GPCW: Gaussian Process Model for Critical Window Estimation

Statistical Model

$$y_i|\boldsymbol{\beta}, \boldsymbol{\theta} \stackrel{\text{ind}}{\sim} \text{Bernoulli}\left(p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)\right), \ i = 1, ..., n;$$

$$\log \left(\frac{p_i\left(\boldsymbol{\beta},\boldsymbol{\theta}\right)}{1 - p_i\left(\boldsymbol{\beta},\boldsymbol{\theta}\right)} \right) = \mathbf{x}_i^{\mathrm{T}} \boldsymbol{\beta} + \sum_{j=1}^{m_i} \mathbf{z}_{ij} \boldsymbol{\theta}\left(j\right);$$

$$\boldsymbol{\theta} = (\theta(1), ..., \theta(m))^{\mathrm{T}} | \sigma_{\theta}^{2}, \phi \sim \text{MVN} (\mathbf{0}_{m}, \sigma_{\theta}^{2} \Sigma(\phi));$$

$$\Sigma \left(\phi\right)_{ij} = \exp\left\{-\phi|i-j|\right\}, \ \phi > 0.$$

- $m = \max\{m_i : i = 1, ..., n\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} \mathcal{N}\left(0, \sigma_\beta^2\right), \ j = 1, ..., p;$$

- p: Length of \mathbf{x}_i vector (same for all i);
- Default setting: $\sigma_{\beta}^2 = 10,000$.

 $\sigma_{\theta}^2 \sim \text{Inverse Gamma}\left(a_{\sigma_{\theta}^2}, b_{\sigma_{\theta}^2}\right);$

• Default setting: $a_{\sigma_{\theta}^2} = 3$, $b_{\sigma_{\theta}^2} = 2$.

 $\phi \sim \text{Uniform}(a_{\phi}, b_{\phi});$

• Default setting: $a_{\phi} = \log(0.9999)/(-(m-1)), b_{\phi} = \log(0.0001)/(-1).$

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma_{\theta}^2 = 1.00;$
- $\phi = 0.01 (b_{\phi} a_{\phi}).$