Problem Set 1

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Question 1

Response

Question 1.32

Response

Question 1.40

Response

Question 1.41

For languages A and B, let the **perfect shuffle** of A and B be the the language

$$\{w : w = a_1 b_1 \cdots a_k b_k, where a_1 \cdots a_k \in A, b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}$$

Show that the class of regular languages is closed under perfect shuffle.

Response

Let $D_1 = (Q_1, \Sigma, \delta_1, q_1, F_1), D_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be two DFA's that recognize A and B respectively. Now, let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing **perfect shuffle** and be constructed as:

$$Q = Q_1 \times Q_2 \times \{odd, even\}$$
$$\Sigma = \Sigma$$
$$q_0 = \{(q_1, q_2, odd)\}$$
$$F = F_1 \times F_2 \times \{odd\}$$
$$\delta = \begin{cases} \{\delta_1(q, \sigma)\} & q \in Q_2 \setminus F_2, \sigma \in \Sigma \\ \{\delta_2(q, \sigma)\} & q \in Q_1 \setminus F_1, \sigma \in \Sigma \\ \emptyset & otherwise \end{cases}$$

Question 1.62

Response

Question 1.66

Response

Question 1.69

Response