

# GPCW: Gaussian Process for Estimating Critical Windows of Susceptibility

## Statistical Model

$$y_i | \beta, \theta \stackrel{\text{iid}}{\sim} \text{Bernoulli}(p_i(\beta, \theta)), \quad i = 1, \dots, n;$$

$$\log \left( \frac{p_i(\beta, \theta)}{1 - p_i(\beta, \theta)} \right) = \mathbf{x}_i^T \beta + \sum_{j=1}^{m_i} z_{ij} \theta(j);$$

$$\theta = (\theta(1), \dots, \theta(m))^T | \sigma_\theta^2, \phi \sim \text{MVN}(\mathbf{0}_m, \sigma_\theta^2 \Sigma(\phi));$$

$$\Sigma(\phi)_{ij} = \exp\{-\phi|i-j|\}, \quad \phi > 0.$$

- $m = \max\{m_i : i = 1, \dots, n\}$ ;
- $\mathbf{0}_m$ : Length  $m$  vector with each entry equal to zero.

## Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} \text{N}(0, \sigma_\beta^2), \quad j = 1, \dots, p;$$

- $p$ : Length of  $\mathbf{x}_i$  vector (same for all  $i$ );
- Default setting:  $\sigma_\beta^2 = 10,000$ .

$$\sigma_\theta^2 \sim \text{Inverse Gamma}(a_{\sigma_\theta^2}, b_{\sigma_\theta^2});$$

- Default setting:  $a_{\sigma_\theta^2} = 3, b_{\sigma_\theta^2} = 2$ .

$$\phi \sim \text{Uniform}(a_\phi, b_\phi);$$

- Default setting:  $a_\phi = \log(0.9999)/(-(m-1)), b_\phi = \log(0.0001)/(-1)$ .

## Default Initial Values

- $\beta_j = 0$  for all  $j$ ;
- $\theta_j = 0$  for all  $j$ ;
- $\sigma_\theta^2 = 1.00$ ;
- $\phi = 1.00$ .