CWVSmix: Critical Window Variable Selection for Mixtures

Statistical Model

$$Y_i|p_i \overset{\text{ind}}{\sim} \text{Bernoulli}\left(p_i\right),\ i = 1, ..., n;$$

$$\ln\left(\frac{p_i}{1-p_i}\right) = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \sum_{t=1}^{m_i}\left[\sum_{j=1}^{q}\lambda_j\left(t\right)\mathrm{z}_{ij}\left(t\right) + \sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\widetilde{\lambda}_{jk}\left(t\right)\mathrm{z}_{ij}\left(t\right)\mathrm{z}_{ik}\left(t\right)\right]\alpha\left(t\right).$$

Time-Varying Weights

$$\begin{aligned}
&\lambda_j\left(t\right) = \frac{\max\left\{\lambda_j^*\left(t\right), 0\right\}}{d\left(t\right)},\\
&\widetilde{\lambda}_{jk}\left(t\right) = \frac{\max\left\{\widetilde{\lambda}_{jk}^*\left(t\right), 0\right\} 1\left\{\lambda_j^*\left(t\right) > 0\right\} 1\left\{\lambda_k^*\left(t\right) > 0\right\}}{d\left(t\right)},\\
&t = 1, ..., m,\ j = 1, ..., q-1,\ k = j+1, ..., q;
\end{aligned}$$

$$d\left(t\right) = \sum_{j=1}^{q}\max\left\{\lambda_j^*\left(t\right), 0\right\} + \sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\max\left\{\widetilde{\lambda}_{jk}^*\left(t\right), 0\right\} 1\left\{\lambda_j^*\left(t\right) > 0\right\} 1\left\{\lambda_k^*\left(t\right) > 0\right\};$$

$$\boldsymbol{\lambda}^* = \left\{\boldsymbol{\lambda}^*\left(1\right)^{\mathrm{T}}, \dots, \boldsymbol{\lambda}^*\left(m\right)^{\mathrm{T}}\right\}^{\mathrm{T}}|\phi_\lambda \sim \text{MVN}\left\{\mathbf{0}_{mq(q+1)/2},\ \Sigma\left(\phi_\lambda\right)\otimes I_{q(q+1)/2}\right\}.$$

- $m = \max\left\{m_i : i = 1, ..., n\right\};$
- $\boldsymbol{\lambda}^*\left(t\right) = \left\{\lambda_1^*\left(t\right), \dots, \lambda_q^*\left(t\right), \widetilde{\lambda}_{12}^*\left(t\right), \dots, \widetilde{\lambda}_{q-1,q}^*\left(t\right)\right\}^{\mathrm{T}};$
- $\mathbf{0}_{mq(q+1)/2}$: Length $mq(q+1)/2$ vector with each entry equal to zero;
- $\boldsymbol{I}_{q(q+1)/2}$: $q(q+1)/2$ by $q(q+1)/2$ identity matrix.

Mixture Risk

$$\alpha\left(t\right) = \theta\left(t\right)\gamma\left(t\right),\ t = 1, ..., m;$$

$$\gamma\left(t\right)|\pi\left(t\right) \overset{\text{ind}}{\sim} \text{Bernoulli}\left\{\pi\left(t\right)\right\},\ \Phi^{-1}\left\{\pi\left(t\right)\right\} = \eta\left(t\right);$$

$$\begin{bmatrix}\theta\left(t\right)\\ \eta\left(t\right)\end{bmatrix} = A\begin{bmatrix}\delta_1\left(t\right)\\ \delta_2\left(t\right)\end{bmatrix},\ A = \begin{bmatrix}A_{11} & 0\\ A_{21} & A_{22}\end{bmatrix};$$

$$\boldsymbol{\delta}_j = \left\{\delta_j\left(1\right), ..., \delta_j\left(m\right)\right\}^{\mathrm{T}}|\phi_j \overset{\text{ind}}{\sim} \text{MVN}\left\{\mathbf{0}_m, \Sigma\left(\phi_j\right)\right\},\ j = 1, 2.$$

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\beta^2\right),\ j = 1, ..., p;$

- p: Length of $\mathbf{x}_{ij}$ vector (same for all i, j);
- Default setting: $\sigma_\beta^2 = 10,000$.

$\ln\left(A_{11}\right), \ln\left(A_{22}\right), A_{21} \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_A^2\right);$

- Default setting: $\sigma_A^2 = 1.00$.

$\phi_\lambda, \phi_j \overset{\text{iid}}{\sim} \text{Gamma}\left(\alpha_{\phi_j}, \beta_{\phi_j}\right),\ j = 1, 2;$

- Default setting: $\alpha_{\phi_j} = 1.00$, $\beta_{\phi_j} = 1.00$, $j = 1, 2$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\gamma\left(t\right) = 1$ for all t;
- $\delta_j\left(t\right) = 0$ for all j, t;
- $\phi_\lambda, \phi_j = -\ln\left(0.05\right) / \left(m - 1\right)$ for all j;
- $A_{jj} = 1$ for all j;
- $A_{21} = 0$.

Interactions

- interaction_indicator = 0:

$$\ln\left(\frac{p_i}{1 - p_i}\right) = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{t=1}^{m_i}\sum_{j=1}^{q}\lambda_j\left(t\right)\text{z}_{ij}\left(t\right)\alpha\left(t\right);$$

- interaction_indicator = 1: Full model with interactions as detailed above.

Alternate Likelihood: Gaussian

$Y_i | \mu_i, \sigma_\epsilon^2 \overset{\text{ind}}{\sim} \text{Normal}\left(\mu_i, \sigma_\epsilon^2\right),\ i = 1, ..., n;$

$\mu_i = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{t=1}^{m_i}\left[\sum_{j=1}^{q}\lambda_j\left(t\right)\text{z}_{ij}\left(t\right) + \sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\widetilde{\lambda}_{jk}\left(t\right)\text{z}_{ij}\left(t\right)\text{z}_{ik}\left(t\right)\right]\alpha\left(t\right).$

- $\sigma_\epsilon^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right);$
- Default setting: $a_{\sigma_\epsilon^2} = 0.01$, $b_{\sigma_\epsilon^2} = 0.01$;
- Default initial value: $\sigma_\epsilon^2 = 1.00$.

Alternate Likelihood: Negative Binomial

$Y_i | r, p_i \overset{\text{ind}}{\sim} \text{Negative Binomial}\left(r, p_i\right),\ i = 1, ..., n;$

$\ln\left(\frac{p_i}{1-p_i}\right) = \text{O}_i + \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{t=1}^{m_i}\left[\sum_{j=1}^{q}\lambda_j\left(t\right)\text{z}_{ij}\left(t\right) + \sum_{j=1}^{q-1}\sum_{k=j+1}^{q}\widetilde{\lambda}_{jk}\left(t\right)\text{z}_{ij}\left(t\right)\text{z}_{ik}\left(t\right)\right]\alpha\left(t\right).$

- $r \sim \text{Discrete Uniform}\left[a_r, b_r\right];$
- Default setting: $a_r = 1$, $b_r = 100$;
- Default initial value: $r = b_r$.

Likelihood Indicator

- likelihood_indicator = 0: Bernoulli;
- likelihood_indicator = 1: Gaussian;
- likelihood_indicator = 2: Negative binomial.