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UAS ALJABAR DAN KOMPUTASI NUMERIK (P)

1. *Hitung* ***Determinan****,* ***Invers Matrix*** *serta tentukan* ***Eigen Value*** *dan* ***Eigen Vector*** *pada Matriks dari Persamaan 3 variabel berikutnya ini;*

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

Jawaban :

(a) ***Determinan***

Mengubah persamaan kedalam bentuk matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left.\begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix}\right\} A$$

Menghitung determinan menggunakan metode sarrus

$$D = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{matrix} 1 & 1 \\ 2 & 4 \\ 3 & 6 \end{matrix}$$

$= (1{*}4{*}{-5}) + (1{*}{-3}{*}3) + (2{*}2{*}6) - (3{*}4{*}2) - (6{*}{-3}{*}1) - (-5{*}2{*}1) =$
$(-20) + (-9) + 24 - 24 - (-18) - (-10)$
$= -1$

Menempatkan A pada kolom I. untuk menghitung Determinan $D_{(x)}$

$$D_{(x)} = \begin{bmatrix} 9 & 1 & 2 \\ 1 & 4 & -3 \\ 0 & 6 & -5 \end{bmatrix} \begin{matrix} 9 & 1 \\ 1 & 4 \\ 0 & 6 \end{matrix}$$

$= (9{*}4{*}{-5}) + (1{*}{-3}{*}0) + (2{*}1{*}6) - (0{*}4{*}2) - (6{*}{-3}{*}9) - (-5{*}1{*}1) =$
$(-180) + 0 + 12 - 0 - (-162) - (-5)$
$= -1$

Menempatkan A pada kolom II. untuk menghitung Determinan$D_{(y)}$

$$D_{(y)} = \begin{bmatrix} 1 & 9 & 2 \\ 2 & 1 & -3 \\ 3 & 0 & -5 \end{bmatrix} \begin{matrix} 1 & 9 \\ 2 & 1 \\ 3 & 0 \end{matrix}$$

$= (1{*}1{*}{-5}) + (9{*}{-3}{*}3) + (2{*}2{*}0) - (3{*}1{*}2) - (0{*}{-3}{*}1) - (-5{*}2{*}9) =$
$(-5) + (-18) + 0 - 6 - 0 - (-90)$
$= -2$

Menempatkan A pada kolom III. untuk menghitung Determinan $D_{(z)}$

$$D_{(z)} = \begin{bmatrix} 1 & 1 & 9 \\ 2 & 4 & 1 \\ 3 & 6 & 0 \end{bmatrix} \begin{matrix} 1 & 1 \\ 2 & 4 \\ 3 & 6 \end{matrix}$$

$= (1*4*0) + (1*1*3) + (9*2*6) - (3*4*9) - (6*1*1) - (0*2*1)$
$= 0 + 3 + 108 - 108 - 6 - 0$
$= -3$
Menentukan nilai x, y, zdengan nilai $D, D_{(x)}, D_{(y)}, D_{(z)}$
$x = \frac{D_{(x)}}{D}$
$= \frac{-1}{-1}$
$= 1$
$y = \frac{D_{(y)}}{D}$
$= \frac{-2}{-1}$
$= 2$
$z = \frac{D_{(z)}}{D}$
$= \frac{-3}{-1}$
$= 3$
Himpunan penyelesaian $\begin{Bmatrix} x & y & z \end{Bmatrix}$dengan nilai $\begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix}$

(b) ***Invers***

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left.\begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix}\right\} B$$

$$Kof\,(A) = \begin{bmatrix} \begin{bmatrix} 4 & -3 \\ 6 & -5 \end{bmatrix} & -\begin{bmatrix} 2 & -3 \\ 3 & -5 \end{bmatrix} & \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \\ -\begin{bmatrix} 1 & 2 \\ 6 & -5 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} & -\begin{bmatrix} 1 & 1 \\ 3 & 6 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} & -\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 17 & -11 & -3 \\ -11 & 7 & 2 \end{bmatrix}$$

$$Adj\,(A) = Kof\,(A)^t = \begin{bmatrix} -2 & 1 & 0 \\ 17 & -11 & -3 \\ -11 & 7 & 2 \end{bmatrix}$$

$$X = \frac{1}{detA} Adj\,(A)\,B$$

$$\frac{1}{-1} \begin{bmatrix} -2 & 1 & 0 \\ 17 & -11 & -3 \\ -11 & 7 & 2 \end{bmatrix} \begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} -18 + 17 + 0 \\ 9 + (-11) + 0 \\ 0 + (-3) + 0 \end{bmatrix} = -1 \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Himpunan penyelesaian $\begin{Bmatrix} x & y & z \end{Bmatrix}$dengan nilai $\begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix}$

(c) ***Eigen Value dan Eigen Vector***

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

dengan $x = 1$,$y = 2$, dan $z = 3$

$$x + y + 2z = 9 \quad \rightarrow \quad (1) + (2) + 2\,(3) = 9$$
$$2x + 4y - 3z = 1 \quad \rightarrow \quad 2\,(1) + 4\,(2) - 3\,(3) = 1$$
$$3x + 6y - 5z = 0 \quad \rightarrow \quad 3\,(1) + 6\,(2) - 5\,(3) = 0$$

$$\left.\begin{array}{c} 1 + 2 + 6 = 9 \\ 2 + 8 - 9 = 1 \\ 3 + 12 - 15 = 0 \end{array}\right\} \begin{bmatrix} 1 & 2 & 6 \\ 2 & 8 & -9 \\ 3 & 12 & -15 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 8 & -9 \\ 3 & 12 & -15 \end{bmatrix}$$

Eigenvectors for the matrix A:

- $v \approx \begin{pmatrix} -0.642 \\ 0.568 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_1 \approx -10.113$
- $v \approx \begin{pmatrix} -24.336 \\ 7.347 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_2 \approx 0.150$
- $v \approx \begin{pmatrix} 2.645 \\ 0.568 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_3 \approx 3.963$

Details :

1. From the definition of eigenvector v corresponding to the eigenvalue λ we have $A\lambda = \lambda v$

Then : $Av - \lambda v = (A - \lambda I) \centerdot v = 0$

Equation has a nonzero solution if and only if $det(A - \lambda I) = 0$

$$det(A - \lambda I) = \begin{bmatrix} 1-\lambda & 2 & 6 \\ 2 & 8-\lambda & -9 \\ 3 & 12 & -15-\lambda \end{bmatrix}$$

$$= -\lambda^3 - 6\lambda^2 + 41\lambda - 6 \approx -(\lambda + (10.113)) \centerdot (\lambda - (0.150)) \centerdot (\lambda - (3.963)) = 0$$

Details using Triangle's Rule, Rule of Sarrus, Montante's method (Bareiss algorithm)), and Gaussian elimination.

So,

1. $\lambda_1 \approx -10.113$
2. $\lambda_2 \approx 0.150$
3. $\lambda_3 \approx 3.963$

2. For every λ we find its own vectors:

1) $\lambda_1 \approx -10.113$

$$A - \lambda_1 I \approx \begin{bmatrix} 11.113 & 2 & 6 \\ 2 & 18.113 & -9 \\ 3 & 12 & -4.887 \end{bmatrix}$$

$Av = \lambda v$

$(A - \lambda I)\, v = 0$

So, we have a homogenous system of linear equations, we solve it by Gaussian Elimination:

$$\left[\begin{array}{ccc|c} 11.113 & 2 & 6 & 0 \\ 2 & 18.113 & -9 & 0 \\ 3 & 12 & -4.887 & 0 \end{array}\right] x\,(0.090)$$

- Divide row 1 by 11.113

$$\frac{R_1}{(11.113)} \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 0.180 & 0.540 & 0 \\ 2 & 18.113 & -9 & 0 \\ 3 & 12 & -4.887 & 0 \end{array}\right] x\,(-2)$$

- Substract 2 x row 1 from row 2 :

$$R_2 - 2\,.\,R_1 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 0.180 & 0.540 & 0 \\ 0 & 17.753 & -10.080 & 0 \\ 3 & 12 & -4.887 & 0 \end{array}\right] x\,(-3)$$

- Substract 3 x row 1 from row 3 :

$$R_3 - 3\,.\,R_1 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 0.180 & 0.540 & 0 \\ 0 & 17.753 & -10.080 & 0 \\ 0 & 11.460 & -6.507 & 0 \end{array}\right] x\,(0.056)$$

- Divine row 1 by 17.753 :

$$\frac{R_2}{(17.753)} \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 0.180 & 0.540 & 0 \\ 0 & 1 & -0.568 & 0 \\ 0 & 11.460 & -6.507 & 0 \end{array}\right] x\,(-11.460)$$

- Substract 11.460 x row 2 from row 3 :

$$R_3 - 11.460\,.\,R_2 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 0.180 & 0.540 & 0 \\ 0 & 1 & -0.568 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] x\,(-0.180)$$

- Substract 0.180 x row 2 from row 1 :

$$R_1 - 0.180\,.\,R_2 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0.642 & 0 \\ 0 & 1 & -0.568 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + 0.642\,.\,x_3 = 0 \\ x_2 - 0.568\,.\,x_3 = 0 \end{cases}$$

- Find the variable x_2 from the equation 2 of the system (1): $x_2 = 0.568 \,.\, x_3$
- Find the variable x_1 from the equation 1 of the system (1): $x_1 = -0.642 \,.\, x_3$

Answer :

$$x_1 = -0.642x_3$$
$$x_2 = 0.568x_3$$
$$x_3 = x_3$$

General Solution: $X = \begin{bmatrix} -0.642x_3 \\ 0.568x_3 \\ x_3 \end{bmatrix}$

The solution set: $\left\{ x_3 \,.\, \begin{bmatrix} -0.642 \\ 0.568 \\ 1 \end{bmatrix} \right\}$

Let $x_3 = 1$, $\nu_1 \approx \begin{bmatrix} -0.642 \\ 0.568 \\ 1 \end{bmatrix}$

2) $\lambda_2 \approx 0.150$

$A - \lambda_2 I \approx \begin{bmatrix} 0.850 & 2 & 6 \\ 2 & 7.850 & -9 \\ 3 & 12 & -15.150 \end{bmatrix}$

$Av = \lambda v$

$(A - \lambda I)\, v = 0$

So, we have a homogenous system of linear equations, we solve it by Gaussian Elimination:

$$\left[\begin{array}{ccc|c} 0.850 & 2 & 6 & 0 \\ 2 & 7.850 & -9 & 0 \\ 3 & 12 & -15.150 & 0 \end{array}\right] x\,(1.176)$$

- Divide row 1 by 0.850

$$\frac{R_1}{(0.850)} \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 2.352 & 7.056 & 0 \\ 2 & 7.850 & -9 & 0 \\ 3 & 12 & -15.150 & 0 \end{array}\right] x\,(-2)$$

- Substract 2 x row 1 from row 2 :

$$R_2 - 2 \,.\, R_1 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 2.352 & 7.056 & 0 \\ 0 & 3.146 & -23.113 & 0 \\ 3 & 12 & -15.150 & 0 \end{array}\right] x\,(-3)$$

- Substract 3 x row 1 from row 3 :

$$R_3 - 3 \,.\, R_1 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 2.352 & 7.056 & 0 \\ 0 & 3.146 & -23.113 & 0 \\ 0 & 4.944 & -36.319 & 0 \end{array}\right] x\,(0.318)$$

- Divine row 2 by 3.146 :

$$\frac{R_2}{(3.146)} \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 2.352 & 7.056 & 0 \\ 0 & 1 & -7.347 & 0 \\ 0 & 4.944 & -36.319 & 0 \end{array}\right] x\,(-4.944)$$

- Substract 4.944 x row 2 from row 3 :

$$R_3 - 4.944\,.\,R_2 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 2.352 & 7.056 & 0 \\ 0 & 1 & -7.347 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] x\,(-2.352)$$

- Substract 2.352 x row 2 from row 1 :

$$R_1 - 2.352\,.\,R_2 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 24.336 & 0 \\ 0 & 1 & -7.347 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + 24.336\,.\,x_3 = 0 \\ x_2 - 7.347\,.\,x_3 = 0 \end{cases}$$

  - Find the variable x_2 from the equation 2 of the system (1): $x_2 = 7.347\,.\,x_3$
  - Find the variable x_1 from the equation 1 of the system (1): $x_1 = -24.336\,.\,x_3$

Answer :

$$x_1 = -24.336x_3$$
$$x_2 = 7.347x_3$$
$$x_3 = x_3$$

General Solution: $X = \begin{bmatrix} -24.336x_3 \\ 7.347x_3 \\ x_3 \end{bmatrix}$

The solution set: $\left\{ x_3\,.\begin{bmatrix} -0.642 \\ 0.568 \\ 1 \end{bmatrix} \right\}$

Let $x_3 = 1$, $\nu_2 \approx \begin{bmatrix} -0.642 \\ 0.568 \\ 1 \end{bmatrix}$

3) $\lambda_3 \approx 3.963$

$A - \lambda_1 I \approx \begin{bmatrix} -2.963 & 2 & 6 \\ 2 & 4.037 & -9 \\ 3 & 12 & -18.963 \end{bmatrix}$

$Av = \lambda v$

$(A - \lambda I)\,v = 0$

So, we have a homogenous system of linear equations, we solve it by Gaussian Elimination:

$$\left[\begin{array}{ccc|c} -2.963 & 2 & 6 & 0 \\ 2 & 4.037 & -9 & 0 \\ 3 & 12 & -18.963 & 0 \end{array}\right] x\,(-0.337)$$

- Divide row 1 by -2.963 :

$$\frac{R_1}{(-2.963)} \to R_1 \quad \left[\begin{array}{ccc|c} 1 & -0.675 & -2.025 & 0 \\ 2 & 7.850 & -9 & 0 \\ 3 & 12 & -15.150 & 0 \end{array}\right] x\,(-2)$$

- Substract 2 x row 1 from row 2 :

$$R_2 - 2\,.\,R_1 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -0.675 & -2.025 & 0 \\ 0 & 5.287 & -4.950 & 0 \\ 3 & 12 & -18.963 & 0 \end{array}\right] x\,(-3)$$

- Substract 3 x row 1 from row 3 :

$$R_3 - 3\,.\,R_1 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -0.675 & -2.025 & 0 \\ 0 & 5.387 & -4.950 & 0 \\ 0 & 14.025 & -12.889 & 0 \end{array}\right] x\,(0.186)$$

- Divine row 1 by 5.387 :

$$\frac{R_2}{(3.146)} \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -0.675 & -2.025 & 0 \\ 0 & 1 & -0.919 & 0 \\ 0 & 14.025 & -12.889 & 0 \end{array}\right] x\,(-14.025)$$

- Substract 14.025 x row 2 from row 3 :

$$R_3 - 14.025.R_2 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -0.675 & -2.025 & 0 \\ 0 & 1 & -0.919 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] x\,(-0.180)$$

- Substract -0.675 x row 2 from row 1 :

$$R_1 = -0.675\,.\,R_2 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & -2.645 & 0 \\ 0 & 1 & -0.919 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 - 2.645\,.\,x_3 = 0 \\ x_2 - 0.919\,.\,x_3 = 0 \end{cases}$$

  - Find the variable x_2 from the equation 2 of the system (1): $x_2 = 0.919\,.\,x_3$
  - Find the variable x_1 from the equation 1 of the system (1): $x_1 = 2.645\,.\,x_3$

  Answer :

$$x_1 = 2.645x_3$$
$$x_2 = 0.919x_3$$
$$x_3 = x_3$$

  General Solution: $X = \begin{bmatrix} 2.645x_3 \\ 0.919x_3 \\ x_3 \end{bmatrix}$

The solution set: $\left\{ x_3 \cdot \begin{bmatrix} 2.645 \\ 0.919 \\ 1 \end{bmatrix} \right\}$

Let $x_3 = 1$, $\nu_1 \approx \begin{bmatrix} 2.645 \\ 0.919 \\ 1 \end{bmatrix}$

2. *Hitung* ***Determinan***, ***Invers Matrix*** *serta tentukan* ***Eigen Value*** *dan* ***Eigen Vector*** *pada Matriks dari Persamaan 3 variabel berikutnya ini;*

$$2x + 3y - z = 6$$

$$x + 2y - 4z = 8$$

$$x + y + 4z = 4$$

Jawaban :

(a) ***Determinan***

Mengubah persamaan kedalam bentuk matrix

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & -4 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left.\begin{bmatrix} 6 \\ 8 \\ 4 \end{bmatrix}\right\} A$$

Menghitung determinan menggunakan metode sarrus

$$D = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & -4 \\ 1 & 1 & 4 \end{bmatrix} \begin{matrix} 2 & 3 \\ 1 & 2 \\ 1 & 1 \end{matrix}$$

$= (2{*}2{*}4) + (3{*}-4{*}1) + (-1{*}1{*}1) - (1{*}2{*}-1) - (1{*}-4{*}2) - (4{*}1{*}3)$
$= 16 + (-12) + (-1) - (-2) - (-8) - 12$
$= 1$

Menempatkan A pada kolom I. untuk menghitung Determinan $D_{(x)}$

$$D_{(x)} = \begin{bmatrix} 6 & 3 & -1 \\ 8 & 2 & -4 \\ 4 & 1 & 4 \end{bmatrix} \begin{matrix} 6 & 3 \\ 8 & 2 \\ 4 & 1 \end{matrix}$$

$= (6{*}2{*}4) + (3{*}-4{*}4) + (-1{*}8{*}1) - (4{*}2{*}-1) - (1{*}-4{*}6) - (4{*}8{*}3)$
$= 48 + (-48) + (-8) - (-8) - (-24) - 96$
$= -72$

Menempatkan A pada kolom II. untuk menghitung Determinan$D_{(y)}$

$$D_{(y)} = \begin{bmatrix} 2 & 6 & -1 \\ 1 & 8 & -4 \\ 1 & 4 & 4 \end{bmatrix} \begin{matrix} 2 & 6 \\ 1 & 8 \\ 1 & 4 \end{matrix}$$

$= (2{*}8{*}4) + (6{*}-4{*}1) + (-1{*}2{*}4) - (1{*}8{*}-1) - (4{*}-4{*}2) - (4{*}1{*}6)$
$= 64 + (-24) + (-4) - (-8) - (-32) - 24$
$= 52$

Menempatkan A pada kolom III. untuk menghitung Determinan $D_{(z)}$

$$D_{(z)} = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 2 & 8 \\ 1 & 1 & 4 \end{bmatrix} \begin{matrix} 2 & 3 \\ 1 & 2 \\ 1 & 1 \end{matrix}$$

$= (2*2*4)+(3*8*1)+(6*1*1)-(1*2*6)-(1*8*2)-(4*1*3)$
$= 16+24+6-12-16-12$
$= 6$
Menentukan nilai x, y, zdengan nilai $D, D_{(x)}, D_{(y)}, D_{(z)}$
$x = \frac{D_{(x)}}{D}$
$= \frac{-72}{1}$
$= -72$
$y = \frac{D_{(y)}}{D}$
$= \frac{52}{1}$
$= 52$
$z = \frac{D_{(z)}}{D}$
$= \frac{6}{1}$
$= 6$
Himpunan penyelesaian $\{x \quad y \quad z\}$dengan nilai $\{-72 \quad 52 \quad 6\}$

(b) ***Invers***

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & -4 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left.\begin{bmatrix} 6 \\ 8 \\ 4 \end{bmatrix}\right\} B$$

$$Kof\,(A) = \begin{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & 4 \end{bmatrix} & -\begin{bmatrix} 1 & -4 \\ 1 & 4 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \\ -\begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix} & \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} & -\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 3 & -1 \\ 2 & -4 \end{bmatrix} & -\begin{bmatrix} 2 & -1 \\ 1 & -4 \end{bmatrix} & \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 12 & -8 & -1 \\ -13 & 9 & 1 \\ -10 & 7 & 1 \end{bmatrix}$$

$$Adj\,(A) = Kof\,(A)^t = \begin{bmatrix} 12 & -13 & -10 \\ -8 & 9 & 7 \\ -1 & 1 & 1 \end{bmatrix}$$

$$X = \frac{1}{detA} Adj\,(A)\,B$$

$$\frac{1}{1} \begin{bmatrix} 12 & -13 & -10 \\ -8 & 9 & 7 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 4 \end{bmatrix} = 1 \begin{bmatrix} 72-104-40 \\ -48+72+28 \\ -6+8+4 \end{bmatrix} = 1 \begin{bmatrix} -72 \\ 52 \\ 6 \end{bmatrix} = \begin{bmatrix} -72 \\ 52 \\ 6 \end{bmatrix}$$

Himpunan penyelesaian $\{x \quad y \quad z\}$dengan nilai $\{-72 \quad 52 \quad 6\}$

(c) ***Eigen Value dan Eigen Vector***

$$2x + 3y - z = 6$$
$$x + 2y - 4z = 8$$
$$x + y + 4z = 4$$

dengan $x = -72$,$y = 52$, dan $z = 6$

$$2x + 3y - z = 6 \quad \to \quad 2(-72) + 3(52) - (6) = 6$$
$$x + 2y - 4z = 8 \quad \to \quad (-72) + 2(52) - 4(6) = 8$$
$$x + y + 4z = 4 \quad \to \quad (-72) + (52) + 4(6) = 4$$

$$\left.\begin{array}{c} -144 + 156 - 6 = 6 \\ -72 + 104 - 24 = 8 \\ -72 + 52 + 24 = 4 \end{array}\right\} \quad \begin{bmatrix} -144 & 156 & -6 \\ -72 & 104 & -24 \\ -72 & 52 & 24 \end{bmatrix}$$

$$A = \begin{bmatrix} -144 & 156 & -6 \\ -72 & 104 & -24 \\ -72 & 52 & 24 \end{bmatrix}$$

Eigenvectors for the matrix A:

- $v \approx \begin{pmatrix} 2.050 \\ 0.968 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_1 \approx -73.258$
- $v \approx \begin{pmatrix} 0.910 \\ 0.913 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_2 \approx 5.980$
- $v \approx \begin{pmatrix} -3.660 \\ -4.543 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_3 \approx 51.278$

Details :

1. From the definition of eigenvector v corresponding to the eigenvalue λ we have $A\lambda = \lambda v$

Then : $Av - \lambda v = (A - \lambda I) \cdot v = 0$

Equation has a nonzero solution if and only if $det(A - \lambda I) = 0$

$$det(A - \lambda I) = \begin{bmatrix} -144 - \lambda & 156 & -6 \\ -72 & 104 - \lambda & -24 \\ -72 & 52 & 24 - \lambda \end{bmatrix}$$

$$= -\lambda^3 - 16\lambda^2 + 3888\lambda - 22464 \approx -(\lambda + (73.258)) \cdot (\lambda - (5.980)) \cdot (\lambda - (51.278)) = 0$$

Details using Triangle's Rule, Rule of Sarrus, Montante's method (Bareiss algorithm)), and Gaussian elimination.

So,

1. $\lambda_1 \approx -73.258$
2. $\lambda_2 \approx 5.980$
3. $\lambda_3 \approx 51.278$

2. For every λ we find its own vectors:

1) $\lambda_1 \approx -73.258$

$$A - \lambda_1 I \approx \begin{bmatrix} -70.742 & 156 & -6 \\ -72 & 177.258 & -24 \\ -72 & 52 & 97.258 \end{bmatrix}$$

$Av = \lambda v$

$(A - \lambda I)\, v = 0$

So, we have a homogenous system of linear equations, we solve it by Gaussian Elimination:

$$\left[\begin{array}{ccc|c} -70.742 & 156 & -6 & 0 \\ -72 & 177.258 & -24 & 0 \\ -72 & 52 & 97.258 & 0 \end{array}\right] x\,(-0.014)$$

- Divide row 1 by 11.113

$$\frac{R_1}{(-70.742)} \to R_1 \quad \left[\begin{array}{ccc|c} 1 & -2.205 & 0.085 & 0 \\ -72 & 177.258 & -24 & 0 \\ -72 & 52 & 97.258 & 0 \end{array}\right] x\,(72)$$

- Substract 2 x row 1 from row 2 :

$$R_2 - (-72)\mathbf{.}R_1 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -2.205 & 0.085 & 0 \\ 0 & 18.484 & -17.893 & 0 \\ -72 & 52 & 97.258 & 0 \end{array}\right] x\,(72)$$

- Substract 3 x row 1 from row 3 :

$$R_3 - (-72)\mathbf{.}R_1 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -2.205 & 0.085 & 0 \\ 0 & 18.484 & -17.893 & 0 \\ 0 & -106.775 & 103.365 & 0 \end{array}\right] x\,(0.054)$$

- Divine row 1 by 18.484 :

$$\frac{R_2}{(18.484)} \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -2.205 & 0.085 & 0 \\ 0 & 1 & -0.968 & 0 \\ 0 & -106.775 & 103.365 & 0 \end{array}\right] x\,(106.775)$$

- Substract 106.775 x row 2 from row 3 :

$$R_3 - 106.775\mathbf{.}R_2 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -2.205 & 0.085 & 0 \\ 0 & 1 & -0.968 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] x\,(2.205)$$

- Substract 2.205 x row 2 from row 1 :

$$R_1 - 2.205\,.\,R_2 \rightarrow R_1 \qquad \left[\begin{array}{ccc|c} 1 & 0 & -2.050 & 0 \\ 0 & 1 & -0.968 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 - 2.050\,.\,x_3 = 0 \\ x_2 - 0.968\,.\,x_3 = 0 \end{cases}$$

  - Find the variable x_2 from the equation 2 of the system (1): $x_2 = 0.968\,.\,x_3$
  - Find the variable x_1 from the equation 1 of the system (1): $x_1 = 2.050\,.\,x_3$

  Answer :

  $x_1 = 2.050x_3$
  $x_2 = 0.968x_3$
  $x_3 = x_3$

  General Solution: $X = \begin{bmatrix} 2.050x_3 \\ 0.968x_3 \\ x_3 \end{bmatrix}$

  The solution set: $\left\{ x_3\,.\begin{bmatrix} 2.050 \\ 0.968 \\ 1 \end{bmatrix} \right\}$

  Let $x_3 = 1$, $\nu_1 \approx \begin{bmatrix} 2.050 \\ 0.968 \\ 1 \end{bmatrix}$

2) $\lambda_2 \approx 5.980$

$A - \lambda_2 I \approx \begin{bmatrix} -149.980 & 156 & -6 \\ -72 & 98.020 & -24 \\ -72 & 52 & 18.020 \end{bmatrix}$

$Av = \lambda v$

$(A - \lambda I)\,v = 0$

So, we have a homogenous system of linear equations, we solve it by Gaussian Elimination:

$$\left[\begin{array}{ccc|c} -149.980 & 156 & -6 & 0 \\ -72 & 98.020 & -24 & 0 \\ -72 & 52 & 18.020 & 0 \end{array}\right] x\,(-0.007)$$

- Divide row 1 by -149.980

$$\frac{R_1}{(-149.980)} \rightarrow R_1 \qquad \left[\begin{array}{ccc|c} 1 & -1.040 & 0.040 & 0 \\ -72 & 98.020 & -24 & 0 \\ -72 & 52 & 18.020 & 0 \end{array}\right] x\,(72)$$

- Substract 2 x row 1 from row 2 :

$$R_2 - (-72)\,.\,R_1 \rightarrow R_2 \qquad \left[\begin{array}{ccc|c} 1 & -1.040 & 0.040 & 0 \\ 0 & 23.130 & -21.120 & 0 \\ -72 & 52 & 18.020 & 0 \end{array}\right] x\,(72)$$

- Substract 3 x row 1 from row 3 :

$$R_3 - (-72).R_1 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -1.040 & 0.040 & 0 \\ 0 & 23.130 & -21.120 & 0 \\ 0 & -22.890 & 20.900 & 0 \end{array}\right] x\,(0.043)$$

- Divine row 2 by 23.130 :

$$\frac{R_2}{(23.130)} \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -1.040 & 0.040 & 0 \\ 0 & 1 & -0.913 & 0 \\ 0 & -22.890 & 20.900 & 0 \end{array}\right] x\,(22.890)$$

- Substract 22.890 x row 2 from row 3 :

$$R_3 - 22.890.R_2 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -1.040 & 0.040 & 0 \\ 0 & 1 & -0.913 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] x\,(1.040)$$

- Substract 1.040 x row 2 from row 1 :

$$R_1 - 1.040 . R_2 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & -0.910 & 0 \\ 0 & 1 & -0.913 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 - 0.910 . x_3 = 0 \\ x_2 - 0.913 . x_3 = 0 \end{cases}$$

  - Find the variable x_2 from the equation 2 of the system (1): $x_2 = 0.913 . x_3$
  - Find the variable x_1 from the equation 1 of the system (1): $x_1 = 0.910 . x_3$

Answer :

$$x_1 = 0.910x_3$$
$$x_2 = 0.913x_3$$
$$x_3 = x_3$$

General Solution: $X = \begin{bmatrix} 0.910x_3 \\ 0.913x_3 \\ x_3 \end{bmatrix}$

The solution set: $\left\{ x_3 . \begin{bmatrix} 0.910 \\ 0.913 \\ 1 \end{bmatrix} \right\}$

Let $x_3 = 1$, $\nu_2 \approx \begin{bmatrix} 0.910 \\ 0.913 \\ 1 \end{bmatrix}$

3) $\lambda_3 \approx 51.278$

$$A - \lambda_1 I \approx \begin{bmatrix} -195.278 & 156 & -6 \\ -72 & 52.722 & -24 \\ -72 & 52 & -27.278 \end{bmatrix}$$

$Av = \lambda v$

$(A - \lambda I)\, v = 0$

So, we have a homogenous system of linear equations, we solve it by Gaussian Elimination:

$$\left[\begin{array}{ccc|c} -195.278 & 156 & -6 & 0 \\ -72 & 52.722 & -24 & 0 \\ -72 & 52 & -27.278 & 0 \end{array}\right] x\,(-0.005)$$

- Divide row 1 by -195.278 :

$$\frac{R_1}{(-195.278)} \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & -0.799 & 0.031 & 0 \\ -72 & 52.722 & -24 & 0 \\ -72 & 52 & -27.278 & 0 \end{array}\right] x\,(72)$$

- Substract 2 x row 1 from row 2 :

$$R_2 - (-72).R_1 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & -0.799 & 0.031 & 0 \\ 0 & -4.796 & -21.788 & 0 \\ -72 & 52 & -27.278 & 0 \end{array}\right] x\,(72)$$

- Substract 3 x row 1 from row 3 :

$$R_3 - (-72).R_1 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & -0.799 & 0.031 & 0 \\ 0 & -4.796 & -21.788 & 0 \\ 0 & -5.518 & -25.066 & 0 \end{array}\right] x\,(0.208)$$

- Divine row 1 by -4.796 :

$$\frac{R_2}{(-4.796)} \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & -0.799 & 0.031 & 0 \\ 0 & 1 & 4.543 & 0 \\ 0 & -5.518 & -25.066 & 0 \end{array}\right] x\,(5.518)$$

- Substract -5.518 x row 2 from row 3 :

$$R_3 - (-5.518).R_2 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & -0.799 & 0.031 & 0 \\ 0 & 1 & 4.543 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] x\,(-0.799)$$

- Substract -0.799 x row 2 from row 1 :

$$R_1 = -\,(-0.799)\,.\,R_2 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 3.660 & 0 \\ 0 & 1 & 4.543 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + 3.660\,.\,x_3 = 0 \\ x_2 + 4.543\,.\,x_3 = 0 \end{cases}$$

  - Find the variable x_2 from the equation 2 of the system (1): $x_2 = -4.543\,.\,x_3$

– Find the variable x_1 from the equation 1 of the system (1):
$x_1 = -3.660 \cdot x_3$

Answer :

$$x_1 = -3.660x_3$$
$$x_2 = -4.543x_3$$
$$x_3 = x_3$$

General Solution: $X = \begin{bmatrix} -3.660x_3 \\ -4.543x_3 \\ x_3 \end{bmatrix}$

The solution set: $\left\{ x_3 \cdot \begin{bmatrix} -3.660 \\ -4.543 \\ 1 \end{bmatrix} \right\}$

Let $x_3 = 1$, $\nu_1 \approx \begin{bmatrix} -3.660 \\ -4.543 \\ 1 \end{bmatrix}$

3. Syntax C++ untuk hitung determinan berikut implementasikan untuk 2 Matriks pada soal sebelumnya.

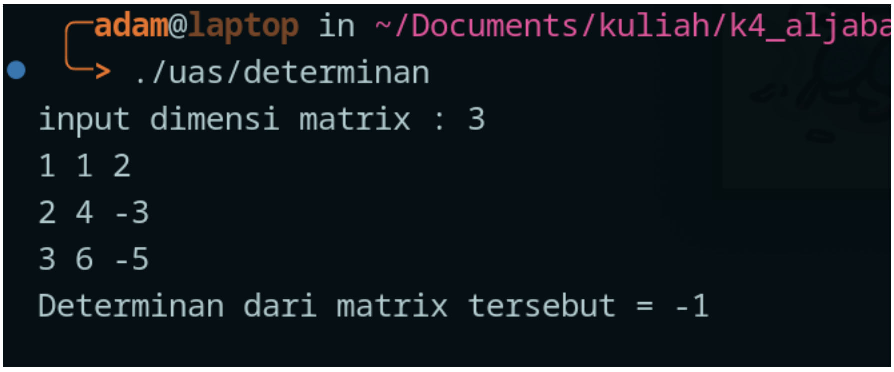
```
#include <iostream>

using std::cin;
using std::cout;
using std::endl;

int **submatrix(int **matrix, unsigned int n,
    unsigned int x, unsigned int y) {
  int **submatrix = new int *[n - 1];
  int subi = 0;
  for (int i = 0; i < n; i++) {
    submatrix[subi] = new int[n - 1];
    int subj = 0;
    if (i == y) {
      continue;
    }
    for (int j = 0; j < n; j++) {
      if (j == x) {
        continue;
      }
      submatrix[subi][subj] = matrix[i][j];
      subj++;
    }
    subi++;
  }
  return submatrix;
}

int determinant(int **matrix, unsigned int n) {
  int det = 0;
  if (n == 2) {
```

```cpp
    return matrix[0][0] * matrix[1][1] - matrix[1][0]
        * matrix[0][1];
  }
  for (int x = 0; x < n; ++x) {
    det += ((x % 2 == 0 ? 1 : -1) * matrix[0][x] *
        determinant(submatrix(matrix, n, x, 0), n - 1
        ));
  }
  return det;
}

int main() {
  int n;
  cout << "input dimensi matrix : ";
  cin >> n;
  int **matrix = new int *[n];
  for (int i = 0; i < n; ++i) {
    matrix[i] = new int[n];
    for (int j = 0; j < n; ++j) {
      cin >> matrix[i][j];
    }
  }

  cout << "Determinan dari matrix tersebut = " <<
      determinant(matrix, n);
  cout << endl << endl;

  return 0;
}
```

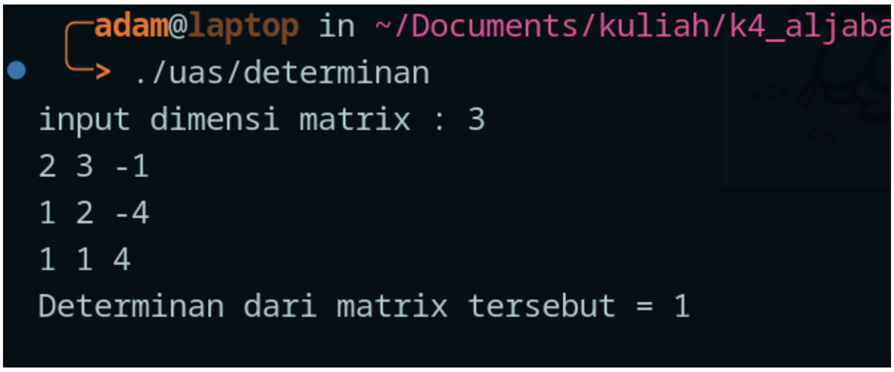
Listing 1: determinan

Hasil output :

(a) soal no 1



(b) soal no 2



4. Syntax C++ untuk Invers matriks dan implementasikan untuk hitung matrik disoal nomor 1 dan 2

```
#include <iostream>
#include <vector>

using namespace std;

// Function to print matrix
void printMatrix(const vector<vector<int>>& matrix) {
  for (const auto& row : matrix) {
    for (int element : row) {
      cout << element << " ";
    }
    cout << endl;
  }
}

// Function to calculate determinant of 2x2 matrix
int determinant2x2(const vector<vector<int>>& matrix)
    {
  return matrix[0][0] * matrix[1][1] - matrix[1][0] *
      matrix[0][1];
}

// Function to find submatrix
vector<vector<int>> submatrix(const vector<vector<int
   >>& matrix, unsigned int x, unsigned int y) {
  vector<vector<int>> submatrix;
  for (unsigned int i = 0; i < matrix.size(); ++i) {
    if (i == y) {
      continue;
    }
    vector<int> row;
    for (unsigned int j = 0; j < matrix.size(); ++j)
        {
      if (j == x) {
        continue;
```

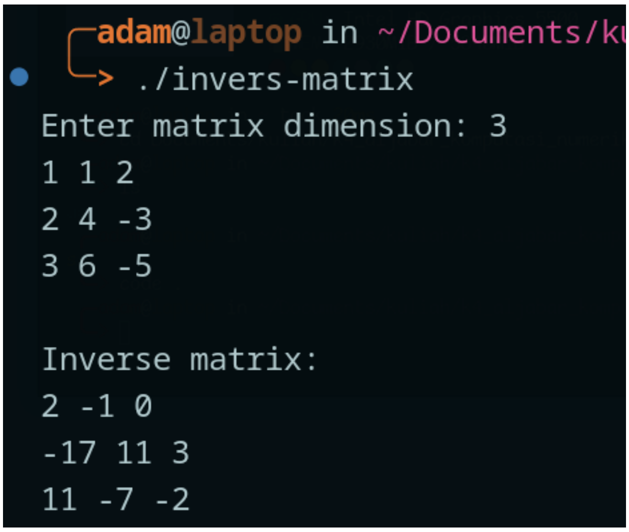
```
      }
      row.push_back(matrix[i][j]);
    }
    submatrix.push_back(row);
  }
  return submatrix;
}

// Function to calculate determinant of matrix
int determinant(const vector<vector<int>>& matrix) {
  int n = matrix.size();
  if (n == 2) {
    return determinant2x2(matrix);
  }
  int det = 0;
  for (unsigned int x = 0; x < matrix.size(); ++x) {
    vector<vector<int>> sub = submatrix(matrix, x, 0)
        ;
    det += ((x % 2 == 0 ? 1 : -1) * matrix[0][x] *
        determinant(sub));
  }
  return det;
}

// Function to find inverse matrix
bool inverseMatrix(const vector<vector<int>>& matrix,
    vector<vector<int>>& inverse) {
  int det = determinant(matrix);
  if (det == 0) {
    cout << "Matrix does not have an inverse." <<
        endl;
    return false;
  }

  int n = matrix.size();
  inverse.resize(n, vector<int>(n));

  vector<vector<int>> cofactor(n, vector<int>(n));
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      vector<vector<int>> sub = submatrix(matrix, j,
          i);
      cofactor[i][j] = ((i + j) % 2 == 0 ? 1 : -1) *
          determinant(sub);
    }
  }

  // Transpose the cofactor matrix
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
```

```cpp
      inverse[i][j] = cofactor[j][i];
    }
  }

  // Divide by the determinant
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      inverse[i][j] /= det;
    }
  }

  return true;
}

int main() {
  unsigned int n;
  cout << "Enter matrix dimension: ";
  cin >> n;

  vector<vector<int>> matrix(n, vector<int>(n));
  for (unsigned int j = 0; j < n; ++j) {
    for (unsigned int i = 0; i < n; ++i) {
      cin >> matrix[i][j];
    }
  }

  cout << endl;

  vector<vector<int>> inverse;
  if (inverseMatrix(matrix, inverse)) {
    cout << "Inverse matrix:" << endl;
    printMatrix(inverse);
  }

  return 0;
}
```

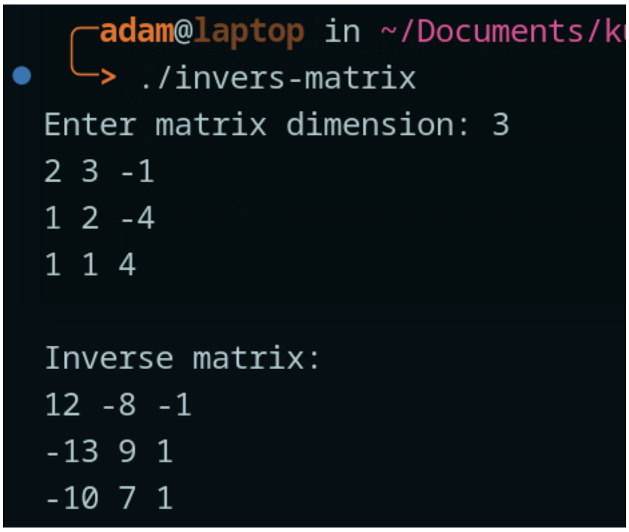
Listing 2: invers-matrix

Hasil Output :

(a) soal no 1 :



(b) soal no 2 :



5. Syntax C++ untuk hitung Eigen Value dan Eigen Vector

```
#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

void eigen(const vector<vector<int>>& matrix, vector<
    double>& eigenvalues, vector<vector<double>>&
    eigenvectors) {
  unsigned int n = matrix.size();

  // Create an identity matrix of the same dimension
     as the input matrix
```

```
vector<vector<double>> identity(n, vector<double>(n
    , 0.0));
for (unsigned int i = 0; i < n; ++i) {
  identity[i][i] = 1.0;
}

// Convert the input matrix to a double precision
    matrix
vector<vector<double>> doubleMatrix(n, vector<
    double>(n, 0.0));
for (unsigned int i = 0; i < n; ++i) {
  for (unsigned int j = 0; j < n; ++j) {
    doubleMatrix[i][j] = static_cast<double>(matrix
        [i][j]);
  }
}

// Perform the power iteration method
unsigned int maxIterations = 100;
double epsilon = 1e-8;

eigenvectors = identity;  // Initial guess for
    eigenvectors

for (unsigned int iteration = 0; iteration <
    maxIterations; ++iteration) {
  // Multiply the matrix with the eigenvectors
  vector<vector<double>> multipliedMatrix(n, vector
      <double>(n, 0.0));
  for (unsigned int i = 0; i < n; ++i) {
    for (unsigned int j = 0; j < n; ++j) {
      for (unsigned int k = 0; k < n; ++k) {
        multipliedMatrix[i][j] += doubleMatrix[i][k
            ] * eigenvectors[k][j];
      }
    }
  }

  // Find the maximum element of the multiplied
      matrix
  double maxElement = 0.0;
  for (unsigned int i = 0; i < n; ++i) {
    for (unsigned int j = 0; j < n; ++j) {
      maxElement = max(maxElement, abs(
          multipliedMatrix[i][j]));
    }
  }

  // Divide the multiplied matrix by the maximum
      element
```

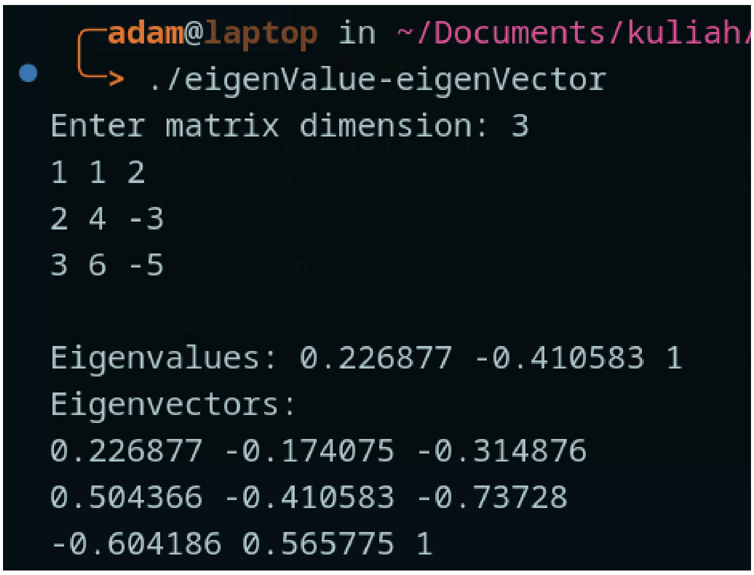
```
    for (unsigned int i = 0; i < n; ++i) {
      for (unsigned int j = 0; j < n; ++j) {
        multipliedMatrix[i][j] /= maxElement;
      }
    }

    // Calculate the difference between the
        multiplied matrix and the eigenvectors
    vector<vector<double>> diff(n, vector<double>(n,
        0.0));
    for (unsigned int i = 0; i < n; ++i) {
      for (unsigned int j = 0; j < n; ++j) {
        diff[i][j] = multipliedMatrix[i][j] -
            eigenvectors[i][j];
      }
    }

    // Check if the difference is within the desired
        epsilon
    double diffNorm = 0.0;
    for (unsigned int i = 0; i < n; ++i) {
      for (unsigned int j = 0; j < n; ++j) {
        diffNorm += diff[i][j] * diff[i][j];
      }
    }
    diffNorm = sqrt(diffNorm);

    if (diffNorm < epsilon) {
      break;  // Converged, exit the loop
    }

    eigenvectors = multipliedMatrix;  // Update the
        eigenvectors for the next iteration
  }

  // Calculate eigenvalues from the final
      eigenvectors
  eigenvalues.resize(n);
  for (unsigned int i = 0; i < n; ++i) {
    eigenvalues[i] = eigenvectors[i][i];
  }
}

int main() {
  unsigned int n;
  cout << "Enter matrix dimension: ";
  cin >> n;

  vector<vector<int>> matrix(n, vector<int>(n));
  for (unsigned int j = 0; j < n; ++j) {
```

```cpp
    for (unsigned int i = 0; i < n; ++i) {
      cin >> matrix[i][j];
    }
  }

  vector<double> eigenvalues;
  vector<vector<double>> eigenvectors;

  eigen(matrix, eigenvalues, eigenvectors);

  cout << endl << "Eigenvalues: ";
  for (double eigenvalue : eigenvalues) {
    cout << eigenvalue << " ";
  }
  cout << endl;

  cout << "Eigenvectors:" << endl;
  for (const auto& eigenvector : eigenvectors) {
    for (double element : eigenvector) {
      cout << element << " ";
    }
    cout << endl;
  }

  return 0;
}
```

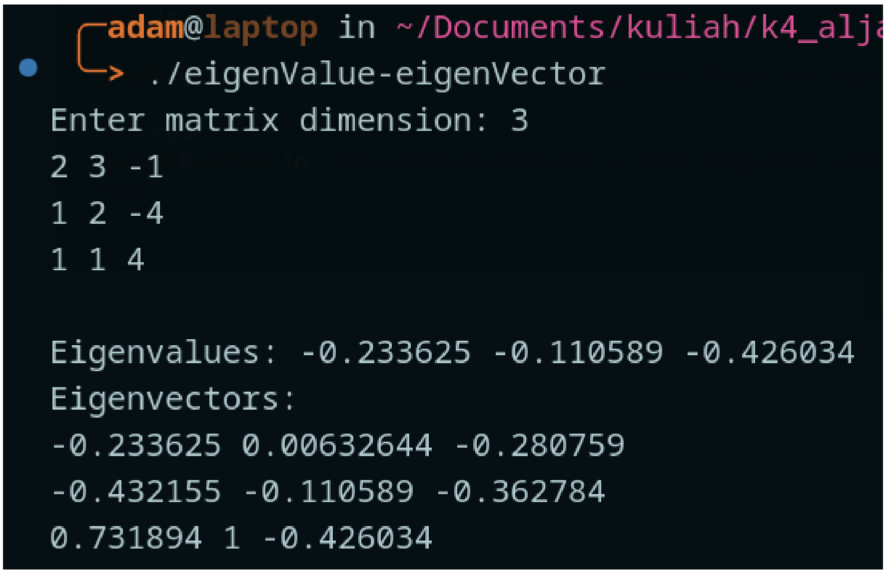
Listing 3: eigenValueNVector

Hasil Output :

(a) soal no 1 :



(b) soal no 2 :



```
╭─adam@laptop in ~/Documents/kuliah/k4_alja
╰─> ./eigenValue-eigenVector
Enter matrix dimension: 3
2 3 -1
1 2 -4
1 1 4

Eigenvalues: -0.233625 -0.110589 -0.426034
Eigenvectors:
-0.233625 0.00632644 -0.280759
-0.432155 -0.110589 -0.362784
0.731894 1 -0.426034
```