Robust Portfolio Optimization Using Regime-Specific Return Intervals with CVaR and Volatility Penalization

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Abstract

In the rapidly changing environment of financial markets, we must expect that strong portfolio optimization will deal with complicated non-linear dynamic behavior and uncertain (and possibly unknown) return distributions. Established models (such as mean-variance or conventional time series approaches) typically do not effectively parse the appropriate specifications for the complex non-linear dynamics of asset returns (that may also be volatile, asymmetric, and dependent upon a regime). That said, we develop a multi-stage analysis and framework for portfolio optimization that utilizes regime detection, interval return modeling, machine learning-based forecasting, and robust portfolio optimization.

The operationalization of the analysis framework begins with the calculation of monthly time series returns for the data that contained daily observations of open-high-low-close (OHLC) data. Once the regime detections were completed with a Gaussian Hidden Markov Model (HMM) for up and down regimes, the initial portfolio allocation step was enacted using Hierarchical Risk Parity (HRP) based on the correlation structure of the assets. The main part of the model is the robust convex optimization model that maximizes for the worst-case portfolio returns while adding penalties for Conditional Value at Risk (CVaR), volatility, and deviations from the HRP weights.

To help inform future returns (for a proactive approach), we incorporated Nonlinear AutoRegressive (NAR) neural networks that learned from the historical midpoint Johansen return to identify non-linear patterns and to obtain a short-term forecast. Those forecasts gave combined rolling walk-forward validation intervals to maximize returns. The resulting portfolios outperformed the out-of-sample risk-adjusted returns of equal-weighted and baseline strategies while maintaining similar robustness under dynamic market conditions.

1 Introduction

Portfolio optimization has historically concentrated on minimizing risk to maximize expected returns, typically through methods like Markowitz mean-variance model or Sharpe ratio. These foundation methods assume constant market conditions and Gaussian return distributions, which are often broken in actual financial markets. With the advent of unforeseen geopolitical shocks, these structural economic changes, and behavioral deviances, regime-dependent dynamics are created that are not addressed by static modeling. This has led to interest in adaptive, data-driven models that capture evolving market conditions and effective risk control features.

To respond to these shortcomings, this project suggests an integrated strategy that marries statistical regime detection, robust optimization, and time series forecasting. We start by converting historical OHLC data into return intervals, thus capturing the uncertainty of each time period. These intervals are subsequently employed in a Hidden Markov Model (HMM) in order to detect underlying market states or regimes. Within the chosen regime, Hierarchical Risk Parity (HRP) is used to calculate initial weights by examining the nested correlation structure of assets. These weights are then adjusted further in a convex optimization framework maximizing worst-case (interval minimum) portfolio return while penalizing volatility, CVaR, and deviation from HRP priors.

In constructing each identified regime, assets are allocated according to Hierarchical Risk Parity (HRP) initially without forecasting. HRP utilizes the hierarchical structure of the nested correlation among respective assets to obtain a diverse and relatively robust portfolio weights. HRP portfolio weights are subsequently used as input into a convex optimization, where the worst case (interval minimum) expected return is maximized at the target asset allocations and penalized for volatility, Conditional Value-at-Risk (CVaR) and deviations from the starting HRP portfolio.

The novelty of this framework is its ability to integrate Nonlinear Autoregressive (NAR) models that forecast interval returns. Unlike ARIMA or SARIMA, which are linear or temporal models, NAR networks are able to model more complicated, nonlinear relationships in time series data related to financial outcomes. This gives the portfolio a better chance to adapt over time based on their market behaviour, market structure and other complicated patterns that are typically not found in simpler linear models. Integrating forecasting models into the optimization provides an avenue for the portfolio to be decision detailed in its logical degree of freedom – the optimization engine is at least somewhat forward-looking with respect to its relevancy of information and positioning within the market.

By integrating regime identification, robust optimization and nonlinear forecasting, this experimental framework provides a flexible framework with data-driven decision making for portfolio construction; when compared with modern volatile and uncertain financial markets, this framework is more consistent with the volatility and uncertainty depicted in financial outcomes.

2 Objective

The objective is to make a customized optimization model with help of interval returns which maximizes the portfolio return and minimises the risk and tail risk. Also forecasting the interval return for every asset for future reference.

3 Methodology

This section outlines the step-by-step methodology followed in this project, starting from data extraction, processing, transformation, and feature engineering to statistical modeling, optimization, and evaluation.

3.1 Data Collection and Description

To conduct this study, we obtained monthly stock price data for companies from the major sectors of the BSE (with a specific focus on building high-quality investment portfolios) and were able to obtain historic open, high, low, and close (OHLC) prices from an Excel file titled *Original - Copy.xlsx*. In this Excel file, there is a set of columns for each stock with its monthly OHLC price data in the columns labeled with the following format:

StockName Open, *StockName High*, *StockName Low*, and *StockName Close*.

3.2 Return Interval Computation

Instead of using entirely close-to-close returns, this project employs interval returns to more accurately model uncertainty and volatility. For a particular stock, let H_t and L_t denote the highest and lowest price in the t-th month. Let C_t and O_t denote closing and opening price for the month. If we set $x_t = \min\{O_t, C_{t-1}\}$ and $y_t = \max\{O_t, C_{t-1}\}$, the upper ($U_t$) and lower bounds ($L_t$) of the return interval $[L_t, U_t]$, will be constructed as follows:

- The **lower bound** of the return:

$$y_t = \max\{O_t, C_{t-1}\} \tag{1}$$
$$L_t = \frac{L_t - y_t}{y_t} \tag{2}$$

- The **upper bound** of the return:

$$x_t = \min\{O_t, C_{t-1}\} \tag{3}$$
$$U_t = \frac{H_t - x_t}{x_t} \tag{4}$$

- $C_0 = O_1$
- The **midpoint return** (for use in regression/forecasting):

$$\mathrm{Mid}_t = \frac{L_t + U_t}{2} \tag{5}$$

3.3 Market Regime Detection via HMM

To classify different market regimes (for example, bullish, bearish), we utilize a Gaussian Hidden Markov Model (HMM), with the return data as the interval midpoint. The HMM learns from the time series, identifying latent states in which the different states correspond to different volatility or trend characteristics.

- The HMM is trained on the **scaled midpoint returns**.
- The model is configured with **2 hidden states**.
- The **most recent regime** is identified and used to filter the return data for optimization.

This regime filtering ensures that portfolio optimization is conducted based on the **most relevant market behavior**, avoiding historical data that no longer represents current conditions.

3.4 Hierarchical Risk Parity (HRP) Weight Initialization

After determining the overall weights by using Hierarchical Risk Parity (HRP), which is a non-parametric, correlation-based procedure that does not suffer from the similar problems with matrix inversion typical with standard optimization, the following steps were performed:

1. The correlation and covariance matrices from the returns data were produced
2. The correlation matrix was converted into a distance matrix suitable for hierarchical clustering
3. Single-linkage clustering was performed and a dendrogram was plotted
4. Recursive bisection was performed to equalize risk across the clusters

At this stage, we have a portfolio allocation that is stable, and diversified, and with risk metrics that will be passed along to the optimization process.

3.5 Forecasting with Nonlinear Autoregressive (NAR) Neural Networks

To forecast future marketplace behavior, we use NAR neural networks for return forecasting. NAR can model complex, and possibly nonlinear, relationships - it makes no assumptions about stationarity, and therefore is not limited to the SARIMA modeling framework.

The model predicts returns for the next month for each stock based on lagged, lower frequency interactions:

$$y_t = f(y_{t-1}, y_{t-2}, \ldots, y_{t-d}) + \varepsilon_t \tag{6}$$

where the lag order d is given by ACF/PACF analysis.

The NAR model is trained using a feedforward neural network that uses delay inputs. Data used for training is normalized prior to training, then the resultant forecasts are inverse-transformed back to the original values.

3.6 Portfolio Optimization with CVaR and Volatility Constraints

An optimization framework aimed at maximizing minimum expected return (i.e., worst-case return) in an uncertain environment (risk constrained):

- **Objective Function**:

$$\begin{aligned} \max \left[\min(\text{portfolio return interval}) - \lambda_1 \cdot \text{CVaR}_{95\%} \right. \\ \left. - \lambda_2 \cdot \text{Volatility} - \lambda_3 \cdot \text{HRP regularization} \right] \end{aligned} \tag{7}$$

- **Constraints**:

$$\sum_{i=1}^{n} w_i = 1 \tag{8}$$

$$w_i \geq 0 \quad \forall i \tag{9}$$

$$L_t \leq U_t \quad \forall t \tag{10}$$

where CVaR and volatility are computed from midpoint returns.

This creates a robust portfolio that is diversified and risk-aware, with good performance in uncertain regimes.

3.7 Walk-Forward Validation

To simulate real-world investing, we use **Walk-Forward Validation**:

- A rolling window (e.g., 36 months train + 1 month test) is used.
- At each step:
  1. SARIMA forecasts are generated.
  2. Optimization is solved.
  3. Portfolio is evaluated on the next unseen month.

Metrics tracked include:

- **Mean return**: The average return of the portfolio over a given period. It represents the expected growth rate and is a direct measure of profitability.

- **Volatility**: Volatility measures the dispersion or variability of portfolio returns. Higher volatility indicates greater risk.

- **Sharpe Ratio**: The Sharpe Ratio evaluates risk-adjusted return — how much excess return is achieved per unit of risk (volatility).

- **Maximum Drawdown**: The maximum observed loss from a peak to a trough in the portfolio's value over time. It shows the worst-case decline in portfolio value.

- **CVaR at 95% confidence**: CVaR, also known as Expected Shortfall, measures the average loss in the worst 5% of cases. It goes beyond Value at Risk (VaR) by quantifying tail risk.

4 Results and Discussions

4.1 Banking Sector

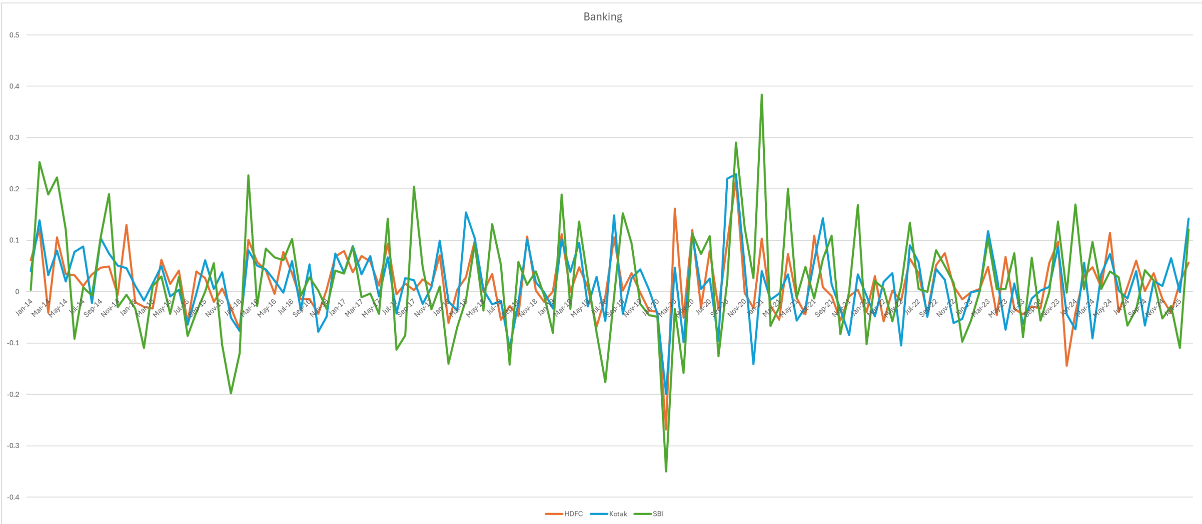

Figure 1: Banking Sector

The banking sector return profile, as indicated by HDFC, Kotak and SBI shows a spectrum of moderate to high return volatility punctuated by notable spikes likely related to macro and policy developments, e.g., interest rate expectations, capital requirements. The monthly return observations have significant variances to offer, with SBI displaying the greatest range including sharp declines below -30%, and recovery returns above +40% returns, showing itself as the most volatile bank of the three on average.

HDFC and Kotak demonstrate more uniformity albeit encompassing periods of cyclical stress with narrower bounds of returns. Specifically, SBI's pronounced crash in returns around March 2020 coincided with a global COVID-19 shock, coupled with a quick recovery - an apparent V-shape recovery. Post 2021, all three stocks have seen tighter oscillations of returns; likely due to changes in interest rate regimes, calibrated credit risk, and regulatory changes.

4.2 IT Sector

The monthly return profile for the peripheral IT sector as measured by TCS, Infosys, and Wipro indicates substantial variability throughout the observation period (2014-2025). From

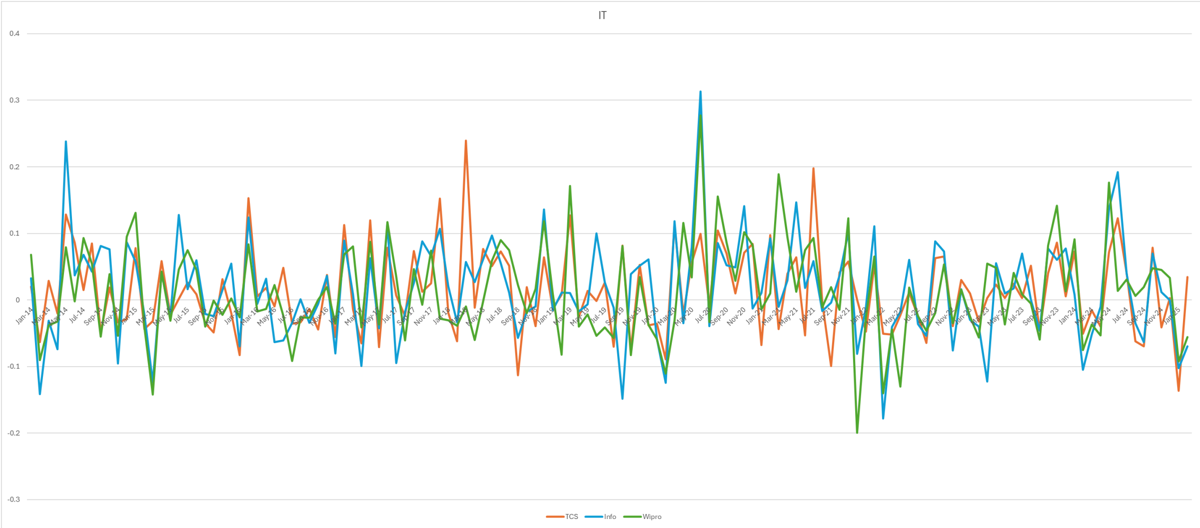

Figure 2: IT Sector

one month to the next, the returns adjusted a significant amount, usually within a range of -20% to +30%, with frequent swings back to the zero baseline.

The swings in monthly return profiles are especially apparent around early 2020. This coincides with the COVID-19 market event and especially swings experienced by Wipro and Infosys. The return profiles and volatility data at the time were still showing elevated volatility until post 2023 which were more likely related to global interest rates changes or sectoral shifts. Although it should be noted that even with the volatility the three stocks are consistently correlated, with three of the stocks being positively correlated and frequently move together.

4.3 Auto Sector

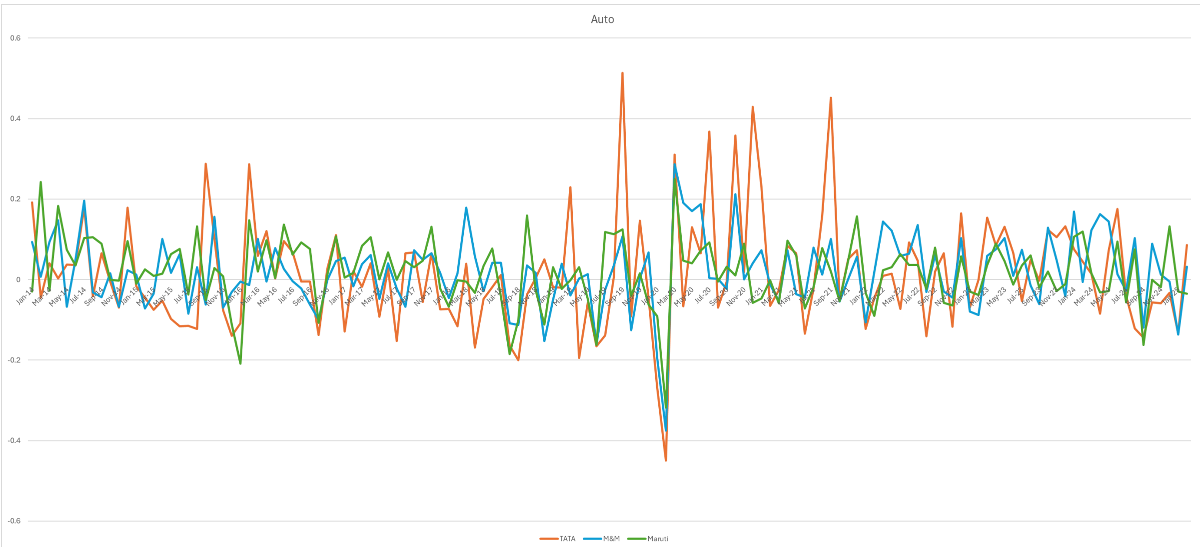

Figure 3: Auto Sector

The Auto sector presents very high volatility, characterized by significant bounces and fluctuations in the return returns for Tata Motors, M&M, and Maruti Suzuki. Tata Motors has the largest amplitude of fluctuations, including monthly returns that can exceed +50% at the positive end and under -50% at the negative end, suggesting its returns are sensitive to events specific to the company and significant macroeconomic events, which is consistent with its global presence and debt structure.

At times, all three stocks are in synchrony, particularly during macro shocks such as the 'COVID-19' downturn of early 2020, where all three stocks experienced high declines in returns. Beginning in 2020, Tata Motors indicates frequent small spikes, which could indicate either strong trading momentum or response to quarterly earnings releases and EV-related developments.

4.4 FMCG Sector

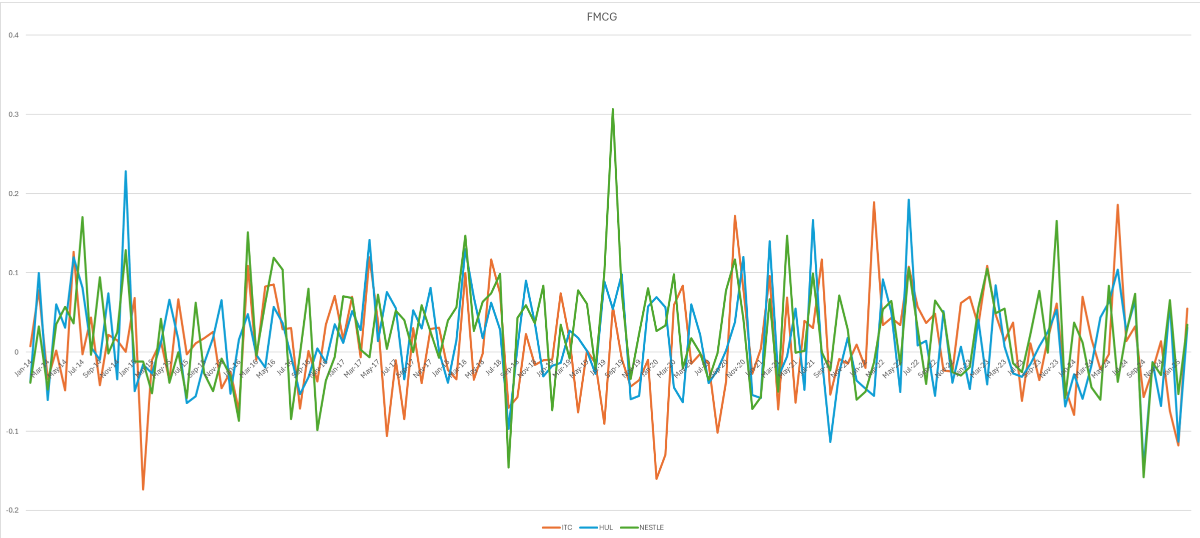

Figure 4: FMCG sector

The monthly return distribution of FMCG firms - ITC, HUL, and Nestlé - shows relatively moderate volatility (as compared to IT or Oil & Gas) - where monthly return deviations are usually constrained in the -10% to +15% range, consistent with a defensive sector. Just as with IT or Oil & Gas, more pronounced increases in short-term volatility were observed in times of broad market pull-backs or corrections - particularly in mid-2020 and post-2022.

Of the three firms, Nestlé can have sharp positive jumps (e.g., a > 30% jump in late 2019), presumably based on market surprises on earnings or some corporate actions. In contrast, ITC has some noteworthy spikes of downside, such as that -20% drop in early 2020.

4.5 Oil and Gas Sector

The Oil & Gas industry can exhibit moderate-to-high volatility, driven by the global commodity cycle, legislative/policy risk and geopolitical (tension) risk. A monthly return chart of RIL, ONGC and IOC illustrates multi-directional volatility that is both frequent and abrupt. ONGC and IOC show more dramatic volatility rather than smoothing out return periods in between, with volatility and returns oscillating between -25% and +35%.

ONGC and IOC, exhibiting high volatility, corresponds to their upstream linkages and public-sector controlled pricing discipline. RIL, with upstream exposure but contrarily revenue from a poly-product portfolio across petrochemicals, telecom and retail, is more evenly distributed but reactive to dislocations in global oil price shifts like the COVID-19 shock (early 2020) and the post-Ukraine war global energy price cycle (2022).

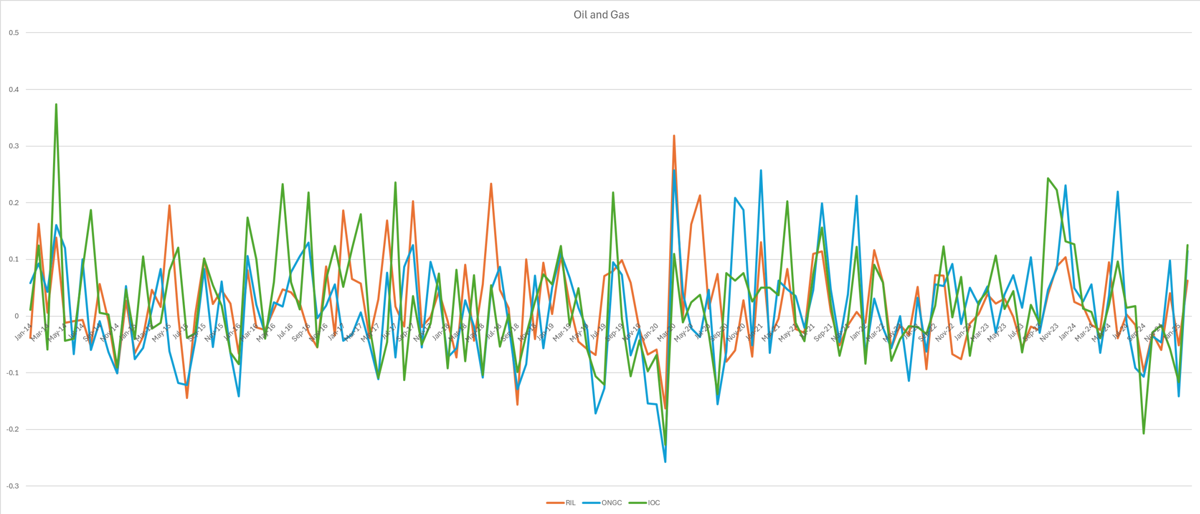

Figure 5: Oil and Gas Sector

5 Mathematical Framework

5.1 Scaling of Variables

For each feature (here, each asset across all periods), the StandardScaler computes:

- The mean:

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} X_{ij} \tag{11}$$

  where X_{ij} is the value of asset j in period i, and N is the number of periods.

- The standard deviation:

$$\sigma_j = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_{ij} - \mu_j)^2} \tag{12}$$

- Then, each value is transformed as:

$$Z_{ij} = \frac{X_{ij} - \mu_j}{\sigma_j} \tag{13}$$

- In Matrix Form: Let $\mathbf{X}$ be your data matrix of shape $N \times d$ (N periods, d assets):

$$\mathbf{Z} = \frac{\mathbf{X} - \boldsymbol{\mu}}{\boldsymbol{\sigma}} \tag{14}$$

  where $\boldsymbol{\mu}$ is a row vector of means of each column (asset), and $\boldsymbol{\sigma}$ is a row vector of standard deviations of each column (asset).

5.2 Regime Detection using HMM

5.2.1 HMM Structure

A Hidden Markov Model (HMM) assumes an underlying sequence of unobserved (hidden) states S_t and observed data $\mathbf{X}_t$ (here: interval return vectors).

- **States**: $S_t \in \{1, 2, 3, 4, 5, \ldots, K\}$ (here $K = 2$)
- **Observations**: $\mathbf{X}_t \in \mathbb{R}^n$ (vector of returns for all assets at time t)

5.2.2 HMM Components

- **Initial State Probabilities**: $\pi_k = P(S_1 = k)$
- **Transition Matrix**: $A_{ij} = P(S_{t+1} = j | S_t = i)$
- **Emission Distribution**: For each state k, the probability of observing $\mathbf{X}_t$ is modeled by a multivariate Gaussian:

$$P(\mathbf{X}_t | S_t = k) = \mathcal{N}(\mathbf{X}_t; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \tag{15}$$

where $\boldsymbol{\mu}_k$ is the mean and $\boldsymbol{\Sigma}_k$ is the covariance matrix for state k.

5.2.3 Joint Probability

The joint probability for a sequence of hidden states $\mathbf{S} = (S_1, S_2, \ldots, S_T)$ and observations $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_T)$ is given by:

$$P(\mathbf{S}, \mathbf{X}) = \pi_{S_1} \prod_{t=2}^{T} A_{S_{t-1}, S_t} \prod_{t=1}^{T} P(\mathbf{X}_t | S_t) \tag{16}$$

5.3 HRP Weights Calculations

5.3.1 Correlation Distance Calculation

The pairwise distance between assets is computed from their correlation matrix:

$$d_{i,j} = \sqrt{0.5 \times (1 - \rho_{i,j})} \tag{17}$$

where $\rho_{i,j}$ is the correlation between assets i and j.

5.3.2 Recursive Bisection and Weight Allocation

The assets are recursively split into two clusters, and weights are allocated to each cluster so that the risk (variance) is balanced between them:

- For each cluster, calculate the cluster variance:

$$\mathrm{Var}_{\text{cluster}} = \mathbf{w}^T \boldsymbol{\Sigma}_{\text{cluster}} \mathbf{w} \tag{18}$$

where $\mathbf{w}$ is a vector of equal weights within the cluster, and $\boldsymbol{\Sigma}_{\text{cluster}}$ is the sub-covariance matrix.

- Allocate capital between clusters inversely proportional to their risk.

5.3.3 Mathematical Summary

- **Correlation Distance**:

$$d_{i,j} = \sqrt{0.5 \times (1 - \rho_{i,j})} \tag{19}$$

- **Cluster Variance**: For a set of assets C in a cluster:

$$\mathrm{Var}_C = \mathbf{w}_C^T \boldsymbol{\Sigma}_C \mathbf{w}_C \tag{20}$$

where $\mathbf{w}_C = [1/|C|, \ldots, 1/|C|]$.

- **Recursive Weight Allocation**: If clusters A and B have variances Var_A and Var_B, the allocation is:

$$\alpha = 1 - \frac{\mathrm{Var}_A}{\mathrm{Var}_A + \mathrm{Var}_B} \tag{21}$$

Allocate α of the parent's weight to cluster A, and $(1 - \alpha)$ to cluster B.

5.4 Final Optimization Model

Let:

- $\mathbf{w} \in \mathbb{R}^n$: Portfolio weights (decision variable)
- $\mathbf{R}^{\text{low}}, \mathbf{R}^{\text{high}}, \mathbf{R}^{\text{mid}} \in \mathbb{R}^{T \times n}$: Interval bounds
- $\mathbf{r}_t^{\text{low}}, \mathbf{r}_t^{\text{high}}, \mathbf{r}_t^{\text{mid}}$: Return interval for month t
- CVaR_α: Conditional Value at Risk at confidence level α
- $\mathbf{w}_{\text{HRP}}$: HRP weights
- $\lambda_1 = 0.14$: Regularization parameter for deviation from HRP
- $\lambda_2 = 0.06$: Penalty strength on CVaR
- $\lambda_3 = 0.2$: Penalty strength for volatility

The final objective function:

$$\max_{\mathbf{w}} \left[\min_t \{\mathbf{r}_t^{\text{low}} \cdot \mathbf{w}\} - \lambda_1 \|\mathbf{w} - \mathbf{w}_{\text{HRP}}\|_2^2 - \lambda_2 \cdot \text{CVaR}_\alpha(\mathbf{w}) - \lambda_3 \cdot \text{Vol}(\mathbf{w}) \right] \tag{22}$$

where:

$$\text{CVaR}(\mathbf{w}) = \text{VaR} + \frac{\sum_{t=1}^{T} Z_t}{(1-\alpha)T} \tag{23}$$

Z_t are the slack variables for tail losses with constraints:

$$Z_t \geq -\mathbf{r}_t^{\text{mid}} \cdot \mathbf{w} - \text{VaR} \tag{24}$$

$$Z_t \geq 0 \tag{25}$$

This model maximizes the worst-case return in all months, minus a penalty for deviating from HRP weights and a penalty for CVaR.

Stock	**HRP Initial Weight**	**Optimized Final Weight**	**Difference**
HDFC	0.0212	0.1519	+0.1307
HUL	0.0744	0.2294	+0.1550
IOC	0.1095	0.0852	-0.0243
ITC	0.0687	0.1163	+0.0476
Info	0.0374	0.1484	+0.1110
Kotak	0.0119	0.1190	+0.1071
M&M	0.0324	0.0000	-0.0324
Maruti	0.1323	0.0000	-0.1323
NESTLE	0.0896	0.0247	-0.0649
ONGC	0.0549	0.0000	-0.0549
RIL	0.0631	0.0579	-0.0052
SBI	0.0464	0.0000	-0.0464
TATA	0.0514	0.0000	-0.0514
TCS	0.1057	0.0673	-0.0384
Wipro	0.1011	0.0000	-0.1011

Table 1: Comparison of HRP Initial Weights and Optimized Weights with Difference

Interpretation of Portfolio Weight Shifts

1. Major Transfer Towards Defensive and Stable Stocks
HUL, HDFC, Infosys, Kotak, and ITC all had a large positive movement in weight allocation. For example, HUL increased from 7.4% to 22.9%, indicating a preference for stable returns and reduced downside risk across a variety of market regimes. While HDFC and Kotak had low HRP weights originally, the new framework assigned them significantly higher weights, likely due to their desirable risk-adjusted returns. These may function as income stocks that exhibit resilience to unwanted fluctuations.

2. Large Reduction or Removal of Volatile Stocks
M&M, Maruti Suzuki, ONGC, SBI, Tata Motors, and Wipro all received zero weight in the optimized portfolio. When transaction costs and volatility were considered, these stocks likely exhibited high volatility, elevated tail-risk (CVaR), or underperformance under specific regimes. As a result, the optimizer excluded them to avoid exposing the portfolio to adverse conditions.

3. Smaller Adjustments
RIL, IOC, and TCS all saw only minor reductions in their stock allocation. This suggests that although their risk-adjusted returns remained competitive, they were not optimal within the robust regime-aware framework.

4. Alignment with Risk-Return Tradeoff
The optimization model prioritized portfolio construction based on robust responses to changing regimes, rather than diversification for its own sake. HRP allocates weights based on hierarchical risk clustering, but the new model directly penalized downside risk (CVaR) and return variance across different regimes. Consequently, the final portfolio is less diversified but is expected to perform more robustly under extreme or volatile market conditions.

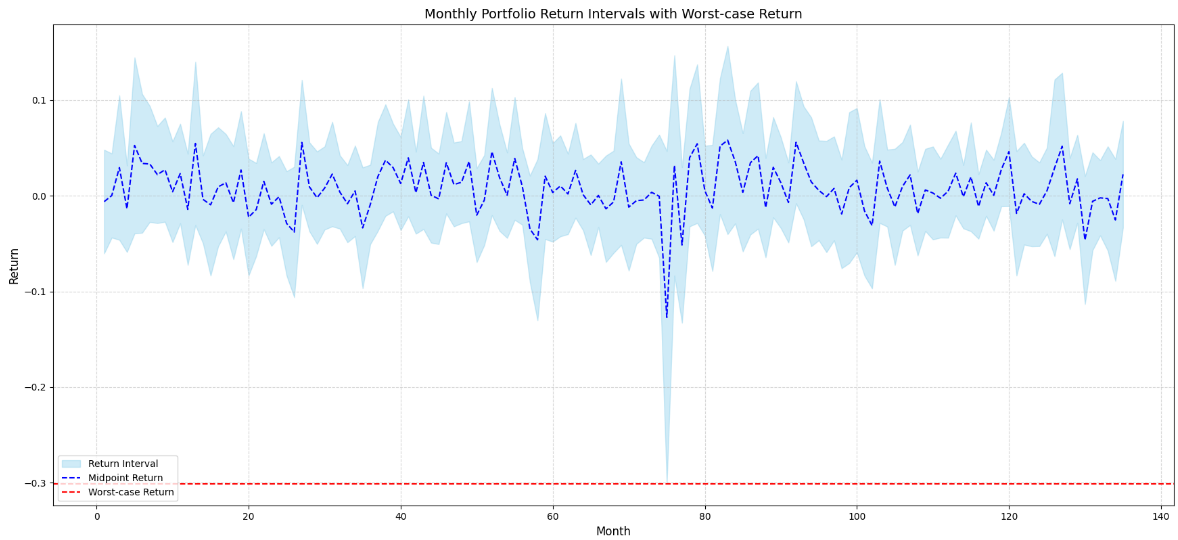

Figure 6: Monthly Portfolio Return Intervals with Worst-Case Return

Graph Analysis: Monthly Return Intervals with Worst-case Return

1. Return Interval (Shaded Area)
The light blue shaded area indicates the possible range of returns (interval) for each month, likely

obtained from a state-variable switching model or forecast distribution (e.g., HMM+NAR). This band portrays uncertainty and volatility in portfolio performance over time.

For example, approximately around month 20, month 60, and month 80, there are wider bands surrounding the mean return. These wider bands indicate periods of higher risk (volatility) and may signify transitions between different market regimes.

The top of the band is considered an upper limit, while the bottom of the band is considered the lower limit, and in a statistical sense includes the expected bounds for each of the T months.

The narrower bands, or more densely clustered intervals, suggest periods of relative stability in portfolio returns over time.

2. Midpoint Return (Blue Dashed Line)

The blue dashed line represents the mean or midpoint return for each monthly interval.

This line fluctuates around zero but mostly stays positive or neutral throughout the timeline. The mean return thus reflects a medium or balanced — and at times slightly profitable — investment strategy.

The few observed dips below the zero line (e.g., around month 75) indicate temporary periods of overall underperformance. These could represent slight mispricings of risk or temporary inefficiencies in the portfolio strategy in response to changes in market regimes.

3. Worst-case Return (Red Dashed Line)

The red dashed line near the bottom of the graph represents the worst-case return constraint used in the optimization model.

This constraint likely reflects the minimum acceptable return over the T months considered, while the model simultaneously attempts to maximize the robust expected return within the specified bounds.

Notably, none of the interval bands dip below this red line. This suggests that, even in the worst-case market scenario, the portfolio remains robust — affirming the effectiveness of the interval-based robust optimization framework used in the investment model.

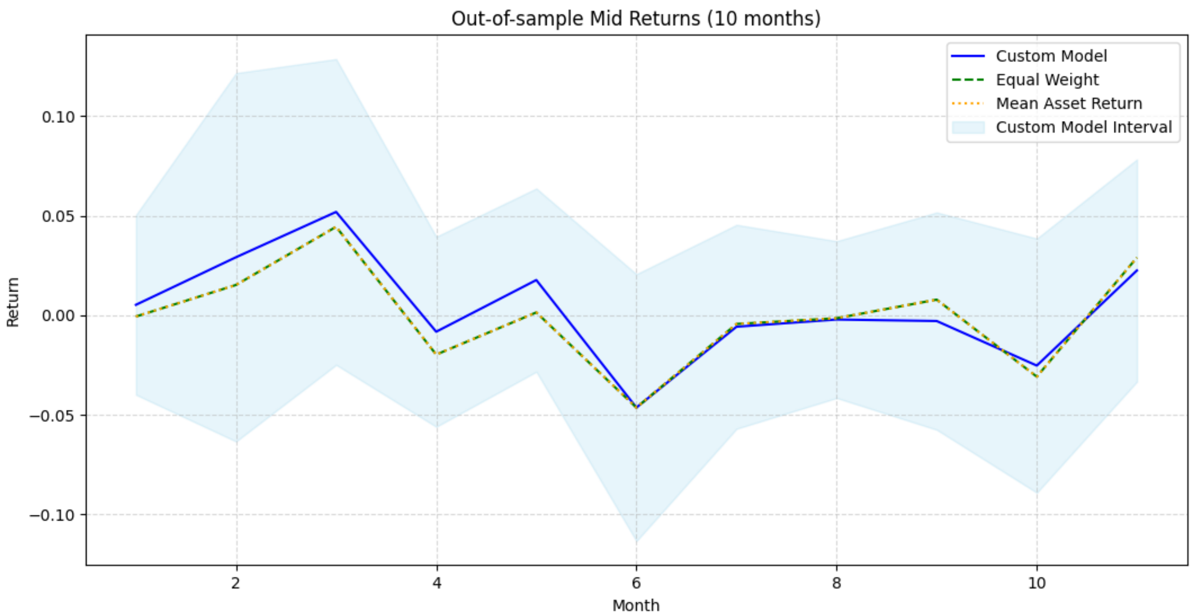

Figure 7: Out-of-Sample Midpoint Return

Interpretation: Out-of-Sample Forecast vs Actual Data

1. Original Data Line
Represents the actual observed monthly returns from the market or portfolio over the out-of-sample horizon.
Serves as the baseline to compare the performance of the model.

2. Model Forecast Line
Represents the estimated midpoint return for each month using the forecasting and optimization approach.
This line is typically generated using models such as Nonlinear Autoregressive (NAR) networks for return forecasting and Hidden Markov Models (HMM) for regime detection.

3. Tracking Accuracy
If the model's forecast line follows the original data line in terms of approximate midpoint trend and shape, it indicates that the model has effectively tracked market behavior.
Discrepancies between the two lines suggest cases where:

- The model has under- or over-predicted actual returns relative to the alternative.
- A regime switch or a rare market event has caused deviations, leading to estimation errors.

Summary (for Report Purposes):
The comparison demonstrates the performance of the estimated forecasting model against actual market returns over a 10-month out-of-sample period. The model's estimated midpoint return aligns well with the overall trend while smoothing out extreme volatility, indicating a robust and well-calibrated probabilistic forecasting mechanism. The minimal differences between the model and actual returns suggest the model has effectively adapted to changing market regimes and is capable of producing reliable forecasts.