Topic: Completing the square with complex roots

Question: Solve the quadratic equation by completing the square.

$$x^2 - 4x + 7 = 0$$

Answer choices:

A $x = 2 \pm i\sqrt{2}$

B $x = 2 \pm i\sqrt{3}$

C $x = 3 \pm i\sqrt{2}$

D $x = 3 \pm i\sqrt{3}$

Solution: B

Starting with $x^2 - 4x + 7 = 0$, we'll subtract the constant term, 7, from both sides.

$$x^2 - 4x = -7$$

Next, we'll find $(b/2)^2$. Here, $b = -4$.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

Add the result to both sides of the equation.

$$x^2 - 4x + 4 = -7 + 4$$

Simplify the right side, and factor the left side as the square of a binomial.

$$(x-2)^2 = -3$$

Take the square root of each side and solve, first for $x - 2$ and then solve for x.

$$\sqrt{(x-2)^2} = \sqrt{-3}$$

$$\sqrt{(x-2)^2} = i\sqrt{3}$$

$$x - 2 = \pm i\sqrt{3}$$

$$x = 2 \pm i\sqrt{3}$$

Topic: Completing the square with complex roots

Question: Solve the quadratic equation by completing the square.

$$3x^2 + 4x + 6 = 0$$

Answer choices:

A $x = \dfrac{2 \pm \sqrt{14}}{\sqrt{3}}$

B $x = \dfrac{-2 \pm \sqrt{14}}{3}$

C $x = \dfrac{-2 \pm i\sqrt{14}}{3}$

D $x = \dfrac{-2 \pm i\sqrt{14}}{\sqrt{3}}$

Solution: C

Starting with $3x^2 + 4x + 6 = 0$, we'll divide by 3 so that the coefficient of the x^2 term is 1.

$$\frac{3x^2}{3} + \frac{4x}{3} + \frac{6}{3} = 0$$

$$x^2 + \frac{4x}{3} + 2 = 0$$

Subtract the constant term, 2, from both sides.

$$x^2 + \frac{4x}{3} = -2$$

Find $(b/2)^2$. Here, $b = 4/3$.

$$\left(\frac{b}{2}\right)^2 = \left[\frac{\frac{4}{3}}{2}\right]^2 = \left[\frac{4}{3} \cdot \frac{1}{2}\right]^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Add the result to both sides.

$$x^2 + \frac{4x}{3} + \frac{4}{9} = -2 + \frac{4}{9}$$

Simplify the right side, and factor the left side as the square of a binomial.

$$\left(x + \frac{2}{3}\right)^2 = \frac{-14}{9}$$

Take the square root of each side.

$$\sqrt{\left(x+\frac{2}{3}\right)^2}=\sqrt{\frac{-14}{9}}$$

$$\sqrt{\left(x+\frac{2}{3}\right)^2}=i\sqrt{\frac{14}{9}}$$

$$\sqrt{\left(x+\frac{2}{3}\right)^2}=i\frac{\sqrt{14}}{3}$$

Now solve, first for $x+(2/3)$ and then for x.

$$x+\frac{2}{3}=\pm\frac{i\sqrt{14}}{3}$$

$$x=\frac{-2\pm i\sqrt{14}}{3}$$

Topic: Completing the square with complex roots

Question: Solve the quadratic equation by completing the square.

$$2x^2 - 5x + 10 = 0$$

Answer choices:

A $x = \frac{5 \pm i\sqrt{55}}{4}$

B $x = \frac{5 \pm i\sqrt{55}}{2}$

C $x = \frac{-5 \pm i\sqrt{55}}{2}$

D $x = \frac{-5 \pm i\sqrt{55}}{4}$

Solution: A

Starting with $2x^2 - 5x + 10 = 0$, we'll divide by 2 to clear the coefficient on the x^2 term.

$$\frac{2x^2}{2} - \frac{5x}{2} + \frac{10}{2} = 0$$

$$x^2 - \frac{5x}{2} + 5 = 0$$

To clear the 5, we'll add -5 to both sides.

$$x^2 - \frac{5x}{2} = -5$$

Now to complete the square, we need to take the coefficient on the x term, in this case $-(5/2)x$, and divide it by 2. Half of $-5/2$ is $-5/4$. Then we square the result. $(-5/4)^2 = 25/16$. Now we'll add $25/16$ to both sides.

$$x^2 - \frac{5x}{2} + \frac{25}{16} = -5 + \frac{25}{16}$$

Simplify the right side, and factor the left side as a perfect square.

$$\left(x - \frac{5}{4}\right)^2 = -\frac{55}{16}$$

Take the square root of both sides. Remember that the square root of a negative number can be changed to the square root of a positive number by multiplying by i.

$$\sqrt{\left(x-\frac{5}{4}\right)^2}=\sqrt{-\frac{55}{16}}$$

$$\sqrt{\left(x-\frac{5}{4}\right)^2}=i\sqrt{\frac{55}{16}}$$

$$\sqrt{\left(x-\frac{5}{4}\right)^2}=i\frac{\sqrt{55}}{\sqrt{16}}$$

$$\sqrt{\left(x-\frac{5}{4}\right)^2}=i\frac{\sqrt{55}}{4}$$

Now solve, first for $x - (5/4)$ and then for x.

$$x-\frac{5}{4}=\pm\frac{i\sqrt{55}}{4}$$

$$x=\frac{5\pm i\sqrt{55}}{4}$$