GPCW: Gaussian Process Model for Critical Window Estimation

Statistical Model

$$Y_i|\boldsymbol{\beta}, \boldsymbol{\theta} \overset{\text{ind}}{\sim} \text{Bernoulli}\left\{p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)\right\},\ i=1,...,n;$$

$$\log\left\{\frac{p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)}{1-p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)}\right\} = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\theta\left(j\right);$$

$$\boldsymbol{\theta} = \left\{\theta\left(1\right), ..., \theta\left(m\right)\right\}^{\text{T}}|\sigma^2_{\theta}, \phi \sim \text{MVN}\left\{\mathbf{0}_m, \sigma^2_{\theta}\Sigma\left(\phi\right)\right\};$$

$$\Sigma\left(\phi\right)_{ij} = \exp\left\{-\phi|i-j|\right\},\ \phi > 0.$$

- $m = \max\left\{m_i : i=1,...,n\right\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma^2_{\beta}\right),\ j=1,...,p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma^2_{\beta} = 10,000$.

$\sigma^2_{\theta} \sim \text{Inverse Gamma}\left(a_{\sigma^2_{\theta}}, b_{\sigma^2_{\theta}}\right);$

- Default setting: $a_{\sigma^2_{\theta}} = 3$, $b_{\sigma^2_{\theta}} = 2$.

$\phi \sim \text{Uniform}\left(a_{\phi}, b_{\phi}\right);$

- Default setting: $a_{\phi} = \log(0.9999)/\left\{-(m-1)\right\}$, $b_{\phi} = \log(0.0001)/(-1)$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma^2_{\theta} = 1.00$;
- $\phi = 0.01\left(b_{\phi} - a_{\phi}\right)$.

Alternate Likelihood

$Y_i|\boldsymbol{\beta}, \boldsymbol{\theta}, \sigma^2_{\epsilon} \overset{\text{ind}}{\sim} \text{Normal}\left(\mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\theta\left(j\right), \sigma^2_{\epsilon}\right),\ i=1,...,n.$

- $\sigma^2_{\epsilon} \sim \text{Inverse Gamma}\left(a_{\sigma^2_{\epsilon}}, b_{\sigma^2_{\epsilon}}\right);$
- Default setting: $a_{\sigma^2_{\epsilon}} = 0.01$, $b_{\sigma^2_{\epsilon}} = 0.01$;
- Default initial value: $\sigma^2_{\epsilon} = 1.00$;
- likelihood_indicator $= 0$: Bernoulli;
- likelihood_indicator $= 1$: Gaussian.