DLfuse: Distributed Lag Data Fusion for Estimating Ambient Air Pollution

DLfuse Statistical Model

$$Y\left(\mathbf{s}_{ij}\right)=\widetilde{\boldsymbol{\beta}}_0\left(\mathbf{s}_{ij}\right)+\widetilde{\boldsymbol{\beta}}_1\left(\mathbf{s}_{ij}\right)\sum_{l=0}^{L}\bar{\mathrm{x}}_{B_i,l}\left(\frac{\pi_{B_i,l}}{\sum_{k=0}^{L}\pi_{B_i,k}}\right)+\epsilon\left(\mathbf{s}_{ij}\right),\ \epsilon\left(\mathbf{s}_{ij}\right)|\sigma_\epsilon^2\overset{\text{iid}}{\sim}\mathrm{N}\left(0,\sigma_\epsilon^2\right);$$

Probit Weights:

$$\pi_{B_i,l}=\Phi\left(\mu+\alpha_{B_i}\right)^l,\ l=0,...,L;$$

Spherical Weights:

$$\pi_{B_i,l}=\left\{1.00-1.50\left(\frac{l}{\exp\left\{\mu+\alpha_{B_i}\right\}}\right)+0.50\left(\frac{l}{\exp\left\{\mu+\alpha_{B_i}\right\}}\right)^3\right\}1\left(l<\exp\left\{\mu+\alpha_{B_i}\right\}\right),\ l=0,...,L;$$

$$\alpha_{B_i}|\boldsymbol{\alpha}_{-B_i},\tau^2\overset{\text{ind}}{\sim}\mathrm{N}\left(\frac{\sum_{j=1}^{m}w_{ij}\alpha_{B_j}}{\sum_{j=1}^{m}w_{ij}},\frac{\tau^2}{\sum_{j=1}^{m}w_{ij}}\right),\ i=1,...,m;$$

$$\widetilde{\beta}_k\left(\mathbf{s}_{ij}\right)=\beta_k+\beta_k\left(\mathbf{s}_{ij}\right),\ k=0,1;$$

$$\begin{pmatrix}\beta_0\left(\mathbf{s}_{ij}\right)\\ \beta_1\left(\mathbf{s}_{ij}\right)\end{pmatrix}=A\begin{pmatrix}w_0\left(\mathbf{s}_{ij}\right)\\ w_1\left(\mathbf{s}_{ij}\right)\end{pmatrix};\ A=\begin{pmatrix}A_{11}&0\\ A_{21}&A_{22}\end{pmatrix};$$

$$\boldsymbol{w}_k=\left\{w_k\left(\mathbf{s}_{11}\right),...,w_k\left(\mathbf{s}_{mn_m}\right)\right\}^{\mathrm{T}}|\phi_k\overset{\text{ind}}{\sim}\mathrm{MVN}\left\{0,\Sigma_k\left(\phi_k\right)\right\},\ k=0,1;$$

$$\mathrm{Corr}\left\{w_k\left(\mathbf{s}_{ij}\right),w_k\left(\mathbf{s}_{i'j'}\right)\right\}=\exp\left\{-\phi_k||\mathbf{s}_{ij}-\mathbf{s}_{i'j'}||\right\};$$

- $i=1,....,m;$
- $j=1,...,n_i$

Prior Information

$\beta_k\overset{\text{iid}}{\sim}\mathrm{N}\left(0,\sigma_\beta^2\right),\ k=0,1;$

- Default setting: $\sigma_\beta^2=10,000.$

$\sigma_\epsilon^2\sim\text{Inverse Gamma}\left(a_{\sigma_\epsilon^2},b_{\sigma_\epsilon^2}\right);$

- Default setting: $a_{\sigma_\theta^2}=0.01,\ b_{\sigma_\theta^2}=0.01.$

$\tau^2\sim\text{Inverse Gamma}\left(a_{\tau^2},b_{\tau^2}\right);$

- Default setting: $a_{\sigma_\theta^2}=3,\ b_{\sigma_\theta^2}=2.$

$\ln\left(A_{11}\right),\ \ln\left(A_{22}\right),\ A_{21}\overset{\text{iid}}{\sim}\mathrm{N}\left(0,\sigma_A^2\right);$

- Default setting: $\sigma_A^2=1.$

$\mu\sim\mathrm{N}\left(0,1\right);$

$\phi_k\overset{\text{iid}}{\sim}\text{Gamma}\left(\alpha_{\phi_k},\beta_{\phi_k}\right),\ k=0,1,$

- Default setting: $a_{\phi_k}=1,\ b_{\phi_k}=1.$

Default Initial Values

- $\beta_k = 0$ for all k;
- $\sigma_\epsilon^2 = 1$;
- $A_{11} = A_{22} = 1,\ A_{21} = 0$;
- $\mu = 0$;
- $\alpha_{B_i} = 0$ for all i;
- $\tau^2 = 1$;
- $w_k(\mathbf{s}_{ij}) = 0$ for all k, i, j;
- $\phi_k = -\ln(0.05) / \max\{||\mathbf{s}_{ij} - \mathbf{s}_{i'j'}||\}$ for all k.