GenePair: Statistical Methods for Modeling Spatially-Referenced Paired Genetic Relatedness Data

Patristic Distances Model

$$\ln(Y_{ij}) = \mathbf{x}_{ij}^{\mathrm{T}}\boldsymbol{\beta} + (\mathbf{d}_i + \mathbf{d}_j)^{\mathrm{T}}\boldsymbol{\gamma} + \theta_i + \theta_j + \epsilon_{ij},\ i = 1, ..., n-1,\ j = i+1, ..., n$$

$$\begin{aligned}
&\theta_i = \eta\{d(\mathbf{s}_i)\} + \zeta_i,\ i = 1, \ldots, n, \\
&\boldsymbol{\eta}^{\mathrm{T}} = \{\eta(\mathbf{s}_1^*), \ldots, \eta(\mathbf{s}_m^*)\} \,|\phi, \tau^2 \sim \mathrm{MVN}\left\{\mathbf{0}_m, \tau^2\Sigma(\phi)\right\}, \text{ and} \\
&\Sigma(\phi)_{ij} = \mathrm{Corr}\left\{\eta(\mathbf{s}_i^*), \eta\left(\mathbf{s}_j^*\right)\right\} = \exp\left\{-\phi\left\|\mathbf{s}_i^* - \mathbf{s}_j^*\right\|\right\}.
\end{aligned}$$

- $\epsilon_{ij}|\sigma_\epsilon^2 \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\epsilon^2\right)$;
- $d(\mathbf{s}_i)$: Maps the spatial location of an individual to an entry within a smaller set of $m < n$ unique locations such that $d(\mathbf{s}_i) \in \{\mathbf{s}_1^*, \ldots, \mathbf{s}_m^*\}$;
- $\zeta_i|\sigma_\zeta^2 \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\zeta^2\right)$;
- m: Number of unique spatial locations ($m \leq n$);
- n: Number of individuals;

Prior Information

$\beta_j, \gamma_k \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_r^2\right),\ j = 1, ..., p_x,\ k = 1, ..., p_d$;

- p_x: Length of $\mathbf{x}_i$ vector (same for all i) which includes an intercept term;
- p_d: Length of $\mathbf{d}_j$ vector (same for all j) which **does not** include an intercept term;
- Default setting: $\sigma_r^2 = 10,000$.

$\sigma_\epsilon^2 \sim$ Inverse Gamma $\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right)$;

- Default setting: $a_{\sigma_\epsilon^2} = 0.01$, $b_{\sigma_\epsilon^2} = 0.01$.

$\sigma_\zeta^2 \sim$ Inverse Gamma $\left(a_{\sigma_\zeta^2}, b_{\sigma_\zeta^2}\right)$;

- Default setting: $a_{\sigma_\zeta^2} = 0.01$, $b_{\sigma_\zeta^2} = 0.01$.

$\tau^2 \sim$ Inverse Gamma $\left(a_{\tau^2}, b_{\tau^2}\right)$;

- Default setting: $a_{\tau^2} = 0.01$, $b_{\tau^2} = 0.01$.

$\phi \sim$ Gamma $\left(a_\phi, b_\phi\right)$;

- Default setting: $a_\phi = 1.00$, $b_\phi = 1.00$.

Default Initial Values

- $\beta_j, \gamma_k = 0$ for all j, k;
- $\theta_i = 0$ for all i;
- $\eta_i = 0$ for all i;
- $\sigma_\epsilon^2 = \mathrm{variance}(\boldsymbol{Y})$;
- $\sigma_\zeta^2 = 1.00$;
- $\tau^2 = 1.00$;
- $\phi = 1.00$.