GPCW: Gaussian Process for Estimating Critical Windows of Susceptibility

Statistical Model

$$y_i|\boldsymbol{\beta},\boldsymbol{\theta} \overset{\text{iid}}{\sim} \text{Bernoulli}\left(p_i\left(\boldsymbol{\beta},\boldsymbol{\theta}\right)\right), \ i = 1,...,n;$$

$$\log\left(\frac{p_i\left(\boldsymbol{\beta},\boldsymbol{\theta}\right)}{1 - p_i\left(\boldsymbol{\beta},\boldsymbol{\theta}\right)}\right) = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} z_{ij}\theta\left(j\right);$$

$$\boldsymbol{\theta} = \left(\theta\left(1\right),...,\theta\left(m\right)\right)^{\text{T}}|\sigma_\theta^2,\phi \sim \text{MVN}\left(\mathbf{0}_m,\sigma_\theta^2\Sigma\left(\phi\right)\right).$$

- $m = \max\left\{m_i : i = 1,...,n\right\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0,\sigma_\beta^2\right), \ j = 1,...,p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma_\beta^2 = 10,000.$

$\sigma_\theta^2 \sim \text{Inverse Gamma}\left(\alpha,\beta\right);$

- Default setting: $\alpha = 3$, $\beta = 2.$

$\phi \sim \text{Uniform}\left(a_\phi,b_\phi\right);$

- Default setting: $a_\phi = \log(0.9999)/(-(m-1))$, $b_\phi = \log(0.0001)/(-1).$

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma_\theta^2 = 1.00;$
- $\phi = 1.00.$