DLfuse: Distributed Lag Data Fusion for Estimating Ambient Air Pollution

DLfuse_Example

[1] Simulate data from the proposed model:

- Setting the reproducibility seed and initializing packages for data simulation:

```
set.seed(2568)

library(DLfuse)
library(geoR)  #Spatial covariance functions
```

```
## ------------------------------------------------------------
##  Analysis of Geostatistical Data
##  For an Introduction to geoR go to http://www.leg.ufpr.br/geoR
##  geoR version 1.7-5.2.1 (built on 2016-05-02) is now loaded
## ------------------------------------------------------------
```

```
library(mnormt)  #Multivariate normal distribution
library(matrixStats)  #colMedians
```

- Setting the global data values:

```
##########################################
#Full CMAQ Grid
##########################################
m<-(12^2)
grid<-matrix(0,
             nrow = m,
             ncol = 2)
counter<-1
for(j in 1:sqrt(m)){
   for(k in 1:sqrt(m)){

      grid[counter,]<-c(j,k)
      counter<-counter +
               1

      }
   }

neighbors<-1/as.matrix(dist(grid,
                            diag = TRUE,
                            upper = TRUE))
diag(neighbors)<-0

CAR<-diag(rowSums(neighbors)) -
     neighbors

#########################################################
#Full AQS Locations
#########################################################
sample_size<-rpois(n = m,
                   lambda = 1)
```

```
sample_size<-pmax(sample_size,
                  rep(1,
                      times = m))

locs<-matrix(runif(n = (2*sum(sample_size)),
                   min = 0,
                   max = 1),
             ncol = 2)
spatial_dists<-as.matrix(dist(locs,
                              diag = TRUE,
                              upper = TRUE))
diag(spatial_dists)<-0

CMAQ_key<-rep(0,
              times = sum(sample_size))
counter<-0
for(j in 1:length(sample_size)){

   CMAQ_key[(1 + counter):(sample_size[j] + counter)]<-j
   counter<-counter +
            sample_size[j]

   }

##############################################################################
#True Spatial Parameter Settings
##############################################################################
sigma2_epsilon_true<-0.05

beta0_true<-0.25
beta1_true<-1.75

A11_true<-0.05
A22_true<-0.05
A21_true<- -0.01

phi0_true<-1.50
Sigma0_true<-cov.spatial(spatial_dists,
                         cov.model="exponential",
                         cov.pars=c(1, (1/phi0_true)))
w0_true<-rmnorm(n=1,
                mean=rep(0, times=sum(sample_size)),
                varcov=Sigma0_true)
w0_true<-w0_true -
         mean(w0_true)
beta0_tilde_true<-A11_true*w0_true

phi1_true<-1.50
Sigma1_true<-cov.spatial(spatial_dists,
                         cov.model="exponential",
                         cov.pars=c(1, (1/phi1_true)))
w1_true<-rmnorm(n=1,
                mean=rep(0, times=sum(sample_size)),
```

```
                varcov=Sigma1_true)
w1_true<-w1_true -
         mean(w1_true)
beta1_tilde_true<-A21_true*w0_true +
                  A22_true*w1_true

tau2_true<-1.00
rho_true<-0.99  #ICAR Model Approximation
CAR_cov_true<-tau2_true*chol2inv(chol(rho_true*CAR + (1 - rho_true)*diag(m)))
alpha_true<-rmnorm(n = 1,
                   mean = rep(0,
                              times = m),
                   varcov = CAR_cov_true)
alpha_true<-(alpha_true - mean(alpha_true))/sd(alpha_true)

mu_true<-1.00

#########################################################
#Creating Lagged Covariates and AQS Data
#########################################################
L<-11
z<-matrix(rgamma(n = (length(sample_size)*L),
                 shape = 1,
                 rate = 1),
          nrow = length(sample_size),
          ncol = L)
covars_true<-construct_lagged_covars_s(z,
                                       mu_true,
                                       alpha_true,
                                       sample_size)[[1]]
```

- Simulating the analysis dataset:

```
y<-rnorm(n = sum(sample_size),
         mean = ((beta0_true + beta0_tilde_true) +
                (beta1_true + beta1_tilde_true)*covars_true),
         sd = sqrt(sigma2_epsilon_true))
```

[2] Fit DLfuse to a Subset of the Data:

```
samples<-11000

test_set<-sort(sample(c(1:m),
                      size = round(0.80*m),
                      replace = FALSE))

CMAQ_key_test_set<-rep(0,
                       times = length(CMAQ_key))
for(j in 1:length(test_set)){
   CMAQ_key_test_set[CMAQ_key == test_set[j]]<-1
   }

dlfuse_results<-
DLfuse_s(mcmc_samples = samples,
         y = y[CMAQ_key_test_set == 1],
```

```
        z = z[test_set,],
        sample_size = sample_size[test_set],
        spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                      (CMAQ_key_test_set == 1)],
        neighbors = neighbors[test_set, test_set],
        metrop_var_A11_trans = (0.70^2),
        metrop_var_A22_trans = (0.80^2),
        metrop_var_mu = (0.20^2),
        metrop_var_alpha = rep((1.00^2),
                               times = nrow(neighbors[test_set, test_set])),
        metrop_var_phi0_trans = (0.50^2),
        metrop_var_phi1_trans = (0.50^2),
        model_type_indicator = 0)
```

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```

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```

[3] Comparing Parameter Estimates to True Values:

```
burnin<-1000
thin<-10
keep_set<-seq((burnin + 1),
              samples,
              thin)

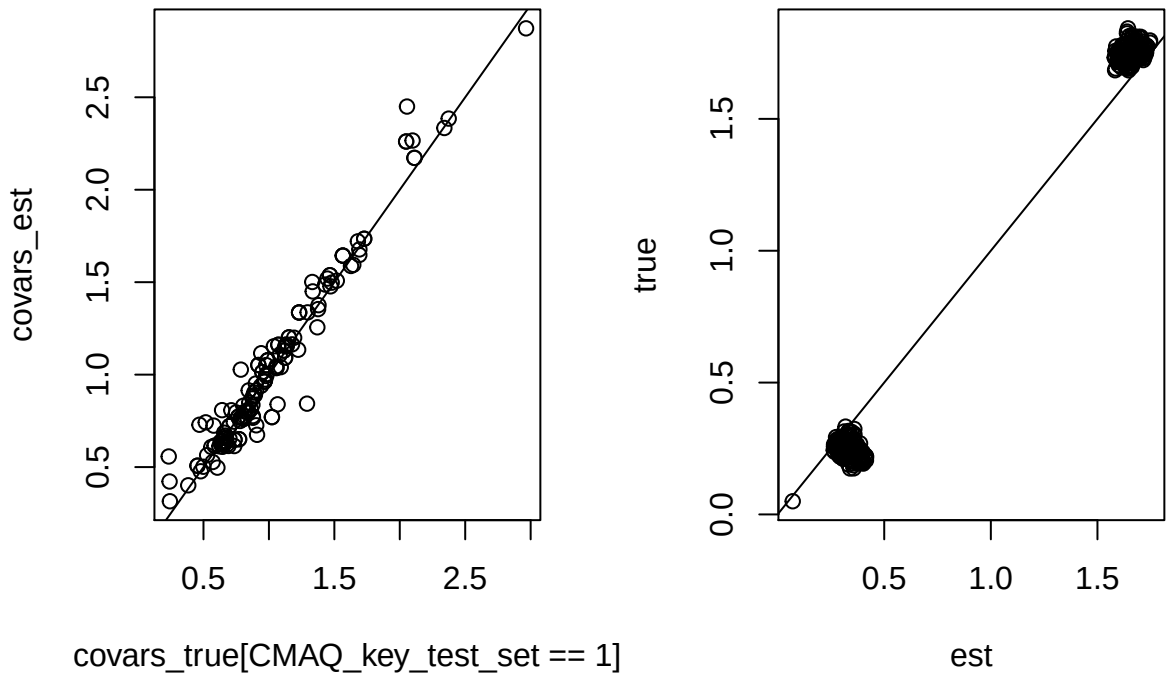
par(mfrow=c(1,2))
covars_est<-construct_lagged_covars_s(z[test_set,],
                                      mean(dlfuse_results$lag_info$mu[keep_set]),
                                      rowMeans(dlfuse_results$lag_info$alpha[,keep_set]),
                                      sample_size[test_set])[[1]]

plot(covars_true[CMAQ_key_test_set == 1],
     covars_est)
abline(0,1)

true<-c(sigma2_epsilon_true,
        (beta0_true + beta0_tilde_true[CMAQ_key_test_set == 1]),
        (beta1_true + beta1_tilde_true[CMAQ_key_test_set == 1]))

est<-c(mean(dlfuse_results$sigma2_epsilon[keep_set]),
       rowMeans(matrix(dlfuse_results$beta0[keep_set],
                       nrow = sum(sample_size[test_set]),
                       ncol = length(keep_set),
                       byrow = TRUE) +
                matrix(dlfuse_results$A11[keep_set],
                       nrow = sum(sample_size[test_set]),
                       ncol = length(keep_set),
                       byrow = TRUE)*dlfuse_results$w0[,keep_set]),
       rowMeans(matrix(dlfuse_results$beta1[keep_set],
                       nrow = sum(sample_size[test_set]),
                       ncol = length(keep_set),
                       byrow = TRUE) +
                matrix(dlfuse_results$A21[keep_set],
                       nrow = sum(sample_size[test_set]),
                       ncol = length(keep_set),
                       byrow = TRUE)*dlfuse_results$w0[,keep_set] +
                matrix(dlfuse_results$A22[keep_set],
                       nrow = sum(sample_size[test_set]),
                       ncol = length(keep_set),
                       byrow = TRUE)*dlfuse_results$w1[,keep_set]))

plot(est, true)
abline(0,1)
```



[4] Spatial Predictions of Validation Data:

```
spatial_dists_full<-as.matrix(dist(rbind(locs[CMAQ_key_test_set == 0,],
                                         locs[CMAQ_key_test_set == 1,]),
                                   diag = TRUE,
                                   upper = TRUE))
diag(spatial_dists_full)<-0
loc_temp<-rbind(locs[CMAQ_key_test_set == 0,], locs[CMAQ_key_test_set == 1,])
for(j in 1:nrow(loc_temp)){
   for(k in 1:nrow(loc_temp)){
      if(prod(loc_temp[j,] == loc_temp[k,]) == 1){
        spatial_dists_full[j,k]<-0
        }
      }
   }

neighbors_full<-1/as.matrix(dist(rbind(unique(grid[-test_set,]), unique(grid[test_set,])),
                                 diag = TRUE,
                                 upper = TRUE))
diag(neighbors_full)<-0
loc_temp<-rbind(unique(grid[-test_set,]), unique(grid[test_set,]))
for(j in 1:nrow(loc_temp)){
   for(k in 1:nrow(loc_temp)){
      if(prod(loc_temp[j,] == loc_temp[k,]) == 1){
        neighbors_full[j,k]<-Inf
        }
      }
```

```
   }
diag(neighbors_full)<-0

dlfuse_pred_results<-ppd_s(modeling_output = dlfuse_results,
                           n_pred = length(y[CMAQ_key_test_set == 0]),
                           m_pred = nrow(z[-test_set, ]),
                           z_pred = z[-test_set,],
                           sample_size_pred = sample_size[-test_set],
                           spatial_dists_full = spatial_dists_full,
                           neighbors_full = neighbors_full,
                           inference_set = keep_set,
                           params_only_indicator = 0,
                           model_type_indicator = 0)
```

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```

[5] Comparison with Other Approaches:

```
slr_results<-
DLfuse_s(mcmc_samples = samples,
         y = y[CMAQ_key_test_set == 1],
         z = z[test_set,],
         sample_size = sample_size[test_set],
         spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                       (CMAQ_key_test_set == 1)],
         neighbors = neighbors[test_set, test_set],
         metrop_var_A11_trans = (0.70^2),
         metrop_var_A22_trans = (0.80^2),
         metrop_var_mu = (0.20^2),
         metrop_var_alpha = rep((2.00^2),
                                times = nrow(neighbors[test_set, test_set])),
         metrop_var_phi0_trans = (0.50^2),
         metrop_var_phi1_trans = (0.50^2),
         model_type_indicator = 3)
```

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```

```
slr_pred_results<-
ppd_s(modeling_output = slr_results,
      n_pred = length(y[CMAQ_key_test_set == 0]),
      m_pred = nrow(z[-test_set, ]),
      z_pred = z[-test_set,],
      sample_size_pred = sample_size[-test_set],
      spatial_dists_full = spatial_dists_full,
      neighbors_full = neighbors_full,
      inference_set = keep_set,
      params_only_indicator = 0,
      model_type_indicator = 3)
```

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```

```
ok_results<-
DLfuse_s(mcmc_samples = samples,
         y = y[CMAQ_key_test_set == 1],
         z = z[test_set,],
         sample_size = sample_size[test_set],
         spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                       (CMAQ_key_test_set == 1)],
         neighbors = neighbors[test_set, test_set],
         metrop_var_A11_trans = (0.70^2),
         metrop_var_A22_trans = (0.80^2),
         metrop_var_mu = (0.20^2),
         metrop_var_alpha = rep((2.00^2),
                                times = nrow(neighbors[test_set, test_set])),
         metrop_var_phi0_trans = (0.50^2),
         metrop_var_phi1_trans = (0.50^2),
         model_type_indicator = 2)
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```

```
ok_pred_results<-
ppd_s(modeling_output = ok_results,
      n_pred = length(y[CMAQ_key_test_set == 0]),
      m_pred = nrow(z[-test_set, ]),
      z_pred = z[-test_set,],
      sample_size_pred = sample_size[-test_set],
      spatial_dists_full = spatial_dists_full,
      neighbors_full = neighbors_full,
      inference_set = keep_set,
      params_only_indicator = 0,
      model_type_indicator = 2)
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```

```
ds_results<-
DLfuse_s(mcmc_samples = samples,
         y = y[CMAQ_key_test_set == 1],
         z = z[test_set,],
         sample_size = sample_size[test_set],
         spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                       (CMAQ_key_test_set == 1)],
         neighbors = neighbors[test_set, test_set],
         metrop_var_A11_trans = (0.70^2),
         metrop_var_A22_trans = (0.80^2),
         metrop_var_mu = (0.20^2),
         metrop_var_alpha = rep((2.00^2),
                                times = nrow(neighbors[test_set, test_set])),
         metrop_var_phi0_trans = (0.50^2),
         metrop_var_phi1_trans = (0.50^2),
         model_type_indicator = 1)
```

```
## Progress: 5%
## A11 Acceptance: 47%
## A22 Acceptance: 30%
## phi0 Acceptance: 31%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 10%
```

```
## A11 Acceptance: 48%
## A22 Acceptance: 30%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 15%
## A11 Acceptance: 48%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 20%
## A11 Acceptance: 50%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 28%
## Original: S
## ********************
## Progress: 25%
## A11 Acceptance: 52%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 28%
## Original: S
## ********************
## Progress: 30%
## A11 Acceptance: 51%
## A22 Acceptance: 31%
## phi0 Acceptance: 30%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 35%
## A11 Acceptance: 53%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 40%
## A11 Acceptance: 52%
## A22 Acceptance: 31%
## phi0 Acceptance: 28%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 45%
## A11 Acceptance: 52%
## A22 Acceptance: 32%
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## phi1 Acceptance: 29%
## Original: S
```

```
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## phi0 Acceptance: 28%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 55%
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## A22 Acceptance: 32%
## phi0 Acceptance: 28%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 60%
## A11 Acceptance: 53%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 65%
## A11 Acceptance: 54%
## A22 Acceptance: 32%
## phi0 Acceptance: 28%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 70%
## A11 Acceptance: 54%
## A22 Acceptance: 31%
## phi0 Acceptance: 28%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 75%
## A11 Acceptance: 54%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 80%
## A11 Acceptance: 54%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 85%
## A11 Acceptance: 54%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
```

```
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 90%
## A11 Acceptance: 53%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 95%
## A11 Acceptance: 53%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
## Progress: 100%
## A11 Acceptance: 53%
## A22 Acceptance: 31%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## ********************
```

```
ds_pred_results<-
ppd_s(modeling_output = ds_results,
      n_pred = length(y[CMAQ_key_test_set == 0]),
      m_pred = nrow(z[-test_set, ]),
      z_pred = z[-test_set,],
      sample_size_pred = sample_size[-test_set],
      spatial_dists_full = spatial_dists_full,
      neighbors_full = neighbors_full,
      inference_set = keep_set,
      params_only_indicator = 0,
      model_type_indicator = 1)
```

```
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## Progress: 25%
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## Progress: 30%
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## Progress: 35%
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## Progress: 40%
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## Progress: 45%
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```

```
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## Progress: 80%
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## Progress: 85%
## **************
## Progress: 90%
## **************
## Progress: 95%
## **************
## Progress: 100%
## **************
```

```
#Results Matrix
results<-matrix(0,
                nrow = 3,
                ncol = 4)
colnames(results)<-c("dlfuse", "slr", "ok", "ds")
rownames(results)<-c("mse", "cover", "length")

#DLfuse
cover<-rep(0,
           times=length(y[CMAQ_key_test_set == 0]))
len<-rep(0,
         times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){

   ci<-quantile(dlfuse_pred_results[[1]][j,], c(0.025, 0.975))
   if((ci[1] <= y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j])){
     cover[j]<-1
     }
   len[j]<-ci[2] -
           ci[1]

   }
results[1,1]<-mean((y[CMAQ_key_test_set == 0] -
                    rowMedians(dlfuse_pred_results[[1]]))^2)
results[2,1]<-mean(cover)
results[3,1]<-mean(len)

#SLR
cover<-rep(0,
           times=length(y[CMAQ_key_test_set == 0]))
len<-rep(0,
```

```
          times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){

   ci<-quantile(slr_pred_results[[1]][j,], c(0.025, 0.975))
   if((ci[1] <= y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j])){
     cover[j]<-1
     }
   len[j]<-ci[2] -
           ci[1]

   }
results[1,2]<-mean((y[CMAQ_key_test_set == 0] -
                    rowMedians(slr_pred_results[[1]]))^2)
results[2,2]<-mean(cover)
results[3,2]<-mean(len)

#OK
cover<-rep(0,
          times=length(y[CMAQ_key_test_set == 0]))
len<-rep(0,
          times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){

   ci<-quantile(ok_pred_results[[1]][j,], c(0.025, 0.975))
   if((ci[1] <= y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j])){
     cover[j]<-1
     }
   len[j]<-ci[2] -
           ci[1]

   }
results[1,3]<-mean((y[CMAQ_key_test_set == 0] -
                    rowMedians(ok_pred_results[[1]]))^2)
results[2,3]<-mean(cover)
results[3,3]<-mean(len)

#DS
cover<-rep(0,
          times=length(y[CMAQ_key_test_set == 0]))
len<-rep(0,
          times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){

   ci<-quantile(ds_pred_results[[1]][j,], c(0.025, 0.975))
   if((ci[1] <= y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j])){
     cover[j]<-1
     }
   len[j]<-ci[2] -
           ci[1]

   }
results[1,4]<-mean((y[CMAQ_key_test_set == 0] -
                    rowMedians(ds_pred_results[[1]]))^2)
```

```
results[2,4]<-mean(cover)
results[3,4]<-mean(len)

results
```

```
##           dlfuse       slr        ok        ds
## mse    0.1708761 0.4106818 0.6111710 0.4965193
## cover  0.9767442 0.9534884 0.8837209 0.9534884
## length 1.6434155 2.7292190 3.2354954 2.6412110
```