Topic: Completing the square

Question: Complete the square to solve for the variable.

$$x^2 + 4x + 2 = 0$$

Answer choices:

$$A \qquad x = -2 \pm \sqrt{2}$$

$$B x = 2 \pm \sqrt{2}$$

$$C x = -2 \pm \sqrt{3}$$

$$D x = 2 \pm \sqrt{3}$$

Solution: A

We have a quadratic polynomial of the form $x^2 + bx + c$ (with b = 4 and c = 2) on the left side of the given equation.

First, we'll subtract c (which is 2) from both sides of the equation.

$$x^2 + 4x + 2 - 2 = 0 - 2$$

$$x^2 + 4x = -2$$

Next, we'll find $(b/2)^2$. Here, b = 4.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

This is the number we have to add to both sides of the equation $x^2 + 4x = -2$ in order to complete the square.

$$x^2 + 4x + 4 = -2 + 4$$

Now we can factor the left-hand side as the square of a binomial.

$$(x+2)(x+2) = 2$$

$$(x+2)^2 = 2$$

$$\sqrt{(x+2)^2} = \sqrt{2}$$

$$x + 2 = \pm \sqrt{2}$$

To solve this equation for x, we subtract 2 from both sides.

$$x + 2 - 2 = -2 \pm \sqrt{2}$$

$$x = -2 \pm \sqrt{2}$$

We can't reduce $\sqrt{2}$ at all, so the roots of our equation are

$$x = -2 + \sqrt{2}$$

and

$$x = -2 - \sqrt{2}$$



Topic: Completing the square

Question: Complete the square to solve for the variable.

$$u^2 - 4u + 3 = 0$$

Answer choices:

A
$$u = -1, -3$$

B
$$u = 1, -3$$

C
$$u = 1, 3$$

D
$$u = -1, 3$$

Solution: C

Find $(b/2)^2$, where b is the coefficient of the u term. Here, b=-4.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

This is the number we have to add to both sides of the equation in order to complete the square.

$$u^2 - 4u + 4 + 3 = 0 + 4$$

$$u^2 - 4u + 4 + 3 = 4$$

$$u^2 - 4u + 4 = 1$$

Factor the left-hand side as the square of a binomial.

$$(u-2)^2 = 1$$

$$u - 2 = \pm \sqrt{1}$$

$$u = 2 \pm 1$$

$$u = 1, 3$$

Topic: Completing the square

Question: Complete the square to solve for the variable.

$$x^2 - 5x - 4 = 0$$

Answer choices:

$$A \qquad x = \frac{5 \pm \sqrt{73}}{2}$$

$$B \qquad x = \frac{-5 \pm \sqrt{73}}{2}$$

$$C x = \frac{5 \pm \sqrt{41}}{2}$$

D
$$x = \frac{-5 \pm \sqrt{41}}{2}$$



Solution: C

To complete the square, we'll add $(b/2)^2$ to both sides of the equation, where b is equal to -5, the coefficient on the first-degree term. We'll also move the other constant term to the right side of the equation.

$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = 4 + \left(-\frac{5}{2}\right)^2$$

$$x^2 - 5x + \frac{25}{4} = \frac{16}{4} + \frac{25}{4}$$

$$x^2 - 5x + \frac{25}{4} = \frac{41}{4}$$

We'll factor the left-hand side and solve for x.

$$\left(x - \frac{5}{2}\right)^2 = \frac{41}{4}$$

$$x - \frac{5}{2} = \pm \sqrt{\frac{41}{4}}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{41}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2}$$

