# C&EE 110 Homework 2

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## Problem 1

The conditional probabilities of the different failure mechanisms given a hazardous event are given below:

- $P(S|E_S) = 0.3$
- $P(S|E_M) = 0.03$
- $P(F|V_H) = 0.1$
- $P(F|V_M) = 0.02$
- $P(F|V_L) = 0.005$
- $P(S|V_H) = P(S|V_M) = P(S|V_L) = P(F|E_S) = P(F|E_M) = 0$
- $P(E_S) = 0.10$
- $P(E_M) = 0.90$
- $P(V_H) = 0.08$
- $P(V_M) = 0.22$
- $P(V_L) = 0.7$

Additionally, the probability of occurrence for a strong or mild earthquake is 0.10 and 0.90 respectively. The probability of occurrence for a high, medium, or low vertical load odue to the trucks is 0.08, 0.22, and 0.7 respectively. Please answer:

**a.** What does the expression

$$P(S|V_H) = P(S|V_M) = P(S|V_L) = P(F|E_S) = P(F|E_M) = 0$$

mean in practice? For the answer, relate the failure mechanism to the different hazardous event.

**b.** Determine the probability that an earthquake causes a shear failure.

**c.** Determine the probability that the traffic of the trucks causes a flexure failure.

**d.** Let's suppose that a flexure failure occurs, what is the probability that the traffic load was medium?

**e.** Now let's suppose that a shear failure occurs, what is the probability that the earthquake was strong?

### Response

**a.** A shear failure can never happen due to vertical load and a flexure failure can never happen due to an earthquake.

**b.**

$$\begin{aligned}
P(S|E) &= P(S|E_S)P(E_S) + P(S|E_M)P(E_M) \\
&= (0.3)(0.1) + (0.03)(0.9) \\
&= 0.03 + 0.027 \\
P(S|E) &= 0.057
\end{aligned}$$

**c.**

$$\begin{aligned}
P(F|V) &= P(F|V_H)P(V_H) + P(F|V_M)P(V_M) + P(F|V_L)P(V_L) \\
&= (0.1)(0.08) + (0.02)(0.22) + (0.005)(0.7) \\
&= 0.008 + 0.0044 + 0.0035 \\
P(F|V) &= 0.0159
\end{aligned}$$

**d.**

$$
\begin{aligned}
P(V_M|F) &= \frac{P(F|V_M)P(V_M)}{P(F)} \\
&= \frac{(0.02)(0.22)}{0.0159} \\
&= \frac{0.0044}{0.0159} \\
P(V_M|F) &= 0.277
\end{aligned}
$$

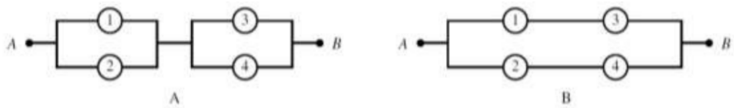
**e.**

$$
\begin{aligned}
P(E_S|S) &= \frac{P(S|E_S)P(E_S)}{P(S)} \\
&= \frac{(0.3)(0.1)}{0.057} \\
&= \frac{0.03}{0.057} \\
P(E_S|S) &= 0.526
\end{aligned}
$$

## Question 2

Relays used in construction of electric circuits function properly if current can flow through them when the circuit is closed.

**a.** Asuming that the circuits are independent, which fo the following ciruit designs yields a higher probability that the current will flow when the relays are activated?




**b.** If we know that the relay elements 1 and 2 are dependent as $P(F_1|F_2) = 0.03)$. How does this change the answer?

The probability of failure of each element is:

| **Relay Element** | **Proability of Failure (F)** |
|---|---|
| 1 | 0.13 |
| 2 | 0.02 |
| 3 | 0.10 |
| 4 | 0.30 |

### Response

**a.**

$$\begin{aligned}
P(F_A) &= P(F_1 \cap F_2) \cup P(F_3 \cap F_4) \\
&= P(F_1)P(F_2) + P(F_3)P(F_4) \\
&= (0.13)(0.02) + (0.1)(0.3) \\
&= 0.0026 + 0.03 \\
P(F_A) &= 0.0326
\end{aligned}$$

$$\begin{aligned}
P(F_B) &= P(F_1 \cap F_2) \cup P(F_1 \cap F_4) \cup P(F_2 \cap F_3) \cup P(F_3 \cup F_4) \\
&= P(F_1)P(F_2) + P(F_1)P(F_4) + P(F_2)P(F_3) + P(F_3)P(F_4) \\
&= (0.13)(0.02) + (013)(0.3) + (0.02)(0.1) + (0.1)(0.3) \\
&= 0.0026 + 0.039 + 0.002 + 0.03 \\
P(F_B) &= 0.0736
\end{aligned}$$

Circuit $A$ is more reliable.

**b.**

$$\begin{aligned}
P(F_A) &= P(F_1 \cap F_2) \cup P(F_3 \cap F_4) \\
&= P(F_1|F_2)P(F_2) + P(F_3)P(F_4) \\
&= (0.03)(0.02) + (0.1)(0.3) \\
&= 0.0006 + 0.03 \\
P(F_A) &= 0.0306
\end{aligned}$$

$$\begin{aligned}
P(F_B) &= P(F_1 \cap F_2) \cup P(F_1 \cap F_4) \cup P(F_2 \cap F_3) \cup P(F_3 \cup F_4) \\
&= P(F_1)P(F_2) + P(F_1)P(F_4) + P(F_2)P(F_3) + P(F_3)P(F_4) \\
&= (0.13)(0.02) + (013)(0.3) + (0.02)(0.1) + (0.1)(0.3) \\
&= 0.0006 + 0.039 + 0.002 + 0.03 \\
P(F_B) &= 0.0716
\end{aligned}$$

Circuit $A$ is more reliable.

## Question 3

- $P(X) = 0.6$
- $P(Y) = 0.3$
- $P(Z) = 0.1$
- $P(R|X) = 0.5$
- $P(R|Y) = 0.6$
- $P(R|Z) = 0.9$

Suppose that we randomly select one coolant tank in the manufacturer's factory. What is the probability that this tank:

**a.** is created from recycled materials?

**b.** is produced in company $Y$ from recycled materials?

**c.** is produced in company $Y$, given that the tank is craeted from recycled materials?

**d.** is created from recycled materials, given that it is produced by company $Z$?

### Response

**a.**

$$\begin{aligned}
P(R) &= P(R|X)P(X) + P(R|Y)P(Y) + P(R|Z)P(Z) \\
&= (0.5)(0.6) + (0.6)(0.3) + (0.1)(0.9) \\
&= 0.3 + 0.18 + 0.09 \\
P(R) &= 0.57
\end{aligned}$$

**b.**

$$\begin{aligned}
P(R \cap Y) &= P(R|Y)P(Y) \\
&= (0.6)(0.3) \\
P(R \cap Y) &= 0.18
\end{aligned}$$

**c.**

$$\begin{aligned}
P(Y|R) &= \frac{P(R|Y)P(Y)}{P(R)} \\
&= \frac{(0.6)(0.3)}{0.57} \\
&= \frac{0.18}{0.57} \\
P(Y|R) &= 0.316
\end{aligned}$$

**d.**

$$P(R|Z) = 0.9$$

## Question 4

Concrete can experience three different types of defects. Let $A_i (i = 1, 2, 3)$ denote the event that the concrete has a defect of type $i$. Suppose that

$$P(A_1) = 0.12, \quad P(A_2) = 0.07, \quad P(A_3) = 0.05$$

$$P(A_1 \cup A_2) = 0.13, \quad P(A_1 \cup A_3) = 0.14, \quad P(A_2 \cup A_3) = 0.10$$

$$P(A_1 \cap A_2 \cap A_3) = 0.01$$

**a.** What is the probability that the concrete does not have a type 1 defect?

**b.** What is the probability that the concrete has both type 1 and type 2 defects?

**c.** What is the probability that the concrete has both type 1 and type 2 defects but not a type 3 defect?

**d.** What is the probability that the concrete has at most two of these defects?

### Response

**a.** $P(\overline{A}_1) = 1 - P(A_1) = 1 - 0.12 = 0.88$

**b.**

$$\begin{aligned} P(A_1 \cap A_2) &= P(A_1) + P(A_2) - P(A_1 \cup A_2) \\ &= 0.12 + 0.07 - 0.13 \\ P(A_1 \cap A_2) &= 0.06 \end{aligned}$$

**c.**

$$\begin{aligned} P(A_1 \cap A_2 \cap \overline{A}_3) &= P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) \\ &= 0.06 - 0.01 \\ P(A_1 \cap A_2 \cap \overline{A}_3) &= 0.05 \end{aligned}$$

**d.** $1 - P(A_1 \cap A_2 \cap A_3) = 1 - 0.01 = 0.99$