GenePair: Statistical Methods for Modeling Spatially-Referenced Paired Genetic Relatedness Data

Transmission Probabilities Model

$$Y_{ij} \overset{\text{ind}}{\sim} f_{y_{ij}}\left(y\right) = \left(1 - \pi_{ij}\right)^{1(y=0)} \left[\pi_{ij} f_{w_{ij}} \left\{\ln\left(\frac{y}{1-y}\right)\right\}\right]^{1(y>0)},\ y \in [0,1),\ i=1,...,n,\ j=1,...,n,\ i \neq j$$

- $$\ln\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) = \mathbf{x}_{ij}^{\mathrm{T}} \boldsymbol{\beta}_z + \mathbf{d}_j^{\mathrm{T}} \boldsymbol{\gamma}_z^{(g)} + \mathbf{d}_i^{\mathrm{T}} \boldsymbol{\gamma}_z^{(r)} + \theta_{zj}^{(g)} + \theta_{zi}^{(r)}$$

- $$f_{w_{ij}}\left(w\right) \equiv \mathrm{N}\left(\mathbf{x}_{ij}^{\mathrm{T}} \boldsymbol{\beta}_w + \mathbf{d}_j^{\mathrm{T}} \boldsymbol{\gamma}_w^{(g)} + \mathbf{d}_i^{\mathrm{T}} \boldsymbol{\gamma}_w^{(r)} + \theta_{wj}^{(g)} + \theta_{wi}^{(r)},\ \sigma_\epsilon^2\right)$$

$$\begin{aligned}
\theta_{zi}^{(g)} &= \eta_z^{(g)}\left\{d\left(\mathbf{s}_i\right)\right\} + \zeta_{zi}^{(g)},\\
\theta_{zi}^{(r)} &= \eta_z^{(r)}\left\{d\left(\mathbf{s}_i\right)\right\} + \zeta_{zi}^{(r)},\\
\theta_{wi}^{(g)} &= \eta_w^{(g)}\left\{d\left(\mathbf{s}_i\right)\right\} + \zeta_{wi}^{(g)},\\
\theta_{wi}^{(r)} &= \eta_w^{(r)}\left\{d\left(\mathbf{s}_i\right)\right\} + \zeta_{wi}^{(r)},\ i=1,...,n,
\end{aligned}$$

$$\begin{aligned}
&\boldsymbol{\eta}|\Omega, \phi \sim \mathrm{MVN}\left\{\mathbf{0}_{4m}, \Sigma\left(\phi\right) \otimes \Omega\right\} \text{ where}\\
&\Sigma\left(\phi\right)_{ij} = \exp\left\{-\phi \left\|\mathbf{s}_i^* - \mathbf{s}_j^*\right\|\right\},\\
&\boldsymbol{\eta}^{\mathrm{T}} = \left\{\boldsymbol{\eta}\left(\mathbf{s}_1^*\right)^{\mathrm{T}}, \ldots, \boldsymbol{\eta}\left(\mathbf{s}_m^*\right)^{\mathrm{T}}\right\}, \text{ and}\\
&\boldsymbol{\eta}\left(\mathbf{s}_i^*\right)^{\mathrm{T}} = \left\{\eta_z^{(g)}\left(\mathbf{s}_i^*\right), \eta_z^{(r)}\left(\mathbf{s}_i^*\right), \eta_w^{(g)}\left(\mathbf{s}_i^*\right), \eta_w^{(r)}\left(\mathbf{s}_i^*\right)\right\}.
\end{aligned}$$

- $d\left(\mathbf{s}_i\right)$: Maps the spatial location of an individual to an entry within a smaller set of $m < n$ unique locations such that $d\left(\mathbf{s}_i\right) \in \{\mathbf{s}_1^*, \ldots, \mathbf{s}_m^*\}$;
- $\zeta_{zi}^{(g)}|\sigma^2_{\zeta_z^{(g)}} \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2_{\zeta_z^{(g)}}\right)$, $\zeta_{zi}^{(r)}|\sigma^2_{\zeta_z^{(r)}} \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2_{\zeta_z^{(r)}}\right)$, $\zeta_{wi}^{(g)}|\sigma^2_{\zeta_w^{(g)}} \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2_{\zeta_w^{(g)}}\right)$, $\zeta_{wi}^{(r)}|\sigma^2_{\zeta_w^{(r)}} \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2_{\zeta_w^{(r)}}\right)$;
- m: Number of unique spatial locations ($m \leq n$);
- n: Number of individuals.

Prior Information

$\beta_{zj}, \beta_{wj}, \gamma_{zk}^{(g)}, \gamma_{zk}^{(r)}, \gamma_{wk}^{(g)}, \gamma_{wk}^{(r)} \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_r^2\right),\ j=1,...,p_x,\ k=1,...,p_d;$

- p_x: Length of $\mathbf{x}_i$ vector (same for all i) which includes an intercept term;
- p_d: Length of $\mathbf{d}_j$ vector (same for all j) which **does not** include an intercept term;
- Default setting: $\sigma_r^2 = 10,000$.

$\sigma_\epsilon^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right);$

- Default setting: $a_{\sigma_\epsilon^2} = 0.01$, $b_{\sigma_\epsilon^2} = 0.01$.

$\sigma^2_{\zeta_z^{(g)}} \sim \text{Inverse Gamma}\left(a_{\sigma^2_{\zeta_z^{(g)}}}, b_{\sigma^2_{\zeta_z^{(g)}}}\right)$, $\sigma^2_{\zeta_z^{(r)}} \sim \text{Inverse Gamma}\left(a_{\sigma^2_{\zeta_z^{(r)}}}, b_{\sigma^2_{\zeta_z^{(r)}}}\right)$, $\sigma^2_{\zeta_w^{(g)}} \sim \text{Inverse Gamma}\left(a_{\sigma^2_{\zeta_w^{(g)}}}, b_{\sigma^2_{\zeta_w^{(g)}}}\right)$,
$\sigma^2_{\zeta_w^{(r)}} \sim \text{Inverse Gamma}\left(a_{\sigma^2_{\zeta_w^{(r)}}}, b_{\sigma^2_{\zeta_w^{(r)}}}\right);$

- Default setting: $a_{\sigma^2_{\zeta_z^{(g)}}} = a_{\sigma^2_{\zeta_z^{(r)}}} = a_{\sigma^2_{\zeta_w^{(g)}}} = a_{\sigma^2_{\zeta_w^{(r)}}} = 0.01$, $b_{\sigma^2_{\zeta_z^{(g)}}} = b_{\sigma^2_{\zeta_z^{(r)}}} = b_{\sigma^2_{\zeta_w^{(g)}}} = b_{\sigma^2_{\zeta_w^{(r)}}} = 0.01$.

$\Omega^{-1} \sim \text{Wishart}\left(\nu, \Omega^*\right);$

- Default setting: $\nu = 5$, $\Omega^* = I_4$.

$\phi \sim \text{Gamma}\left(a_\phi, b_\phi\right);$

- Default setting: $a_\phi = 1.00$, $b_\phi = 1.00$.

Default Initial Values

- $\beta_{zj}, \beta_{wj}, \gamma_{zk}^{(g)}, \gamma_{zk}^{(r)}, \gamma_{wk}^{(g)}, \gamma_{wk}^{(r)} = 0$ for all j, k;
- $\theta_{zi}^{(g)}, \theta_{zi}^{(r)}, \theta_{wi}^{(g)}, \theta_{wi}^{(r)} = 0$ for all i;
- $\eta_{zi}^{(g)}, \eta_{zi}^{(r)}, \eta_{wi}^{(g)}, \eta_{wi}^{(r)} = 0$ for all i;
- $\sigma_\epsilon^2 = \text{variance}\left(\boldsymbol{Y}\right)$;
- $\sigma^2_{\zeta_z^{(g)}}, \sigma^2_{\zeta_z^{(r)}}, \sigma^2_{\zeta_w^{(g)}}, \sigma^2_{\zeta_w^{(r)}} = 1.00$;
- $\Omega = I_4$;
- $\phi = 1.00$.