C&EE 110 Homework 2

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Problem 1

The conditional probabilities of the different failure mechanisms given a hazardous event are given below:

- $P(S|E_S) = 0.3$
- $P(S|E_M) = 0.03$
- $P(F|V_H) = 0.1$
- $P(F|V_M) = 0.02$
- $P(F|V_L) = 0.005$
- $P(S|V_H) = P(S|V_M) = P(S|V_L) = P(F|E_S) = P(F|E_M) = 0$
- $P(E_S) = 0.10$
- $P(E_M) = 0.90$
- $P(V_H) = 0.08$
- $P(V_M) = 0.22$
- $P(V_L) = 0.7$

Additionally, the probability of occurrence for a strong or mild earthquake is 0.10 and 0.90 respectively. The probability of occurrence for a high, medium, or low vertical load odue to the trucks is 0.08, 0.22, and 0.7 respectively. Please answer:

a. What does the expression

$$P(S|V_H) = P(S|V_M) = P(S|V_L) = P(F|E_S) = P(F|E_M) = 0$$

mean in practice? For the answer, relate the failure mechanism to the different hazardous event.

b. Determine the probability that an earthquake causes a shear failure.

c. Determine the probability that the traffic of the trucks causes a flexure failure.

d. Let's suppose that a flexure failure occurs, what is the probability that the traffic load was medium?

e. Now let's suppose that a shear failure occurs, what is the probability that the earthquake was strong?

Response

a. A shear failure can never happen due to vertical load and a flexure failure can never happen due to an earthquake.

b.

$$\begin{aligned} P(S|E) &= P(S|E_S)P(E_S) + P(S|E_M)P(E_M) \\ &= (0.3)(0.1) + (0.03)(0.9) \\ &= 0.03 + 0.027 \\ P(S|E) &= 0.057 \end{aligned}$$

c.

$$\begin{aligned} P(F|V) &= P(F|V_H)P(V_H) + P(F|V_M)P(V_M) + P(F|V_L)P(V_L) \\ &= (0.1)(0.08) + (0.02)(0.22) + (0.005)(0.7) \\ &= 0.008 + 0.0044 + 0.0035 \\ P(F|V) &= 0.0159 \end{aligned}$$

d.

$$\begin{aligned} P(V_M|F) &= \frac{P(F|V_M)P(V_M)}{P(F)} \\ &= \frac{(0.02)(0.22)}{0.0159} \\ &= \frac{0.0044}{0.0159} \\ P(V_M|F) &= 0.277 \end{aligned}$$

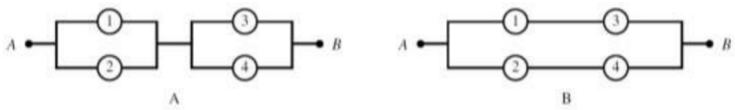
e.

$$\begin{aligned} P(E_S|S) &= \frac{P(S|E_S)P(E_S)}{P(S)} \\ &= \frac{(0.3)(0.1)}{0.057} \\ &= \frac{0.03}{0.057} \\ P(E_S|S) &= 0.526 \end{aligned}$$

Question 2

Relays used in construction of electric circuits function properly if current can flow through them when the circuit is closed.

a. Asuming that the circuits are independent, which fo the following ciruit designs yields a higher probability that the current will flow when the relays are activated?



b. If we know that the relay elements 1 and 2 are dependent as $P(F_1|F_2) = 0.03)$. How does this change the answer?

The probability of failure of each element is:

Relay Element	Proability of Failure (F)
1	0.13
2	0.02
3	0.10
4	0.30

Response

a.

$$\begin{aligned}
P(F_A) &= P(F_1 \cap F_2) \cup P(F_3 \cap F_4) \\
&= P(F_1)P(F_2) + P(F_3)P(F_4) - P(F_1)P(F_2)P(F_3)P(F_4) \\
&= (0.13)(0.02) + (0.10)(0.30) - (0.13)(0.02)(0.10)(0.30) \\
&= 0.0026 + 0.03 - 0.0000078 \\
P(F_A) &= 0.0327 \implies P(S_A) = 0.967
\end{aligned}$$

$$\begin{aligned}
P(S_B) &= P(S_1 \cap S_3) \cup P(S_2 \cap S_4) \\
&= P(S_1)P(S_3) + P(S_2)P(S_4) - P(S_1 \cap S_2 \cap S_3 \cap S_4) \\
&= P(S_1)(P(S_3) + P(S_2)P(S_4) - P(S_1)P(S_2)P(S_3)P(S_4) \\
&= (0.87)(0.90) + (0.98)(0.70) - (0.87)(0.98)(0.90)(0.70) \\
P(S_B) &= 0.932
\end{aligned}$$

Circuit A is more reliable.

b.

$$\begin{aligned}
P(F_A) &= P(F_1 \cap F_2) \cup P(F_3 \cap F_4) \\
&= P(F_1|F_2)P(F_2) + P(F_3)P(F_4) - P(F_1|F_2)P(F_2)P(F_3)P(F_4) \\
&= (0.03)(0.02) + (0.10)(0.30) - (0.03)(0.02)(0.10)(0.30) \\
&= 0.0006 + 0.03 - 0.000018 \\
P(F_A) &= 0.0306 \implies P(S_A) = 0.969
\end{aligned}$$

$$\begin{aligned}
P(F_B) &= P(F_1 \cup F_3) \cap P(F_2 \cup F_4) \\
&= [P(F_1) + P(F_3) - P(F_1)P(F_3)][P(F_2) + P(F_4) - P(F_2)P(F_4)] \\
&= [0.13 + 0.10 - (0.13)(0.10)][0.02 + 0.30 - (0.02)(0.30)] \\
P(F_B) &= 0.0681 \implies P(S_B) = 0.932
\end{aligned}$$

Circuit A is more reliable.

Question 3

- $P(X) = 0.6$
- $P(Y) = 0.3$
- $P(Z) = 0.1$
- $P(R|X) = 0.5$
- $P(R|Y) = 0.6$
- $P(R|Z) = 0.9$

Suppose that we randomly select one coolant tank in the manufacturer's factory. What is the probability that this tank:

a. is created from recycled materials?

b. is produced in company Y from recycled materials?

c. is produced in company Y, given that the tank is craeted from recycled materials?

d. is created from recycled materials, given that it is produced by company Z?

Response

a.

$$\begin{aligned} P(R) &= P(R|X)P(X) + P(R|Y)P(Y) + P(R|Z)P(Z) \\ &= (0.5)(0.6) + (0.6)(0.3) + (0.1)(0.9) \\ &= 0.3 + 0.18 + 0.09 \\ P(R) &= 0.57 \end{aligned}$$

b.

$$\begin{aligned} P(R \cap Y) &= P(R|Y)P(Y) \\ &= (0.6)(0.3) \\ P(R \cap Y) &= 0.18 \end{aligned}$$

c.

$$\begin{aligned} P(Y|R) &= \frac{P(R|Y)P(Y)}{P(R)} \\ &= \frac{(0.6)(0.3)}{0.57} \\ &= \frac{0.18}{0.57} \\ P(Y|R) &= 0.316 \end{aligned}$$

d.

$$P(R|Z) = 0.9$$

Question 4

Concrete can experience three different types of defects. Let $A_i (i = 1, 2, 3)$ denote the event that the concrete has a defect of type i. Suppose that

$$P(A_1) = 0.12, \quad P(A_2) = 0.07, \quad P(A_3) = 0.05$$

$$P(A_1 \cup A_2) = 0.13, \quad P(A_1 \cup A_3) = 0.14, \quad P(A_2 \cup A_3) = 0.10$$

$$P(A_1 \cap A_2 \cap A_3) = 0.01$$

a. What is the probability that the concrete does not have a type 1 defect?

b. What is the probability that the concrete has both type 1 and type 2 defects?

c. What is the probability that the concrete has both type 1 and type 2 defects but not a type 3 defect?

d. What is the probability that the concrete has at most two of these defects?

Response

a. $P(\overline{A}_1) = 1 - P(A_1) = 1 - 0.12 = 0.88$

b.

$$\begin{aligned} P(A_1 \cap A_2) &= P(A_1) + P(A_2) - P(A_1 \cup A_2) \\ &= 0.12 + 0.07 - 0.13 \\ P(A_1 \cap A_2) &= 0.06 \end{aligned}$$

c.

$$\begin{aligned} P(A_1 \cap A_2 \cap \overline{A}_3) &= P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) \\ &= 0.06 - 0.01 \\ P(A_1 \cap A_2 \cap \overline{A}_3) &= 0.05 \end{aligned}$$

d. $1 - P(A_1 \cap A_2 \cap A_3) = 1 - 0.01 = 0.99$