
    i - j][s];
  }
 }
 for (int i = 0; i <= n; ++i) {
  for (int j = 0; j < n; ++j) {
   for (int s = 0; s < m; ++s) {
    if (s >> j & 1) c[i][s] -= c[i][s ^ (1 << j)];
   }
  }
 }
 vector<int> res(m);
 for (int i = 0; i < m; ++i) res[i] = c[
    __builtin_popcount(i)][i];
 return res;
}
```

7.14 SqrtMod

```
int get_root(int n, int P) { // ensure 0 <= n < p
 if (P == 2 or n == 0) return n;
 auto check = [&](int x) {
  return modpow(x, (P - 1) / 2, P); };
 if (check(n) != 1) return -1;
 mt19937 rnd(7122); lld z = 0, w;
 while (check(w = (z * z - n + P) % P) != P - 1)
  z = rnd() % P;
 const auto M = [P, w](auto &u, auto &v) {
  auto [a, b] = u; auto [c, d] = v;
  return make_pair((a * c + b * d % P * w) % P,
    (a * d + b * c) % P);
 };
 pair<lld, lld> r(1, 0), e(z, 1);
 for (int w = (P + 1) / 2; w; w >>= 1, e = M(e, e))
  if (w & 1) r = M(r, e);
 return r.first; // sqrt(n) mod P where P is prime
}
```

7.15 FloorSum

```
// sigma 0 ~ n-1: (a * i + b) / m
i64 floor_sum(i64 n, i64 m, i64 a, i64 b) {
  u64 ans = 0;
  if (a < 0) {
    u64 a2 = (a % m + m) % m;
    ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
    a = a2;
  }
  if (b < 0) {
    u64 b2 = (b % m + m) % m;
    ans -= 1ULL * n * ((b2 - b) / m);
    b = b2;
  }
  while (true) {
    if (a >= m) {
      ans += n * (n - 1) / 2 * (a / m);
      a %= m;
    }
    if (b >= m) {
      ans += n * (b / m);
      b %= m;
    }
    u64 y_max = a * n + b;
    if (y_max < m) break;
    n = y_max / m;
    b = y_max % m;
    swap(m, a);
  }
  return ans;
}
```

8 Geometry

8.1 2D Point

```
using Pt = pair<i64, i64>;
constexpr double eps = 1e-9;
Pt operator+(Pt a, Pt b) { return {a.ff + b.ff, a.ss +
    b.ss}; }
Pt operator-(Pt a, Pt b) { return {a.ff - b.ff, a.ss -
    b.ss}; }
Pt operator*(Pt a, double b) { return {a.ff * b, a.ss *
     b}; }
Pt operator/(Pt a, double b) { return {a.ff / b, a.ss /
     b}; }
double operator*(Pt a, Pt b) { return a.ff * b.ff + a.
    ss * b.ss; }
double operator^(Pt a, Pt b) { return a.ff * b.ss - a.
    ss * b.ff; }
double abs(Pt a) { return sqrt(a * a); }
double cro(Pt a, Pt b, Pt c) { return (b - a) ^ (c - a)
    ; }
int sig(double x) { return (x > -eps) - (x < eps); }
Pt Inter(Pt a, Pt b, Pt c, Pt d) {
```

```
  double s = cro(c, d, a), t = -cro(c, d, b);
  return (a * t + b * s) / (s + t);
}
struct Line {
  Pt a{}, b{};
  Line() {}
  Line(Pt _a, Pt _b) : a{_a}, b{_b} {}
};
Pt Inter(Line L, Line R) {
  return Inter(L.a, L.b, R.a, R.b);
}
```

8.2 Convex Hull

```
vector<Pt> Hull(vector<Pt> P) {
  sort(all(P));
  P.erase(unique(all(P)), P.end());
  P.insert(P.end(), rall(P));
  vector<Pt> stk;
  for (auto p : P) {
    while (stk.size() >= 2 and \
        cro(*++stk.rbegin(), stk.back(), p) <= 0 and \
        (*++stk.rbegin() < stk.back()) == (stk.back() <
     p)) {
      stk.pop_back();
    }
    stk.push_back(p);
  }
  stk.pop_back();
  return stk;
}
```

8.3 Convex Hull trick

```
vector<Pt> Hull(vector<Pt> P) {
  sort(all(P));
  P.erase(unique(all(P)), P.end());
  P.insert(P.end(), rall(P));
  vector<Pt> stk;
  for (auto p : P) {
    while (stk.size() >= 2 and \
        cro(*++stk.rbegin(), stk.back(), p) <= 0 and \
        (*++stk.rbegin() < stk.back()) == (stk.back() <
     p)) {
      stk.pop_back();
    }
    stk.push_back(p);
  }
  stk.pop_back();
  return stk;
}
```

8.4 Dynamic Convex Hull

```
template<class T, class Comp = less<T>>
struct DynamicHull {
  set<T, Comp> H;
  DynamicHull() {}
  void insert(T p) {
    if (inside(p)) return;
    auto it = H.insert(p).ff;
    while (it != H.begin() and prev(it) != H.begin() \
        and cross(*prev(it, 2), *prev(it), *it) <= 0) {
      it = H.erase(--it);
    }
    while (it != --H.end() and next(it) != --H.end() \
        and cross(*it, *next(it), *next(it, 2)) <= 0) {
      it = --H.erase(++it);
    }
  }
  bool inside(T p) {
    auto it = H.lower_bound(p);
    if (it == H.end()) return false;
    if (it == H.begin()) return p == *it;
    return cross(*prev(it), p, *it) <= 0;
  }
};
```

8.5 Half Plane Intersection

```
vector<Pt> HPI(vector<Line> P) {
  const int n = P.size();
  sort(all(P), [&](Line L, Line R) -> bool {
    Pt u = L.b - L.a, v = R.b - R.a;
    bool f = Pt(sig(u.ff), sig(u.ss)) < Pt{};
    bool g = Pt(sig(v.ff), sig(v.ss)) < Pt{};
    if (f != g) return f < g;
    return (sig(u ^ v) ? sig(u ^ v) : sig(cro(L.a, R.a,
     R.b))) > 0;
  });
  auto Same = [&](Line L, Line R) {
    Pt u = L.b - L.a, v = R.b - R.a;
    return sig(u ^ v) == 0 and sig(u * v) == 1;
  };
  deque <Pt> inter;
  deque <Line> seg;
  for (int i = 0; i < n; i++) if (i == 0 or !Same(P[i -
     1], P[i])) {
    while (seg.size() >= 2 and sig(cro(inter.back(), P[
    i].b, P[i].a)) == 1) {
      seg.pop_back(), inter.pop_back();
    }
    while (seg.size() >= 2 and sig(cro(inter[0], P[i].b
    , P[i].a)) == 1) {
      seg.pop_front(), inter.pop_front();
    }
    if (!seg.empty()) inter.push_back(Inter(seg.back(),
     P[i]));
    seg.push_back(P[i]);
  }
  while (seg.size() >= 2 and sig(cro(inter.back(), seg
    [0].b, seg[0].a)) == 1) {
    seg.pop_back(), inter.pop_back();
  }
  inter.push_back(Inter(seg[0], seg.back()));
  return vector<Pt>(all(inter));
}
```

8.6 Minimal Enclosing Circle

```
using circle = pair<Pt, double>;
struct MES {
  MES() {}
  bool inside(const circle &c, Pt p) {
    return abs(p - c.ff) <= c.ss + eps;
  };
  circle get_cir(Pt a, Pt b) {
    return circle((a + b) / 2., abs(a - b) / 2.);
  }
  circle get_cir(Pt a, Pt b, Pt c) {
    Pt p = (b - a) / 2.;
    p = Pt(-p.ss, p.ff);
    double t = ((c - a) * (c - b)) / (2 * (p * (c - a))
    );
    p = ((a + b) / 2.) + (p * t);
    return circle(p, abs(p - a));
  }
  circle get_mes(vector<Pt> P) {
    if (P.empty()) return circle{Pt(0, 0), 0};
    mt19937 rng(random_device{}());
    shuffle(all(P), rng);
    circle C{P[0], 0};
    for (int i = 1; i < P.size(); i++) {
      if (inside(C, P[i])) continue;
      C = get_cir(P[i], P[0]);
      for (int j = 1; j < i; j++) {
        if (inside(C, P[j])) continue;
        C = get_cir(P[i], P[j]);
        for (int k = 0; k < j; k++) {
          if (inside(C, P[k])) continue;
          C = get_cir(P[i], P[j], P[k]);
        }
      }
    }
    return C;
  }
};
```

8.7 Minkowski

```
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
  auto reorder = [&](auto &R) -> void {
    auto cmp = [&](Pt a, Pt b) -> bool {
      return Pt(a.ss, a.ff) < Pt(b.ss, b.ff);
    };
    rotate(R.begin(), min_element(all(R), cmp), R.end()
    );
    R.push_back(R[0]), R.push_back(R[1]);
  };
```

```
  const int n = P.size(), m = Q.size();
  reorder(P), reorder(Q);
  vector<Pt> R;
  for (int i = 0, j = 0, s; i < n or j < m; ) {
    R.push_back(P[i] + Q[j]);
    s = sig((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
    i += (s >= 0), j += (s <= 0);
  }
  return R;
}
```

9 Stringology

9.1 Z-algorithm

```
vector<int> zalgo(string s) {
  if (s.empty()) return {};
  int len = s.size();
  vector<int> z(len);
  z[0] = len;
  for (int i = 1, l = 1, r = 1; i < len; i++) {
    z[i] = i < r ? min(z[i - l], r - i) : 0;
    while (i + z[i] < len and s[i + z[i]] == s[z[i]]) z
    [i]++;
    if (i + z[i] > r) l = i, r = i + z[i];
  }
  return z;
}
```

9.2 Manacher

```
vector<int> manacher(const string &s) {
  string p = "@#";
  for (char c : s) p += c + '#';
  p += '$';
  vector<int> dp(p.size());
  int mid = 0, r = 1;
  for (int i = 1; i < p.size() - 1; i++) {
    auto &k = dp[i];
    k = i < mid + r ? min(dp[mid * 2 - i], mid + r - i)
     : 0;
    while (p[i + k + 1] == p[i - k - 1]) k++;
    if (i + k > mid + r) mid = i, r = k;
  }
  return vector<int>(dp.begin() + 2, dp.end() - 2);
}
```

9.3 SuffixArray

```
namespace sfx {
#define fup(a, b) for (int i = a; i < b; i++)
#define fdn(a, b) for (int i = b - 1; i >= a; i--)
  constexpr int N = 5e5 + 5;
  bool _t[N * 2];
  int H[N], RA[N], x[N], _p[N];
  int SA[N * 2], _s[N * 2], _c[N * 2], _q[N * 2];
  void pre(int *sa, int *c, int n, int z) {
    fill_n(sa, n, 0), copy_n(c, z, x);
  }
  void induce(int *sa, int *c, int *s, bool *t, int n,
    int z) {
    copy_n(c, z - 1, x + 1);
    fup(0, n) if (sa[i] and !t[sa[i] - 1])
      sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
    copy_n(c, z, x);
    fdn(0, n) if (sa[i] and t[sa[i] - 1])
      sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
  }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
    int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
    last = -1;
    fill_n(c, z, 0);
    fup(0, n) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) { fup(0, n) sa[--c[s[i]]] = i; return; }
    fdn(0, n - 1)
      t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i
    + 1]);
    pre(sa, c, n, z);
    fup(1, n) if (t[i] and !t[i - 1])
      sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    fup(0, n) if (sa[i] and t[sa[i]] and !t[sa[i] - 1])
     {
      bool neq = last < 0 or !equal(s + sa[i], s + p[q[
    sa[i]] + 1], s + last);
      ns[q[last = sa[i]]] = nmxz += neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
     + 1);
    pre(sa, c, n, z);
    fdn(0, nn) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
  }
  vector<int> build(vector<int> s, int n) {
    copy_n(begin(s), n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    vector<int> sa(n);
    fup(0, n) sa[i] = SA[i + 1];
    return sa;
  }
  vector<int> lcp_array(vector<int> &s, vector<int> &sa
    ) {
    int n = int(s.size());
    vector<int> rnk(n);
    fup(0, n) rnk[sa[i]] = i;
    vector<int> lcp(n - 1);
    int h = 0;
    fup(0, n) {
      if (h > 0) h--;
      if (rnk[i] == 0) continue;
      int j = sa[rnk[i] - 1];
      for (; j + h < n and i + h < n; h++)
        if (s[j + h] != s[i + h]) break;
      lcp[rnk[i] - 1] = h;
    }
    return lcp;
  }
}
```

9.4 PalindromicTree

```
struct PAM {
  struct Node {
    int fail, len, dep;
    array<int, 26> ch;
    Node(int _len) : len{_len}, fail{}, ch{}, dep{} {};
  };
  vector<Node> g;
  vector<int> id;
  int odd, even, lst;
  string S;
  int new_node(int len) {
    g.emplace_back(len);
    return g.size() - 1;
  }
  PAM() : odd(new_node(-1)), even(new_node(0)) {
    lst = g[even].fail = odd;
  }
  int up(int p) {
    while (S.rbegin()[g[p].len + 1] != S.back())
      p = g[p].fail;
    return p;
  }
  int add(char c) {
    S += c;
    lst = up(lst);
    c -= 'a';
    if (!g[lst].ch[c]) g[lst].ch[c] = new_node(g[lst].
    len + 2);
    int p = g[lst].ch[c];
    g[p].fail = (lst == odd ? even : g[up(g[lst].fail)
    ].ch[c]);
    lst = p;
    g[lst].dep = g[g[lst].fail].dep + 1;
    id.push_back(lst);
    return lst;
  }
  void del() {
    S.pop_back();
    id.pop_back();
    lst = id.empty() ? odd : id.back();
  }
};
```

9.5 SmallestRotation

```
string Rotate(const string &s) {
 int n = s.length();
 string t = s + s;
 int i = 0, j = 1;
 while (i < n && j < n) {
  int k = 0;
  while (k < n && t[i + k] == t[j + k]) ++k;
  if (t[i + k] <= t[j + k]) j += k + 1;
  else i += k + 1;
  if (i == j) ++j;
 }
 int pos = (i < n ? i : j);
 return t.substr(pos, n);
}
```

10 Misc

10.1 HilbertCurve

```
long long hilbert(int n, int x, int y) {
 long long res = 0;
 for (int s = n / 2; s; s >>= 1) {
  int rx = (x & s) > 0;
  int ry = (y & s) > 0;
  res += s * 1ll * s * ((3 * rx) ^ ry);
  if (ry == 0) {
   if (rx == 1) x = s - 1 - x, y = s - 1 - y;
   swap(x, y);
  }
 }
 return res;
}
```

10.2 DLX

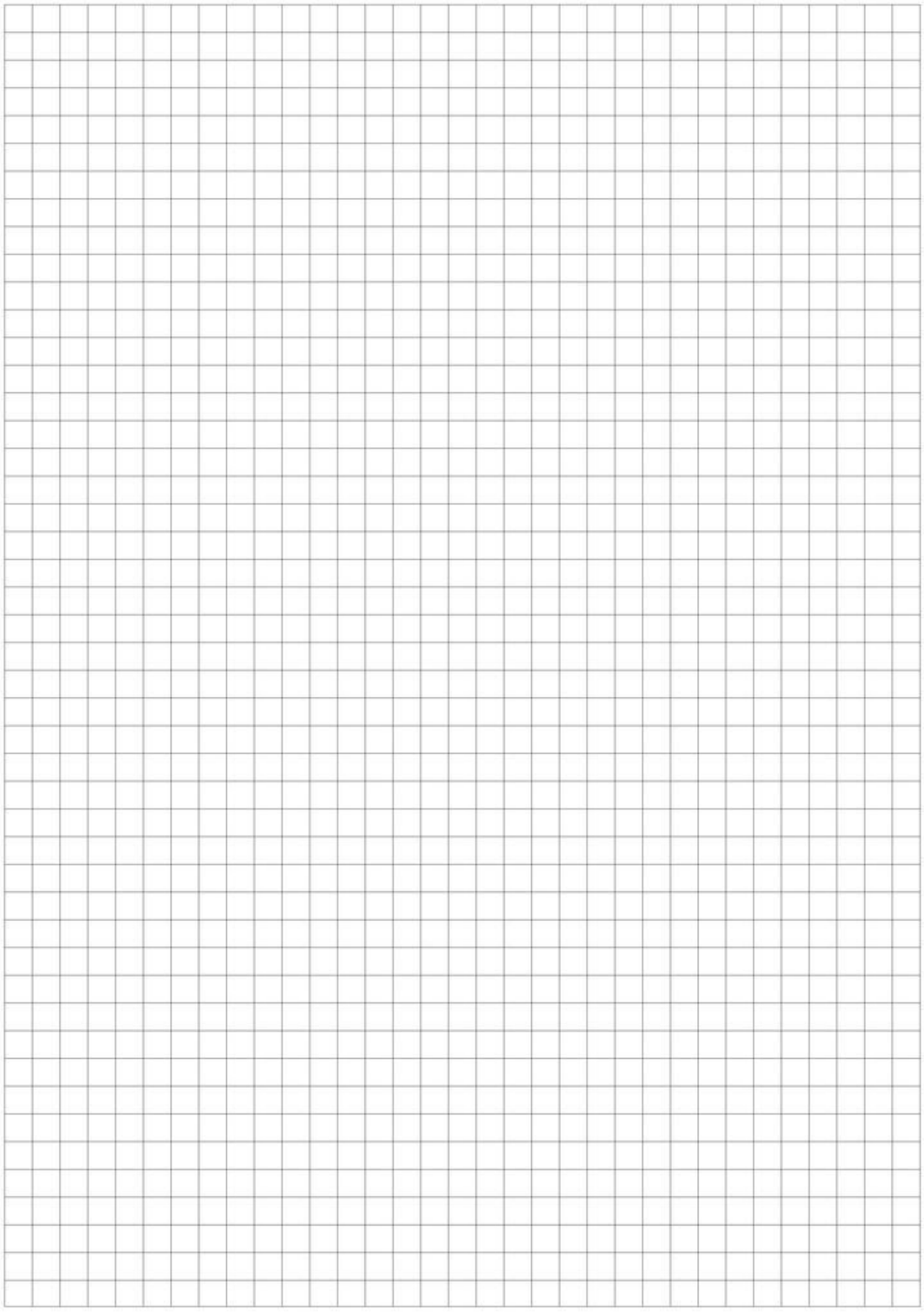
```
namespace dlx {
int lt[maxn], rg[maxn], up[maxn], dn[maxn], cl[maxn],
    rw[maxn], bt[maxn], s[maxn], head, sz, ans;
void init(int c) {
 for (int i = 0; i < c; ++i) {
  up[i] = dn[i] = bt[i] = i;
  lt[i] = i == 0 ? c : i - 1;
  rg[i] = i == c - 1 ? c : i + 1;
  s[i] = 0;
 }
 rg[c] = 0, lt[c] = c - 1;
 up[c] = dn[c] = -1;
 head = c, sz = c + 1;
}
void insert(int r, const vector<int> &col) {
 if (col.empty()) return;
 int f = sz;
 for (int i = 0; i < (int)col.size(); ++i) {
  int c = col[i], v = sz++;
  dn[bt[c]] = v;
  up[v] = bt[c], bt[c] = v;
  rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
  rw[v] = r, cl[v] = c;
  ++s[c];
  if (i > 0) lt[v] = v - 1;
 }
 lt[f] = sz - 1;
}
void remove(int c) {
 lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
 for (int i = dn[c]; i != c; i = dn[i]) {
  for (int j = rg[i]; j != i; j = rg[j])
   up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[j]];
 }
}
void restore(int c) {
 for (int i = up[c]; i != c; i = up[i]) {
  for (int j = lt[i]; j != i; j = lt[j])
   ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
 }
 lt[rg[c]] = c, rg[lt[c]] = c;
}
// Call dlx::make after inserting all rows.
void make(int c) {
 for (int i = 0; i < c; ++i)
  dn[bt[i]] = i, up[i] = bt[i];
}
void dfs(int dep) {
 if (dep >= ans) return;
 if (rg[head] == head) return ans = dep, void();
 if (dn[rg[head]] == rg[head]) return;
 int c = rg[head];
 int w = c;
 for (int x = c; x != head; x = rg[x]) if (s[x] < s[w])
     w = x;
 remove(w);
 for (int i = dn[w]; i != w; i = dn[i]) {
  for (int j = rg[i]; j != i; j = rg[j]) remove(cl[j]);
  dfs(dep + 1);
  for (int j = lt[i]; j != i; j = lt[j]) restore(cl[j])
    ;
 }
 restore(w);
}
int solve() {
 ans = 1e9, dfs(0);
 return ans;
}}
```

10.3 NextPerm

```
i64 next_perm(i64 x) {
  i64 y = x | (x - 1);
  return (y + 1) | (((~y & -~y) - 1) >> (__builtin_ctz(
    x) + 1));
}
```

10.4 FastIO

```
struct FastIO {
  const static int ibufsiz = 4<<20, obufsiz = 18<<20;
  char ibuf[ibufsiz], *ipos = ibuf, obuf[obufsiz], *
    opos = obuf;
  FastIO() { fread(ibuf, 1, ibufsiz, stdin); }
  ~FastIO() { fwrite(obuf, 1, opos - obuf, stdout); }
  template<class T> FastIO& operator>>(T &x) {
    bool sign = 0; while (!isdigit(*ipos)) { if (*ipos
    == '-') sign = 1; ++ipos; }
    x = *ipos++ & 15;
    while (isdigit(*ipos)) x = x * 10 + (*ipos++ & 15);
    if (sign) x = -x;
    return *this;
  }
  template<class T> FastIO& operator<<(T n) {
    static char _buf[18];
    char* _pos = _buf;
    if (n < 0) *opos++ = '-', n = -n;
    do *_pos++ = '0' + n % 10; while (n /= 10);
    while (_pos != _buf) *opos++ = *--_pos;
    return *this;
  }
  FastIO& operator<<(char ch) { *opos++ = ch; return *
    this; }
} FIO;
#define cin FIO
#define cout FIO
```



INSPIRATION HUT - 0.6CM GRID

$$x = r \sin\theta \cos\varphi$$

$$y = r \sin\theta \sin\varphi$$

$$z = r \cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}(z/r)$$

$$\varphi = \tan^{-1}(y/x)$$