GPCW: Gaussian Process Model for Critical Window Estimation

Statistical Model

$$Y_i|\boldsymbol{\beta}, \boldsymbol{\theta} \overset{\text{ind}}{\sim} \text{Bernoulli}\left\{p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)\right\},\ i = 1, ..., n;$$

$$\log\left\{\frac{p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)}{1 - p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)}\right\} = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\theta\left(j\right);$$

$$\boldsymbol{\theta} = \left\{\theta\left(1\right), ..., \theta\left(m\right)\right\}^{\text{T}}|\sigma^2_\theta, \phi \sim \text{MVN}\left\{\mathbf{0}_m, \sigma^2_\theta\Sigma\left(\phi\right)\right\};$$

$$\Sigma\left(\phi\right)_{ij} = \exp\left\{-\phi|i-j|\right\},\ \phi > 0.$$

- $m = \max\left\{m_i : i = 1, ..., n\right\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma^2_\beta\right),\ j = 1, ..., p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma^2_\beta = 10,000$.

$\sigma^2_\theta \sim \text{Inverse Gamma}\left(a_{\sigma^2_\theta}, b_{\sigma^2_\theta}\right);$

- Default setting: $a_{\sigma^2_\theta} = 3$, $b_{\sigma^2_\theta} = 2$.

$\phi \sim \text{Uniform}\left(a_\phi, b_\phi\right);$

- Default setting: $a_\phi = \log(0.9999)/\left\{-(m-1)\right\}$, $b_\phi = \log(0.0001)/(-1)$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma^2_\theta = 1.00$;
- $\phi = 0.01\left(b_\phi - a_\phi\right)$.

Alternate Likelihood: Gaussian

$Y_i|\boldsymbol{\beta}, \boldsymbol{\theta}, \sigma^2_\epsilon \overset{\text{ind}}{\sim} \text{Normal}\left(\mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\theta\left(j\right), \sigma^2_\epsilon\right),\ i = 1, ..., n.$

- $\sigma^2_\epsilon \sim \text{Inverse Gamma}\left(a_{\sigma^2_\epsilon}, b_{\sigma^2_\epsilon}\right);$
- Default setting: $a_{\sigma^2_\epsilon} = 0.01$, $b_{\sigma^2_\epsilon} = 0.01$;
- Default initial value: $\sigma^2_\epsilon = 1.00$;

Alternate Likelihood: Negative Binomial

$Y_i|\boldsymbol{\beta}, \boldsymbol{\theta}, r \overset{\text{ind}}{\sim} \text{Negative Binomial}\left\{r, \lambda_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)\right\},\ i = 1, ..., n;$

$\ln\left\{\frac{\lambda_i(\boldsymbol{\beta},\boldsymbol{\theta})}{1-\lambda_i(\boldsymbol{\beta},\boldsymbol{\theta})}\right\} = \text{O}_i + \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\theta\left(j\right).$

- $r \sim \text{Discrete Uniform}\left[a_r, b_r\right];$
- Default setting: $a_r = 1$, $b_r = 50$;
- Default initial value: $r = a_r$;

Likelihood Indicator

- likelihood_indicator = 0: Bernoulli;
- likelihood_indicator = 1: Gaussian;
- likelihood_indicator = 2: Negative binomial.