DLfuse: Distributed Lag Data Fusion for Estimating Ambient Air Pollution

$DLfuse_Example$

- [1] Simulate data from the proposed model:
 - Setting the reproducibility seed and initializing packages for data simulation:

• Setting the global data values:

```
#Full CMAQ Grid
m<-(12^2)
grid<-matrix(0,</pre>
         nrow = m,
         ncol = 2)
counter<-1
for(j in 1:sqrt(m)){
  for(k in 1:sqrt(m)){
    grid[counter,]<-c(j,k)</pre>
    counter<-counter +</pre>
           1
  }
neighbors<-1/as.matrix(dist(grid,</pre>
                     diag = TRUE,
                     upper = TRUE))
diag(neighbors)<-0</pre>
CAR<-diag(rowSums(neighbors)) -</pre>
   neighbors
*************************
#Full AQS Locations
sample_size<-rpois(n = m,</pre>
              lambda = 1)
```

```
sample_size<-pmax(sample_size,</pre>
                rep(1,
                    times = m)
locs<-matrix(runif(n = (2*sum(sample_size)),</pre>
                 min = 0,
                 max = 1),
           ncol = 2)
spatial_dists<-as.matrix(dist(locs,</pre>
                            diag = TRUE,
                            upper = TRUE))
diag(spatial_dists)<-0</pre>
CMAQ_key<-rep(0,</pre>
             times = sum(sample_size))
counter<-0
for(j in 1:length(sample_size)){
  CMAQ_key[(1 + counter):(sample_size[j] + counter)]<-j</pre>
  counter<-counter +</pre>
           sample_size[j]
  }
#True Spatial Parameter Settings
sigma2_epsilon_true<-0.05
beta0_true<-0.25
beta1_true<-1.75
A11_true<-0.05
A22_true<-0.05
A21_true<- -0.01
phi0_true<-1.50
Sigma0_true<-cov.spatial(spatial_dists,</pre>
                       cov.model = "exponential",
                       cov.pars = c(1, (1/phi0_true)))
w0_{true} < -rmnorm(n = 1,
              mean = rep(0,
                        times=sum(sample_size)),
              varcov = Sigma0_true)
w0_true<-w0_true -</pre>
        mean(w0_true)
beta0_tilde_true<-A11_true*w0_true
phi1_true<-1.50
Sigma1_true<-cov.spatial(spatial_dists,</pre>
                       cov.model = "exponential",
                       cov.pars = c(1, (1/phi1_true)))
w1_{true}<-rmnorm(n = 1,
```

```
mean = rep(0,
                        times = sum(sample_size)),
              varcov=Sigma1_true)
w1_true<-w1_true -
        mean(w1_true)
beta1_tilde_true<-A21_true*w0_true +
                A22_true*w1_true
tau2_true<-1.00
rho_true<-0.99 #ICAR Model Approximation</pre>
CAR_cov_true<-tau2_true*chol2inv(chol(rho_true*CAR + (1 - rho_true)*diag(m)))</pre>
alpha_true < -rmnorm(n = 1,
                 mean = rep(0,
                           times = m),
                 varcov = CAR_cov_true)
alpha_true<-alpha_true -
           mean(alpha_true)
mu_true<-1.00
#Creating Lagged Covariates
L<-11
z<-matrix(rgamma(n = (length(sample_size)*L),</pre>
               shape = 1,
               rate = 1),
         nrow = length(sample_size),
         ncol = L)
covars_true<-construct_lagged_covars_s(z,</pre>
                                   mu_true,
                                   alpha_true,
                                   sample_size,
                                   0)[[1]] #Probit Weights (1 for Spherical)
  • Simulating the analysis dataset:
y<-rnorm(n = sum(sample_size),</pre>
        mean = ((beta0_true + beta0_tilde_true) +
               (beta1_true + beta1_tilde_true)*covars_true),
        sd = sqrt(sigma2_epsilon_true))
[2] Fit DLfuse to a subset of the data:
samples < -11000
test_set<-sort(sample(c(1:m),</pre>
                    size = round(0.80*m),
                    replace = FALSE))
```

times = length(CMAQ_key))

CMAQ_key_test_set[CMAQ_key == test_set[j]]<-1</pre>

CMAQ key test set <- rep(0,

for(j in 1:length(test_set)){

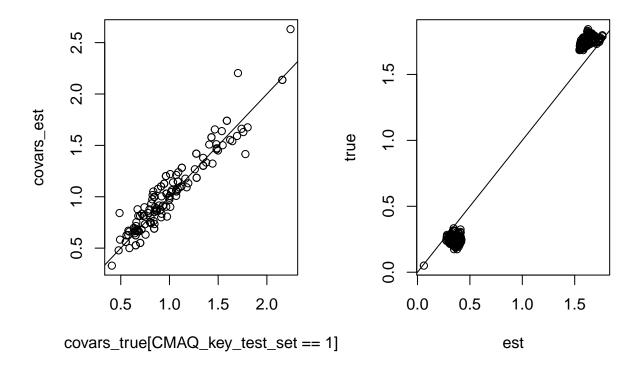
```
dlfuse_results<-
DLfuse_s(mcmc_samples = samples,
        y = y[CMAQ_key_test_set == 1],
        z = z[test set,],
        sample_size = sample_size[test_set],
        spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                      (CMAQ_key_test_set == 1)],
        neighbors = neighbors[test set, test set],
        metrop_var_A11_trans = (0.70^2),
        metrop_var_A22_trans = (0.80^2),
        metrop_var_mu = (0.20^2),
        metrop_var_alpha = rep((1.00^2),
                               times = nrow(neighbors[test_set, test_set])),
        metrop_var_phi0_trans = (0.50^2),
        metrop_var_phi1_trans = (0.50^2),
        weights_definition_indicator = 0, #Probit (1 for Spherical)
        model_type_indicator = 0)
## Progress: 5%
## A11 Acceptance: 27%
## A22 Acceptance: 30%
## mu Acceptance: 36%
## alpha Acceptance (min): 22%
## alpha Acceptance (max): 72%
## phi0 Acceptance: 28%
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## *********
## Progress: 10%
## A11 Acceptance: 30%
## A22 Acceptance: 32%
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## alpha Acceptance (min): 21%
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## mu Acceptance: 32%
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## DLfuse: S
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## mu Acceptance: 33%
## alpha Acceptance (min): 21%
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## alpha Acceptance (max): 74%
## phi0 Acceptance: 28%
## phi1 Acceptance: 28%
## DLfuse: S
## **********
## Progress: 25%
## A11 Acceptance: 31%
## A22 Acceptance: 36%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
## alpha Acceptance (max): 74%
## phi0 Acceptance: 29%
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## DLfuse: S
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## Progress: 30%
## A11 Acceptance: 33%
## A22 Acceptance: 38%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
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## phi0 Acceptance: 28%
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## DLfuse: S
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## A22 Acceptance: 38%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
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## alpha Acceptance (max): 74%
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## mu Acceptance: 33%
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```

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## A22 Acceptance: 40%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
## alpha Acceptance (max): 74%
## phi0 Acceptance: 28%
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## A11 Acceptance: 35%
## A22 Acceptance: 40%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
## alpha Acceptance (max): 74%
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## DLfuse: S
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## A11 Acceptance: 34%
## A22 Acceptance: 40%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
## alpha Acceptance (max): 74%
## phi0 Acceptance: 28%
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## DLfuse: S
## ************
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## A11 Acceptance: 34%
## A22 Acceptance: 40%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
## alpha Acceptance (max): 74%
## phi0 Acceptance: 28%
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## Progress: 70%
## A11 Acceptance: 34%
## A22 Acceptance: 40%
## mu Acceptance: 33%
## alpha Acceptance (min): 20%
## alpha Acceptance (max): 74%
## phi0 Acceptance: 28%
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## phi0 Acceptance: 28%
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```
## phi1 Acceptance: 28%
## DLfuse: S
[3] Comparing parameter estimates to true values:
burnin<-1000
thin < -10
keep_set<-seq((burnin + 1),</pre>
              samples,
              thin)
par(mfrow=c(1,2))
covars_est<-construct_lagged_covars_s(z[test_set,],</pre>
                                       mean(dlfuse_results$lag_info$mu[keep_set]),
                                       rowMeans(dlfuse_results$lag_info$alpha[,keep_set]),
                                       sample_size[test_set],
                                       0)[[1]] #Probit Weights (1 for Spherical)
plot(covars_true[CMAQ_key_test_set == 1],
     covars_est)
abline(0,1)
true<-c(sigma2_epsilon_true,</pre>
        (beta0_true + beta0_tilde_true[CMAQ_key_test_set == 1]),
        (beta1_true + beta1_tilde_true[CMAQ_key_test_set == 1]))
est<-c(mean(dlfuse_results\sigma2_epsilon[keep_set]),
       rowMeans(matrix(dlfuse_results$beta0[keep_set],
                       nrow = sum(sample_size[test_set]),
                        ncol = length(keep_set),
                        byrow = TRUE) +
                matrix(dlfuse_results$A11[keep_set],
                       nrow = sum(sample_size[test_set]),
                        ncol = length(keep_set),
                       byrow = TRUE)*dlfuse_results$w0[,keep_set]),
       rowMeans(matrix(dlfuse_results$beta1[keep_set],
                       nrow = sum(sample_size[test_set]),
                       ncol = length(keep_set),
                        byrow = TRUE) +
                matrix(dlfuse_results$A21[keep_set],
                        nrow = sum(sample size[test set]),
                        ncol = length(keep_set),
                        byrow = TRUE)*dlfuse_results$w0[,keep_set] +
                matrix(dlfuse_results$A22[keep_set],
                        nrow = sum(sample_size[test_set]),
                        ncol = length(keep_set),
                        byrow = TRUE)*dlfuse_results$w1[,keep_set]))
plot(est, true)
abline(0,1)
```



[4] Spatial predictions of validation data:

```
spatial_dists_full<-as.matrix(dist(rbind(locs[CMAQ_key_test_set == 0,],</pre>
                                            locs[CMAQ_key_test_set == 1,]),
                                      diag = TRUE,
                                      upper = TRUE))
diag(spatial dists full)<-0</pre>
loc_temp<-rbind(locs[CMAQ_key_test_set == 0,], locs[CMAQ_key_test_set == 1,])</pre>
for(j in 1:nrow(loc_temp)){
   for(k in 1:nrow(loc_temp)){
      if(prod(loc_temp[j,] == loc_temp[k,]) == 1){
        spatial_dists_full[j,k]<-0</pre>
        }
      }
   }
neighbors_full<-1/as.matrix(dist(rbind(unique(grid[-test_set,]), unique(grid[test_set,])),</pre>
                                   diag = TRUE,
                                   upper = TRUE))
diag(neighbors_full)<-0</pre>
loc_temp<-rbind(unique(grid[-test_set,]), unique(grid[test_set,]))</pre>
for(j in 1:nrow(loc_temp)){
   for(k in 1:nrow(loc_temp)){
      if(prod(loc_temp[j,] == loc_temp[k,]) == 1){
        neighbors_full[j,k]<-Inf</pre>
        }
```

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slr results<-

[5] Comparison with other approaches:

```
DLfuse s(mcmc samples = samples,
       y = y[CMAQ_key_test_set == 1],
        z = z[test_set,],
        sample_size = sample_size[test_set],
        spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                  (CMAQ_key_test_set == 1)],
        neighbors = neighbors[test_set, test_set],
        metrop_var_A11_trans = (0.70^2),
        metrop_var_A22_trans = (0.80^2),
        metrop_var_mu = (0.20^2),
        metrop_var_alpha = rep((2.00^2),
                            times = nrow(neighbors[test_set, test_set])),
        metrop_var_phi0_trans = (0.50^2),
        metrop_var_phi1_trans = (0.50^2),
       model_type_indicator = 3)
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## Progress: 100%
## Simple Linear Regression: S
## **********
slr_pred_results<-
ppd_s(modeling_output = slr_results,
     n_pred = length(y[CMAQ_key_test_set == 0]),
     m_pred = nrow(z[-test_set, ]),
     z_pred = z[-test_set,],
     sample_size_pred = sample_size[-test_set],
     spatial_dists_full = spatial_dists_full,
     neighbors_full = neighbors_full,
     inference_set = keep_set,
     params_only_indicator = 0,
     model_type_indicator = 3)
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## *******
ok_results<-
DLfuse_s(mcmc_samples = samples,
        y = y[CMAQ_key_test_set == 1],
        z = z[test_set,],
        sample_size = sample_size[test_set],
        spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                     (CMAQ_key_test_set == 1)],
        neighbors = neighbors[test_set, test_set],
        metrop_var_A11_trans = (0.70^2),
        metrop_var_A22_trans = (0.80^2),
        metrop_var_mu = (0.20^2),
        metrop_var_alpha = rep((2.00^2),
                              times = nrow(neighbors[test_set, test_set])),
        metrop_var_phi0_trans = (0.50^2),
        metrop_var_phi1_trans = (0.50^2),
        model_type_indicator = 2)
## Progress: 5%
## A11 Acceptance: 32%
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## Ordinary Kriging: S
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## A11 Acceptance: 33%
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## Ordinary Kriging: S
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## Progress: 15%
## A11 Acceptance: 31%
## phi0 Acceptance: 28%
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## Ordinary Kriging: S
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## Progress: 100%
## A11 Acceptance: 32%
## phi0 Acceptance: 29%
## Ordinary Kriging: S
## ********
ok_pred_results<-
ppd_s(modeling_output = ok_results,
     n_pred = length(y[CMAQ_key_test_set == 0]),
     m_pred = nrow(z[-test_set, ]),
     z_pred = z[-test_set,],
     sample_size_pred = sample_size[-test_set],
     spatial_dists_full = spatial_dists_full,
     neighbors_full = neighbors_full,
     inference_set = keep_set,
     params_only_indicator = 0,
     model_type_indicator = 2)
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ds_results<-
DLfuse_s(mcmc_samples = samples,
        y = y[CMAQ_key_test_set == 1],
        z = z[test_set,],
        sample_size = sample_size[test_set],
        spatial_dists = spatial_dists[(CMAQ_key_test_set == 1),
                                    (CMAQ_key_test_set == 1)],
        neighbors = neighbors[test_set, test_set],
        metrop_var_A11_trans = (0.70^2),
        metrop_var_A22_trans = (0.80^2),
        metrop_var_mu = (0.20^2),
        metrop_var_alpha = rep((2.00^2),
                              times = nrow(neighbors[test_set, test_set])),
        metrop_var_phi0_trans = (0.50^2),
        metrop_var_phi1_trans = (0.50^2),
        model_type_indicator = 1)
## Progress: 5%
## A11 Acceptance: 41%
## A22 Acceptance: 36%
## phi0 Acceptance: 31%
## phi1 Acceptance: 29%
## Original: S
```

```
## **********
## Progress: 10%
## A11 Acceptance: 35%
## A22 Acceptance: 39%
## phi0 Acceptance: 30%
## phi1 Acceptance: 29%
## Original: S
## *******
            *****
## Progress: 15%
## A11 Acceptance: 36%
## A22 Acceptance: 39%
## phi0 Acceptance: 30%
## phi1 Acceptance: 28%
## Original: S
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## A11 Acceptance: 39%
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## Original: S
## **********
## Progress: 30%
## A11 Acceptance: 38%
## A22 Acceptance: 40%
## phi0 Acceptance: 30%
## phi1 Acceptance: 28%
## Original: S
## **********
## Progress: 35%
## A11 Acceptance: 39%
## A22 Acceptance: 42%
## phi0 Acceptance: 29%
## phi1 Acceptance: 28%
## Original: S
## **********
## Progress: 40%
## A11 Acceptance: 39%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## *********
## Progress: 45%
## A11 Acceptance: 38%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
```

```
## phi1 Acceptance: 29%
## Original: S
## **********
## Progress: 50%
## A11 Acceptance: 38%
## A22 Acceptance: 41%
## phi0 Acceptance: 28%
## phi1 Acceptance: 28%
## Original: S
## *********
## Progress: 55%
## A11 Acceptance: 40%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
## phi1 Acceptance: 28%
## Original: S
## *********
## Progress: 60%
## A11 Acceptance: 40%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
## phi1 Acceptance: 28%
## Original: S
## *********
## Progress: 65%
## A11 Acceptance: 41%
## A22 Acceptance: 41%
## phi0 Acceptance: 28%
## phi1 Acceptance: 28%
## Original: S
## ***********
## Progress: 70%
## A11 Acceptance: 41%
## A22 Acceptance: 41%
## phi0 Acceptance: 28%
## phi1 Acceptance: 29%
## Original: S
## **********
## Progress: 75%
## A11 Acceptance: 41%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
## phi1 Acceptance: 28%
## Original: S
## *********
## Progress: 80%
## A11 Acceptance: 41%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## *******
## Progress: 85%
## A11 Acceptance: 41%
```

```
## A22 Acceptance: 40%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## **********
## Progress: 90%
## A11 Acceptance: 41%
## A22 Acceptance: 40%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## *********
## Progress: 95%
## A11 Acceptance: 41%
## A22 Acceptance: 40%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## **********
## Progress: 100%
## A11 Acceptance: 41%
## A22 Acceptance: 41%
## phi0 Acceptance: 29%
## phi1 Acceptance: 29%
## Original: S
## **********
ds_pred_results<-
ppd_s(modeling_output = ds_results,
     n_pred = length(y[CMAQ_key_test_set == 0]),
     m_pred = nrow(z[-test_set, ]),
     z_pred = z[-test_set,],
     sample_size_pred = sample_size[-test_set],
     spatial_dists_full = spatial_dists_full,
     neighbors_full = neighbors_full,
     inference_set = keep_set,
     params_only_indicator = 0,
     model_type_indicator = 1)
## Progress: 5%
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## Progress: 10%
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## Progress: 15%
## ********
## Progress: 20%
## *******
## Progress: 25%
## ********
## Progress: 30%
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## Progress: 35%
## ********
## Progress: 40%
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```

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## Progress: 45%
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## Progress: 50%
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## Progress: 55%
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## Progress: 60%
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## Progress: 70%
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## Progress: 80%
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## Progress: 85%
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## Progress: 90%
## *******
## Progress: 95%
## ********
## Progress: 100%
## *******
#Results Matrix
results <- matrix (0,
               nrow = 3,
               ncol = 4)
colnames(results)<-c("dlfuse", "slr", "ok", "ds")</pre>
rownames(results)<-c("mse", "cover", "length")</pre>
#DLfuse
cover<-rep(0,
          times=length(y[CMAQ_key_test_set == 0]))
len < -rep(0,
        times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){
  ci<-quantile(dlfuse_pred_results[[1]][j,], c(0.025, 0.975))</pre>
  if((ci[1] \le y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j]))
    cover[j]<-1
    }
  len[j] < -ci[2] -
          ci[1]
  }
results[1,1] <-mean((y[CMAQ_key_test_set == 0] -
                  rowMedians(dlfuse_pred_results[[1]]))^2)
results[2,1] <-mean(cover)
results[3,1] <-mean(len)
#SLR
cover<-rep(0,
```

```
times=length(y[CMAQ_key_test_set == 0]))
len < -rep(0,
           times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){
   ci<-quantile(slr_pred_results[[1]][j,], c(0.025, 0.975))
   if((ci[1] \le y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j]))
     cover[j]<-1
     }
   len[j]<-ci[2] -
           ci[1]
   }
results[1,2] <-mean((y[CMAQ_key_test_set == 0] -
                   rowMedians(slr_pred_results[[1]]))^2)
results[2,2]<-mean(cover)
results[3,2] <-mean(len)
#OK
cover<-rep(0,
           times=length(y[CMAQ_key_test_set == 0]))
len < -rep(0,
           times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){
   ci<-quantile(ok_pred_results[[1]][j,], c(0.025, 0.975))</pre>
   if((ci[1] <= y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j])){
     cover[j]<-1
   len[j] < -ci[2] -
           ci[1]
results[1,3] <-mean((y[CMAQ_key_test_set == 0] -
                   rowMedians(ok_pred_results[[1]]))^2)
results[2,3]<-mean(cover)
results[3,3] <-mean(len)
#DS
cover<-rep(0,</pre>
           times=length(y[CMAQ_key_test_set == 0]))
len < -rep(0,
           times=length(y[CMAQ_key_test_set == 0]))
for(j in 1:length(y[CMAQ_key_test_set == 0])){
   ci<-quantile(ds_pred_results[[1]][j,], c(0.025, 0.975))</pre>
   if((ci[1] \le y[CMAQ_key_test_set == 0][j]) & (ci[2] >= y[CMAQ_key_test_set == 0][j]))
     cover[j]<-1
     }
   len[j] < -ci[2] -
           ci[1]
   }
```