

GenePair: Statistical Methods for Modeling Spatially-Referenced Paired Genetic Relatedness Data

SNP Distances Model

$$Y_{ij}|r, p_{ij} \stackrel{\text{iid}}{\sim} \text{Negative Binomial}(r, p_{ij}), \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n$$

$$\text{logit}(p_{ij}) = \mathbf{x}_{ij}^T \boldsymbol{\beta} + (\mathbf{d}_i + \mathbf{d}_j)^T \boldsymbol{\gamma} + \theta_i + \theta_j$$

$$\theta_i = \eta\{d(\mathbf{s}_i)\} + \zeta_i, \quad i = 1, \dots, n,$$

$$\boldsymbol{\eta}^T = \{\eta(\mathbf{s}_1^*), \dots, \eta(\mathbf{s}_m^*)\} | \phi, \tau^2 \sim \text{MVN}\{\mathbf{0}_m, \tau^2 \Sigma(\phi)\}, \quad \text{and}$$

$$\Sigma(\phi)_{ij} = \text{Corr}\{\eta(\mathbf{s}_i^*), \eta(\mathbf{s}_j^*)\} = \exp\{-\phi \|\mathbf{s}_i^* - \mathbf{s}_j^*\|\}.$$

- $d(\mathbf{s}_i)$: Maps the spatial location of an individual to an entry within a smaller set of $m < n$ unique locations such that $d(\mathbf{s}_i) \in \{\mathbf{s}_1^*, \dots, \mathbf{s}_m^*\}$;
- $\zeta_i | \sigma_\zeta^2 \stackrel{\text{iid}}{\sim} N(0, \sigma_\zeta^2)$;
- m : Number of unique spatial locations ($m \leq n$);
- n : Number of individuals.

Prior Information

$$\beta_j, \gamma_k \stackrel{\text{iid}}{\sim} N(0, \sigma_r^2), \quad j = 1, \dots, p_x, \quad k = 1, \dots, p_d;$$

- p_x : Length of \mathbf{x}_i vector (same for all i) which includes an intercept term;
- p_d : Length of \mathbf{d}_j vector (same for all j) which **does not** include an intercept term;
- Default setting: $\sigma_r^2 = 10,000$.

$$r \sim \text{Discrete Uniform}[a_r, b_r];$$

- Default setting: $a_r = 1, b_r = 100$.

$$\sigma_\zeta^2 \sim \text{Inverse Gamma}(a_{\sigma_\zeta^2}, b_{\sigma_\zeta^2});$$

- Default setting: $a_{\sigma_\zeta^2} = 0.01, b_{\sigma_\zeta^2} = 0.01$.

$$\tau^2 \sim \text{Inverse Gamma}(a_{\tau^2}, b_{\tau^2});$$

- Default setting: $a_{\tau^2} = 0.01, b_{\tau^2} = 0.01$.

$$\phi \sim \text{Gamma}(a_\phi, b_\phi);$$

- Default setting: $a_\phi = 1.00, b_\phi = 1.00$.

Default Initial Values

- $\beta_j, \gamma_k = 0$ for all j, k ;
- $\theta_i = 0$ for all i ;
- $\eta_i = 0$ for all i ;
- $r = a_r$;
- $\sigma_\zeta^2 = 1.00$;
- $\tau^2 = 1.00$;
- $\phi = 1.00$.