DLfuse: Distributed Lag Data Fusion for Estimating Ambient Air Pollution

DLfuseST_Example

[1] Simulate data from the proposed model:

- Setting the reproducibility seed and initializing packages for data simulation:

```r
set.seed(3146)

library(DLfuse)
library(geoR)   #Spatial covariance functions
```

```
## Warning: package 'geoR' was built under R version 4.1.1
```

```
## --------------------------------------------------------------
##  Analysis of Geostatistical Data
##  For an Introduction to geoR go to http://www.leg.ufpr.br/geoR
##  geoR version 1.8-1 (built on 2020-02-08) is now loaded
## --------------------------------------------------------------
```

```r
library(mnormt)   #Multivariate normal distribution
library(matrixStats)   #colMedians
```

- Setting the global data values:

```r
##########################################
#Full CMAQ Grid
##########################################
m<-(5^2)
grid<-matrix(0,
             nrow = m,
             ncol = 2)
counter<-1
for(j in 1:sqrt(m)){
   for(k in 1:sqrt(m)){

      grid[counter,]<-c(j,k)
      counter<-counter +
              1

   }
}

neighbors<-1/as.matrix(dist(grid,
                            diag = TRUE,
                            upper = TRUE))
diag(neighbors)<-0

CAR<-diag(rowSums(neighbors)) -
     neighbors

###################################################################
#Full AQS Locations
###################################################################
sample_size<-list(0)
```

```r
CMAQ_key<-list(0)
AQS_key<-list(0)

sample_size[[1]]<-rpois(n = m,
                        lambda = 1)
sample_size[[1]]<-pmax(sample_size[[1]],
                       rep(1,
                           times = m))

locs<-matrix(runif(n = (2*sum(sample_size[[1]])),
                   min = 0,
                   max = 1),
             ncol = 2)
spatial_dists<-as.matrix(dist(locs,
                              diag = TRUE,
                              upper = TRUE))
diag(spatial_dists)<-0

CMAQ_key[[1]]<-rep(0,
                   times = sum(sample_size[[1]]))
counter<-0
for(j in 1:length(sample_size[[1]])){

   CMAQ_key[[1]][(1 + counter):(sample_size[[1]][j] + counter)]<-j
   counter<-counter +
            sample_size[[1]][j]

   }

CMAQ_key[[1]]<-c(1:nrow(neighbors))
AQS_key[[1]]<-c(1:nrow(spatial_dists))

######################################################################
#Creating the Spatiotemporal Information
######################################################################
d<-16   #Must be larger than one
if(d > 1){
  for(j in 2:d){

     CMAQ_key[[j]]<-sample(c(1:m),
                           size = round((4/5)*m),
                           replace = FALSE)

     sample_size[[j]]<-rpois(n = length(CMAQ_key[[j]]),
                             lambda = 1)
     sample_size[[j]]<-pmax(sample_size[[j]],
                            rep(1,
                                times=length(CMAQ_key[[j]])))

     while(sum(sample_size[[j]]) > nrow(spatial_dists)){

          sample_size[[j]]<-rpois(n = length(CMAQ_key[[j]]),
                                  lambda = 1)
```

```r
            sample_size[[j]]<-pmax(sample_size[[j]],
                                   rep(1,
                                       times = length(CMAQ_key[[j]])))

        }

    AQS_key[[j]]<-sample(c(1:nrow(spatial_dists)),
                         size = sum(sample_size[[j]]),
                         replace = FALSE)

    }
  }

############################################################################
#True Spatial Parameter Settings
############################################################################
sigma2_epsilon_true<-0.05

beta0_true<-0.25
beta1_true<-1.75

A11_true<-0.05
A22_true<-0.05
A21_true<- -0.01

phi0_true<-1.50
Sigma0_true<-cov.spatial(spatial_dists,
                         cov.model = "exponential",
                         cov.pars=c(1, (1/phi0_true)))
w0_true<-rmnorm(n = 1,
                mean = rep(0,
                           times = sum(sample_size[[1]])),
                varcov = Sigma0_true)
w0_true<-w0_true -
         mean(w0_true)
beta0_tilde_true<-A11_true*w0_true

phi1_true<-1.50
Sigma1_true<-cov.spatial(spatial_dists,
                         cov.model = "exponential",
                         cov.pars = c(1, (1/phi1_true)))
w1_true<-rmnorm(n = 1,
                mean = rep(0,
                           times=sum(sample_size[[1]])),
                varcov = Sigma1_true)
w1_true<-w1_true -
         mean(w1_true)
beta1_tilde_true<-A21_true*w0_true +
                  A22_true*w1_true

tau2_true<-1.00
rho_true<-0.99  #ICAR Model Approximation
CAR_cov_true<-tau2_true*chol2inv(chol(rho_true*CAR + (1 - rho_true)*diag(m)))
```

```r
alpha_true<-rmnorm(n = 1,
                   mean = rep(0,
                              times=m),
                   varcov = CAR_cov_true)
alpha_true<-alpha_true -
            mean(alpha_true)

mu_true<-1.00

####################################################
#True Temporal Parameters
####################################################
V_true<-matrix(c(0.03, -0.01, -0.01, 0.03),
               nrow = 2,
               ncol = 2)

rho1_true<-0.90
rho2_true<-0.10
Omega_true<-matrix(0,
                   nrow = 2,
                   ncol = 2)
Omega_true[1,1]<-rho1_true
Omega_true[2,2]<-rho2_true

betat_true<-matrix(0,
                   nrow = d,
                   ncol = 2)
betat_true[1,]<-rmnorm(n = 1,
                       mean = rep(0,
                                  times = 2),
                       varcov = V_true)

rho3_true<-0.75

sigma2_delta_true<-0.01

mut_true<-rep(0,
              times = d)
mut_true[1]<-rnorm(n = 1,
                   mean = 0,
                   sd = sqrt(sigma2_delta_true))

for(j in 2:d){

   betat_true[j,]<-Omega_true%*%betat_true[(j-1),] +
                   rmnorm(n = 1,
                          mean = rep(0,
                                     times = 2),
                          varcov = V_true)

   mut_true[j]<-rho3_true*mut_true[j-1] +
               rnorm(n = 1,
                     mean = 0,
```

```r
                             sd = sqrt(sigma2_delta_true))

   }

betat_true[,1]<-betat_true[,1] -
              mean(betat_true[,1])
betat_true[,2]<-betat_true[,2] -
              mean(betat_true[,2])

mut_true<-mut_true -
         mean(mut_true)

###########################################################################
#Creating Lagged Covariates and Observed Data
###########################################################################
L<-11
y<-list(0)
z<-list(0)
covars_true<-list(0)
for(j in 1:d){

   z[[j]]<-matrix(rgamma(n = (length(CMAQ_key[[j]])*L),
                        shape = 1,
                        rate = 1),
                  nrow = length(CMAQ_key[[j]]),
                  ncol = L)

   covars_true[[j]]<-construct_lagged_covars_st(
                  z[[j]],
                  mu_true,
                  mut_true[j],
                  alpha_true,
                  sample_size[[j]],
                  CMAQ_key[[j]],
                  0)[[1]]  #Probit Weights (1 for Spherical)

   y[[j]]<-rnorm(n = sum(sample_size[[j]]),
               mean = ((beta0_true + betat_true[j,1] +
                      beta0_tilde_true[AQS_key[[j]]]) +
                    (beta1_true + betat_true[j,2] +
                     beta1_tilde_true[AQS_key[[j]]])*covars_true[[j]]),
               sd = sqrt(sigma2_epsilon_true))

   }

sample_size_validation<-sample_size[[d]]
AQS_key_validation<-AQS_key[[d]]
CMAQ_key_validation<-CMAQ_key[[d]]
y_validation<-y[[d]]
z_validation<-z[[d]]

sample_size[[d]]<-NULL
AQS_key[[d]]<-NULL
```

```
CMAQ_key[[d]]<-NULL
y[[d]]<-NULL
z[[d]]<-NULL
```

[2] Fit DLfuse to a subset of the data:

```
samples<-11000

dlfuse_st_results<-DLfuse_st(mcmc_samples = samples,
                             y = y,
                             z = z,
                             sample_size = sample_size,
                             AQS_key = AQS_key,
                             CMAQ_key = CMAQ_key,
                             spatial_dists = spatial_dists,
                             AQS_unique_total = nrow(spatial_dists),
                             neighbors = neighbors,
                             CMAQ_unique_total = nrow(neighbors),
                             metrop_var_rho1_trans = (3.00^2),
                             metrop_var_rho2_trans = (3.00^2),
                             metrop_var_A11_trans = (0.40^2),
                             metrop_var_A22_trans = (0.60^2),
                             metrop_var_mu = (0.20^2),
                             metrop_var_mut = rep(0.75^2, times=length(y)),
                             metrop_var_rho3_trans = (3.00^2),
                             metrop_var_alpha = rep(1.40^2, times=nrow(neighbors)),
                             metrop_var_phi0_trans = (0.60^2),
                             metrop_var_phi1_trans = (0.62^2),
                             weights_definition_indicator = 0,  #Probit (1 for Spherical)
                             model_type_indicator = 0)
```

```
## Progress: 5%
## rho1 Acceptance: 43%
## rho2 Acceptance: 46%
## A11 Acceptance: 47%
## A22 Acceptance: 50%
## mu Acceptance: 37%
## mut Acceptance (min): 31%
## mut Acceptance (max): 57%
## rho3 Acceptance: 38%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 75%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## DLfuse: ST
## *************************
## Progress: 10%
## rho1 Acceptance: 42%
## rho2 Acceptance: 45%
## A11 Acceptance: 48%
## A22 Acceptance: 50%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 57%
## rho3 Acceptance: 40%
```

```
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 78%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## DLfuse: ST
## *************************
## Progress: 15%
## rho1 Acceptance: 42%
## rho2 Acceptance: 44%
## A11 Acceptance: 44%
## A22 Acceptance: 49%
## mu Acceptance: 36%
## mut Acceptance (min): 33%
## mut Acceptance (max): 57%
## rho3 Acceptance: 39%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 78%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## DLfuse: ST
## *************************
## Progress: 20%
## rho1 Acceptance: 42%
## rho2 Acceptance: 44%
## A11 Acceptance: 47%
## A22 Acceptance: 50%
## mu Acceptance: 36%
## mut Acceptance (min): 34%
## mut Acceptance (max): 56%
## rho3 Acceptance: 39%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 78%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## *************************
## Progress: 25%
## rho1 Acceptance: 42%
## rho2 Acceptance: 44%
## A11 Acceptance: 48%
## A22 Acceptance: 50%
## mu Acceptance: 36%
## mut Acceptance (min): 33%
## mut Acceptance (max): 56%
## rho3 Acceptance: 40%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 78%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## *************************
## Progress: 30%
## rho1 Acceptance: 42%
## rho2 Acceptance: 44%
```

```
## A11 Acceptance: 47%
## A22 Acceptance: 51%
## mu Acceptance: 36%
## mut Acceptance (min): 33%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 78%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## DLfuse: ST
## *************************
## Progress: 35%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 46%
## A22 Acceptance: 52%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 78%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## *************************
## Progress: 40%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 45%
## A22 Acceptance: 52%
## mu Acceptance: 35%
## mut Acceptance (min): 32%
## mut Acceptance (max): 57%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 77%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## *************************
## Progress: 45%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 46%
## A22 Acceptance: 52%
## mu Acceptance: 35%
## mut Acceptance (min): 32%
## mut Acceptance (max): 57%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 77%
## phi0 Acceptance: 45%
```

```
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 50%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 47%
## A22 Acceptance: 52%
## mu Acceptance: 35%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 77%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 55%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 47%
## A22 Acceptance: 52%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 60%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 47%
## A22 Acceptance: 52%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 65%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 47%
## A22 Acceptance: 52%
## mu Acceptance: 36%
```

```
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 70%
## rho1 Acceptance: 43%
## rho2 Acceptance: 43%
## A11 Acceptance: 46%
## A22 Acceptance: 53%
## mu Acceptance: 35%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 44%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 75%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 46%
## A22 Acceptance: 54%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
## Progress: 80%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 47%
## A22 Acceptance: 54%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 44%
## phi1 Acceptance: 44%
## DLfuse: ST
## **************************
```

```
## Progress: 85%
## rho1 Acceptance: 42%
## rho2 Acceptance: 43%
## A11 Acceptance: 48%
## A22 Acceptance: 54%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## ************************
## Progress: 90%
## rho1 Acceptance: 42%
## rho2 Acceptance: 42%
## A11 Acceptance: 47%
## A22 Acceptance: 53%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 44%
## phi1 Acceptance: 44%
## DLfuse: ST
## ************************
## Progress: 95%
## rho1 Acceptance: 42%
## rho2 Acceptance: 42%
## A11 Acceptance: 47%
## A22 Acceptance: 53%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 44%
## phi1 Acceptance: 44%
## DLfuse: ST
## ************************
## Progress: 100%
## rho1 Acceptance: 42%
## rho2 Acceptance: 42%
## A11 Acceptance: 47%
## A22 Acceptance: 53%
## mu Acceptance: 36%
## mut Acceptance (min): 32%
## mut Acceptance (max): 56%
## rho3 Acceptance: 41%
```

```
## alpha Acceptance (min): 16%
## alpha Acceptance (max): 76%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## DLfuse: ST
## ***************************
```

[3] Comparing parameter estimates to true values:
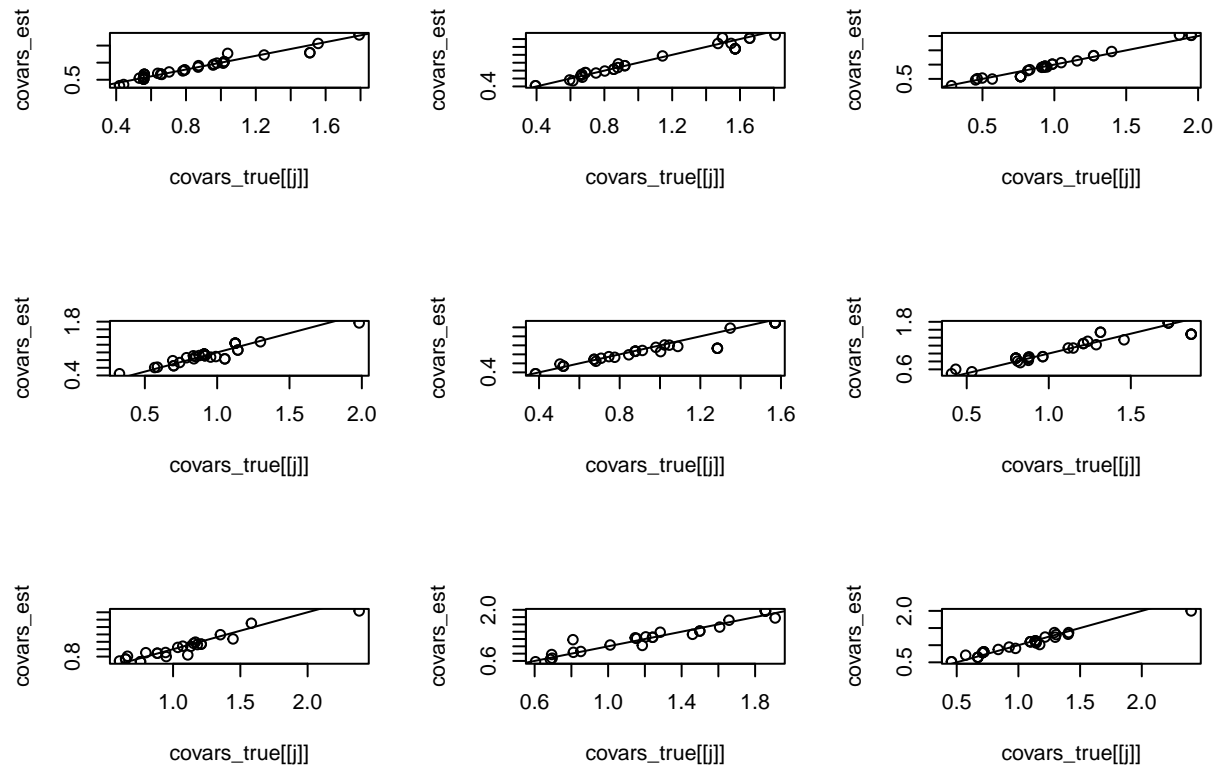
```r
burnin<-1000
thin<-10
keep_set<-seq((burnin + 1),
              samples,
              thin)
par(mfrow=c(3,3))

for(j in 1:(d-1)){

   covars_est<-construct_lagged_covars_st(
              z[[j]],
              mean(dlfuse_st_results$lag_info$mu[keep_set]),
              mean(dlfuse_st_results$lag_info$mut[j,keep_set]),
              rowMeans(dlfuse_st_results$lag_info$alpha[,keep_set]),
              sample_size[[j]],
              CMAQ_key[[j]],
              0)[[1]]  #Probit Weights (1 for Spherical)

   plot(covars_true[[j]],
        covars_est)
   abline(0,1)

   }
```
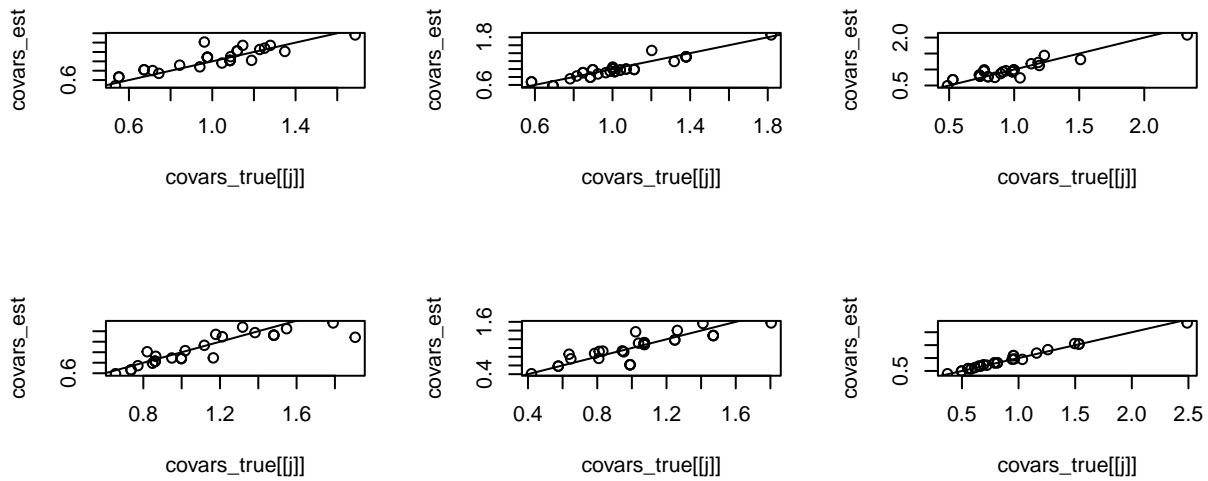
```
par(mfrow=c(3,3))
```

```
betat<-simplify2array(dlfuse_st_results$betat)[,,keep_set]
for(j in 1:(d-1)){

   true<-c(sigma2_epsilon_true,
           (beta0_true + betat_true[j,1] + beta0_tilde_true[AQS_key[[j]]]),
           (beta1_true + betat_true[j,2] + beta1_tilde_true[AQS_key[[j]]]))

   est<-c(mean(dlfuse_st_results$sigma2_epsilon[keep_set]),
          rowMeans(matrix(dlfuse_st_results$beta0[keep_set],
                          nrow = sum(sample_size[[j]]),
                          ncol = length(keep_set),
                          byrow = TRUE) +
                   matrix(betat[1,j,],
                          nrow = sum(sample_size[[j]]),
                          ncol = length(keep_set),
                          byrow = TRUE) +
                   matrix(dlfuse_st_results$A11[keep_set],
                          nrow = sum(sample_size[[j]]),
                          ncol = length(keep_set),
                          byrow = TRUE)*dlfuse_st_results$w0[AQS_key[[j]] ,keep_set]),
          rowMeans(matrix(dlfuse_st_results$beta1[keep_set],
                          nrow = sum(sample_size[[j]]),
                          ncol = length(keep_set),
                          byrow = TRUE) +
                   matrix(betat[2,j,],
                          nrow = sum(sample_size[[j]]),
```

```
                  ncol = length(keep_set),
                  byrow = TRUE) +
          matrix(dlfuse_st_results$A21[keep_set],
                  nrow = sum(sample_size[[j]]),
                  ncol = length(keep_set),
                  byrow = TRUE)*dlfuse_st_results$w0[AQS_key[[j]], keep_set] +
          matrix(dlfuse_st_results$A22[keep_set],
                  nrow = sum(sample_size[[j]]),
                  ncol = length(keep_set),
                  byrow = TRUE)*dlfuse_st_results$w1[AQS_key[[j]], keep_set]))

plot(est, true)
abline(0,1)

}
```
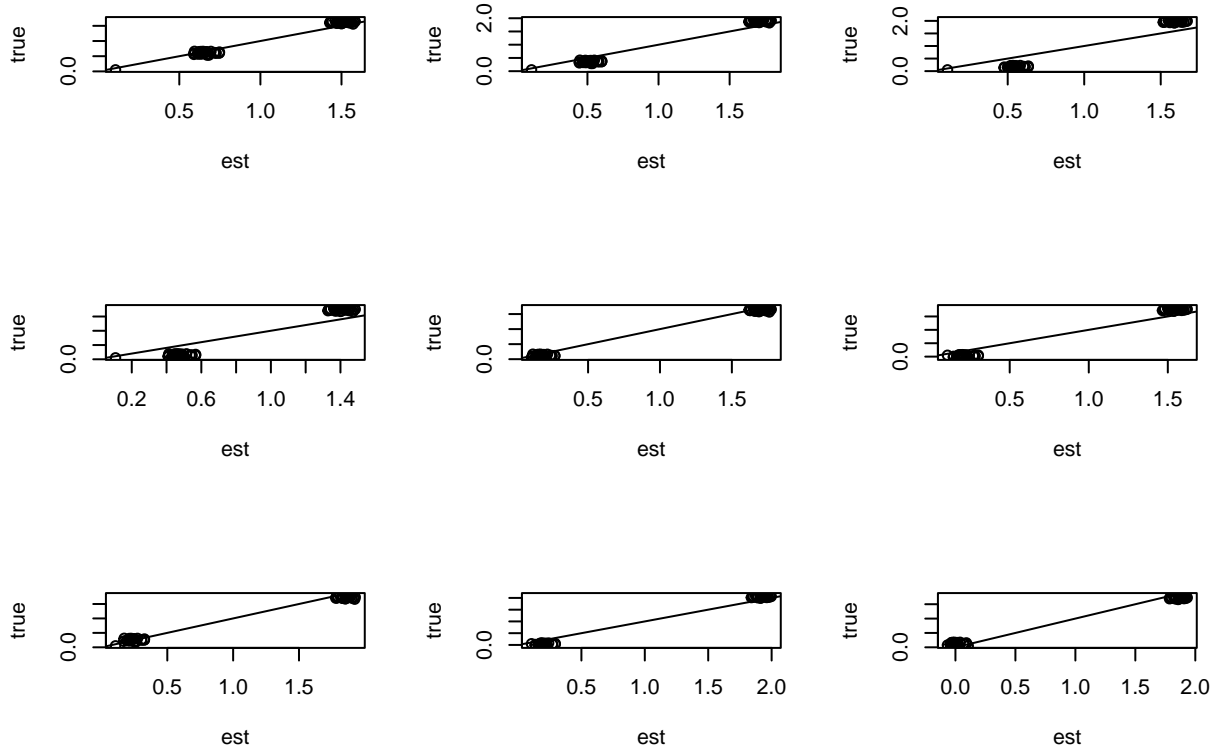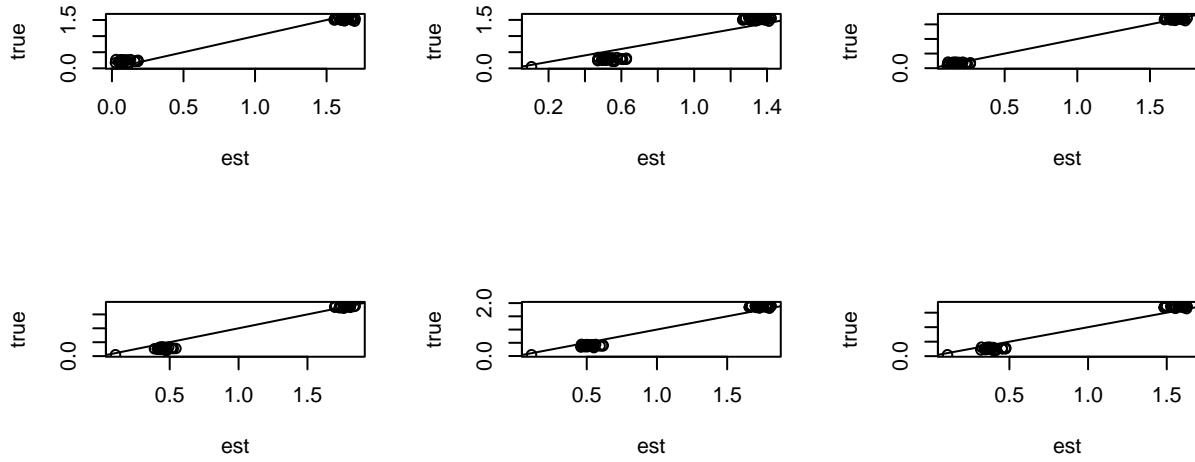
[4] Spatiotemporal predictions of validation data:

```r
spatial_dists_full<-as.matrix(dist(rbind(locs[AQS_key_validation,], locs),
                                   diag = TRUE,
                                   upper = TRUE))
diag(spatial_dists_full)<-0
loc_temp<-rbind(locs[AQS_key_validation,], locs)
for(j in 1:nrow(loc_temp)){
   for(k in 1:nrow(loc_temp)){
      if(prod(loc_temp[j,] == loc_temp[k,]) == 1){
        spatial_dists_full[j,k]<-0
        }
      }
   }

neighbors_full<-1/as.matrix(dist(rbind(grid[CMAQ_key_validation,], grid),
                                 diag = TRUE,
                                 upper = TRUE))
diag(neighbors_full)<-0
loc_temp<-rbind(grid[CMAQ_key_validation,], grid)
for(j in 1:nrow(loc_temp)){
   for(k in 1:nrow(loc_temp)){
      if(prod(loc_temp[j,] == loc_temp[k,]) == 1){
        neighbors_full[j,k]<-Inf
        }
      }
   }
```

```r
diag(neighbors_full)<-0

dlfuse_st_pred_results<-
ppd_st(modeling_output = dlfuse_st_results,
       n_pred = length(y_validation),
       m_pred = nrow(z_validation),
       z_pred = z_validation,
       sample_size_pred = sample_size_validation,
       spatial_dists_full = spatial_dists_full,
       neighbors_full = neighbors_full,
       inference_set = keep_set,
       params_only_indicator = 0,
       weights_definition_indicator = 0,  #Probit Weights (1 for Spherical)
       model_type_indicator = 0)
```

```
## Progress: 5%
## **************
## Progress: 10%
## **************
## Progress: 15%
## **************
## Progress: 20%
## **************
## Progress: 25%
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## Progress: 30%
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## Progress: 35%
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## Progress: 40%
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## Progress: 45%
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## Progress: 50%
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## Progress: 55%
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## Progress: 60%
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## Progress: 65%
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## Progress: 70%
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## Progress: 75%
## **************
## Progress: 80%
## **************
## Progress: 85%
## **************
## Progress: 90%
## **************
## Progress: 95%
## **************
## Progress: 100%
```

```
## **************
```

[5] Comparison with other approaches:

```
slr_st_results<-DLfuse_st(mcmc_samples = samples,
                          y = y,
                          z = z,
                          sample_size = sample_size,
                          AQS_key = AQS_key,
                          CMAQ_key = CMAQ_key,
                          spatial_dists = spatial_dists,
                          AQS_unique_total = nrow(spatial_dists),
                          neighbors = neighbors,
                          CMAQ_unique_total = nrow(neighbors),
                          metrop_var_rho1_trans = (3.00^2),
                          metrop_var_rho2_trans = (3.00^2),
                          metrop_var_A11_trans = (0.30^2),
                          metrop_var_A22_trans = (0.30^2),
                          metrop_var_mu = (0.10^2),
                          metrop_var_mut = rep(0.75^2, times=length(y)),
                          metrop_var_rho3_trans = (3.00^2),
                          metrop_var_alpha = rep(1.40^2, times=nrow(neighbors)),
                          metrop_var_phi0_trans = (0.60^2),
                          metrop_var_phi1_trans = (0.62^2),
                          model_type_indicator = 3)
```

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## Progress: 5%
## rho1 Acceptance: 43%
## rho2 Acceptance: 53%
## Simple Linear Regression: ST
## **************************
## Progress: 10%
## rho1 Acceptance: 44%
## rho2 Acceptance: 50%
## Simple Linear Regression: ST
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## rho2 Acceptance: 50%
## Simple Linear Regression: ST
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## Simple Linear Regression: ST
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## rho2 Acceptance: 48%
## Simple Linear Regression: ST
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## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## Simple Linear Regression: ST
```

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## Simple Linear Regression: ST
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## Progress: 85%
## rho1 Acceptance: 44%
## rho2 Acceptance: 48%
```

```
## Simple Linear Regression: ST
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## Progress: 90%
## rho1 Acceptance: 44%
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## Simple Linear Regression: ST
## ***************************
## Progress: 95%
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## rho2 Acceptance: 48%
## Simple Linear Regression: ST
## ***************************
## Progress: 100%
## rho1 Acceptance: 44%
## rho2 Acceptance: 48%
## Simple Linear Regression: ST
## ***************************
```

```r
slr_st_pred_results<-ppd_st(modeling_output = slr_st_results,
                            n_pred = length(y_validation),
                            m_pred = nrow(z_validation),
                            z_pred = z_validation,
                            sample_size_pred = sample_size_validation,
                            spatial_dists_full = spatial_dists_full,
                            neighbors_full = neighbors_full,
                            inference_set = keep_set,
                            params_only_indicator = 0,
                            model_type_indicator = 3)
```

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## Progress: 95%
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## Progress: 100%
## **************
```

```r
ok_st_results<-DLfuse_st(mcmc_samples = samples,
                         y = y,
                         z = z,
                         sample_size = sample_size,
                         AQS_key = AQS_key,
                         CMAQ_key = CMAQ_key,
                         spatial_dists = spatial_dists,
                         AQS_unique_total = nrow(spatial_dists),
                         neighbors = neighbors,
                         CMAQ_unique_total = nrow(neighbors),
                         metrop_var_rho1_trans = (3.00^2),
                         metrop_var_rho2_trans = (3.00^2),
                         metrop_var_A11_trans = (0.60^2),
                         metrop_var_A22_trans = (0.30^2),
                         metrop_var_mu = (0.10^2),
                         metrop_var_mut = rep(0.75^2, times=length(y)),
                         metrop_var_rho3_trans = (3.00^2),
                         metrop_var_alpha = rep(1.40^2, times=nrow(neighbors)),
                         metrop_var_phi0_trans = (0.60^2),
                         metrop_var_phi1_trans = (0.62^2),
                         model_type_indicator = 2)
```

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## Progress: 5%
## rho1 Acceptance: 49%
## A11 Acceptance: 57%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
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## Progress: 10%
## rho1 Acceptance: 47%
## A11 Acceptance: 56%
## phi0 Acceptance: 44%
## Ordinary Kriging: ST
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## rho1 Acceptance: 45%
## A11 Acceptance: 56%
## phi0 Acceptance: 44%
## Ordinary Kriging: ST
## *******************
```

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## phi0 Acceptance: 44%
## Ordinary Kriging: ST
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## phi0 Acceptance: 44%
## Ordinary Kriging: ST
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## Ordinary Kriging: ST
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## phi0 Acceptance: 44%
## Ordinary Kriging: ST
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## Ordinary Kriging: ST
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## phi0 Acceptance: 45%
## Ordinary Kriging: ST
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## Ordinary Kriging: ST
## *******************
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## A11 Acceptance: 57%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 60%
## rho1 Acceptance: 45%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
```

```
## Progress: 65%
## rho1 Acceptance: 45%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 70%
## rho1 Acceptance: 45%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 75%
## rho1 Acceptance: 46%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 80%
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## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 85%
## rho1 Acceptance: 46%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 90%
## rho1 Acceptance: 46%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 95%
## rho1 Acceptance: 46%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
## Progress: 100%
## rho1 Acceptance: 46%
## A11 Acceptance: 56%
## phi0 Acceptance: 45%
## Ordinary Kriging: ST
## *******************
```

```r
ok_st_pred_results<-ppd_st(modeling_output = ok_st_results,
                           n_pred = length(y_validation),
                           m_pred = nrow(z_validation),
                           z_pred = z_validation,
                           sample_size_pred = sample_size_validation,
```

```
                                 spatial_dists_full = spatial_dists_full,
                                 neighbors_full = neighbors_full,
                                 inference_set = keep_set,
                                 params_only_indicator = 0,
                                 model_type_indicator = 2)
```

```
## Progress: 5%
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## Progress: 80%
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## Progress: 85%
## **************
## Progress: 90%
## **************
## Progress: 95%
## **************
## Progress: 100%
## **************
```

```
ds_st_results<-DLfuse_st(mcmc_samples = samples,
                         y = y,
                         z = z,
                         sample_size = sample_size,
                         AQS_key = AQS_key,
                         CMAQ_key = CMAQ_key,
                         spatial_dists = spatial_dists,
```

```
                          AQS_unique_total = nrow(spatial_dists),
                          neighbors = neighbors,
                          CMAQ_unique_total = nrow(neighbors),
                          metrop_var_rho1_trans = (3.00^2),
                          metrop_var_rho2_trans = (3.00^2),
                          metrop_var_A11_trans = (0.60^2),
                          metrop_var_A22_trans = (0.60^2),
                          metrop_var_mu = (0.10^2),
                          metrop_var_mut = rep(0.75^2, times=length(y)),
                          metrop_var_rho3_trans = (3.00^2),
                          metrop_var_alpha = rep(1.40^2, times=nrow(neighbors)),
                          metrop_var_phi0_trans = (0.60^2),
                          metrop_var_phi1_trans = (0.62^2),
                          model_type_indicator = 1)
```

```
## Progress: 5%
## rho1 Acceptance: 43%
## rho2 Acceptance: 46%
## A11 Acceptance: 63%
## A22 Acceptance: 60%
## phi0 Acceptance: 42%
## phi1 Acceptance: 41%
## Original: ST
## *******************
## Progress: 10%
## rho1 Acceptance: 43%
## rho2 Acceptance: 47%
## A11 Acceptance: 61%
## A22 Acceptance: 59%
## phi0 Acceptance: 43%
## phi1 Acceptance: 44%
## Original: ST
## *******************
## Progress: 15%
## rho1 Acceptance: 43%
## rho2 Acceptance: 49%
## A11 Acceptance: 59%
## A22 Acceptance: 57%
## phi0 Acceptance: 44%
## phi1 Acceptance: 44%
## Original: ST
## *******************
## Progress: 20%
## rho1 Acceptance: 44%
## rho2 Acceptance: 49%
## A11 Acceptance: 58%
## A22 Acceptance: 56%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 25%
## rho1 Acceptance: 45%
## rho2 Acceptance: 49%
```

```
## A11 Acceptance: 57%
## A22 Acceptance: 56%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## Original: ST
## ******************
## Progress: 30%
## rho1 Acceptance: 45%
## rho2 Acceptance: 49%
## A11 Acceptance: 56%
## A22 Acceptance: 56%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## Original: ST
## ******************
## Progress: 35%
## rho1 Acceptance: 45%
## rho2 Acceptance: 49%
## A11 Acceptance: 57%
## A22 Acceptance: 55%
## phi0 Acceptance: 45%
## phi1 Acceptance: 44%
## Original: ST
## ******************
## Progress: 40%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 56%
## A22 Acceptance: 55%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## ******************
## Progress: 45%
## rho1 Acceptance: 45%
## rho2 Acceptance: 47%
## A11 Acceptance: 55%
## A22 Acceptance: 56%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## ******************
## Progress: 50%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 56%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## ******************
## Progress: 55%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
```

```
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 60%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 65%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 56%
## A22 Acceptance: 55%
## phi0 Acceptance: 45%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 70%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 75%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 80%
## rho1 Acceptance: 45%
## rho2 Acceptance: 48%
## A11 Acceptance: 56%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 85%
## rho1 Acceptance: 44%
## rho2 Acceptance: 48%
```

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## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 90%
## rho1 Acceptance: 44%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 95%
## rho1 Acceptance: 44%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
## Progress: 100%
## rho1 Acceptance: 44%
## rho2 Acceptance: 48%
## A11 Acceptance: 55%
## A22 Acceptance: 55%
## phi0 Acceptance: 44%
## phi1 Acceptance: 43%
## Original: ST
## *******************
```

```r
ds_st_pred_results<-ppd_st(modeling_output = ds_st_results,
                           n_pred = length(y_validation),
                           m_pred = nrow(z_validation),
                           z_pred = z_validation,
                           sample_size_pred = sample_size_validation,
                           spatial_dists_full = spatial_dists_full,
                           neighbors_full = neighbors_full,
                           inference_set = keep_set,
                           params_only_indicator = 0,
                           model_type_indicator = 1)
```

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## Progress: 95%
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## Progress: 100%
## **************
```

```r
#Results Matrix
results<-matrix(0,
                nrow = 3,
                ncol = 4)
colnames(results)<-c("dlfuse_st", "slr", "ok", "ds")
rownames(results)<-c("mse", "cover", "length")

#DLfuse_st
cover<-rep(0,
           times=length(y_validation))
len<-rep(0,
         times=length(y_validation))
for(j in 1:length(y_validation)){

  ci<-quantile(dlfuse_st_pred_results[[1]][j,], c(0.025, 0.975))
  if((ci[1] <= y_validation[j]) & (ci[2] >= y_validation[j])){
    cover[j]<-1
    }
  len[j]<-ci[2] -
          ci[1]


  }
results[1,1]<-mean((y_validation -
```

```
                      rowMedians(dlfuse_st_pred_results[[1]]))^2)
results[2,1]<-mean(cover)
results[3,1]<-mean(len)

#SLR_st
cover<-rep(0,
          times=length(y_validation))
len<-rep(0,
        times=length(y_validation))
for(j in 1:length(y_validation)){

  ci<-quantile(slr_st_pred_results[[1]][j,], c(0.025, 0.975))
  if((ci[1] <= y_validation[j]) & (ci[2] >= y_validation[j])){
    cover[j]<-1
    }
  len[j]<-ci[2] -
          ci[1]

  }
results[1,2]<-mean((y_validation -
                   rowMedians(slr_st_pred_results[[1]]))^2)
results[2,2]<-mean(cover)
results[3,2]<-mean(len)


#OK_st
cover<-rep(0,
          times=length(y_validation))
len<-rep(0,
        times=length(y_validation))
for(j in 1:length(y_validation)){

  ci<-quantile(ok_st_pred_results[[1]][j,], c(0.025, 0.975))
  if((ci[1] <= y_validation[j]) & (ci[2] >= y_validation[j])){
    cover[j]<-1
    }
  len[j]<-ci[2] -
          ci[1]

  }
results[1,3]<-mean((y_validation -
                   rowMedians(ok_st_pred_results[[1]]))^2)
results[2,3]<-mean(cover)
results[3,3]<-mean(len)

#DS_st
cover<-rep(0,
          times=length(y_validation))
len<-rep(0,
        times=length(y_validation))
for(j in 1:length(y_validation)){

  ci<-quantile(ds_st_pred_results[[1]][j,], c(0.025, 0.975))
```

```
   if((ci[1] <= y_validation[j]) & (ci[2] >= y_validation[j])){
     cover[j]<-1
     }
  len[j]<-ci[2] -
         ci[1]

  }
results[1,4]<-mean((y_validation -
                rowMedians(ds_st_pred_results[[1]]))^2)
results[2,4]<-mean(cover)
results[3,4]<-mean(len)

results
```

```
##          dlfuse_st       slr        ok        ds
## mse     0.07232132 0.3608366 0.3750603 0.3814659
## cover   1.00000000 0.9696970 0.9696970 0.9393939
## length  1.98989554 2.3808350 2.8806060 2.4060989
```