Problem Set 3

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Question 4

Let $x_n = \frac{2n+1}{3n+7}$.

(a) Prove directly, using the definition, that $\lim_{n\to\infty} x_n = \frac{2}{3}$

(b) Prove, using the algebraic limit theorem, that $\lim_{n\to\infty} x_n = \frac{2}{3}$

Response

Question 10

(a) Let (x_n) be bounded (not necessarily convergent) and assume that $y_n \to 0$ as $n \to \infty$. Show that $x_n y_n \to 0$ as $n \to \infty$. (Why can we not just use the Algebraic limit theorem?)

(b) Let (x_n) be bounded and $y_n \to y$ with $y \neq 0$. Does $(x_n y_n)$ converge? If yes, show it. If not, give a counter-example.

Response

Question 12

For the following sequences, provide an example or prove that no souch request is possible. You may appeal to results from lectures.

(a) Sequences (x_n) and (y_n) which both diverge, but whose sum $(x_n + y_n)$ converges.

(b) Sequences (x_n), which converges, and (y_n), which diverges, but whose sum $(x_n + y_n)$ converges.

(c) A convergent sequence (x_n), such that $x_n \neq 0$ for all $n \in \mathbb{N}$ and $(1/x_n)$ diverges.

(d) An unbounded sequence (x_n) and (y_n), where $(x_n y_n)$ and (x_n) converge, but (y_n) does not converge.

Response