CS 181

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Preface

In this course, we want to answer two questions:

(1) What is computation?

(2) Are there problems that computers cannot solve?

1 Deterministic Finite Automata

1.1 Definitions

Alphabet: Any **nonempty finite** set of symbols.

(β) $\{0, 1\}$ is the binary alphabet.

(γ) $+, -, \cdot, /$ is the alphabet of arithmetic operators.

(α) $a, b, \ldots, z$ is the alphabet of lowercase English letters.

String: Any **finite** sequence of symbols from a given alphabet.
Note: The empty string (ϵ) is the only string contained in **all** alphabets.

$101101 \in \beta$

$++-/--\cdot-+ \in \gamma$

$abcad \in \alpha$

Language: A set of strings over a given alphabet. More specifically, the language of a Discrete Finite Automata (DFA) is a set of strings that the DFA accepts.

(1) $\{0, 001, 010, 100, \ldots\}$ is the set containing all odd length binary strings over β

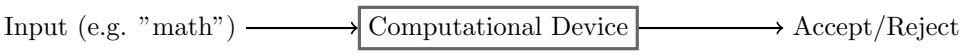
(2) $\{aim, claim, denim, \ldots\}$ is the set containing all English words that end in "im" over γ

(3) $\{\epsilon\}$ is the set containing the empty string over **all** alphabets.

(4) $\emptyset$ is the empty set over **all** alphabets.

Note: (1) and (2) are infinite while (3) and (4) are finite languages.

Computational Device: Any mechanism that imports a string and either accepts or rejects it.



1.2 Formulating Automata

Automata abides by the following rules:

- Choose an alphabet.
- Draw states.
- Choose a start[1] state.
- Choose accept[2] states.
- Draw transitions from **every** state to **every** symbol.

[1] Start states are required
[2] Accept states are **not** required

1.2.1 Automata Examples

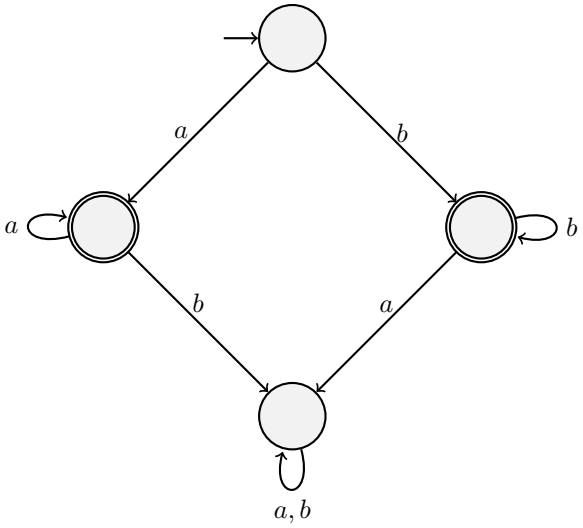


Figure 1: Accepts the set of strings $W = \{w : w \text{ is nonempty and contains either all } a\text{'s or all } b\text{'s}\}$ over the alphabet $\{a, b\}$

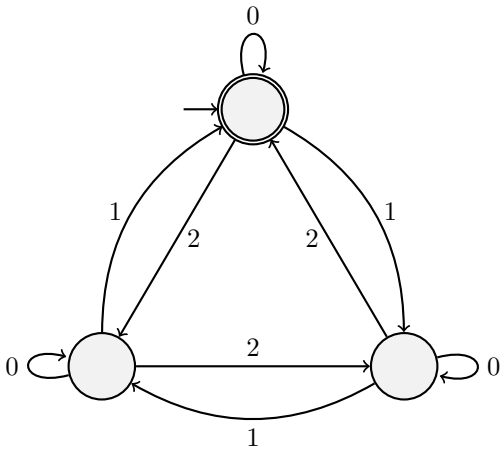


Figure 2: Accepts the set of strings $W = \{w : w \text{ is nonempty and } \sum_{i=1}^{k=|w|} w_i \text{ is divisible by 3, } 1 \leq i \leq n\}$ over the alphabet $\{0, 1, 2\}$

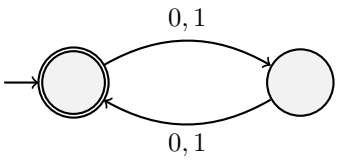


Figure 3: Accepts the set of strings $W = \{w : |w| \text{ is even}, 1 \leq i \leq n\}$ over the alphabet $\{0, 1\}$

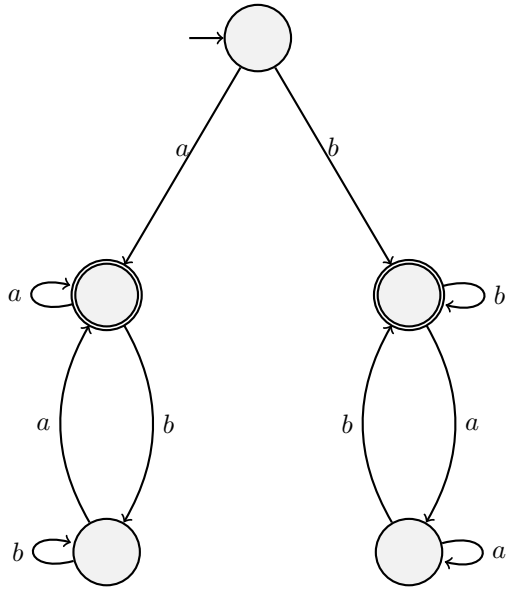


Figure 4: Accepts the set of strings $W = \{w : w \text{ is nonempty and begins and ends with the same letter}, 1 \leq i \leq n\}$ over the alphabet $\{a, b\}$