

# EpiBuffer: Modeling Spatial Heterogeneity in Exposure Buffers and Risk

## SingleBuffer Statistical Model

$$Y_i(\mathbf{s}_j) | \mu_i(\mathbf{s}_j), \zeta \stackrel{\text{ind}}{\sim} f(y | \mu_i(\mathbf{s}_j), \zeta), \quad j = 1, \dots, m, \quad i = 1, \dots, n_j;$$

$$g\{\mu_i(\mathbf{s}_j)\} = O_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta.$$

## Likelihood Options

- Binomial likelihood with logit link function (likelihood\_indicator = 0):

$$Y_i(\mathbf{s}_j) | p_i(\mathbf{s}_j) \stackrel{\text{ind}}{\sim} \text{Binomial}(\tilde{n}_i(\mathbf{s}_j), p_i(\mathbf{s}_j)), \quad \text{logit}\{p_i(\mathbf{s}_j)\} = \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta;$$

- Gaussian likelihood with identity link function (likelihood\_indicator = 1):

$$Y_i(\mathbf{s}_j) = \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta + \epsilon_i, \quad \epsilon_i | \sigma_\epsilon^2 \stackrel{\text{iid}}{\sim} N(0, \sigma_\epsilon^2);$$

- Negative binomial likelihood with logit link function (likelihood\_indicator = 2):

$$Y_i(\mathbf{s}_j) | r, p_i(\mathbf{s}_j) \stackrel{\text{ind}}{\sim} \text{Negative Binomial}(r, p_i(\mathbf{s}_j)), \quad \text{logit}\{p_i(\mathbf{s}_j)\} = O_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta.$$

## Exposure Definitions

- Counts (exposure\_definition\_indicator = 0):

$$z(\mathbf{s}_j; \delta) = \sum_{k=1}^h 1(d_{jk} \leq \delta);$$

- Spherical (exposure\_definition\_indicator = 1):

$$z(\mathbf{s}_j; \delta) = \sum_{k=1}^h 1(d_{jk} \leq \delta) \left( 1 - 1.5 \left\{ \frac{d_{jk}}{\delta} \right\} + 0.5 \left\{ \frac{d_{jk}}{\delta} \right\}^3 \right);$$

- Presence/absence (exposure\_definition\_indicator = 2):

$$z(\mathbf{s}_j; \delta) = \min \left( \sum_{k=1}^h 1(d_{jk} \leq \delta), 1 \right);$$

- $d_{jk}$ : Distance between location  $\mathbf{s}_j$  and point source  $\mathbf{c}_k$ ;
- $h$ : Total number of point sources.

## Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} N(\mathbf{0}, \sigma_\beta^2), \quad j = 1, \dots, p_x;$$

- $p_x$ : Length of  $\mathbf{x}_i$  vector (same for all  $i$ ), including an intercept;
- Default setting:  $\sigma_\beta^2 = 100^2$ ;

$$\delta \sim \text{Uniform}(a_\delta, b_\delta);$$

$$\eta \sim N(0, \sigma_\eta^2);$$

- Default setting:  $\sigma_\eta^2 = 100^2$ ;

Gaussian specific:  $\sigma_\epsilon^2 \sim \text{Inverse Gamma}(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2})$ ;

- Default setting:  $a_{\sigma_\epsilon^2} = b_{\sigma_\epsilon^2} = 0.01$ ;

Negative binomial specific:  $r \sim \text{Discrete Uniform}[a_r, b_r]$ ;

- Default setting:  $a_r = 1, b_r = 100$ .

## Default Initial Values

- $\beta_j = 0$  for all  $j$ ;
- $\delta = \frac{b_\delta - a_\delta}{2}$ ;
- $\eta = 0$ ;
- Gaussian specific:  $\sigma_\epsilon^2 = \text{variance}(\mathbf{Y})$ ;
- Negative binomial specific:  $r = 100$ .

## Additional Information

- `waic_info_indicator = 1`: Provides output needed to calculate Watanabe-Akaike information criterion;
- `fitted_info_indicator = 1`: Provides posterior samples of fitted values.