

EpiBuffer: Spatially-Varying Buffer Radii Model for Estimating Spatial Heterogeneity in Exposure Buffers and Risk

SpatialBuffers Statistical Model

$$Y_i(\mathbf{s}_j) | \mu_i(\mathbf{s}_j), \zeta \stackrel{\text{ind}}{\sim} f(y | \mu_i(\mathbf{s}_j), \zeta), \quad j = 1, \dots, m, \quad i = 1, \dots, n_j;$$

$$g\{\mu_i(\mathbf{s}_j)\} = O_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta(\mathbf{s}_j)) \theta\{\delta(\mathbf{s}_j)\}.$$

Likelihood Options

- Gaussian likelihood with identity link function:

$$Y_i(\mathbf{s}_j) = \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta(\mathbf{s}_j)) \theta\{\delta(\mathbf{s}_j)\} + \epsilon_i, \quad \epsilon_i | \sigma_\epsilon^2 \stackrel{\text{iid}}{\sim} N(0, \sigma_\epsilon^2);$$

- Binomial likelihood with logit link function:

$$Y_i(\mathbf{s}_j) | p_i(\mathbf{s}_j) \stackrel{\text{ind}}{\sim} \text{Binomial}(\tilde{n}_i(\mathbf{s}_j), p_i(\mathbf{s}_j)), \quad \text{logit}\{p_i(\mathbf{s}_j)\} = \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta(\mathbf{s}_j)) \theta\{\delta(\mathbf{s}_j)\};$$

- Negative binomial likelihood with logit link function:

$$Y_i(\mathbf{s}_j) | r, p_i(\mathbf{s}_j) \stackrel{\text{ind}}{\sim} \text{Negative Binomial}(r, p_i(\mathbf{s}_j)), \quad \text{logit}\{p_i(\mathbf{s}_j)\} = O_i(\mathbf{s}_j) + \mathbf{x}_i(\mathbf{s}_j)^T \boldsymbol{\beta} + z(\mathbf{s}_j; \delta(\mathbf{s}_j)) \theta\{\delta(\mathbf{s}_j)\}.$$

Exposure Definitions

- Counts:

$$z(\mathbf{s}_j; \delta(\mathbf{s}_j)) = \sum_{k=1}^h 1(d_{jk} \leq \delta(\mathbf{s}_j));$$

- Spherical:

$$z(\mathbf{s}_j; \delta(\mathbf{s}_j)) = \sum_{k=1}^h 1(d_{jk} \leq \delta(\mathbf{s}_j)) \left(1 - 1.5 \left\{ \frac{d_{jk}}{\delta(\mathbf{s}_j)} \right\} + 0.5 \left\{ \frac{d_{jk}}{\delta(\mathbf{s}_j)} \right\}^3 \right);$$

- Presence/absence:

$$z(\mathbf{s}_j; \delta(\mathbf{s}_j)) = \min \left(\sum_{k=1}^h 1(d_{jk} \leq \delta(\mathbf{s}_j)), 1 \right);$$

- d_{jk} : Distance between location \mathbf{s}_j and point source \mathbf{c}_k ;
- h : Total number of point sources.

Spatially-varying Radii and Exposure Effect Parameters

- Radii:

$$\Phi^{-1} \left(\frac{\delta(\mathbf{s}_j) - a}{b - a} \right) = \mathbf{w}(\mathbf{s}_j)^T \boldsymbol{\gamma} + \phi(\mathbf{s}_j);$$

- $\boldsymbol{\phi}^T = (\phi(\mathbf{s}_1), \dots, \phi(\mathbf{s}_m)) | \rho_\phi \sim \text{MVN}(\mathbf{0}_m, \Sigma(\rho_\phi)), \Sigma_{ij}(\rho_\phi) = \exp\{-\rho_\phi \|\mathbf{s}_i - \mathbf{s}_j\|\};$
- Gaussian predictive process option also available for large m ;

- Effects:

$$\theta\{\delta(\mathbf{s}_j)\} = \sum_{l=0}^p \left\{ \frac{\delta_j(\mathbf{s}_j) - a}{b - a} \right\}^l \eta_l.$$

Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} \text{N}(\mathbf{0}, \sigma_\beta^2), \quad j = 1, \dots, p_x;$$

- p_x : Length of \mathbf{x}_i vector (same for all i), including an intercept;
- Default setting: $\sigma_\beta^2 = 100^2$;

$$\gamma_j \stackrel{\text{iid}}{\sim} \text{N}(\mathbf{0}, 1), \quad j = 1, \dots, p_w;$$

- p_w : Length of $\mathbf{w}(\mathbf{s}_j)$ vector (same for all j), including an intercept;

$$\eta_j \sim \text{N}(0, \sigma_\eta^2), \quad j = 1, \dots, p;$$

- Default setting: $\sigma_\eta^2 = 100^2$;

$$\rho_\phi \sim \text{Gamma}(a_{\rho_\phi}, b_{\rho_\phi});$$

- Default setting: $a_{\rho_\phi} = b_{\rho_\phi} = 1$;

$$\text{Gaussian specific: } \sigma_\epsilon^2 \sim \text{Inverse Gamma}(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2});$$

- Default setting: $a_{\sigma_\epsilon^2} = b_{\sigma_\epsilon^2} = 0.01$;

$$\text{Negative binomial specific: } r \sim \text{Discrete Uniform}[a_r, b_r];$$

- Default setting: $a_r = 1, b_r = 100$.

Default Initial Values

- $\beta_j = 0$ for all j ;
- $\gamma_j = 0$ for all j ;
- $\eta_j = 0$ for all j ;
- $\rho_\phi = -\ln(0.05) / \max\{\|\mathbf{s}_i - \mathbf{s}_j\|\}$;
- Gaussian specific: $\sigma_\epsilon^2 = \text{variance}(\mathbf{Y})$;
- Negative binomial specific: $r = 100$.