NE 565
Applied Thermal Hydraulics
Assignment#2

The transport of ϕ through a given pipe with no sources is governed by the following equation for steady-state convection and diffusion.

$$\int_S \rho u_x \phi \cdot \bar{n} dS = \int_S \Gamma \frac{\partial \phi}{\partial x} \cdot \bar{n} dS \qquad (1)$$

We saw in assignment #1 that this equation can be solved directly using an appropriate discretization scheme and matrix solver. This "steady-state" problem can also be solved by marching the following transient equation in time until the solution converges.

$$\int_{\Delta t} \frac{\partial}{\partial t}\left(\int_{CV} \rho \phi dV \right) dt + \int_{\Delta t} \int_S \rho u_x \phi \cdot \bar{n} dS dt = \int_{\Delta t} \int_S \Gamma \frac{\partial \phi}{\partial x} \cdot \bar{n} dS dt \qquad (2)$$

a) Derive the interior-node and boundary-node discretized equations for the transient convection-diffusion of ϕ (equation 2) for the following approximation schemes:

Scheme 1: Upwind in space, Explicit Euler in time.
Scheme 2: Upwind in space, Implicit Euler in time.
Scheme 3: Upwind in space, Trapezoidal in time. (complete for extra credit)

b) Using the above three schemes, calculate the distribution of ϕ(x, t) using the following given conditions and K = 0.2, 2.0, and 20.0 (for use in equation 3). Graph ϕ(x) at n = 0, 4, 16, 64 and 256.

$$\Delta t = \frac{K \Delta x}{u_x} \qquad (3)$$

Given
Pipe length = 1.0 m
ρ = 1.0 kg/m^3 (constant)L
Γ = 0.1 kg-s/m (constant)
Dirichlet boundary conditions, φ_L = 100, φ_R = 50
Initial conditions, $\varphi(x, 0)$ = 50
u_x = 2.5 m/s (constant)
20 control volumes

c) At n = 256, compare your transient solutions to the steady-state solution (numerical) from the problem set#1 by calculating the norm using the following equation:

$$\varepsilon = \frac{\sum_i \left| \varphi_i^{transient} - \varphi_i^{steadystate} \right|}{N} \qquad (4)$$