DLfuse: Distributed Lag Data Fusion for Estimating Ambient Air Pollution

DLfuseST Statistical Model

$$Y_t(\mathbf{s}_{ij}) = \widetilde{\boldsymbol{\beta}}_{0t}(\mathbf{s}_{ij}) + \widetilde{\boldsymbol{\beta}}_{1t}(\mathbf{s}_{ij}) \sum_{l=0}^{L} \bar{\mathrm{x}}_{B_i,t,l} \left(\frac{\pi_{B_i,t,l}}{\sum_{k=0}^{L} \pi_{B_i,t,k}} \right) + \epsilon_t(\mathbf{s}_{ij}),\ \epsilon_t(\mathbf{s}_{ij}) | \sigma_\epsilon^2 \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\epsilon^2\right)$$

$$\pi_{B_i,t,l} = \Phi\left(\mu + \alpha_{B_i} + \mu_t\right)^l,\ 0 = 1, ..., L;$$

$$\alpha_{B_i} | \boldsymbol{\alpha}_{-B_i}, \tau^2 \overset{\text{ind}}{\sim} \mathrm{N}\left(\frac{\sum_{j=1}^{m} w_{ij} \alpha_{B_j}}{\sum_{j=1}^{m} w_{ij}}, \frac{\tau^2}{\sum_{j=1}^{m} w_{ij}} \right),\ i = 1, ..., m;$$

$$\mu_t = \kappa \mu_{t-1} + \delta_t,\ \delta_t | \sigma_\delta^2 \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\delta^2\right),\quad t = 1, ..., d;$$

$$\widetilde{\beta}_{kt}(\mathbf{s}_{ij}) = \beta_k + \beta_k(\mathbf{s}_{ij}) + \beta_{kt},\ k = 0, 1;$$

$$\begin{pmatrix} \beta_0(\mathbf{s}_{ij}) \\ \beta_1(\mathbf{s}_{ij}) \end{pmatrix} = A \begin{pmatrix} w_0(\mathbf{s}_{ij}) \\ w_1(\mathbf{s}_{ij}) \end{pmatrix};\ A = \begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix};$$

$$\begin{pmatrix} \beta_{0t} \\ \beta_{1t} \end{pmatrix} = \Omega \begin{pmatrix} \beta_{0,t-1} \\ \beta_{1,t-1} \end{pmatrix} + \boldsymbol{\eta}_t,\ \Omega_{ii} = \rho_i,\ \boldsymbol{\eta}_t | V \overset{\text{iid}}{\sim} \mathrm{MVN}\left(\mathbf{0}_2, V\right),\ t = 1, ..., d$$

$$\boldsymbol{w}_k = \{w_k(\mathbf{s}_{11}), ..., w_k(\mathbf{s}_{mn_m})\}^{\mathrm{T}} | \phi_k \overset{\text{ind}}{\sim} \mathrm{MVN}\left\{0, \Sigma_k(\phi_k)\right\},\ k = 0, 1$$

$$\mathrm{Corr}\left\{w_k(\mathbf{s}_{ij}), w_k(\mathbf{s}_{i'j'})\right\} = \exp\left\{-\phi_k ||\mathbf{s}_{ij} - \mathbf{s}_{i'j'}||\right\};$$

- $i = 1, ..., m$;
- $j = 1, ..., n_i$

Prior Information

$\beta_k \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\beta^2\right),\ k = 0, 1;$

- Default setting: $\sigma_\beta^2 = 10,000.$

$\sigma_\epsilon^2 \sim$ Inverse Gamma $\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right)$;

- Default setting: $a_{\sigma_\theta^2} = 0.01$, $b_{\sigma_\theta^2} = 0.01$.

$\tau^2 \sim$ Inverse Gamma $\left(a_{\tau^2}, b_{\tau^2}\right)$;

- Default setting: $a_{\sigma_\theta^2} = 3$, $b_{\sigma_\theta^2} = 2$.

$\sigma_\delta^2 \sim$ Inverse Gamma $\left(a_{\sigma_\delta^2}, b_{\sigma_\delta^2}\right)$;

- Default setting: $a_{\sigma_\delta^2} = 3$, $b_{\sigma_\delta^2} = 2$.

$\ln(A_{11})$, $\ln(A_{22})$, $A_{21} \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_A^2\right)$;

- Default setting: $\sigma_A^2 = 1$.

$\mu \sim \mathrm{N}(0, 1)$;

$\phi_k \overset{\text{iid}}{\sim} \text{Gamma}(\alpha_{\phi_k}, \beta_{\phi_k}),\ k = 0, 1,$

- Default setting: $a_{\phi_k} = 1$, $b_{\phi_k} = 1$.

$\kappa, \rho_1, \rho_2 \overset{\text{iid}}{\sim} \text{Uniform}(0, 1)$;

$V^{-1} \sim \text{Wishart}(\Omega^*, \rho^*)$;

- Default setting: $\Omega^* = I_2$, $\rho^* = 3$.

Default Initial Values

- $\beta_k = 0$ for all k;
- $\sigma_\epsilon^2 = 1$;
- $A_{11} = A_{22} = 1,\ A_{21} = 0$;
- $\mu = 0$;
- $\alpha_{B_i} = 0$ for all i;
- $\tau^2 = 1$;
- $w_k(\mathbf{s}_{ij}) = 0$ for all k, i, j;
- $\phi_k = -\ln(0.05) / \max\{||\mathbf{s}_{ij} - \mathbf{s}_{i'j'}||\}$ for all k;
- V a two-by-two identity matrix;
- $\kappa = \rho_1 = \rho_2 = 0.50$;
- $\sigma_\delta^2 = 1$;
- $\beta_{kt} = 0$ for all k, t;
- $\mu_t = 0$ for all t.