Homework 5

Warren Kim

March 7, 2023

Question 1

Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

(a) $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

(b) $(Smoke \implies Fire) \implies (Smoke \vee Heat) \implies Fire)$

(c) $((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$

Justify your answer using truth tables (worlds).

Response

(a)

$Smoke$	$Fire$	$(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$
F	F	T
F	T	F
T	F	T
T	T	T

Neither

(b)

$Smoke$	$Fire$	$Heat$	$(Smoke \implies Fire) \implies (Smoke \vee Heat) \implies Fire)$
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Neither

(c)

$Smoke$	$Fire$	$Heat$	$((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Valid

Question 2

Consider the following:
If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

(b) Convert the knowledge base into CNF

(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Justify your answers by deriving a contradiction for the augmented knowledge base. Use resolution and provide corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations

Response

(a)
- $(mythical \implies \neg mortal) \wedge$
- $(\neg mythical \implies (mortal \wedge mammal)) \wedge$
- $((\neg mortal \vee mammal) \implies horned) \wedge$
- $(horned \implies magical)$

(b)
(a) $(mythical \implies \neg mortal) \equiv \neg mythical \vee \neg mortal$

(b) $(\neg mythical \implies (mortal \wedge mammal)) \equiv \neg(\neg mythical) \vee (mortal \wedge mammal)$
$\equiv mythical \vee (mortal \wedge mammal)$
$\equiv (mythical \vee mortal) \wedge (mythical \vee mammal)$

(c) $(\neg mortal \vee mammal \implies horned) \equiv \neg(\neg mortal \vee mammal) \vee horned$
$\equiv (mortal \wedge \neg mammal) \vee horned$
$\equiv (horned \vee mortal) \wedge (horned \vee \neg mammal)$

(d) $(horned \implies magical) \equiv \neg horned \vee magical$

(e) $(\neg mythical \vee \neg mortal) \wedge$
$(mythical \vee mortal) \wedge (mythical \vee mammal) \wedge$
$((horned \vee mortal) \wedge (horned \vee \neg mammal)) \wedge$
$(\neg horned \vee magical)$

(c)
(a) $(\neg mythical \vee \neg mortal)$

(b) $(mythical \vee mortal)$

(c) $(mythical \vee mammal)$

(d) $(horned \vee mortal)$

(e) $(horned \vee \neg mammal)$

(f) $(\neg horned \vee magical)$

We cannot prove that the unicorn is mythical given the knowledge base. (resolving (a) and (b) results in a contradiction).

(1)	$(mammal \vee \neg mortal)$	resolve (a) and (c)
(2)	$(horned \vee \neg mythical)$	resolve (a) and (d)
(3)	$(horned \vee mythical)$	resolve (c) and (e)
(4)	$(mortal \vee magical)$	resolve (d) and (f)
(5)	$(\neg mammal \vee magical)$	resolve (e) and (f)
(6)	$(mammal \vee magical)$	resolve (1) and (4)
(7)	$(horned \vee horned)$	resolve (2) and (3)
(8)	$(magical \vee magical)$	resolve (5) and (6)

Therefore, from (7), the unicorn is horned and from (8) we have that the unicorn is magical.

Question 3

For each pair of atomic sentences, give the most general unifier if it exists:

(a) P(A, B, B), P(x, y, z).

(b) Q(y, G(A, B)), Q(G(x, x), y).

(c) Older(Father(y), y), Older(Father(x), John).

(d) Knows(Father(y),y), Knows(x,x).

Response

(a) {x/A, y/B, z/B}

(b) Doesn't exist

(c) {x/y, y/John} $\equiv$ {x/John, y/John}

(d) Doesn't exist

Question 4

Consider the following sentences:

- John likes all kinds of food.
- Apples are food.
- Chicken is food.
- Anything anyone eats and isn't killed by is food.
- If you are killed by something, you are not alive.
- Bill eats peanuts and is still alive. *
- Sue eats everything Bill eats.

For first-order syntax, feel free to use the following text file notation: | (for disjunction), & (for conjunction), - (for negation), => (for implication), <=> (for equivalence), E (for existential quantification, e.g., E x, y, Loves(x, y)), and A (for universal quantification, e.g., A x, y, Loves(x, y)).

(a) Translate these sentences into formulas in first-order logic.

(b) Convert the formulas of part (a) into CNF (also called clausal form).

(c) Prove that John likes peanuts using resolution.

(d) Use resolution to answer the question, "What food does Sue eat?"

(e) Use resolution to answer the question, "What food does Sue eat?" if, instead of the axiom marked with an asterisk above, we had:

- If you don't eat, you die.
- If you die, you are not alive.
- Bill is alive.

Response

- John likes all kinds of food.
  (a) $\forall x,\ Food(x),\ Likes(John, x)$
  (b) $\neg Food(x) \vee Likes(John, x)$
- Apples are food.
  (a) $\forall x,\ Apple(x) \implies Food(x)$
  (b) $\neg Apple(x) \vee Food(x)$
- Chicken is food.
  (a) $\forall x,\ Chicken(x) \implies Food(x)$
  (b) $\neg Chicken(x) \vee Food(x)$
- Anything anyone eats and isn't killed by is food.
  (a) $\forall x,\ Something(x), [\exists y,\ Someone(y),\ (Eat(y, x) \wedge \neg Kill(x, y)) \implies Food(x)]$
  (b) $\neg Eat(y, x) \vee Kill(x, y) \vee Food(x)$
- If you are killed by something, you are not alive.
  (a) $\forall x[\exists y\ Someone(x),\ Something(y)\ Kill(y, x) \implies \neg Alive(x)]$
  (b) $\neg Kill(y, x) \vee \neg Alive(x)$

- Bill eats peanuts and is still alive. *
  (a) $Eat(Bill, Peanuts) \wedge Alive(Bill)$
  (b) $Eat(Bill, Peanuts) \wedge Alive(Bill)$
- Sue eats everything Bill eats.
  (a) $\forall x,\ Food(x),\ Eat(Bill, x) \implies Eat(Sue, x)$
  (b) $\neg Eat(Bill, x) \vee Eat(Sue, x)$

(c) Prove that John likes peanuts using resolution.

(a) $\neg Food(x) \vee Likes(John, x)$
(b) $\neg Apple(x) \vee Food(x)$
(c) $\neg Chicken(x) \vee Food(x)$
(d) $\neg Eat(y, x) \vee Kill(x, y) \vee Food(x)$
(e) $\neg Kill(y, x) \vee \neg Alive(x)$
(f) $Eat(Bill, Peanuts)$
(g) $Alive(Bill)$
(h) $\neg Eat(Bill, x) \vee Eat(Sue, x)$

(1)	$Kill(Peanuts, Bill) \vee Food(Peanuts), \{x/Bill, y/Peanuts\}$	resolve (d) and (f)
(2)	$Food(Peanuts) \vee \neg Alive(Bill), \{x/Peanuts, y/Bill\}$	resolve (1) and (e)
(3)	$Food(Peanuts), \{x/Peanuts\}$	(2) and (g)
(4)	$Likes(John, Peanuts), \{x/Peanuts\}$	resolve (a) and (3)

Therefore, John likes peanuts.

(d) Use resolution to answer the question, "What food does Sue eat?"

(1)	$Eat(Sue, Peanuts), \{x/Peanuts\}$	resolve (f) and (h)

(e) Use resolution to answer the question, "What food does Sue eat?" if, instead of the axiom marked with an asterisk above, we had:

(a) $\neg Food(x) \vee Likes(John, x)$
(b) $\neg Apple(x) \vee Food(x)$
(c) $\neg Chicken(x) \vee Food(x)$
(d) $\neg Eat(y, x) \vee Kill(x, y) \vee Food(x)$
(e) $\neg Kill(y, x) \vee \neg Alive(x)$
(f) $\neg Eat(Bill, x) \vee Eat(Sue, x)$
(g) $Eat(y, x) \vee Die(x)$
(h) $\neg Die(x) \vee \neg Alive(x)$
(i) $Alive(Bill)$

(1)	$Eat(y, x) \vee \neg Alive(x)$	resolve (g) and (h)
(2)	$Eat(Bill, x), \{y/Bill\}$	resolve (1) and (i)
(3)	$Eat(Sue, x), \{y/Sue\}$	resolve (2) and (f)

Therefore, based on (f), we know that Sue eats everything Bill eats, but we don't know if Sue eats other food since we conclude from (3) that Sue eats food.