Homework 1 Solutions

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Question 1

1. $S = \{aaf, afa, faa, aff, ffa, faf, aaa, fff\}$
2. The two sets are given by:

$$Z_f = \{aaf, aff, faf, fff\}$$
$$X_a = \{aaf, afa, aff, aaa\}$$

3. No, because $Z_f \cap X_a \neq \emptyset$
4. No, because $Z_f \cup X_a \neq S$
5. The two sets are given by:

$$C = \{aaf, afa, faa, aaa\}$$
$$D = \{aff, ffa, faf, fff\}$$

6. Yes, because $C \cap D = \emptyset$
7. Yes, because $C \cup D = S$

Question 2

1. $P[H_0] = P[L \cap H_0] + P[B \cap H_0] = 0.1 + 0.3 = 0.4$
2. $P[B] = P[B \cap H_0] + P[B \cap H_1] + P[B \cap H_2] = 0.3 + 0.1 + 0.1 = 0.5$

3. From the additive property of probabilities

$$\begin{aligned} P[L \cup H_2] &= P[L] + P[H_2] - P[L \cap H_2] \\ &= P[L] + P[H_2 \cap B] \\ &= 0.1 + 0.1 + 0.3 + 0.1 = 0.6 \end{aligned}$$

4. They ARE NOT Independent

$$\begin{aligned} P[B] &= 0.5 \\ P[H_0] &= 0.4 \\ P[B \cap H_0] &\neq P[B]P[H_0] \\ 0.3 &\neq (0.5)(0.4) \end{aligned}$$

5. They ARE Independent

$$\begin{aligned} P[B] &= 0.5 \\ P[H_1] &= 0.2 \\ P[B \cap H_1] &= P[B]P[H_1] \\ 0.1 &\neq (0.5)(0.2) \end{aligned}$$

Question 3

1. The Conditional Probability is given by:

$$\begin{aligned} P(E_2|E_1) &= \frac{P(E_2 \cap E_1)}{P(E_1)} \\ &= \frac{|\{243, 423\}|}{|\{234, 243, 423, 432\}|} \\ &= \frac{2}{4} = \frac{1}{2} \end{aligned}$$

2. This event cannot occur. There are only 3 cards, and one of the cards is even. Thus, the probability that all 3 cards are even is 0.

3. The probability that the second card is even given the first card is odd:

$$\begin{aligned} P(E_2|O_1) &= \frac{P(E_2 \cap O_1)}{P(O_1)} \\ &= \frac{|\{324, 342\}|}{|\{324, 342\}|} \\ &= \frac{2}{2} = 1 \end{aligned}$$

4. Similar to part b, this event cannot occur because there is only 1 odd card. The probability that the second card is odd given the first card is odd must be 0.

Question 4

$$\begin{aligned} P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1)P(E_2)P(E_3) \\ E_1 = [1 \cap 2] \rightarrow P(E_1) = \frac{1}{4} \\ E_2 = [2 \cap 3] \rightarrow P(E_2) = \frac{1}{4} \\ E_3 = [3 \cap 4] \rightarrow P(E_3) = \frac{1}{4} \end{aligned}$$

Question 5

1. The probability that exactly one photo detector of a pair is acceptable is:

$$\begin{aligned} P[\mathit{One\ Pair\ Acceptable}] &= P[A_1, D_2] + P[D_1, A_2] - P[(A_1, D_2) \cap (D_1, A_2)] \\ &= P(A_1)P(D_2|A_1) + P(D_1)P(A_2|D_1) + 0 \\ &= \left(\frac{3}{5}\right)\left(1 - \frac{4}{5}\right) + \left(1 - \frac{3}{5}\right)\left(\frac{2}{5}\right) \\ &= \frac{3}{25} + \frac{4}{25} = \frac{7}{25} \end{aligned}$$

2. The probability that both photo detectors in a pair are defective:

$$\begin{aligned} P[Both\ Defective] &= P[D_1, D_2] \\ &= P(D_1)P(D_2|D_1) = \left(\frac{2}{5}\right)\left(\frac{3}{5}\right) = \frac{6}{25} \end{aligned}$$

Question 6

Swingman at Gaurd

$$\binom{3}{1}\binom{4}{1}\binom{4}{2} = 72$$

Swingman at Forward

$$\binom{3}{1}\binom{4}{2}\binom{4}{1} = 72$$

Swingman does not play

$$\binom{3}{1}\binom{4}{2}\binom{4}{2} = 108$$

72 + 72 + 108 = 252

Question 7

1. The probability that a field goal is kicked and made is:

$$P(K) = P(G_1)P(|G_1) + P(G_2)P(K|G_2)$$
$$P(K) = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$$
$$P(K) = \frac{7}{18}$$

2. The events are NOT independent

$$\begin{aligned}
P(K_1 \cap K_2) &= P(K_1 \cap K_2|G_1, G_1)P(G_1, G_1) + P(K_1 \cap K_2|G_1, G_2)P(G_1, G_2) \\
&\quad + P(K_1 \cap K_2|G_2, G_1)P(G_2, G_1) + P(K_1 \cap K_2|G_2, G_2)P(G_2, G_2) \\
&= \left(\frac{1}{4}\right)\left(\frac{1}{12}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{9}\right)\left(\frac{5}{12}\right) \\
&= \left(\frac{15}{96}\right) \\
Thus & \\
P(K_1, K_2) &\neq P(K_1)P(K_2) \\
\left(\frac{15}{96}\right) &\neq \left(\frac{7}{18}\right)\left(\frac{7}{18}\right)
\end{aligned}$$