

GPCW: Gaussian Process Model for Critical Window Estimation

Statistical Model

$$Y_i | \beta, \theta \stackrel{\text{ind}}{\sim} \text{Binomial} \{c_i, p_i(\beta, \theta)\}, \quad i = 1, \dots, n;$$

$$\log \left\{ \frac{p_i(\beta, \theta)}{1 - p_i(\beta, \theta)} \right\} = \mathbf{x}_i^T \beta + \sum_{j=1}^{m_i} z_{ij} \theta(j);$$

$$\theta = \{\theta(1), \dots, \theta(m)\}^T | \sigma_\theta^2, \phi \sim \text{MVN} \{ \mathbf{0}_m, \sigma_\theta^2 \Sigma(\phi) \};$$

$$\Sigma(\phi)_{ij} = \exp \{ -\phi |i - j| \}, \quad \phi > 0.$$

- $m = \max \{m_i : i = 1, \dots, n\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} \text{N} \left(0, \sigma_\beta^2 \right), \quad j = 1, \dots, p;$$

- p : Length of \mathbf{x}_i vector (same for all i);
- Default setting: $\sigma_\beta^2 = 10,000$.

$$\sigma_\theta^2 \sim \text{Inverse Gamma} \left(a_{\sigma_\theta^2}, b_{\sigma_\theta^2} \right);$$

- Default setting: $a_{\sigma_\theta^2} = 3, b_{\sigma_\theta^2} = 2$.

$$\phi \sim \text{Uniform} (a_\phi, b_\phi);$$

- Default setting: $a_\phi = \log(0.9999) / \{-(m-1)\}, b_\phi = \log(0.0001) / (-1)$.

Default Initial Values

- $\beta_j = 0$ for all j ;
- $\theta_j = 0$ for all j ;
- $\sigma_\theta^2 = 1.00$;
- $\phi = 0.01 (b_\phi - a_\phi)$.

Alternate Likelihood: Gaussian

$$Y_i | \beta, \theta, \sigma_\epsilon^2 \stackrel{\text{ind}}{\sim} \text{Normal} \left(\mathbf{x}_i^T \beta + \sum_{j=1}^{m_i} z_{ij} \theta(j), \sigma_\epsilon^2 \right), \quad i = 1, \dots, n.$$

- $\sigma_\epsilon^2 \sim \text{Inverse Gamma} (a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2});$
- Default setting: $a_{\sigma_\epsilon^2} = 0.01, b_{\sigma_\epsilon^2} = 0.01$;
- Default initial value: $\sigma_\epsilon^2 = 1.00$.

Alternate Likelihood: Negative Binomial

$Y_i | \boldsymbol{\beta}, \boldsymbol{\theta}, r \stackrel{\text{ind}}{\sim} \text{Negative Binomial} \{r, \lambda_i(\boldsymbol{\beta}, \boldsymbol{\theta})\}, i = 1, \dots, n;$

$$\ln \left\{ \frac{\lambda_i(\boldsymbol{\beta}, \boldsymbol{\theta})}{1 - \lambda_i(\boldsymbol{\beta}, \boldsymbol{\theta})} \right\} = \mathbf{O}_i + \mathbf{x}_i^T \boldsymbol{\beta} + \sum_{j=1}^{m_i} z_{ij} \theta(j).$$

- $r \sim \text{Discrete Uniform} [a_r, b_r];$
- Default setting: $a_r = 1, b_r = 100;$
- Default initial value: $r = b_r.$

Likelihood Indicator

- likelihood_indicator = 0: Binomial;
- likelihood_indicator = 1: Gaussian;
- likelihood_indicator = 2: Negative binomial.