EpiBuffer: Modeling Spatial Heterogeneity in Exposure Buffers and Risk

SingleBuffer Statistical Model

$$Y_i\left(\mathbf{s}_j\right)|\mu_i\left(\mathbf{s}_j\right), \boldsymbol{\zeta} \overset{\text{ind}}{\sim} f\left(y \mid \mu_i\left(\mathbf{s}_j\right), \boldsymbol{\zeta}\right),\ j = 1, ..., m,\ i = 1, ..., n_j;$$
$$g\{\mu_i(\mathbf{s}_j)\} = \mathrm{O}_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}(\mathbf{s}_j;\delta)\eta.$$

Likelihood Options

- Binomial likelihood with logit link function (likelihood_indicator = 0):

$$Y_i(\mathbf{s}_j)|p_i(\mathbf{s}_j) \overset{\text{ind}}{\sim} \text{Binomial}\left(\tilde{n}_i(\mathbf{s}_j), p_i(\mathbf{s}_j)\right), \text{logit}\left\{p_i(\mathbf{s}_j)\right\} = \mathbf{x}_i(\mathbf{s}_j)^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}\left(\mathbf{s}_j;\delta\right)\eta;$$

- Gaussian likelihood with identity link function (likelihood_indicator = 1):

$$Y_i(\mathbf{s}_j) = \mathbf{x}_i(\mathbf{s}_j)^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}\left(\mathbf{s}_j;\delta\right)\eta + \epsilon_i,\ \ \epsilon_i|\sigma^2_\epsilon \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2_\epsilon\right);$$

- Negative binomial likelihood with logit link function (likelihood_indicator = 2):

$$Y_i(\mathbf{s}_j)|r, p_i(\mathbf{s}_j) \overset{\text{ind}}{\sim} \text{Negative Binomial}\left(r, p_i(\mathbf{s}_j)\right), \text{logit}\left\{p_i(\mathbf{s}_j)\right\} = \mathrm{O}_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}\left(\mathbf{s}_j;\delta\right)\eta.$$

Exposure Definitions

- Counts (exposure_definition_indicator = 0):

$$\mathrm{z}\left(\mathbf{s}_j;\delta\right) = \sum_{k=1}^{h} 1(\mathrm{d}_{jk} \le \delta);$$

- Spherical (exposure_definition_indicator = 1):

$$\mathrm{z}\left(\mathbf{s}_j;\delta\right) = \sum_{k=1}^{h} 1(\mathrm{d}_{jk} \le \delta)\left(1 - 1.5\left\{\frac{\mathrm{d}_{jk}}{\delta}\right\} + 0.5\left\{\frac{\mathrm{d}_{jk}}{\delta}\right\}^3\right);$$

- Presence/absence (exposure_definition_indicator = 2):

$$\mathrm{z}\left(\mathbf{s}_j;\delta\right) = \min\left(\sum_{k=1}^{h} 1\left(d_{jk} \le \delta\right), 1\right);$$

  - d_{jk}: Distance between location $\mathbf{s}_j$ and point source $\mathbf{c}_k$;
  - h: Total number of point sources.

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \mathrm{N}\left(\mathbf{0}, \sigma^2_\beta\right),\ j = 1, \dots, p_x;$

- p_x: Length of $\mathbf{x}_i$ vector (same for all i), including an intercept;
- Default setting: $\sigma^2_\beta = 100^2$;

$\delta \sim \text{Uniform}\left(a_\delta, b_\delta\right);$

$\eta \sim \mathrm{N}\left(0, \sigma^2_\eta\right);$

- Default setting: $\sigma_\eta^2 = 100^2$;

Gaussian specific: $\sigma_\epsilon^2 \sim$ Inverse Gamma $\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right)$;

- Default setting: $a_{\sigma_\epsilon^2} = b_{\sigma_\epsilon^2} = 0.01$;

Negative binomial specific: $r \sim$ Discrete Uniform $[a_r, b_r]$;

- Default setting: $a_r = 1, b_r = 100$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\delta = \frac{b_\delta - a_\delta}{2}$;
- $\eta = 0$;
- Gaussian specific: $\sigma_\epsilon^2 = \text{variance}(\mathbf{Y})$;
- Negative binomial specific: $r = 100$.

Additional Information

- waic_info_indicator = 1: Provides output needed to calculate Watanabe-Akaike information criterion;
- fitted_info_indicator = 1: Provides posterior samples of fitted values.

Interpretation

Within the function, exposure is scaled by the factor `exposure_scale`. As a result, the fitted θ and η values returned by the function depend on the exposure scaling factor used. The function returns the value used for scaling (`exposure_scale`), allowing the user to unscale as desired: $\theta_{unscaled} = \theta_{scaled}/\text{exposure\_scale}$; $\eta_{unscaled} = \eta_{scaled}/\text{exposure\_scale}$.