Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(x+3y)(4x^2+2xy-1)$$

Answer choices:

A
$$4x^3 + 14x^2y + 6xy^2 - x - 3y$$

B
$$2x^3 + 24x^2y - 6xy + x - 3y$$

C
$$4x^3 + 12x^2 + 6xy + x - 3y$$

D
$$2x^4 - 6xy^2 - x + 3y$$

Solution: A

When we multiply one polynomial by another, we need to make sure we multiply every term in the first polynomial by every term in the second polynomial.

$$(x+3y)(4x^2+2xy-1)$$
$$x(4x^2+2xy-1)+3y(4x^2+2xy-1)$$
$$4x^3+2x^2y-x+12x^2y+6xy^2-3y$$

Rearrange the terms in descending order of powers of x.

$$4x^3 + 2x^2y + 12x^2y + 6xy^2 - x - 3y$$

Group like terms together, and then combine like terms.

$$4x^3 + (2x^2y + 12x^2y) + 6xy^2 - x - 3y$$

$$4x^3 + 14x^2y + 6xy^2 - x - 3y$$

Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(3x + 3y)(x + y) + (x + y)(2x - 2y)$$

Answer choices:

- A $5x^2 5y^2$
- B $6x^2 6xy 6y^2$
- C $5x^2 5xy 5y^2$
- D $5x^2 + 6xy + y^2$

Solution: D

Since we're starting with

$$(3x + 3y)(x + y) + (x + y)(2x - 2y)$$

we can use FOIL on each pair of binomials.

$$\left[3x(x) + 3x(y) + 3y(x) + 3y(y)\right] + \left[x(2x) + x(-2y) + y(2x) + y(-2y)\right]$$

$$3x^2 + 3xy + 3xy + 3y^2 + 2x^2 - 2xy + 2xy - 2y^2$$

Rearrange for clarity, then group like terms together and combine like terms.

$$3x^2 + 2x^2 + 3xy + 3xy - 2xy + 2xy + 3y^2 - 2y^2$$

$$(3x^2 + 2x^2) + (3xy + 3xy - 2xy + 2xy) + (3y^2 - 2y^2)$$

$$5x^2 + 6xy + y^2$$

Topic: Multiplying multivariable polynomials

Question: Simplify the expression.

$$(a+2b-c)(a-2b+c)$$

Answer choices:

$$A = a^2 - 4b^2 + 4bc - c^2$$

B
$$a^2 + 4ab - 4b^2 + 4bc - c^2$$

C
$$a^2 - 4ab - 4b^2 + 2ac - c^2$$

D
$$a^2 - 4b^2 + 4ac - c^2$$

Solution: A

If we start with

$$(a+2b-c)(a-2b+c)$$

then we can distribute each term in the first trinomial over each term in the second trinomial.

$$a(a) + a(-2b) + a(c) + 2b(a) + 2b(-2b) + 2b(c) + (-c)(a) + (-c)(-2b) + (-c)(c)$$

 $a^2 - 2ab + ac + 2ab - 4b^2 + 2bc - ac + 2bc - c^2$

Rearrange for clarity, and then collect like terms.

$$a^{2} - 2ab + 2ab - 4b^{2} + ac - ac + 2bc + 2bc - c^{2}$$

 $a^{2} - 4b^{2} + 4bc - c^{2}$