

Partial Order: $\forall x, y, z \in A$: Reflexive: xRx , Anti-symmetric: $xRy, yRx \implies x = y$, Transitive: $xRy, yRz \implies xRz$. **Total:** $\forall x, y \in A, xRy \vee yRx$

Equivalence Relation: $\forall x, y, z \in A$: Reflexive: xRx , Symmetric: $xRy = yRx$, Transitive: $xRy, yRz \implies xRz$. **Eq. Class:** $[x] := \{y \in A : x \sim y\}$

Induction: Base step: (i) P_1 is true. **Inductive Hypothesis:** (ii) Assume P_n is true for some $n \in \mathbb{N}$. Prove P_{n+1} is true. Then, P_n is true $\forall n \in \mathbb{N}$.

Ordered Fields: A field with a partial order (\leq) s.t.: (i) If $x, y, z \in \mathbb{F}$, $x < y \implies x + z < x + y$, (ii) $x, y \in \mathbb{F}$, $x, y > 0 \implies xy > 0$

Algebraic Number: a is algebraic if it solves $c_n x^n + \dots + c_1 x + c_0 = 0$ for some $n \in \mathbb{N}, c_0, c_n \in \mathbb{Z}, c_n \neq 0$ (e.g. $\sqrt[3]{2}$). **Note:** $\mathbb{Q} \subset \{\text{algebraic numbers}\}$

RZT: Suppose $c_0, \dots, c_n \in \mathbb{Z}$, $r \in \mathbb{Q}$ satisfies $c_n r^n + \dots + c_1 r + c_0 = 0$ for some $n \in \mathbb{N}$, $c_n \neq 0$. Let $r = \frac{c}{d}$, $c, d \in \mathbb{Z}, d \neq 0$, be coprime. Then c, d divides c_0, c_n .

LUBP: Given $A \in \mathbb{E}$ where \mathbb{E} is an ordered field, $\exists \sup A \in \mathbb{E} \iff A \neq \emptyset, A \subseteq \mathbb{E}, A$ is bounded above. $\sup A := \alpha, \exists \alpha, \beta \in \mathbb{E}$ s.t. $\forall a \in A, a \leq \alpha \leq \beta$.

GLBP: Given $A \in \mathbb{E}$ where \mathbb{E} is an ordered field, $\exists \sup A \in \mathbb{E} \iff A \neq \emptyset, A \subseteq \mathbb{E}, A$ is bounded below. $\inf A := \alpha, \exists \alpha, \beta \in \mathbb{E}$ s.t. $\forall a \in A, a \leq \beta \leq \alpha$.