

$$\frac{\partial}{\partial x_\mu} \frac{\partial}{\partial x^\mu} \phi(\hat{x}) = g^{\sigma\nu} \frac{\partial}{\partial x^\sigma} \frac{\partial}{\partial x^\nu} \phi(\hat{x}) = (\star)$$

$$\left[\frac{\partial}{\partial x^\nu} \phi(\hat{x}) = \frac{\partial}{\partial x^\nu} \phi\left(x^\varepsilon \frac{\partial \hat{x}^\varepsilon}{\partial x^\varepsilon}\right) = \partial_\eta \phi(\hat{x}) \frac{\partial \hat{x}^\eta}{\partial x^\nu} \right]$$

$$(\star) = g^{\sigma\nu} (\partial_{\eta_1} \partial_{\eta_2} \phi)(\hat{x}) \frac{\partial \hat{x}^{\eta_1}}{\partial x^\nu} \frac{\partial \hat{x}^{\eta_2}}{\partial x^\sigma} = g^{\sigma\nu} \Lambda_{\nu}^{\hat{\eta}_1} \Lambda_{\sigma}^{\hat{\eta}_2} (\partial_{\eta_1} \partial_{\eta_2} \phi)(\hat{x}) =$$

$$= \hat{g}^{\eta_1 \eta_2} (\partial_{\eta_1} \partial_{\eta_2} \phi)(\hat{x}) = (\partial^\mu \partial_\mu \phi)(\hat{x})$$