Question 1

Response: Given $R(A, B, C, D, E, G)$ and the following decomposition: $R_1 = (A, B, C, G), R_2 = (A, D, E)$ with functional dependencies $F = \{A \to B, A \to C, CD \to E, B \to D, E \to A\}$, the decomposition is lossless if it satisfies the following:

1. $R_1 \cup R_2 = R$: $R_1 \cup R_2 = (A, B, C, D, E, G) = R$ ✓
2. $R_1 \cap R_2 \neq \emptyset$: $R_1 \cap R_2 = (A) \neq \emptyset$ ✓
3. $(R_1 \cap R_2)^+$ forms a superkey for either R_1 or R_2: Looking at $(R_1 \cap R_2)^+$, we have $A^+ = ABCDE$, which forms a superkey for $R_2 = (A, D, E)$.
   $B \in A^+$ by $A \to B$.
   $C \in A^+$ by $A \to C$.
   $D \in A^+$ by $A \to B \to D$.
   $E \in A^+$ by $A \to C, A \to B \to D \implies A \to CD \to E$. ✓

The decomposition is **lossless**.

Question 2

Response: Given the following relation

A	B	C
a_1	b_1	c_2
a_1	b_1	c_1
a_2	b_1	c_1
a_2	b_1	c_3

The non-trivial functional dependencies satisfied by the relation are:
$A \to B$ since $a_1 \mapsto b_1$.
$C \to B$ since $c_1 \mapsto b_1, c_2 \mapsto b_1, c_3 \mapsto b_1$.
$AC \to B$ since $(a_1, c_1) \mapsto b_1, (a_1, c_2) \mapsto b_1, (a_2, c_1) \mapsto b_1, (a_2, c_3) \mapsto b_1$.

Question 3

Response: Given the following relation:

A	B	C	D
Name	Class	Score	Grade
Ted E. Bear	CS111	65	B
Ted E. Bear	CS143	78	B
Wile E. Coyote	CS111	91	A
Joe Bruin	CS118	31	F
Josie Bruin	CS131	89	A

The candidate key in the context of the problem is AB. The functional dependencies are
$AB \rightarrow C$, $AB \rightarrow D$ since AB is a superkey.
$C \rightarrow D$ since there is only one score/grade per class.
R is in $2NF$ if for every attribute in R, either (1) it is part of a candidate key or (2) the attribute depends on the entire candidate key. Clearly, A, B are in the candidate key AB. The non-prime attributes C, D both depend on the entire candidate key AB, so **this relation is in 2NF**.
This relation is *not* in 3NF because D can be determined through C, so there is a transitive dependency $AB \rightarrow C \rightarrow D$.

Question 4

Response: If we add another column called `CourseName` that is determined by the `Class` column, it would no longer be 2NF since there would be a partial dependency $B \to E$, assuming `CourseName` column is named E.

Question 5

Response: $R(A,B,C,D,E,G)$ with functional dependencies $F = \{A \to B, A \to C, C \to E, B \to D\}$.

Note that $A^+ = ABCDE$, $B^+ = BD$, $C^+ = CE$. By the union property, we have $F = \{A \to BC, C \to E, B \to D\}$.

- $A \to BC$: *(i)* $A \to BC$ is not trivial. *(ii)* A is not a superkey. **x**
- $C \to E$: *(i)* $C \to E$ is not trivial. *(ii)* C is not a superkey. **x**
- $B \to D$: *(i)* $B \to D$ is not trivial. *(ii)* B is not a superkey. **x**

All three dependencies violate BCNF. Decomposing on $B \to D$, we get $R_1(A,B,C,E,G), R_2(B,D)$.

Looking at R_1:

- $A \to BC$: *(i)* $A \to BC$ is not trivial. *(ii)* A is not a superkey. **x**
- $C \to E$: *(i)* $C \to E$ is not trivial. *(ii)* C is not a superkey. **x**

Both dependencies violate BCNF. Decomposing on $C \to E$, we get $R_3(A,B,C,G), R_4(C,E), R_2(B,D)$.

Looking at R_3:

- $A \to BC$: *(i)* $A \to BC$ is not trivial. *(ii)* A is not a superkey. **x**

$A \to BC$ violates BCNF. Decomposing on $A \to BC$, we get $R_5(A,B,C), R_6(A,G), R_4(C,E), R_2(B,D)$.

Reindexing the decomposition, we get $R_1(A,G), R_2(C,E), R_3(B,D), R_4(A,B,C)$. Recall the following attribute closures: $A^+ = ABCDE, B^+ = BD, C^+ = CE, D^+ = D, E^+ = E, G^+ = G$. Then,

- $B \to D$:
  Iteration 1:
  $$\begin{aligned} &result = B \\ &\quad t = (B \cap R_1)^+ \cap R_1 = (B \cap AG)^+ \cap AG = \emptyset \to result = B \\ &\quad t = (B \cap R_2)^+ \cap R_2 = (B \cap CE)^+ \cap CE = \emptyset \to result = B \\ &\quad t = (B \cap R_3)^+ \cap R_2 = (B \cap BD)^+ \cap BD = BD \cap BD = BD \to result = BD \checkmark \\ &short\ circuit \end{aligned}$$
  We derived D, so $B \to D$ is preserved.
- $C \to E$:
  Iteration 1:
  $$\begin{aligned} &result = C \\ &\quad t = (C \cap R_1)^+ \cap R_1 = (C \cap AG)^+ \cap AG = \emptyset \to result = C \\ &\quad t = (C \cap R_2)^+ \cap R_2 = (C \cap CE)^+ \cap CE = CE \cap CE = CE \to result = CE \checkmark \\ &short\ circuit \end{aligned}$$
  We derived E, so $C \to E$ is preserved.
- $A \to BC$:
  Iteration 1:
  $$\begin{aligned} &result = A \\ &\quad t = (A \cap R_1)^+ \cap R_1 = (A \cap AG)^+ \cap AG = ABCDE \cap AG = A \to result = A \\ &\quad t = (A \cap R_2)^+ \cap R_2 = (A \cap CE)^+ \cap CE = \emptyset \to result = A \\ &\quad t = (A \cap R_3)^+ \cap R_3 = (A \cap BD)^+ \cap BD = \emptyset \to result = A \\ &\quad t = (A \cap R_4)^+ \cap R_4 = (A \cap ABC)^+ \cap ABC = ABCDE \cap ABC = ABC \to result = ABC \\ &short\ circuit \end{aligned}$$
  We derived BC, so $A \to BC$ is preserved.

Because every functional dependency was preserved, this decomposition is dependency preserving.