GPCW: Gaussian Process Model for Critical Window Estimation

Statistical Model

$$Y_i | \boldsymbol{\beta}, \boldsymbol{\theta} \overset{\text{ind}}{\sim} \text{Bernoulli} \left\{ p_i \left(\boldsymbol{\beta}, \boldsymbol{\theta} \right) \right\}, \ i = 1, ..., n;$$

$$\log \left\{ \frac{p_i \left(\boldsymbol{\beta}, \boldsymbol{\theta} \right)}{1 - p_i \left(\boldsymbol{\beta}, \boldsymbol{\theta} \right)} \right\} = \mathbf{x}_i^{\text{T}} \boldsymbol{\beta} + \sum_{j=1}^{m_i} \mathbf{z}_{ij} \theta \left(j \right);$$

$$\boldsymbol{\theta} = \left\{ \theta \left(1 \right), ..., \theta \left(m \right) \right\}^{\text{T}} | \sigma_\theta^2, \phi \sim \text{MVN} \left\{ \mathbf{0}_m, \sigma_\theta^2 \Sigma \left(\phi \right) \right\};$$

$$\Sigma \left(\phi \right)_{ij} = \exp \left\{ -\phi | i - j | \right\}, \ \phi > 0.$$

- $m = \max \left\{ m_i : i = 1, ..., n \right\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N} \left(0, \sigma_\beta^2 \right), \ j = 1, ..., p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma_\beta^2 = 10,000.$

$\sigma_\theta^2 \sim \text{Inverse Gamma} \left(a_{\sigma_\theta^2}, b_{\sigma_\theta^2} \right);$

- Default setting: $a_{\sigma_\theta^2} = 3$, $b_{\sigma_\theta^2} = 2.$

$\phi \sim \text{Uniform} \left(a_\phi, b_\phi \right);$

- Default setting: $a_\phi = \log(0.9999) / \left\{ -(m-1) \right\}$, $b_\phi = \log(0.0001)/(-1).$

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma_\theta^2 = 1.00$;
- $\phi = 0.01 \left(b_\phi - a_\phi \right).$