GenePair: Statistical Methods for Modeling Spatially-Referenced Paired Genetic Relatedness Data

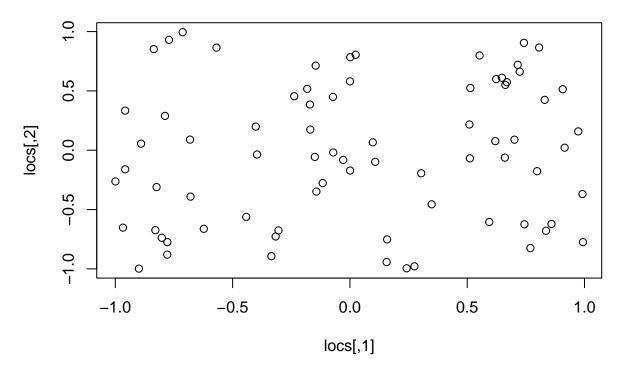
Trans_Prob_Example

- [1] Simulate data from the proposed model:
 - Setting the reproducibility seed and initializing packages for data simulation:

```
set.seed(2411)
library(GenePair)
library(mnormt) #Multivariate normal distribution
```

• Setting the global data values:

```
n<-75 #Number of individuals
n_star<-0 #Number of paired responses
for(j in 1:n){
  for(k in 1:n){
      if(j != k){
        n_star < -n_star +
                1
        }
      }
#Unique locations (one for each individual)
locs<-matrix(runif((2*n),</pre>
                   min = -1.00,
                   \max = 1.00),
             nrow = n,
             ncol = 2)
plot(locs)
```



```
v<-diag(n)</pre>
spatial_dists<-as.matrix(dist(locs,</pre>
                                  diag = TRUE,
                                  upper = TRUE))
#x_pair (intercept included)
x_pair<-matrix(1,</pre>
                 nrow = n_star,
                 ncol = 2)
x < -rnorm(n = n)
counter<-1
for(j in 1:n){
   for(k in 1:n){
       if(j != k){
         x_pair[counter, 2]<-abs(x[j] - x[k])</pre>
         counter<-counter +</pre>
         }
      }
x_pair[,2]<-scale(x_pair[,2])</pre>
\#x\_ind\_g, x\_ind\_r (no intercept)
```

```
x1 < -rnorm(n = n)
x2 < -rnorm(n = n)
x_ind_g<-
x_{ind_r<-matrix(0.00,
                  nrow = n_star,
                  ncol = 2)
counter<-1
for(j in 1:n){
   for(k in 1:n){
       if(j != k){
         x_{ind}g[counter, 1] < -x1[k]
         x_ind_g[counter, 2]<-x2[k]</pre>
         x_ind_r[counter, 1]<-x1[j]</pre>
         x_ind_r[counter, 2]<-x2[j]</pre>
         counter<-counter +</pre>
                    1
       }
x_ind_g<-scale(x_ind_g)</pre>
x_ind_r<-scale(x_ind_r)</pre>
\#z_g, z_r
z_g<-matrix(0,
              nrow = n_star,
              ncol = n)
z_r<-matrix(0,</pre>
              nrow = n_star,
              ncol = n
counter<-1
for(j in 1:n){
   for(k in 1:n){
       if(j != k){
         z_g[counter, k]<-1</pre>
         z_r[counter, j]<-1</pre>
         counter<-counter +</pre>
                   1
         }
       }
   }
```

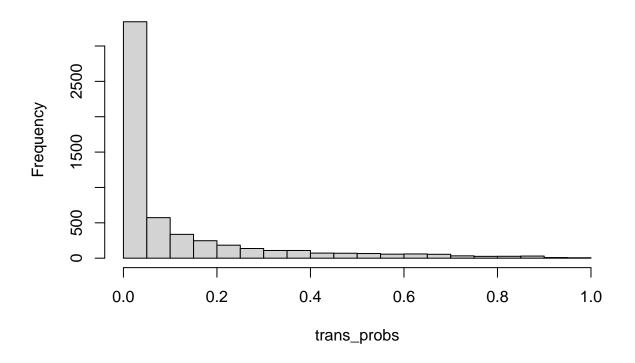
• Setting the values for the statistical model parameters:

```
beta_z_true<- c(1.00,
		 -0.30)
gamma_z_g_true<-c(2.10,
		 -1.24)
gamma_z_r_true<-c(1.05,
		 0.56)
sigma2_zeta_z_g_true<-0.01
```

```
sigma2_zeta_z_r_true<-0.01
sigma2 epsilon true<-0.01
beta_w_true<-c(-2.40,
               0.50)
gamma_w_g_true < -c(0.80,
                   -0.35)
gamma_w_r_true < -c(-1.10,
                   -0.75)
sigma2_zeta_w_g_true<-0.01
sigma2_zeta_w_r_true<-0.01
Sigma_true<-chol2inv(chol(rWishart(n = 1,</pre>
                                    df = 5,
                                    Sigma = diag(4))[,,1]))
phi_true<-5.00
Sigma_full_true<-Sigma_true%x%exp(-phi_true*spatial_dists)
eta_full_true<-rmnorm(n = 1,</pre>
                      mean = rep(0.00,
                                  times = (n*4)),
                      varcov = Sigma_full_true)
theta_z_g_true<-eta_full_true[1:n] +</pre>
                rnorm(n = n,
                      mean = 0.00,
                      sd = sqrt(sigma2_zeta_z_g_true))
theta_z_g_true<-theta_z_g_true -
                mean(theta_z_g_true)
theta_z_r_true\leftarroweta_full_true[(n + 1):(2*n)] +
                rnorm(n = n,
                      mean = 0.00,
                      sd = sqrt(sigma2_zeta_z_r_true))
theta_z_r_true<-theta_z_r_true -
                mean(theta_z_r_true)
theta_w_g_true<-eta_full_true[(2*n + 1):(3*n)] +
                rnorm(n = n,
                      mean = 0.00,
                      sd = sqrt(sigma2_zeta_w_g_true))
theta_w_g_true<-theta_w_g_true -
                mean(theta_w_g_true)
theta_w_r_true<-eta_full_true[(3*n + 1):(4*n)] +
                rnorm(n = n,
                      mean = 0.00,
                      sd = sqrt(sigma2_zeta_w_r_true))
theta_w_r_true<-theta_w_r_true -
                mean(theta_w_r_true)
mu_z<-x_pair%*%beta_z_true +</pre>
      x_ind_g%*%gamma_z_g_true +
      x_ind_r%*%gamma_z_r_true +
```

```
z_g%*%theta_z_g_true +
      z_r%*%theta_z_r_true
probs_z<-1.00/(1.00 + exp(-mu_z))
bin < -rbinom(n = n_star,
            size = 1,
            prob = probs_z)
mu_w<-x_pair%*%beta_w_true +</pre>
      x_ind_g%*%gamma_w_g_true +
      x_{ind_r}*%gamma_w_r_true +
      z_g%*%theta_w_g_true +
      z_r%*%theta_w_r_true
w < -rnorm(n = n_star,
         mean = mu_w,
         sd = sqrt(sigma2_epsilon_true))
w<-1.00/(1.00 + exp(-w))
trans_probs<-bin*w
hist(trans_probs)
```

Histogram of trans_probs



[2] Apply the Trans_Prob Function within the GenePair Package:

```
x_pair = x_pair,
                  x_{ind}g = x_{ind}g,
                  x_{ind}r = x_{ind}r,
                  z_g = z_g
                  z_r = z_r
                  spatial_dists = spatial_dists,
                  v = v,
                  metrop_var_phi_trans = 0.20)
## Progress: 10%
## phi Acceptance: 28%
## *********
## Progress: 20%
## phi Acceptance: 28%
## *********
## Progress: 30%
## phi Acceptance: 29%
## **********
## Progress: 40%
## phi Acceptance: 29%
## *********
## Progress: 50%
## phi Acceptance: 29%
## **********
## Progress: 60%
## phi Acceptance: 30%
## *********
## Progress: 70%
## phi Acceptance: 29%
## *********
## Progress: 80%
## phi Acceptance: 29%
## *********
## Progress: 90%
## phi Acceptance: 29%
## *********
## Progress: 100%
## phi Acceptance: 29%
## *********
[3] Analyzing Output:
#Compare random effects
par(mfrow = c(2,2))
plot(rowMeans(results$theta_z_g[,2001:10000]),
    theta_z_g_true)
abline(0,1)
plot(rowMeans(results$theta_z_r[,2001:10000]),
    theta_z_r_true)
abline(0,1)
plot(rowMeans(results$theta_w_g[,2001:10000]),
    theta_w_g_true)
abline(0,1)
```

```
plot(rowMeans(results$theta_w_r[,2001:10000]),
      theta_w_r_true)
abline(0,1)
theta_z_g_true
                                                     theta_z_r_true
                                                          0.5
      0
                                                          ι
      4
                                          3
                                     2
                                                                                0.0
                                                                                       0.5
                                                                        -0.5
                                                                                               1.0
       rowMeans(results$theta_z_g[, 2001:10000])
                                                            rowMeans(results$theta_z_r[, 2001:10000])
 theta_w_g_true
                                                     theta_w_r_true
      0.5
      -1.0
                 -0.5
                          0.0
                                   0.5
                                                                         -0.5
                                                                                   0.5
                                                                                            1.5
         -1.0
                                           1.0
                                                               -1.5
      rowMeans(results$theta_w_g[, 2001:10000])
                                                           rowMeans(results$theta_w_r[, 2001:10000])
#Compare \beta
round(rowMeans(results$beta_z[,2001:10000]), 2)
## [1] 0.86 -0.35
round(beta_z_true, 2)
## [1] 1.0 -0.3
round(rowMeans(results$beta_w[,2001:10000]), 2)
## [1] -2.39 0.50
round(beta_w_true, 2)
## [1] -2.4 0.5
\#Compare \setminus gamma\_z\_g
round(rowMeans(results$gamma_z_g[,2001:10000]), 2)
```

[1] 2.10 -1.24

[1] 1.92 -1.21

round(gamma_z_g_true, 2)

```
round(rowMeans(results$gamma_z_r[,2001:10000]), 2)
## [1] 0.96 0.59
round(gamma_z_r_true, 2)
## [1] 1.05 0.56
round(rowMeans(results$gamma_w_g[,2001:10000]), 2)
## [1] 0.71 -0.31
round(gamma_w_g_true, 2)
## [1] 0.80 -0.35
round(rowMeans(results$gamma_w_r[,2001:10000]), 2)
## [1] -1.09 -0.65
round(gamma_w_r_true, 2)
## [1] -1.10 -0.75
#Compare \sigma2_epsilon
round(mean(results$sigma2_epsilon[2001:10000]), 2)
## [1] 0.01
sigma2_epsilon_true
## [1] 0.01
#Compare \Sigma
Sigma<-array(0.00,
             dim = c(10000, 4, 4))
for(j in 1:10000){
   Sigma[j,,]<-results$Sigma[[j]]</pre>
round(colMeans(Sigma[2001:10000,,]), 2)
         [,1] [,2] [,3] [,4]
## [1,] 1.15 -0.16 0.28 -0.55
## [2,] -0.16 0.17 -0.01 0.02
## [3,] 0.28 -0.01 0.22 -0.18
## [4,] -0.55 0.02 -0.18 0.56
Sigma_true
              [,1]
##
                          [,2]
                                     [,3]
                                                 [,4]
## [1,] 3.4165470 -0.73126656 0.3379183 -0.95289423
## [2,] -0.7312666  0.75024506  0.0862960  0.05214396
## [3,] 0.3379183 0.08629600 0.2399986 -0.16795824
## [4,] -0.9528942 0.05214396 -0.1679582 0.51692972
```