

EpiBuffer: Modeling Spatial Heterogeneity in Exposure Buffers and Risk

FixedBuffer Statistical Model

$$Y_i(\mathbf{s}_j) | \mu_i(\mathbf{s}_j), \zeta \stackrel{\text{ind}}{\sim} f(y | \mu_i(\mathbf{s}_j), \zeta), \quad j = 1, \dots, m, \quad i = 1, \dots, n_j;$$

$$g\{\mu_i(\mathbf{s}_j)\} = O_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta.$$

* δ is selected by the user and treated as fixed.

Likelihood Options

- Binomial likelihood with logit link function (likelihood_indicator = 0):

$$Y_i(\mathbf{s}_j) | p_i(\mathbf{s}_j) \stackrel{\text{ind}}{\sim} \text{Binomial}(\tilde{n}_i(\mathbf{s}_j), p_i(\mathbf{s}_j)), \quad \text{logit}\{p_i(\mathbf{s}_j)\} = \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta;$$

- Gaussian likelihood with identity link function (likelihood_indicator = 1):

$$Y_i(\mathbf{s}_j) = \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta + \epsilon_i, \quad \epsilon_i | \sigma_\epsilon^2 \stackrel{\text{iid}}{\sim} N(0, \sigma_\epsilon^2);$$

- Negative binomial likelihood with logit link function (likelihood_indicator = 2):

$$Y_i(\mathbf{s}_j) | r, p_i(\mathbf{s}_j) \stackrel{\text{ind}}{\sim} \text{Negative Binomial}(r, p_i(\mathbf{s}_j)), \quad \text{logit}\{p_i(\mathbf{s}_j)\} = O_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta) \eta.$$

Exposure Definitions

- Counts (exposure_definition_indicator = 0):

$$z(\mathbf{s}_j; \delta) = \sum_{k=1}^h 1(d_{jk} \leq \delta);$$

- Spherical (exposure_definition_indicator = 1):

$$z(\mathbf{s}_j; \delta) = \sum_{k=1}^h 1(d_{jk} \leq \delta) \left(1 - 1.5 \left\{ \frac{d_{jk}}{\delta} \right\} + 0.5 \left\{ \frac{d_{jk}}{\delta} \right\}^3 \right);$$

- Presence/absence (exposure_definition_indicator = 2):

$$z(\mathbf{s}_j; \delta) = \min \left(\sum_{k=1}^h 1(d_{jk} \leq \delta), 1 \right);$$

- d_{jk} : Distance between location \mathbf{s}_j and point source \mathbf{c}_k ;
- h : Total number of point sources.

Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} N(\mathbf{0}, \sigma_\beta^2), \quad j = 1, \dots, p_x;$$

- p_x : Length of \mathbf{x}_i vector (same for all i), including an intercept;
- Default setting: $\sigma_\beta^2 = 100^2$;

$$\eta \sim N(0, \sigma_\eta^2);$$

- Default setting: $\sigma_\eta^2 = 100^2$;

Gaussian specific: $\sigma_\epsilon^2 \sim \text{Inverse Gamma}(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2})$;

- Default setting: $a_{\sigma_\epsilon^2} = b_{\sigma_\epsilon^2} = 0.01$;

Negative binomial specific: $r \sim \text{Discrete Uniform}[a_r, b_r]$;

- Default setting: $a_r = 1, b_r = 100$.

Default Initial Values

- $\beta_j = 0$ for all j ;
- $\eta = 0$;
- Gaussian specific: $\sigma_\epsilon^2 = \text{variance}(\mathbf{Y})$;
- Negative binomial specific: $r = 100$.

Additional Information

- `waic_info_indicator = 1`: Provides output needed to calculate Watanabe-Akaike information criterion;
- `fitted_info_indicator = 1`: Provides posterior samples of fitted values.

Interpretation

Within the function, exposure is scaled by the factor `exposure_scale`. As a result, the fitted θ and η values returned by the function depend on the exposure scaling factor used. The function returns the value used for scaling (`exposure_scale`), allowing the user to unscale as desired: $\theta_{unscaled} = \theta_{scaled}/\text{exposure_scale}$; $\eta_{unscaled} = \eta_{scaled}/\text{exposure_scale}$.