110A HW7

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Throughout this section, F is a field and $F[x]$ is the ring of polynomials with F coefficients.

Question 1

Let $f, g, h \in F[x]$, and suppose f and g are relatively prime. Show that if $f|h$ and $g|h$, we have $fg|h$.

Response

Proof: Let $f, g, h \in F[x]$ and suppose f and g are coprime. Suppose that $f \mid h$ and $g \mid h$. $\square$

Question 2

Let $a, b \in F$ be distinct (i.e., $a \neq b$). Show that $x - a$ and $x - b$ (viewed as elements of $F[x]$) are relatively prime.

Response

Question 3

Let $f, g \in F[x]$ and suppose $g \neq 0$. Consider the set $S = \{f - gs | s \in F[x]\}$. Let $r \in S$ be of lowest degree. Show that $\deg(r) < \deg(s)$. (yes, we did this in class.)

Response

Question 4

Let $f \in F[x]$, $a \in F$, and suppose $f(a) = 0$ (that is, when plugging in a for x in f, we obtain 0). Show that $x - a$ divides f.

Response

Question 5

Let $p \in F[x]$, and suppose whenever $p = ab$ for $a, b \in F[x]$, we either have $p|a$ or $p|b$. Show that p is irreducible (i.e., its only factors are units and associates).

Response