C&EE 110 Homework 1

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April 18, 2023

Problem 1

a) Using Sturge's rule, compute the proper number of intervals for this data sample.

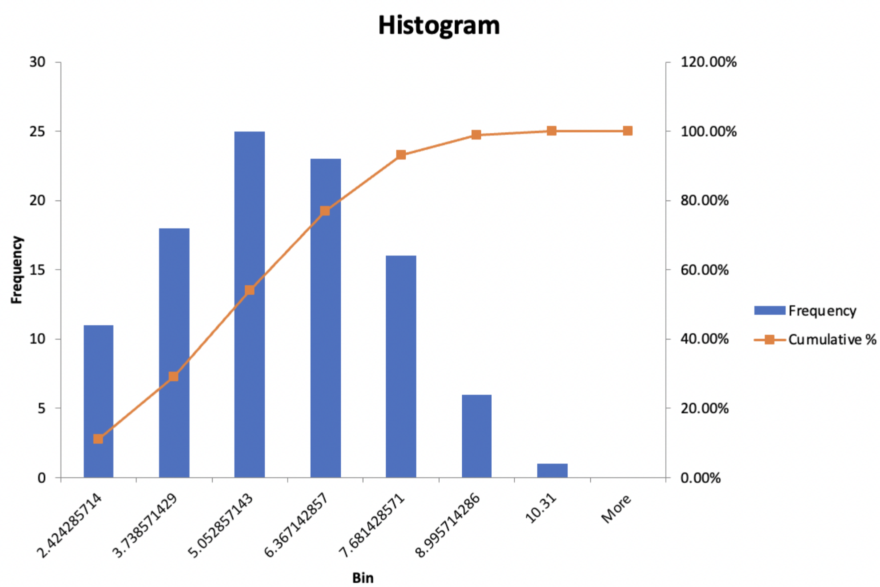
b) Using the number of bins from part a), report the histogram for this data sample.

c) Compute the sample mean and sample standard deviation.

Response

a) $k = \lfloor 1 + 3.3log_{10}(n) \rfloor = \lfloor log_{10}(1 + 3.3(100)) \rfloor = 7$

b) The histogram has $k = 7$ bins and a bin width of $\frac{max-min}{bins} = \frac{10.31-1.11}{7} = 1.314$



c) The formula for sample mean $\overline{y}$ and sample standard deviation s are:

$$\overline{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \overline{y})^2}{n-1}}$$

Sample mean:

$$\begin{aligned}\overline{y} &= \frac{\sum_{i=1}^{n} y_i}{n} \\ &= \frac{\sum_{i=1}^{100} y_i}{100} \\ \overline{y} &= 4.888\end{aligned}$$

Sample standard deviation:

$$\begin{aligned}s &= \sqrt{\frac{\sum_{i=1}^{n}(y_i - \overline{y})^2}{n-1}} \\ &= \sqrt{\frac{\sum_{i=1}^{100}(y_i - 4.889)^2}{100-1}} \\ s &= 1.884\end{aligned}$$

Problem 2

The time taken by college-age students to complete an obstacle course is approximately normally distributed with a mean of 45 seconds and a standard deviation of 7.2 seconds. What fraction of all students finished the obstacle course in the following intervals?

a) 37.8 to 52.2 seconds

b) More than 30.6 and less than 66.6 seconds

c) Less than 59.4 seconds

d) Less than 23.4 or more than 66.6 seconds

e) What is the largest standard deviation acceptable to assume the data is normally distributed? (Hint: the time should always be positive or equal to zero.)

Response

Let $\overline{y} = 45,\ s = 7.2$ be the sample mean and standard deviation respectively.

a) $37.8 = \overline{y} - s,\ 52.2 = \overline{y} + s$. By the empirical rule, the interval captures 68%.

b) $30.6 = \overline{y} - 2s,\ 66.6 = \overline{y} + 3s$. By the empirical rule, the interval captures $47.5\% + 49.85\% = 97.35\%$.

c) $59.4 = \overline{y} - 2s$. By the empirical rule, the interval captures $\frac{95}{2}\% + 50\% = 97.5\%$.

d) $23.4 = \overline{y} - 3s,\ 66.6 = \overline{y} + 3s$. By the empiricle rule, the interval captures $100\% - 99.7\% = 0.3\%$.

e)

$$\begin{aligned} 45 - 3s &= 0 \\ 3s &= 45 \\ s &= 15 \end{aligned}$$

The largest standard deviation acceptable to assume the data is normally distributed is 15 seconds.

Problem 3

Show that

$$\sum_{i=1}^{n}(y_i - \overline{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2$$

And calculate the sample variance s^2 based on the following information:

$$\sum_{i=1}^{n} y_i^2 = 40,\ \sum_{i=1}^{n} y_i = 14,\ n = 6$$

Hint:

$$\sum_{i=1}^{n}(x_i \pm y_i) = \sum_{i=1}^{n} x_i \pm \sum_{i=1}^{n} y_i \tag{1}$$

$$\sum_{i=1}^{n} cy_i = c\sum_{i=1}^{n} y_i \tag{2}$$

$$\sum_{i=1}^{n} c = nc \tag{3}$$

$$\sum_{i=1}^{n} y_i = n\overline{y} \tag{4}$$

Response

Proof.

$$\begin{aligned}
\sum_{i=1}^{n}(y_i - \overline{y})^2 &= \sum_{i=1}^{n}(y_i^2 - 2y_i\overline{y} + \overline{y}^2) \\
&= \left(\sum_{i=1}^{n} y_i^2\right) - \left(\sum_{i=1}^{n} 2n\overline{y}^2\right) + \left(\sum_{i=1}^{n} \overline{y}^2\right) && \text{from (1) and (4)} \\
&= \left(\sum_{i=1}^{n} y_i^2\right) - 2n\overline{y}^2 + n\overline{y}^2 && \text{from (2) and (3)} \\
&= \sum_{i=1}^{n} y_i^2 - n\overline{y}^2 \\
&= \sum_{i=1}^{n} y_i^2 - n\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)^2 \\
&= \sum_{i=1}^{n} y_i^2 - n\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n} y_i\right)^2 \\
&= \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2 && \text{from (4)}
\end{aligned}$$

□

$$\begin{aligned}
s^2 &= \frac{\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2}{n-1} \\
&= \frac{40 - \frac{1}{6}(14)^2}{6-1} \\
&= \frac{7.333}{5} \\
s^2 &= 1.467
\end{aligned}$$

Problem 4

a) Calculate the sample mean and mediean.

b) In one of the experiments the operator made a mistake in registering the temperature. Which value seems to be registered wrongly?

c) Remove the wrong data type from the dataset, and calculate the sample mean and median again.

d) Between mean and median, which one is sensitive to outliers and why?

Response

a) The formula for sample mean $\overline{y}$ is:

$$\overline{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

Sample mean:

$$\begin{aligned} \overline{y} &= \frac{\sum_{i=1}^{n} y_i}{n} \\ &= \frac{\sum_{i=1}^{20} y_i}{20} \\ \overline{y} &= 33.6 \end{aligned}$$

$\overline{y} = 33.6,\ median = 31.$

b) 112, because $\overline{y} \pm 3s = 33.6 \pm 3(18.723) = -22.57, 89.77$, and 112 does not fall within the interval -22.57 to 89.77 which captures 99.7% of the data assuming a normal distribution.

c) The formula for sample mean $\overline{y}$ is:

$$\overline{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

Sample mean:

$$\begin{aligned} \overline{y} &= \frac{\sum_{i=1}^{n} y_i}{n} \\ &= \frac{\sum_{i=1}^{19} y_i}{19} \\ \overline{y} &= 29.474 \end{aligned}$$

$\overline{y} = 29.474,\ median = 31.$

d) The mean is more sensitive to outliers because all data points in the dataset have equal weight (of 1) when calculating the mean.