

# GPCW: Gaussian Process Model for Critical Window Estimation

## Statistical Model

$$Y_i | \beta, \theta \stackrel{\text{ind}}{\sim} \text{Bernoulli} \{p_i(\beta, \theta)\}, \quad i = 1, \dots, n;$$

$$\log \left\{ \frac{p_i(\beta, \theta)}{1 - p_i(\beta, \theta)} \right\} = \mathbf{x}_i^T \beta + \sum_{j=1}^{m_i} z_{ij} \theta(j);$$

$$\theta = \{\theta(1), \dots, \theta(m)\}^T | \sigma_\theta^2, \phi \sim \text{MVN} \{ \mathbf{0}_m, \sigma_\theta^2 \Sigma(\phi) \};$$

$$\Sigma(\phi)_{ij} = \exp \{ -\phi |i - j| \}, \quad \phi > 0.$$

- $m = \max \{m_i : i = 1, \dots, n\};$
- $\mathbf{0}_m$ : Length  $m$  vector with each entry equal to zero.

## Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} \text{N} \left( 0, \sigma_\beta^2 \right), \quad j = 1, \dots, p;$$

- $p$ : Length of  $\mathbf{x}_i$  vector (same for all  $i$ );
- Default setting:  $\sigma_\beta^2 = 10,000$ .

$$\sigma_\theta^2 \sim \text{Inverse Gamma} \left( a_{\sigma_\theta^2}, b_{\sigma_\theta^2} \right);$$

- Default setting:  $a_{\sigma_\theta^2} = 3, b_{\sigma_\theta^2} = 2$ .

$$\phi \sim \text{Uniform} (a_\phi, b_\phi);$$

- Default setting:  $a_\phi = \log(0.9999) / \{-(m-1)\}, b_\phi = \log(0.0001) / (-1)$ .

## Default Initial Values

- $\beta_j = 0$  for all  $j$ ;
- $\theta_j = 0$  for all  $j$ ;
- $\sigma_\theta^2 = 1.00$ ;
- $\phi = 0.01 (b_\phi - a_\phi)$ .

## Alternate Likelihood

$$Y_i | \beta, \theta, \sigma_\epsilon^2 \stackrel{\text{ind}}{\sim} \text{Normal} \left( \mathbf{x}_i^T \beta + \sum_{j=1}^{m_i} z_{ij} \theta(j), \sigma_\epsilon^2 \right), \quad i = 1, \dots, n$$

- $\sigma_\epsilon^2 \sim \text{Inverse Gamma} (a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2});$
- Default setting:  $a_{\sigma_\epsilon^2} = 0.01, b_{\sigma_\epsilon^2} = 0.01$ ;
- Default initial value:  $\sigma_\epsilon^2 = 1.00$ ;
- likelihood\_indicator = 0: Bernoulli;
- likelihood\_indicator = 1: Gaussian.