PSEUDODYNAMIC METHOD FOR SEISMIC TESTING

By Stephen A. Mahin,[1] M. ASCE and
Pui-shum B. Shing,[2] A. M. ASCE

ABSTRACT: The pseudodynamic method is a relatively new experimental technique for evaluating the seismic performance of structural models in a laboratory by means of on-line computer controlled testing. During such a test, the displacement response of a structure to a specified dynamic excitation is numerically computed and quasi-statically imposed on the structure, based on analytically prescribed inertia and viscous damping characteristics for the structure and the experimentally measured structural restoring forces. This paper presents the basic approach of the method, describing the numerical and experimental techniques. Based on current studies, the capabilities and limitations of the method are examined, and possible improvement methods are mentioned. In spite of certain numerical and experimental errors, recent verification tests show that the method can be as reliable and realistic as shaking table testing and that it can be readily implemented in many structural laboratories. The capabilities of the method can be further expanded to test specimens under various load and structural boundary conditions.

INTRODUCTION

Buildings and other structures are generally designed to dissipate some of the energy input during severe earthquake ground motions by means of inelastic deformations. Experimental research remains the most efficient and reliable method for assessing the ability of various types of structural systems to develop these deformations and for devising structural details to improve seismic performance of critical components. Various experimental techniques are available to perform such studies, such as shaking table, forced-vibration, and quasi-static test methods. Recently, an on-line computer control (or pseudodynamic) method has been developed (21,24) for testing the inelastic seismic performance of large-scale structural models that cannot be tested realistically or efficiently by conventional methods.

The pseudodynamic test method attempts to combine the realism of shaking table tests with the economy and convenience of quasi-static testing. The method is similar to conventional quasi-static tests except that the displacements imposed on a specimen are determined by a computer during a test. The computation of the displacement response is based on the restoring forces measured directly from the deformed test specimen, the analytically prescribed inertia and damping characteristics for the specimen, and a numerically specified earthquake excitation record. Thus, by accounting for the dynamic characteristics of the test structure in the controlling computer software, the quasi-statically imposed

[1]Assoc. Prof., Dept. of Civ. Engrg., Univ. of California, Berkeley, Calif.
[2]Asst. Research Engr., Dept. of Civ. Engrg., Univ. of California, Berkeley, Calif.

Note.—Discussion open until December 1, 1985. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on May 23, 1984. This paper is part of the *Journal of Structural Engineering*, Vol. 111, No. 7, July, 1985. ©ASCE, ISSN 0733-9445/85/0007-1482/$01.00. Paper No. 19867.

J. Struct. Eng., 1985, 111(7): 1482-1503

displacement histories closely resemble those that would occur if the structure were tested dynamically.

Pseudodynamic tests have been successfully performed in Japan by researchers at the Institute of Industrial Science of the University of Tokyo (16,24,25) and the Building Research Institute (BRI) of the Ministry of Construction (18). The results of tests performed on one- and two-story steel and reinforced concrete structures have correlated well with equivalent analytical and shaking table test results. In spite of some technical difficulties, tests on a seven-story reinforced concrete structure (17) and six-story steel braced frames have also been completed at BRI. As part of the U.S.-Japan Cooperative Earthquake Program, extensive development and evaluation work has been carried out in the U.S., notably at the University of California, Berkeley (13,20–23) and the University of Michigan, Ann Arbor (9,14). These studies indicate the practicability of the test method, as well as the necessity for further research to improve its reliability in testing multi-story structures and to expand its capabilities to more general applications.

The objectives of this paper are: (1) To present the basic approach, numerical techniques, and implementation scheme used in the pseudodynamic test method; and (2) to examine its reliability and limitations in general. The paper provides a brief overview of the studies completed at Berkeley, presenting the results of some pseudodynamic tests and correlating them with those of analytical simulations and shaking table tests. Potential applications and current development of the method will be examined as well.

Development Background

The inelastic behavior of most structural systems is sensitive to the loading histories applied. Consequently, a key question in laboratory tests is whether the loads or displacements imposed on the test specimen are representative of those that would occur during a severe earthquake. Possibly the most realistic method for assessing the nonlinear dynamic response of a particular structure is to place it on a shaking table and subject it to appropriately selected ground motion time histories. However, currently available shaking tables have significant limitations on the size, weight, and strength of specimens that can be tested (2,10). These limitations often necessitate testing of structures at substantially reduced scales, raising problems of material and dynamic similitude. Moreover, such tests require specialized and costly equipment, data acquisition systems, and personnel.

Forced-vibration tests of full-scale buidings into the inelastic range have also been proposed, and one such test has been successfully completed (6). It is difficult with such methods, however, to realistically simulate the distribution and history of forces developed during seismic excitations. While these and other innovative dynamic test methods, such as pulse generators (19) and blast-induced ground motions (11), show considerable promise, the high costs involved would likely limit their use to special structures or to complex problems involving soil-foundation-structure interaction.

The most economical and thus the most common method for obtaining information on the inelastic behavior of structures is through quasi-static tests in which prescribed histories of load or displacement are imposed on small structural systems or basic subassemblages of larger systems. Such tests utilize conventional test equipment and permit detailed observation of specimens during testing. To determine appropriate loading histories in such tests, an inelastic dynamic computer analysis can be performed for the structure in question. The displacements computed at various locations in the structure can then be used to control the experiment (26). Unfortunately, most mathematical idealizations are much simpler than actual structural behavior so that computed loading histories are not likely to be realistic. Because of this, it is more common to assume a priori a highly idealized deformation history that is characteristic of the general cyclic nature of seismic response (12). Such prescribed displacement histories can be particularly valuable in: (1) Assessing the effect of different structural details on the inelastic behavior of structures by subjecting different specimens to identical deformation histories; and (2) studying the basic mechanisms that affect the inelastic behavior of a particular structure by varying the amplitude, rate, or pattern of the applied deformation histories. However, it is not possible to directly relate the energy-dissipation capacity of a specimen measured in this type of test with that required for seismic safety. Consequently, questions continually arise with such experiments as to whether the specimen is under- or over-tested.

To facilitate the formulation of more rational and reliable seismic-resistance design methods, it is desirable to develop more realistic methods for prescribing displacement histories for quasi-static seismic performance tests. One potential method for doing this would be pseudodynamic testing.

Basic Approach

The basic assumption in the pseudodynamic test method, as with most computer-based dynamic analysis procedures, is that the dynamic behavior of a structure can be accurately represented by a discrete-parameter model that has only a finite number of degrees of freedom [see Figs. 1(*a*–*c*)]. The equations of motion for such an idealized multiple-degree-of-freedom model can be expressed in terms of a family of second-order ordinary differential equations, which, in matrix form, is

$$\mathbf{m}\mathbf{a} + \mathbf{c}\mathbf{v} + \mathbf{r} = \mathbf{f} \qquad (1)$$

in which $\mathbf{m}$ and $\mathbf{c}$ = the mass and viscous damping matrices of the system; $\mathbf{v}$ and $\mathbf{a}$ = the nodal velocity and acceleration vectors that are, respectively, the first- and second-order derivatives of the nodal displacement vector, $\mathbf{d}$; $\mathbf{r}$ = the restoring force vector, and it is equal to $\mathbf{k} \cdot \mathbf{d}$ for a linear elastic system having a stiffness matrix, $\mathbf{k}$; and $\mathbf{f}$ = the external force excitation applied to the system. This equation can be conveniently solved by means of a direct step-by-step integration method to obtain the displacement response, $\mathbf{d}$, for any arbitrary external excitation, $\mathbf{f}$. For a planar structure, with mass lumped at floor levels, subjected to a co-

J. Struct. Eng., 1985, 111(7): 1482-1503

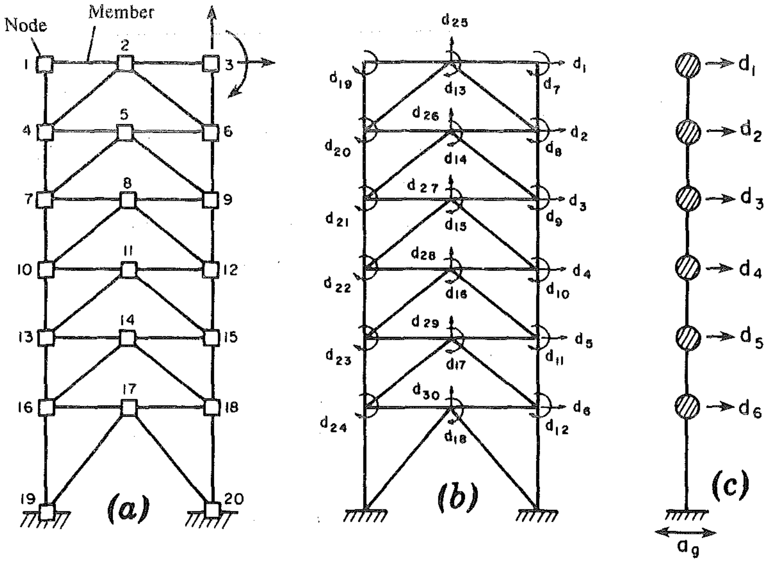

FIG. 1.—Frame Idealization for Dynamic Analyses: (*a*) Discrete-Parameter Frame Model; (*b*) Degrees of Freedom Considered; (*c*) Static Condensation

planar horizontal ground acceleration, a_g [like that shown in Fig. 2(*a*)], $\mathbf{f} = -\mathbf{m}\,\{1\}\,a_g$.

For dynamic analyses, the mass and stiffness matrices of a system can be formulated with a standard finite element procedure (3), and the damping matrix is usually determined based on some idealized modal damping properties. The principal difference between pseudodynamic testing and well-established dynamic analysis methods is that the computed structural displacements, **d**, are actually imposed on the test specimen, and the restoring forces, **r**, are measured experimentally from the deformed specimen [see Fig. 2(*b*)]. Therefore, the uncertainties associated with the analytical modeling of the nonlinear stiffness or restoring-force characteristics of a structure are not present in a pseudodynamic test.

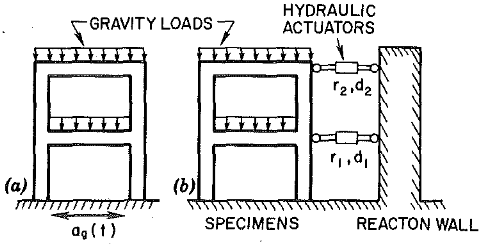

FIG. 2.—Pseudodynamic Test Idealization: (*a*) Actual Structure; (*b*) Pseudodynamic Test

A pseudodynamic test proceeds in a stepwise manner under a step-by-step integration procedure. In each step, the computed displacements, **d**, are *quasi-statically imposed on the test specimen by means of* computer controlled electrohydraulic actuators. The restoring forces, **r**, measured at the end of a step are then used to compute the displacement response in the next step, based on the analytically prescribed values of **m** and **c** as well as on a numerically specified excitation history, **f**. This process is repeated until the entire response history is obtained. The quasi-static approach not only allows a detailed inspection of structural damages during a test, but also permits the use of test equipment and instrumentation available in many structural laboratories for conventional quasi-static testing.

In theory, the method should be applicable to any structure whose equations of motion can be adequately represented by Eq. 1. However, in order to minimize the complexity of the loading apparatus, it is desirable that structures have mass concentrated at discrete locations and only a few degrees of freedom are needed to describe the overall dynamic response. Thus, an ideal system would be a multi-story structure, with mass concentrated at floor levels, subjected to one or two horizontal components of ground motion. In such a case, a static condensation procedure (as shown in Fig. 1) can be adequately applied to reduce the dynamic degrees of freedom (3).

In general, structures with significant distributed masses (or where distributed mass can influence the local failure mode) or constructed of materials that are highly sensitive to loading rate may not be suitable for pseudodynamic testing. In addition, dynamic effects not induced by external excitations, such as those due to brittle fracture of members occurring in real time between steps, and shock responses of local members may not be modeled properly in a pseudodynamic test.

Numerical Integration Methods

Direct Step-By-Step Integration.—The pseudodynamic test method requires the direct step-by-step solution of the equations of motion (Eq. 1) for a discrete-parameter model of the test specimen. In this procedure, the duration, τ, for which the structural response is to be evaluated, is divided into N equal time intervals Δt, i.e., $\Delta t = \tau/N$. By considering the equilibrium conditions specified in Eq. 1 at time equal to 0, Δt, $2\Delta t$, ..., and $N\Delta t$, and assuming that the solution in each step is a function of those in the previous step or steps, we can obtain an approximate numerical solution of the structural response. These numerical integration methods can, generally, be classified into two types: explicit and implicit methods (1). We adopt the definition that an integration method is explicit if the displacement solution in each step is assumed to be a function of previous step solutions only. Otherwise, the method is considered to be implicit.

The major considerations in selecting an integration method are its stability and accuracy. A method is stable if the numerical solution of a free-vibration response will not grow without bound for any arbitrary initial conditions (1). Many implicit methods are unconditionally stable, i.e., they remain stable for any value of $\omega\Delta t$, in which ω = a natural

J. Struct. Eng., 1985, 111(7): 1482-1503

angular frequency of the system analyzed. Explicit methods are, generally, conditionally stable, such that stable solutions can be obtained only if the $\omega\Delta t$ value is within a certain range. In general, the smaller the value of $\omega\Delta t$ is, the more accurate will be a numerical solution. A method is defined as convergent if the numerical solution approaches the exact solution as Δt goes to zero.

One of the most general integration methods in structural dynamics is the Newmark algorithm (15), which assumes that

$$\mathbf{m}\mathbf{a}_{i+1} + \mathbf{c}\mathbf{v}_{i+1} + \mathbf{r}_{i+1} = \mathbf{f}_{i+1} \quad (2)$$

$$\mathbf{d}_{i+1} = \mathbf{d}_i + \Delta t \mathbf{v}_i + \Delta t^2 \left[\left(\frac{1}{2} - \beta \right) \mathbf{a}_i + \beta \mathbf{a}_{i+1} \right] \quad (3)$$

$$\mathbf{v}_{i+1} = \mathbf{v}_i + \Delta t [(1 - \gamma)\, \mathbf{a}_i + \gamma \mathbf{a}_{i+1}] \quad (4)$$

in which $\mathbf{a}_{i+1}$, $\mathbf{v}_{i+1}$, and $\mathbf{d}_{i+1}$ = the acceleration, velocity, and displacement vectors, respectively, at time equal to $(i + 1)\Delta t$; and β and γ = parameters selected by users to obtain desirable stability and accuracy properties. For example, by letting β equal to 1/4 and γ equal to 1/2, we have the constant-average-acceleration method, which is implicit and unconditionally stable. When β is zero, the $\mathbf{a}_{i+1}$ term in Eq. 3 disappears and the method becomes explicit.

Explicit Integration Methods.—From Eqs. 2–4, we can observe that the determination of $\mathbf{d}_{i+1}$ in terms of $\mathbf{d}_i$, $\mathbf{v}_i$, and $\mathbf{a}_i$ by an implicit integration method requires knowledge of stiffness matrix, $\mathbf{k}$. Nevertheless, in pseudodynamic testing, only the product $\mathbf{k} \cdot \mathbf{d}_{i+1}$ is measured experimentally as a restoring force vector, $\mathbf{r}_{i+1}$, after the displacement $\mathbf{d}_{i+1}$ has been computed and physically imposed on the test structure. Where a method for determining the tangent stiffness of a nonlinear system during a test can be devised (24), the resulting values may be overly sensitive to errors in experimental measurements. Furthermore, due to the change of tangent stiffness from one integration step to the next, the solution of nonlinear differential equations by an implicit method usually requires iterative corrections, which are highly undesirable for strain-history dependent inelastic systems. To avoid these problems, explicit integration methods are usually recommended for pseudodynamic testing:

1. *Explicit Newmark Method.*—An explicit form of the Newmark algorithm can be obtained by letting $\beta = 0$ and $\gamma = 1/2$ in Eqs. 3 and 4. Further, by substituting $\mathbf{v}_{i+1}$ in Eq. 2 with Eq. 4, we can solve for $\mathbf{a}_{i+1}$ as

$$\mathbf{a}_{i+1} = \left(\mathbf{m} + \frac{\Delta t}{2} \mathbf{c} \right)^{-1} \left(\mathbf{f}_{i+1} - \mathbf{r}_{i+1} - \mathbf{c}\mathbf{v}_i - \frac{\Delta t}{2} \mathbf{c}\mathbf{a}_i \right) \quad (5)$$

which, with Eqs. 3 and 4, constitutes the basic algorithm for pseudodynamic testing as shown in Fig. 3. In general, the value of γ should be greater than or equal to 1/2. Otherwise, the algorithm is unstable. The solution is most accurate when $\gamma = 1/2$.

2. *Central Difference Method.*—The central difference method is one of the most widely used explicit integration techniques in structural dy-

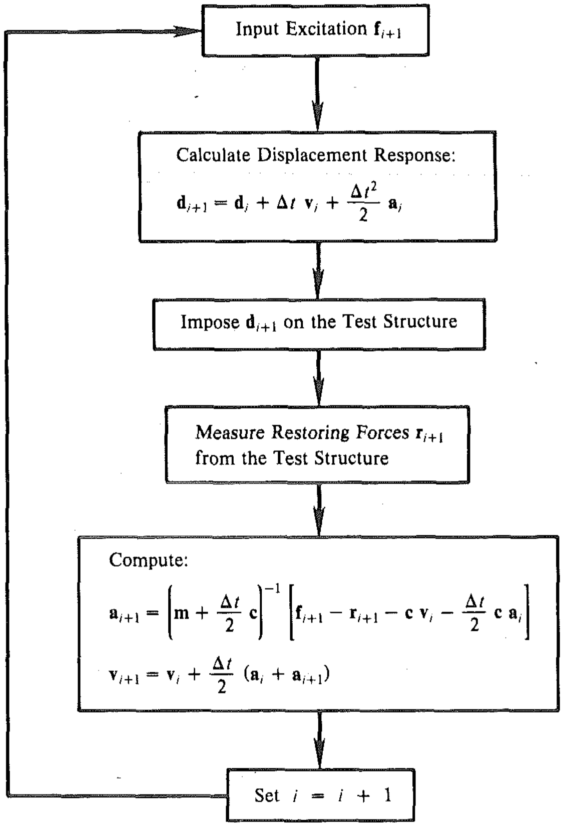

FIG. 3.—Explicit Newmark Algorithm ($\gamma = 1/2$)

namics. It has been applied to pseudodynamic testing in many previous studies (16,25). The basic central difference method assumes that the velocity and acceleration can be represented by the following difference equations:

$$\mathbf{v}_i = \frac{\mathbf{d}_{i+1} - \mathbf{d}_{i-1}}{2\Delta t} \quad \text{(6)}$$

$$\mathbf{a}_i = \frac{\mathbf{d}_{i+1} - 2\mathbf{d}_i + \mathbf{d}_{i-1}}{\Delta t^2} \quad \text{(7)}$$

By substituting these equations into the equilibrium equation (Eq. 1) at time equal to $i\Delta t$, we can solve for $\mathbf{d}_{i+1}$ as

$$\mathbf{d}_{i+1} = \left(\mathbf{m} + \frac{\Delta t}{2}\mathbf{c}\right)^{-1}\left[\Delta t^2(\mathbf{f}_i - \mathbf{r}_i) + \left(\frac{\Delta t}{2}\mathbf{c} - \mathbf{m}\right)\mathbf{d}_{i-1} + 2\,\mathbf{m}\mathbf{d}_i\right] \quad \text{(8)}$$

which can be readily used for displacement computations in a pseudodynamic test, since the restoring forces $\mathbf{r}_i$ in the previous step are measured experimentally.

The basic central difference method examined earlier may cause significant round-off errors in a digital computer when a very small value of Δt is used in the computations. An alternative formulation, the summed form, has been suggested (5) to improve that condition. In this formulation, it is defined that $\mathbf{z}_i = (\mathbf{d}_i - \mathbf{d}_{i-1})/\Delta t$. Thus, we have $\mathbf{z}_{i+1} - \mathbf{z}_i = \Delta t \mathbf{a}_i$ and $\mathbf{z}_{i+1} + \mathbf{z}_i = 2\mathbf{v}_i$, according to Eqs. 6 and 7. By substituting these relations into the equilibrium equation instead of Eqs. 6 and 7, we obtain

$$\mathbf{z}_{i+1} = \left(\frac{\mathbf{m}}{\Delta t} + \frac{\mathbf{c}}{2}\right)^{-1}\left[\mathbf{f}_i - \mathbf{r}_i + \mathbf{z}_i\left(\frac{\mathbf{m}}{\Delta t} - \frac{\mathbf{c}}{2}\right)\right] \quad \text{(9)}$$

$$\mathbf{d}_{i+1} = \mathbf{d}_i + \Delta t \mathbf{z}_{i+1} \quad \text{(10)}$$

which can be applied to pseudodynamic testing as the basic central difference method (14).

The explicit Newmark method (with $\gamma = 1/2$) and the central difference method are, actually, mathematically equivalent to each other (20) and, consequently, have identical numerical properties (21). They have a stability limit of $\omega\Delta t \leq 2$, which is governed by the highest frequency of a multiple-degree-of-freedom system. However, to ensure the accuracy of a solution that is dominated by a natural frequency, ω, the Δt selected should be much smaller than $2/\omega$. Otherwise, a significant frequency distortion will occur in the numerical results. This will be examined later.

Consequently, the explicit Newmark and central difference methods are equally reliable. However, since the central difference method is a two-step method, it requires a special initiation step to start the numerical computation. Furthermore, it will be shown later that under certain conditions, the basic central difference method is more sensitive to experimental errors than the explicit Newmark method in pseudodynamic testing.

Implementation Scheme

Experimental System and Procedure.—A typical implementation scheme, which is currently adopted at Berkeley, for the pseudodynamic method is shown in Fig. 4. The test algorithm is implemented in a minicomputer, which performs the following major functions in each step of a test: (1) Reading the data channels of a data acquisition unit and storing the data in a disk file; (2) calculating the response of the test structure using step-by-step integration; and (3) sending the displacement signals to actuator controllers. The structural displacements computed in each step are converted to voltage signals by a multi-channel ramp generator (digital-to-analog or D/A converter). These signals are then sent to actuator controllers, each of which commands a hydraulic actuator to impose the specified displacement at each structural degree of freedom. The restoring forces developed by the test structure, as well as other measurements of structural behavior, are returned to the computer as digital signals after sampling and conversion by a high speed data acquisition unit (analog-to-digital or A/D converter).

The time interval, ΔT, taken by each step of a test consists of two

J. Struct. Eng., 1985, 111(7): 1482-1503

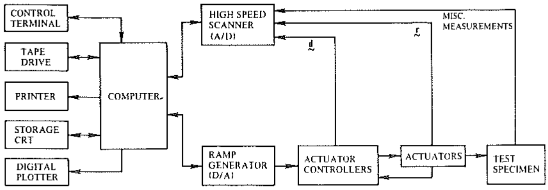

FIG. 4.—On-Line Computer Control System

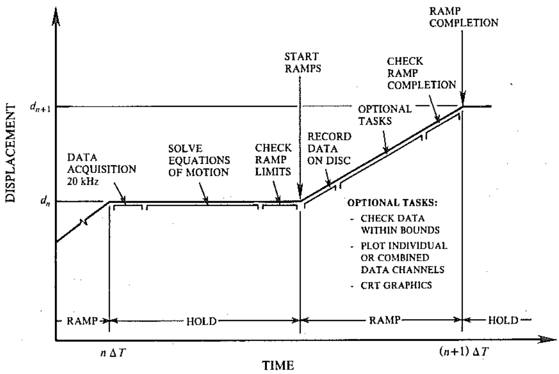

FIG. 5.—Tasks Performed in each Step of Pseudodynamic Test

operational phases: the "hold" and "ramp" phases, as shown in Fig. 5. During the hold period, data acquisition and numerical computations are carried out. Data acquisition starts immediately after the ramps of the previous step are completed. The computed displacements are checked so that they will not exceed the specified ramp limits. The hold period is usually a small fraction of a second if a high speed data acquisition system is used. The ramps start if all the computed displacements are within the limits. During this phase, several auxiliary tasks can also be performed: (1) Recording the acquired data in a disk file; (2) reading the external excitation record from a disk file; (3) checking all data to see if they are within the calibrated limits; and (4) plotting the selected data channels for graphical display. Therefore, the ramp speeds should be so selected that all these tasks can be completed before the ramps stop. This minimizes the hold period, ensuring more or less continuous displacement increments. In addition, individual ramp speeds are appropriately proportioned so that the displacement increments for all channels are reached simultaneously. The ramps can take a fraction of a second to a few seconds. However, the maximum speed is also limited by the sensitivity of an actuator-controller system in responding to the ramp signals. On the whole, each step of a pseudodynamic test can be performed in less than one second.

Experimental Equipment.—As shown earlier, the pseudodynamic test method uses almost the same equipment as conventional quasi-static testing. However, since experimental data are used in numerical com-

J. Struct. Eng., 1985, 111(7): 1482-1503

putations, pseudodynamic testing requires high precision control and measurement instruments. The performance characteristics of the major instruments are briefly mentioned in the following:

1. *Measurement Transducers.*—The displacements imposed on a structure and the restoring forces developed are measured by displacement and load transducers, respectively. These are electronic devices that correlate the displacement or force variations with voltage changes. A perfect device will give an exact linear correlation. The accuracy of these devices depends on their qualities, the calibration techniques, and their installations. Since the output of a displacement transducer is fed back to an actuator controller in a closed-loop displacement control, and the force measurement from a load transducer is used for the computation of displacement response, the accuracy of these devices is directly related to the reliability of pseudodynamic test results.

2. *Actuator-Controller System.*—The displacements of a structure are controlled by means of actuator-controller systems during a pseudodynamic test. An electronic servo-controller is used to command the displacement of a hydraulic actuator in response to the difference between the command signal and the measured displacement. The response of an actuator-controller system to a displacement signal from a computer depends on the quality and flow capacity of the servo-valve, which drives the hydraulic actuator, and the gain setting on the controller. If the gain is low, the system may respond sluggishly. If the gain is too high, the system may become unstable, and the actuator will overshoot and oscillate about the commanded displacement. Therefore, an optimal gain should be selected if an actuator is to respond sensitively and stably to a command signal. The maximum response speed of an actuator is limited by the capacity of the servo-valve, which is specified in terms of gallons of fluid flow per minute. In general, the selection of servo-valve capacity depends on the size of the actuator and the velocity requirement.

3. *Ramp Generator and Data Acquisition Unit.*—The ramp generator is a D/A converter, which transmits displacement commands from the computer to actuator controllers. Floating-point displacement values computed by the pseudodynamic algorithm are first converted into integer numbers before being sent to the D/A converter, which translates binary integer numbers to analog voltage signals. This device has a resolution limit, e.g., a 12-bit binary D/A converter neglects fractions smaller than 1/2,048 ($2/2^{12}$) of the maximim calibrated value. The data acquisition unit is an A/D converter that translates analog feedback from measurement instruments to digital signals, which are returned to the main computer. Again, this instrument has a resolution limit. In addition to the resolution errors, random electical noise is usually inevitable in these systems.

Reliability of Test Method

As mentioned previously, various errors can enter into a pseudodynamic test, such as errors related to idealizing the inertia and viscous damping characteristics of the test specimen, numerical errors in step-

J. Struct. Eng., 1985, 111(7): 1482-1503

by-step integration, and experimental inaccuracies caused by test equipment. However, as with most experimental methods, it is possible to reduce these errors to tolerable levels and obtain reliable results by realizing the existence of the limitations and by exercising reasonable care and consideration in performing a pseudodynamic test. As a summary of current investigations (20,21), the factors that might affect the reliability of a pseudodynamic test are examined here.

Structural Idealizations.—To perform a pseudodynamic test, we have to idealize the test structure as a discrete-parameter system having mass concentrated at a limited number of degrees of freedom. To simplify the loading apparatus, it is always desirable to assume a simple lumped-mass model for the structure. By selecting a n-degree-of-freedom model, the vibration modes higher than the n-th are truncated and the lower mode vibration characteristics that are retained in the model may be distorted. The accuracy of such a model depends entirely on the mechanical properties and, especially, the mass distribution of the original structure considered and the characteristics of external excitations applied. In general, the lower frequency modes can be more accurately represented than the higher ones in a discrete-parameter model; and the accuracy increases with the number of degrees of freedom considered. However, this does not imply that a large number of degrees of freedom is always necessary in a pseudodynamic test for reliable results. Under seismic excitations, the higher mode participations are generally insignificant. Furthermore, a model of sufficient accuracy can very often be obtained by selecting the dynamic degrees of freedom at locations where structural mass is actually concentrated. Therefore, the pseudodynamic method is especially efficient and reliable for load carrying structures, such as multi-story buildings, with heavy floor systems at story levels.

Structural damping is most conveniently modeled by a viscous damping mechanism. However, other forms of energy-dissipation exist in real systems, such as Coulomb damping due to friction and hysteretic damping caused by inelastic material deformations. Both Coulomb and hysteretic damping mechanisms are automatically taken into account in a pseudodynamic test by using the actual restoring-force measurements in the numerical computations, whereas viscous damping coefficients have to be analytically specified. The damping properties of an elastic system can be measured with reasonable accuracy by vibration tests. Based on these measurements, appropriate viscous damping coefficients can be selected for pseudodynamic testing. While the elastic response of a structure may be very sensitive to viscous damping, the energy-dissipation in an inelastic response is usually dominated by hysteretic damping. For this reason, the uncertainties associated with the selection of viscous damping properties are usually immaterial in a test where considerable inelastic deformations take place.

Since pseudodynamic testing is performed quasi-statically, the consideration of strain-rate effects leads to the question whether the results of such tests can realistically represent the inelastic behavior of a structure under actual seismic excitations. Strain-rate effects vary from one structural material to another. In general, the larger the strain-rate is, the higher will be the inelastic strength. Therefore, we can expect that a structure tested quasi-statically will exhibit a lower inelastic strength than one under dynamic excitations. However, by assuming a minium fundamental response period of 0.1 sec, the strain-rate effects can be neglected for most steel structures (8,21). For reinforced concrete structures, this phenomenon should be carefully considered, because cracking and other failure mechanisms occur in real time. In any case, a pseudodynamic test should be performed at a reasonable rate. If the time of a test is too long, strain-aging can occur in some materials. On the other hand, if a test is too fast, the real inertial effects enter into the restoring force measurements. The pause between displacement increments should be small so that no significant stress relaxation will occur.

Numerical Integration.—The accuracy of a numerical solution depends on the value of Δt selected. If a method is convergent, the numerical solution should approach the exact solution as Δt goes to zero. On the other hand, to reduce computational efforts, we should always select the largest possible value of Δt for which the solution is still reliable.

Errors in numerical solutions are often manifested in the form of frequency distortion and energy dissipation. To examine these effects, the free-vibration response of a linear single-degree-of-freedom system (with a natural period T of 1 sec) was evaluated by the explicit Newmark method with various Δt values. As shown in Fig. 6, at Δt equal to 0.01 sec, the numerical solution could be considered as the exact solution. At Δt equal to 0.02 sec, a small period shrinkage appeared in the numerical result. The period shrinkage increased when Δt was increased to 0.2 sec, and the solution became unstable, as expected, at Δt equal to 0.32 sec. Since no artificial amplitude decay was observed in the free-vibration response, the method is energy conserving. In addition, it can be shown that the frequency distortion approaches zero as Δt goes to zero (see Fig. 7). Therefore, the explicit Newmark method is convergent.

Under seismic excitations, the displacement amplitudes of numerical solutions can be very sensitive to the value of Δt used because of the frequency distortion introduced (21). This is especially true for linear elastic systems without viscous damping. For this reason, a very small Δt value (preferably less than $0.05T$) should be used in such analyses. Fortunately, the sensitivity of response amplitudes to frequency distor-

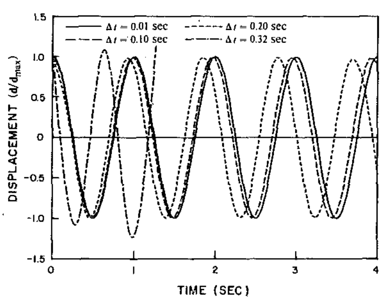

FIG. 6.—Free-Vibration Response of Single-Degree-of-Freedom System (T = 1 sec; Explicit Newmark Method)

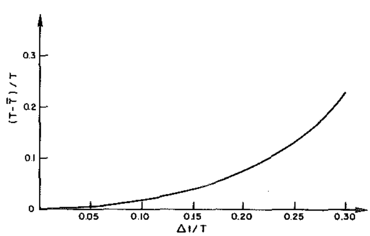

FIG. 7.—Period Shrinkage by Explicit Newmark Method

J. Struct. Eng., 1985, 111(7): 1482-1503

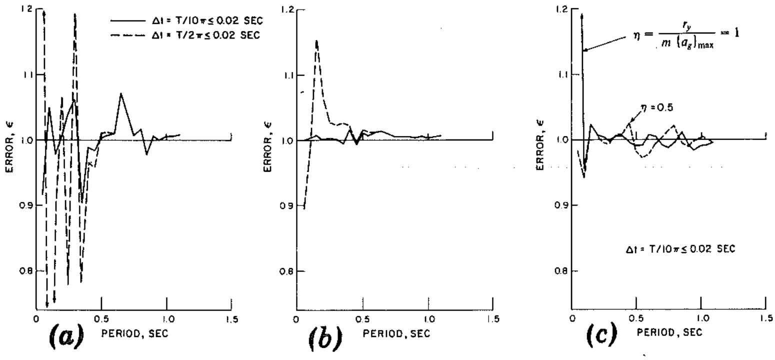

FIG. 8.—Error Spectra for Single-Degree-of-Freedom Systems Subjected to El Centro 1940 (NS) Earthquake: (*a*) Linear Elastic, No Damping; (*b*) Linear Elastic, 5% Damping; (*c*) Elastoplastic, No Damping

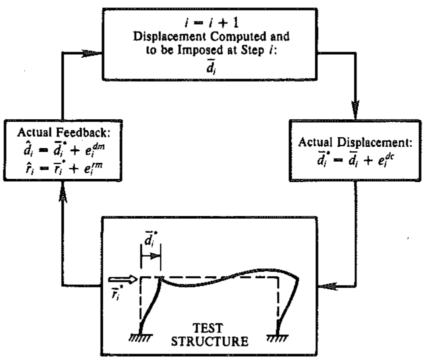

FIG. 9.—Sources of Experimental Errors in Pseudodynamic Testing

tion is greatly reduced in most pseudodynamic tests where viscous damping and inelastic deformations exist, as shown by the error spectra in Figs. 8(*a–c*), where the error index, ϵ, represents the ratio of the maximum displacement amplitude numerically computed over the exact value.

In general, the stability and accuracy characteristics of explicit integration methods, based on linear elastic systems, are still valid for nonlinear systems by the fact that a nonlinear system can always be considered as piece-wise linear. Based on this reasoning, the Δt selected for a linear system will remain conservative if the nonlinearity is of the softening type, and the opposite will be true for a hardening system. In either case, the Δt selected should be sufficiently small so that the nonlinear behavior of a system can be accurately traced by the discretized displacement increments. Otherwise, the stability and accuracy of an algorithm can be imparied (21). This is an additional consideration for nonlinear systems.

Experimental Errors.—In addition to the numerical errors mentioned earlier, experimental errors are introduced into numerical computations during the control and feedback processes of a pseudodynamic test, as shown in Fig. 9. A computed displacement usually cannot be exactly imposed on a test structure, due to errors caused by instability or lack of sensitivity of the actuator-controller system, mis-calibration of displacement transducers, or digital-to-analog conversions of displacement signals as mentioned before. The displacement or restoring forces measured from a test structure may also be different from the actual quantities due to measurement errors, e.g., the frictional force in actuator connections may influence the restoring-force measurements. The cumulative effect of these errors in a pseudodynamic test can be realized by the fact that incorrect displacements imposed on a test structure will result in erroneous force feedback, and the errors in force feedback will in turn lead to erroneous displacements being computed in the next step. Due to this error cumulation, a pseudodynamic test result can be rendered unreliable even though the experimental errors introduced in each step are relatively small.

The error-propagation problem can be illustrated by a numerical simulation using a single-degree-of-freedom linear elastic system. In each step of the simulation, random errors were numerically generated and introduced into the computed displacements and restoring forces, respectively. The erroneous displacement and force values were then used to compute the response in the next step. The errors introduced into the displacement feedback had a standard deviation of σ_d and those into the force feedback had a standard deviation of $k \cdot \sigma_d$ in which k was the stiffness of the system. Without any external excitation to the system, spurious displacement responses (i.e., cumulative errors) due to random feedback errors were obtained by using the central difference and explicit Newmark methods, respectively, as shown in Fig. 10. It can be observed that the central difference method is very sensitive to random errors as the maximum cumulative errors obtained were approximately $100\sigma_d$ in this example. However, the cumulative errors produced by the explicit Newmark method were much smaller than that.

The preceding example illustrates that mathematically identical methods can have different error-propagation characteristics due to different numerical procedures. However, if the computed displacements are used

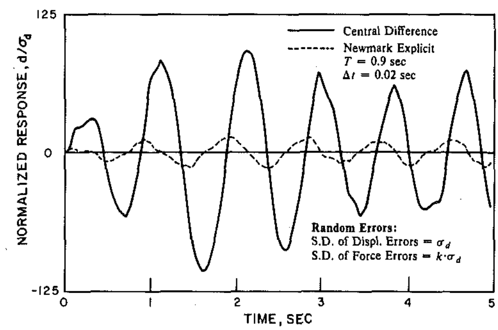

FIG. 10.—Spurious Displacement Response of Single-Degree-of-Freedom System Subjected to Random Feedback Errors

J. Struct. Eng., 1985, 111(7): 1482-1503

in numerical computations, i.e., the displacement feedback errors are eliminated, the two aforementioned methods have identical error-propagation characteristics (20). This is actually recommended for all pseudodynamic tests regardless of the numerical method used. Even if this is done, displacement control errors are still introduced into the numerical computations through the resulting force feedback errors. Force feedback errors can have significant effects on experimental results depending on whether the errors are random or systematic in nature. Systematic errors can induce a significant cumulative error growth due to resonance-like effects. The cumulative growth of systematic errors within a fixed computational time span cannot be effectively reduced by decreasing the value of Δt. Random errors may result from electrical noise or other less well-defined sources. Their effects are less severe and can be mitigated by reducing the Δt value. In both cases, the larger the value of $\omega \Delta t$ is, the faster will be the rate of cumulative error growth with respect to the number of integration steps. Therefore, the higher modes of a system will be more sensitive to experimental errors than the lower ones.

Due to these error-propagation effects, experimental errors should always be eliminated or reduced to insignificant levels in any test. This may not be always possible, especially in systems having many degrees of freedom. In such systems, even small errors can propagate very rapidly in the higher modes. Under that circumstance, numerical techniques, such as frequency proportional numerical damping, can be used to mitigate the error effects (20).

Verification Tests

A series of experimental tests were performed to verify the practicability and reliability of the pseudodynamic method. With the current system at Berkeley, tests involving 2,000 time steps (20 sec of earthquake with $\Delta t = 0.01$ sec) generally took less than 30 min in real time. Some of the test results are briefly presented in the following.

One-Degree-of-Freedom Cantilever System.—A single-degree-of-freedom cantilever system [see Fig. 11(*a*)] was selected for initial verification tests. The cantilever column was fabricated from a W 6 × 16 section. One end of the column was attached to a concrete reaction block, and the other end was attached to a hydraulic actuator through a clevis, representing the tip which carried the concentrated mass [see Fig. 11(*b*)]. The system was tested horizontally, and both the viscous damping and gravity effects of the mass were neglected.

The linear elastic response of the system to the El Centro ground motion with 0.1 g peak acceleration is shown in Fig. 12(*a*). Due to the Coulomb damping introduced by the experimental apparatus (such as the clevis and the support apparatus for the specimen), some energy dissipation could be observed in the pseudodynamic response. It had an equivalent viscous damping ratio of 1.6%. Fig. 12(*a*) shows that the pseudodynamic test result closely matched that of an analytical simulation using 1.6% viscous damping. The inelastic responses of the system subjected to the 0.8 g El Centro and 0.45 *g* Miyagi Ken Oki ground accelerations are shown in Figs. 12(*b*) and 12(*c*), respectively. Significant

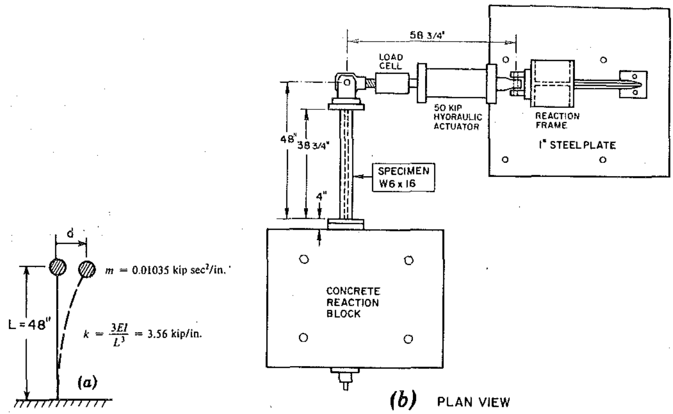

FIG. 11.—Pseudodynamic Tests of Single-Degree-of-Freedom System (1 in. = 25.4 mm, 1 kip = 4.45 kN): (*a*) Test Model; (*b*) Test Setup

yielding was developed at the fixed end of the column, as can be observed from the period elongations and displacement drifts in the displacement time histories. Again, the results of these tests showed good correlations with analytical simulations, where the inelastic force-defor-

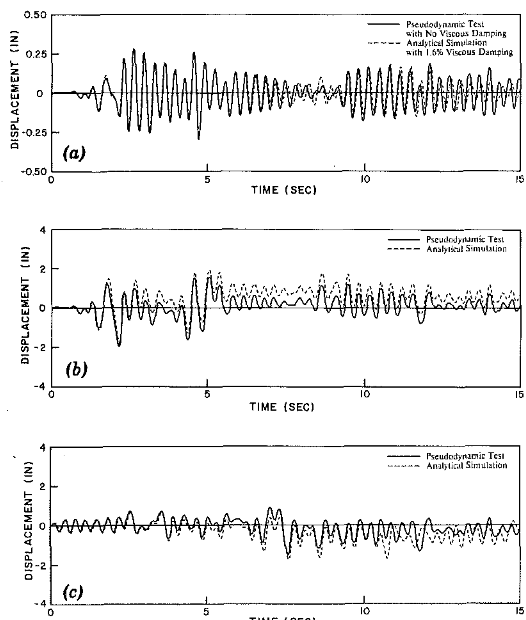

FIG. 12.—Seismic Response of Single-Degree-of-Freedom System (1 in. = 25.4 mm): (*a*) 0.1 g El Centro 1940 (NS); (*b*) 0.8 g El Centro 1940 (NS); (*c*) 0.45 Miyagi Ken Oki 1978 (NS)

J. Struct. Eng., 1985, 111(7): 1482-1503

mation relation was modeled by the Menegotto-Pinto relation (21). The analytical results showed slightly more significant displacement drifts due to the sensitivity of residual deformations to the precise details of the inelastic hysteresis model. Coulomb damping was relatively insignificant when compared to the hysteretic energy dissipation. Therefore, no viscous damping was required in the analyses to match the experimental results.

Tubular Braced Frame.—A tubular x-braced frame specimen previously tested on a shaking table (7) was repaired and tested pseudodynamically (23). The tubular frame specimen was a 5/48 scale planar model of a representative offshore platform located in Southern California. The shaking table test specimen was subjected to three levels of excitation based on the 1952 Taft (S69E) record. These corresponded to strength level (0.28 g), ductility level (0.58 g), and maximum credible (1.228 g) events. The recorded motions of the table in these tests were used as the input for the pseudodynamic tests. Since 99% of the mass was concentrated at the top of the frame as service loads in the shaking table tests (see Fig. 13), the specimen could be considered as a single-degree-of-freedom system. A 1.5% viscous damping ratio, which was measured from the shaking table tests, was numerically specified in the pseudodynamic tests. The pseudodynamic test setup is shown in Fig. 14.

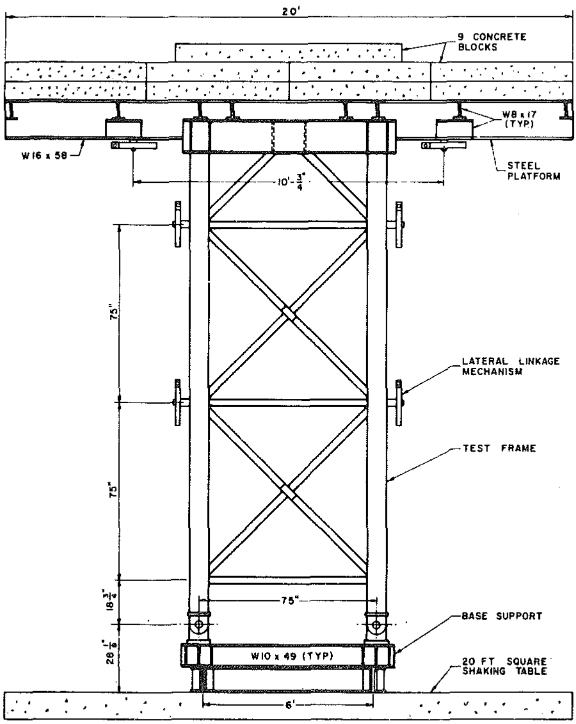

FIG. 13.—Shaking Table Tests of Tubular Steel Frame (from Ref. 7; 1 in. = 25.4 mm)

J. Struct. Eng., 1985, 111(7): 1482-1503

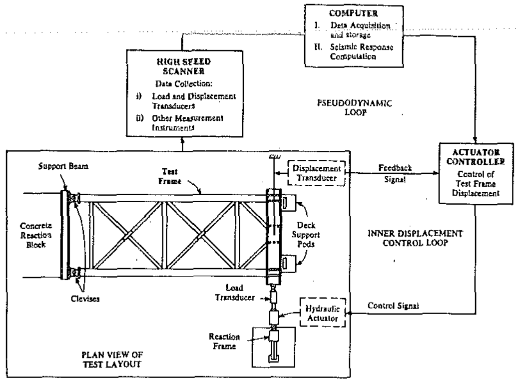

FIG. 14.—Pseudodynamic Tests of Tubular Steel Frame

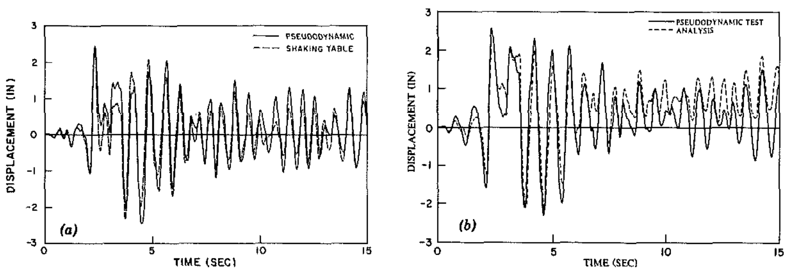

FIG. 15.—Seismic Response of Tubular Steel Frame Subjected to 1.23 g Taft 1952 Ground Motions (1 in. = 25.4 mm): (*a*) 1st Maximum Credible Event; (*b*) 2nd Maximum Credible Event

Due to the flexibilities of the specimen's base support and of the table itself, the frame stiffness measured on the table was 31% lower than that in the pseudodynamic tests. Because of this, the frame responses in the low level events were different in the two experiments. However, the inelastic seismic behaviors and the failure modes of the specimens in the two experiments were very similar. In the maximum credible event, the pseudodynamically tested frame had a deteriorated stiffness nearly identical to that of the shaking table specimen. Consequently, the two experimental results were almost identical during that event [see Fig. 15(*a*)]. Good correlations were also obtained by analytical simulations using an appropriate inelastic brace model, as shown in Fig. 15(*b*).

Potential Applications

The pseudodynamic method can be readily applied to test three-dimensional structural systems under multi-component ground excitations. By including additional load conditions in the basic equations of motion, the pseudodynamic method can be extended to other dynamic

J. Struct. Eng., 1985, 111(7): 1482-1503

problems, such as cases with hydrodynamic or aeroelastic excitations. One foreseeable application would be the testing of offshore platforms subjected to both seismic and hydrodynamic loads.

Only complete structural systems can be currently tested by the method. However, in many large structural systems, severe inelastic deformations will only occur in certain localized regions, which are of the major concern to most researchers and designers. Consequently, significant costs can be saved by testing only those portions of large systems where severe inelastic deformations will develop. This can be done in pseudodynamic testing by means of analytical substructuring concepts (4), so that part of the structure can be tested experimentally and the rest of it is modeled analytically. The technique is currently under development at Berkeley. By the substructuring method, structural subassemblages and components can be tested economically under realistic load histories and boundary conditions. This can also be applied to test special systems, such as equipment mounted in structures, and to include the effects of soil-structure interaction by modeling the soil media as finite element meshes.

Conclusions

This paper presents the basic approach of the pseudodynamic test method, indicating the numerical techniques and implementation method. The capabilities and limitations of the method are also examined with the following conclusions:

1. The pseudodynamic test method combines well-established analytical techniques in structural dynamics with quasi-static testing. This hybrid approach attains the realism of experimental testing (such as shaking table tests), while retaining the versatility of analytical techniques. In addition, since it uses almost the same equipment and instrumentation as utilized in conventional quasi-static tests, the method can be readily implemented in many structural laboratories.
2. Based on the dual nature of the method, both numerical and experimental errors can be introduced into the test results. However, the numerical errors are identical to those that are generally encountered in computer-based dynamic analyses and should be relatively insignificant if proper numerical criteria, as examined in this paper, are followed.
3. Experimental errors appear to be the major source of inaccuracies in pseudodynamic testing, due to the error-cumulation effect in the step-by-step integration procedure. Nevertheless, most experimental errors can be reduced to insignificant levels by using high performance test equipment and appropriate instrumentation techniques. These special needs, however, are limited to critical instruments used for displacement control and force measurement. Numerical methods are also available to mitigate the effects of experimental errors.
4. The reliability of the pseudodynamic method is verified by correlating the results of pseudodynamic tests with those of analytical simulations and shaking table tests, as presented in this paper. However, these tests are relatively simple, involving only a single degree of freedom. In general, as with all testing and analytical techniques, users must

J. Struct. Eng., 1985, 111(7): 1482-1503

have a clear understanding of its limitations and the factors which might influence its accuracy.

5. According to current analytical and experimental studies, we can expect certain difficulties in testing stiff systems which have large numbers of degrees of freedom. Such systems are extremely sensitive to experimental errors. Therefore, further research is required to improve the performance of experimental equipment, such as actuator-controller systems, and to develop error-resistant numerical algorithms.

6. Because of well-controlled experimental conditions, pseudodynamic testing can provide valuable data for increasing our understanding of nonlinear behavior of structures subjected to seismic excitations and to assess and improve current analytical modeling techniques for analyzing such systems. Furthermore, the analytical basis of the pseudodynamic test method can be greatly expanded (e.g., by using substructuring methods) to permit testing of many types of structures not previously tested.

Acknowledgments

The writers are grateful to acknowledge that this study is conducted under the U.S.-Japan Cooperative Earthquake Research Program and is financially supported by the National Science Foundation (NSF). However, opinions expressed in this paper do not necessarily represent those of other participants in the program or of NSF. The writers are particularly in debt to Dr. Jerry Dimsdale for his assistance in the development of the on-line computer control system. The assistance of Stavros Dermitzakis, Amir Javadian-Gilani, and Chris Thewalt in system development and verification tests is also appreciated.

Appendix I.—References

1. Bathe, K., and Wilson, E. L., *Numerical Methods in Finite Element Analysis*, Prentice Hall, Englewood Cliffs, N.J., 1976.
2. Bertero, V. V., Clough, R. W., and Oliva, M., "Use of Earthquake Simulators and Large-Scale Loading Facilities," *Proceedings*, Workshop on ERCBC, University of California, Berkeley, Calif., 1978.
3. Clough, R. W., and Penzien, J., *Dynamics of Structures*, McGraw-Hill, New York, N.Y., 1975.
4. Clough, R. W., and Wilson, E. L., "Dynamic Analysis of Large Structural Systems with Local Nonlinearities," *Computer Methods in Applied Mechanics and Engineering*, Vol. 17/18, Jan., 1979, pp. 107–129.
5. Dahlquist, G., Björck, A., and Anderson, N., *Numerical Methods*, Prentice Hall, Englewood Cliffs, N.J., 1974.
6. Galambos, T. V., and Mayes, R. L., "Dynamic Tests of a Reinforced Concrete Building," *Report No. 51*, Washington University, St. Louis, Mo., 1978.
7. Ghanaat, Y., and Clough, R. W., "Shaking Table Tests of a Tubular Steel Frame Model," *UCBEERC-8202*, Earthquake Engineering Research Center, University of California, Berkeley, Calif., Jan., 1982.
8. Hanson, R. D., "Comparison of Static and Dynamic Hysteresis Curves," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 92, No. EM5, Oct., 1966, pp. 87–113.
9. Hanson, R. D., and McClamroch, N. H., "Pseudo Dynamic Test Method for Inelastic Building Response," *Proceedings, Eighth World Conference on Earth-*

quake Engineering, San Francisco, Calif., July, 1984.
10. Karwinkler, H., "Experimental Research Needs," *Proceedings*, Workshop on ERCBC, University of California, Berkeley, Calif., 1978.
11. Bruce, J., and Lindberg, H., "Earthquake Simulation using Contained Explosion," presented at the Jan., 1981, *Second ASCE-EMD Specialty Conference on Dynamic Response of Structures*, held at Atlanta, Ga.
12. Mahin, S. A., Popov, E. P., and Zayas, V. A., "Seismic Behavior of Tubular Steel Offshore Platforms," *Proceedings, Offshore Technology Conference*, Houston, Tex., May, 1980.
13. Mahin, S. A., and Williams, M. E., "Computer Controlled Seismic Performance Testing," presented at the Jan., 1981, *Second ASCE-EMD Specialty Conference on Dynamic Response of Structures*, held at Atlanta, Ga.
14. McClamroch, N. H., Serakos, J., and Hanson, R. D., "Design and Analysis of the Pseudo-Dynamic Test Method," *UMEE 81R3*, University of Michigan, Ann Arbor, Mich., Sept., 1981.
15. Newmark, N. M., "A Method of Computation for Structural Dynamics," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 85, No. EM3, July, 1959, pp. 67–94.
16. Okada, T., Seki, M., and Park, Y. J., "A Simulation of Earthquake Response of Reinforced Concrete Building Frames to Bi-Directional Ground Motion by IIS Computer-Actuator On-Line System," *Proceedings, Seventh World Conference on Earthquake Engineering*, Istanbul, Turkey, Sept., 1980.
17. Okamoto, S., Nakata, S., Kitagawa, Y., Yoshimura, M., and Kaminosono, T., "A Progress Report on the Full-Scale Seismic Experiment on a Seven Story Reinforced Concrete Building—Part of the U.S.-Japan Cooperative Program," *Research Paper No. 94*, Building Research Institute, Ministry of Construction, Japan, Mar., 1982.
18. Okamoto, S., Yamazaki, Y., Kaminosono, T., and Nakashima, M., "Correlation between Shaking Table Test and Pseudodynamic Test on Steel Structures," presented at the May, 1983, Fifteenth Joint Meeting, U.S.-Japan Panel on Wind and Seismic Effects, held at Tsukuba, Japan.
19. Safford, B., "Development of Force Pulse Train Generators for the Study of Dynamic Response," presented at the Jan., 1981, *Second ASCE-EMD Specialty Conference on Dynamic Response of Structures*, held at Atlanta, Ga.
20. Shing, P. B., and Mahin, S. A., "Experimental Error Propagation in Pseudodynamic Testing," *UCB/EERC-83/12*, Earthquake Engineering Research Center, University of California, Berkeley, Calif., June, 1983.
21. Shing, P. B., and Mahin, S. A., "Pseudodynamic Test Method for Seismic Performance Evaluation: Theory and Implementation," *UCB/EERC-84/01*, Earthquake Engineering Research Center, University of California, Berkeley, Calif., Jan., 1984.
22. Shing, P. B., Mahin, S. A., and Demitzakis, S. N., "Evaluation of On-Line Computer Control Methods for Seismic Testing," *Proceedings, Eighth World Conference on Earthquake Engineering*, San Francisco, Calif., July, 1984.
23. Shing, P. B., Javadian-Gilani, A. S., and Mahin, S. A., "Evaluation of Seismic Behavior of a Braced Tubular Steel Structure by Pseudodynamic Testing," *Journal of Energy Resources Technology*, ASME Transactions, Sept., 1984.
24. Takanashi, K., Udagawa, K., Seki, M., Okada, T., and Tanaka, H., "Nonlinear Earthquake Response Analysis of Structures by a Computer-Actuator On-Line System," *Bulletin of Earthquake Resistant Structure Research Center*, No. 8, Institute of Industrial Science, University of Tokyo, Tokyo, Japan, 1975.
25. Takanashi, K., Taniguchi, H., and Tanaka, H., "Inelastic Response of H-Shaped Columns to Two Dimensional Earthquake Motions," *Bulletin of Earthquake Resistant Structure Research Center*, No. 13, Institute of Industrial Science, University of Tokyo, Tokyo, Japan, 1980.
26. Townsend, W. H., and Anderson, J. C., "Analytical/Experimental Modeling of Exterior Beam-Column Connections," presented at the Nov., 1978, ACI Convention.

Appendix II.—Notation

The following symbols are used in this paper:

a = vector of nodal accelerations;
a_g = earthquake ground accelerations;
c = viscous damping matrix;
d = vector of nodal displacements;
f = vector of external force excitations;
k = stiffness matrix;
m = mass matrix;
r = vector of nodal restoring forces;
T = natural period of structure;
v = vector of nodal velocities;
β, γ = parameters of Newmark algorithm;
ΔT = real time taken by each step of test;
Δt = integration time interval;
ϵ = ratio of maximum displacement amplitude computed by numerical integration over exact value;
σ_d = standard deviation of displacement feedback errors; and
ω = natural angular frequency of structure.

Subscript

i = time equal to $i\Delta t$.

J. Struct. Eng., 1985, 111(7): 1482-1503