NE 565
Applied Thermal Hydraulics
Assignment#1

The transport of ϕ through a given pipe is governed by the following equation for steady-state convection and diffusion.

$$\int_S \rho \bar{V} \phi \cdot \bar{n} dS = \int_S \Gamma \frac{\partial \phi}{\partial x} \cdot \bar{n} dS + \int_V q_\phi dV \qquad (1)$$

(a) Using the central differencing scheme, calculate the distribution of $\phi(x)$ for the following three cases. GRAPH. Assume constant velocity along pipe.

Case 1: 5 control volumes and $\bar{V}$ = 0.1 m/s,

Case 2: 5 control volume and $\bar{V}$ = 2.5 m/s, and

Case 3: 20 control volumes and $\bar{V}$ = 2.5 m/s.

(b) For all three cases, compare your numerical solution to the following analytical solution. GRAPH.

$$\frac{\phi - \phi_L}{\phi_R - \phi_L} = \frac{\exp(\rho \bar{V} x / \Gamma) - 1}{\exp(\rho \bar{V} L / \Gamma) - 1} \qquad (2)$$

(c) Calculate the average error for each of the three cases using the following formula.

$$\varepsilon = \frac{\sum_i \left| \phi_i^{exact} - \phi_i \right|}{N} \qquad (3)$$

(d) Are the numerical results that you obtained what you expected? Why or why not?

<u>Given</u>
Pipe length = 1.0 m
ρ = 1.0 kg/m^3 (constant)
Γ = 0.1 kg-s/m (constant)
Q_ϕ = 0.0
Dirichlet boundary conditions, ϕ_L = 100, ϕ_R = 50