C&EE 110 Homework 9

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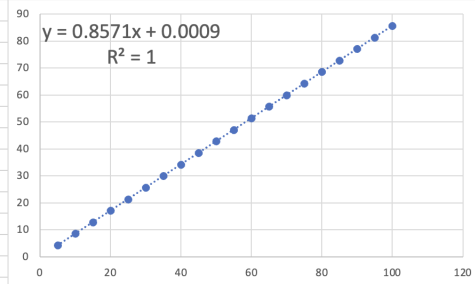
Question 1

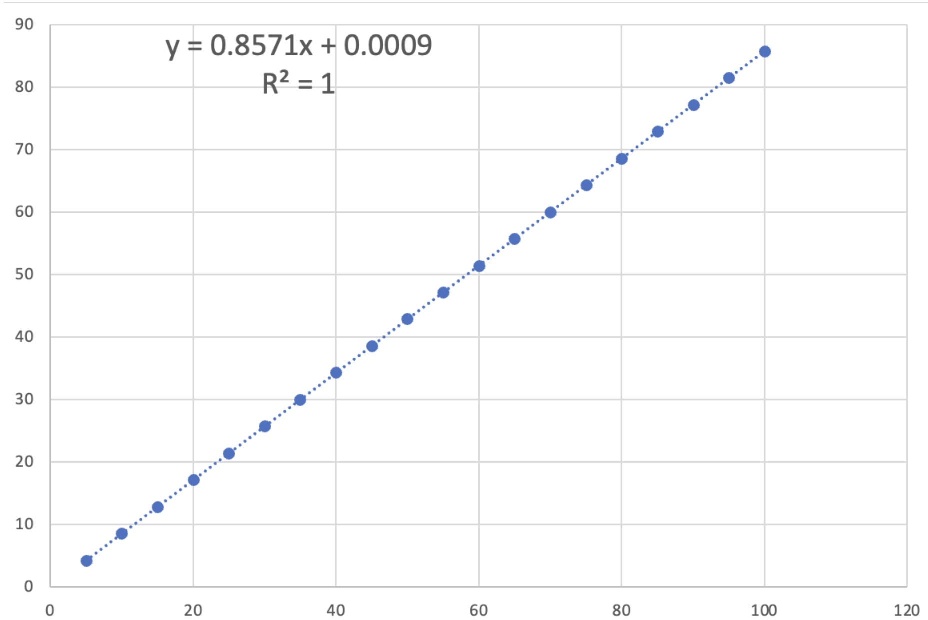
Using the Table, we get:

Sxy	14249.7		**x**	**y**		**xiyi**	**xi^2**
Sxx	16625		5	4.29		21.45	25
			10	8.57		85.7	100
Beta1	0.8571248		15	12.86		192.9	225
			20	17.14		342.8	400
Xbar	52.5		25	21.43		535.75	625
ybar	45		30	25.71		771.3	900
			35	30		1050	1225
Beta0	0.0009474		40	34.29		1371.6	1600
			45	38.57		1735.65	2025
			50	42.86		2143	2500
			55	47.14		2592.7	3025
			60	51.43		3085.8	3600
			65	55.71		3621.15	4225
			70	60		4200	4900
			75	64.29		4821.75	5625
			80	68.57		5485.6	6400
			85	72.86		6193.1	7225
			90	77.14		6942.6	8100
			95	81.43		7735.85	9025
			100	85.71		8571	10000



Then,

(a)



(b) Given:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{N}(x_i - \overline{x})^2}$$
$$= \frac{S_{xy}}{S_{xx}}$$
$$= 0.8571$$

and $\hat{\beta}_0 = \overline{y} - \hat{\beta}_1\overline{x} = 0.8571$ from excel. So, we get $\hat{\beta}_0 = 0.0009,\ \hat{\beta}_1 = 0.8571$.

β_0 is the deflection when a load of 0KN is applied to the beam.
β_1 is the rate of change of the deflection when the load is increased on the beam.

Question 2

Proof. We want to show that $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ always goes through the point $(\overline{x}, \overline{y})$. Given

$$
\begin{aligned}
\hat{y} &= \hat{\beta}_0 + \hat{\beta}_1 x \\
\overline{y} &= \hat{\beta}_0 + \hat{\beta}_1 \overline{x} && = \overline{y} - \hat{\beta}_1 \overline{x} + \hat{\beta}_1 \overline{x} && \text{by definition} \\
&= \overline{y} - \hat{\beta}_1 (\overline{x} - \overline{x}) \\
\overline{y} &= \overline{y}
\end{aligned}
$$

So, $\hat{y}$ always goes through the point $(\overline{x}, \overline{y})$. $\square$

Question 3

Proof.

$$\begin{aligned}
E(\beta_1) &= E\left(\frac{\sum_{i=1}^{N}(x_i - \overline{x})Y_i}{S_{xx}}\right) \\
&= \frac{\sum_{i=1}^{N}(x_i - \overline{x})E(Y_i)}{S_{xx}} \\
&= \frac{\sum_{i=1}^{N}(x_i - \overline{x})(\beta_0 + \beta_1 x_i)}{S_{xx}} \\
&= \beta_0 \frac{\sum_{i=1}^{N}(x_i - \overline{x})}{S_{xx}} + \beta_1 \frac{\sum_{i=1}^{N}(x_i - \overline{x})x_i}{S_{xx}} \\
&= 0 + \beta_1 \frac{S_{xx}}{S_{xx}} && \sum_{i=1}^{N}(x_i - \overline{x}) = 0 \\
E(\beta_1) &= \beta_1 && (*)
\end{aligned}$$

and

$$\begin{aligned}
E(\hat{\beta}_0) &= E(\overline{Y}) - E(\hat{\beta}_1)\overline{x} \\
&= \beta_0 + \beta_1 \overline{x} && \text{from } (*) \\
E(\hat{\beta}_0) &= \beta_0
\end{aligned}$$

Therefore, $\hat{\beta}_0,\ \hat{\beta}_1$ represent unbiased estimators of $\beta_0,\ \beta_1$ $\square$