1. **Linear algebra refresher.**

   (a) Let $\mathbf{Q}$ be a real orthogonal matrix.

      i. If $\mathbf{Q}$ is orthogonal, then $\mathbf{Q}\mathbf{Q}^{\mathbf{T}} = \mathbf{Q}^{\mathbf{T}}\mathbf{Q} = \mathbf{I}$. Consider $\mathbf{Q}^{\mathbf{T}}$. We want to show

$$\mathbf{Q}^{\mathbf{T}}\left(\mathbf{Q}^{\mathbf{T}}\right)^{\mathbf{T}} = \mathbf{I}$$

      Recall that $\left(\mathbf{Q}^{\mathbf{T}}\right)^{\mathbf{T}} = \mathbf{Q}$. Then substituting $\left(\mathbf{Q}^{\mathbf{T}}\right)^{\mathbf{T}}$ with $\mathbf{Q}$, we get

$$\mathbf{Q}^{\mathbf{T}}\mathbf{Q} = \mathbf{Q}\mathbf{Q}^{\mathbf{T}} = \mathbf{I}$$

      Note that if $\mathbf{Q}$ is orthogonal, then $\mathbf{Q}^{\mathbf{T}} = \mathbf{Q}^{\mathbf{-1}}$. Then, since $\mathbf{Q}^{\mathbf{T}}$ is orthogonal, $\mathbf{Q}^{\mathbf{-1}}$ is orthogonal.

      ii.

$$\begin{aligned}
\mathbf{Q}\mathbf{x} &= \lambda\mathbf{x} \\
\mathbf{Q}^{\mathbf{T}}\mathbf{Q}\mathbf{x} &= \mathbf{Q}^{\mathbf{T}}\lambda\mathbf{x} \\
\mathbf{I}\mathbf{x} &= \mathbf{Q}^{\mathbf{T}}\lambda\mathbf{x} \\
\mathbf{x} &= \mathbf{Q}^{\mathbf{T}}\lambda\mathbf{x} \\
\mathbf{x}^*\mathbf{Q}\mathbf{x} &= \mathbf{x}^*\lambda\mathbf{x} \\
&= \lambda\mathbf{x}^*\mathbf{x} \\
&= \lambda\|\mathbf{x}\|^2
\end{aligned}$$