BPTR: Bernstein Polynomial Temporal Realignment

Statistical Model

$$\frac{Y_{ijk} - d_0}{d_1 - Y_{ijk}} = \beta_{ijk} \left\{ \sum_{l=0}^{d} \theta_l b_{l,d} \left(\frac{t_{ijk} + \delta_{ij}}{\max\limits_{i,j} \{c_{ij}\}} \right) \right\} \epsilon_{ijk},\ i = 1, ..., n;\ j = 1, ..., r_i;\ k = 1, ..., m_{ij};$$

- $\ln(\epsilon_{ijk}) | \sigma_\epsilon^2 \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\epsilon^2\right)$;
- $c_{ij} = t_{ijm_{ij}} + a_{0ij} - a_1$;
- Progression parameters:

$$\ln(\beta_{ijk}) = \mathbf{x}_{ijk}^{\text{T}} \boldsymbol{\gamma} + \zeta_{0i} + \zeta_{1ij},\ \zeta_{0i} | \sigma_{\zeta_0}^2 \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_{\zeta_0}^2\right),\ \zeta_{1ij} | \sigma_{\zeta_1}^2 \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_{\zeta_1}^2\right);$$

- Disease onset parameters:

$$\ln\left(\frac{a_{0ij} - \delta_{ij} - a_1}{\delta_{ij}} \right) = \mathbf{z}_{ij}^{\text{T}} \boldsymbol{\eta} + \phi_{0i} + \phi_{1ij},\ \phi_{0i} | \sigma_{\phi_0}^2 \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_{\phi_0}^2\right),\ \phi_{1ij} | \sigma_{\phi_1}^2 \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_{\phi_1}^2\right);$$

- Bernstein polynomial regression parameters

$$\theta_0 \equiv 0;\ \theta_j = \theta_{j-1} + \psi_j,\ j = 1, \ldots, d,$$

  - $\psi_1 = V_1$;
  - $\psi_j = V_j \prod_{k=1}^{j-1} (1 - V_k)$ for $2 \leq j < d$;
  - $\psi_d = \prod_{k=1}^{d-1} (1 - V_k)$;
  - $V_j | \alpha \overset{\text{iid}}{\sim} \text{Beta}(1, \alpha)$ for $j = 1, ..., d-1$.

Prior Information

$\gamma_l \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\gamma^2\right),\ l = 1, ..., p_x$;

- p_x: Length of $\mathbf{x}_{ijk}$ vector (same for all i, j, k);
- Default setting: $\sigma_\gamma^2 = 10,000$.

$\sigma_{\zeta_0}^2 \sim \text{Inverse Gamma}\left(a_{\sigma_{\zeta_0}^2}, b_{\sigma_{\zeta_0}^2}\right)$;

- Default setting: $a_{\sigma_{\zeta_0}^2} = 0.01$, $b_{\sigma_{\zeta_0}^2} = 0.01$.

$\sigma_{\zeta_1}^2 \sim \text{Inverse Gamma}\left(a_{\sigma_{\zeta_1}^2}, b_{\sigma_{\zeta_1}^2}\right)$;

- Default setting: $a_{\sigma_{\zeta_1}^2} = 0.01$, $b_{\sigma_{\zeta_1}^2} = 0.01$.

$\alpha \sim \text{Gamma}(a_\alpha, b_\alpha)$;

- Default setting: $a_\alpha = 0.01$, $b_\alpha = 0.01$.

$\eta_l \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\eta^2\right),\ j = 1, ..., p_z$;

- p_z: Length of $\mathbf{z}_{ij}$ vector (same for all i, j);
- Default setting: $\sigma_\eta^2 = 10,000$.

$\sigma_{\phi_0}^2 \sim \text{Inverse Gamma}\left(a_{\sigma_{\phi_0}^2}, b_{\sigma_{\phi_0}^2}\right)$;

- Default setting: $a_{\sigma_{\phi_0}^2} = 0.01$, $b_{\sigma_{\phi_0}^2} = 0.01$.

$\sigma^2_{\phi_1} \sim$ Inverse Gamma $\left(a_{\sigma^2_{\phi_1}}, b_{\sigma^2_{\phi_1}}\right)$;

- Default setting: $a_{\sigma^2_{\phi_1}} = 0.01$, $b_{\sigma^2_{\phi_1}} = 0.01$.

$\sigma^2_\epsilon \sim$ Inverse Gamma $\left(a_{\sigma^2_\epsilon}, b_{\sigma^2_\epsilon}\right)$;

- Default setting: $a_{\sigma^2_\epsilon} = 0.01$, $b_{\sigma^2_\epsilon} = 0.01$.

Default Initial Values

- $\gamma_l = 0$ for all l;
- $\zeta_{0i} = 0$ for all i;
- $\sigma^2_{\zeta_0} = 0.01$;
- $\zeta_{1ij} = 0$ for all i, j;
- $\sigma^2_{\zeta_1} = 0.01$;
- $\alpha = 1.00$;
- $V_l = 0.50$ for all l;
- $\eta_l = 0$ for all l;
- $\phi_{0i} = 0$ for all i;
- $\sigma^2_{\phi_0} = 0.01$;
- $\sigma^2_{\phi_1} = 0.01$;
- $\delta_{ij} = \frac{a_{0ij} - a_1}{2}$;
- $\sigma^2_\epsilon = 0.01$.

Notes

- d: Selected degree of Bernstein polynomial;
- a_{0ij}: Age of individual at first visit (specific to j);
- a_1: Minimum age that any individual can develop the disease;
- r_i must be > 1 for all i;
- m_{ij} must be > 1 for all i, j.