Unit 2: Multivariate Calculus Solutions

Question 1

Find the partial derivatives if the following functions:

A) $f(x, y) = x^4 y + 2x$

$\frac{df(x,y)}{dx} = 4x^3 y + 2$
$\frac{df(x,y)}{dy} = x^4$

B) $f(x, y) = 2y + 3x^2$

$\frac{df(x,y)}{dx} = 6x$
$\frac{df(x,y)}{dy} = 2$

Question 2

The direction of maximum **decrease** for an objective function $f(\mathbf{x})$ is given by the:

A. Gradient (∇)

B. Negative Gradient ($-\nabla$)

C. Jacobian Matrix ($\mathbf{J}$)

D. Laplacian (∇^2)

Question 3

Fill in the blank: The **definite** integral of $f(x)$ is a number and represents the area under the curve from $x = a$ to $x = b$. The **indefinite** integral of $f(x)$ has no limits and returns a function.

Question 4

The critical point of a convex function is guaranteed to be a **global** minimum.

Question 5

Find the global minimum for the objective function, $f(x) = 2x^2 - 3$.

$$\frac{df(x)}{dx} = 2(2)x^1 = 4x$$

Set 1st derivative equal to 0

$$0 = 4x$$
$x = 0$ is a critical point

2nd derivative test

$$\frac{d^2f(x)}{dx} = 4$$

Since the 2nd derivative is positive and there are no other critical points, $x = 0$ is a global minimum.