GenePair: Statistical Methods for Modeling Spatially-Referenced Paired Genetic Relatedness Data

Patristic Distances Model

$$\ln\left(Y_{ij}\right) = \mathbf{x}_{ij}^{\mathrm{T}}\boldsymbol{\beta} + \left(\mathbf{d}_i + \mathbf{d}_j\right)^{\mathrm{T}}\boldsymbol{\gamma} + \theta_i + \theta_j + \epsilon_{ij},\ i = 1, ..., n-1,\ j = i+1, ..., n$$

$$\theta_i = \eta\left\{d\left(\mathbf{s}_i\right)\right\} + \zeta_i,\ i = 1, \ldots, n,$$
$$\boldsymbol{\eta}^{\mathrm{T}} = \left\{\eta\left(\mathbf{s}_1^*\right), \ldots, \eta\left(\mathbf{s}_m^*\right)\right\}|\phi, \tau^2 \sim \mathrm{MVN}\left\{\mathbf{0}_m, \tau^2\Sigma\left(\phi\right)\right\},\ \text{and}$$
$$\Sigma\left(\phi\right)_{ij} = \mathrm{Corr}\left\{\eta\left(\mathbf{s}_i^*\right), \eta\left(\mathbf{s}_j^*\right)\right\} = \exp\left\{-\phi\left\|\mathbf{s}_i^* - \mathbf{s}_j^*\right\|\right\}.$$

- $\epsilon_{ij}|\sigma_\epsilon^2 \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\epsilon^2\right)$;
- $d\left(\mathbf{s}_i\right)$: Maps the spatial location of an individual to an entry within a smaller set of $m < n$ unique locations such that $d\left(\mathbf{s}_i\right) \in \left\{\mathbf{s}_1^*, \ldots, \mathbf{s}_m^*\right\}$;
- $\zeta_i|\sigma_\zeta^2 \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_\zeta^2\right)$;
- m: Number of unique spatial locations ($m \leq n$);
- n: Number of individuals.

Prior Information

$\beta_j, \gamma_k \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_r^2\right),\ j = 1, ..., p_x,\ k = 1, ..., p_d$;

- p_x: Length of $\mathbf{x}_i$ vector (same for all i);
- p_d: Length of $\mathbf{d}_j$ vector (same for all j);
- Default setting: $\sigma_r^2 = 10,000$.

$\sigma_\epsilon^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right)$;

- Default setting: $a_{\sigma_\epsilon^2} = 0.01$, $b_{\sigma_\epsilon^2} = 0.01$.

$\sigma_\zeta^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\zeta^2}, b_{\sigma_\zeta^2}\right)$;

- Default setting: $a_{\sigma_\zeta^2} = 0.01$, $b_{\sigma_\zeta^2} = 0.01$.

$\tau^2 \sim \text{Inverse Gamma}\left(a_{\tau^2}, b_{\tau^2}\right)$;

- Default setting: $a_{\tau^2} = 0.01$, $b_{\tau^2} = 0.01$.

$\phi \sim \text{Gamma}\left(a_\phi, b_\phi\right)$;

- Default setting: $a_\phi = 1.00$, $b_\phi = 1.00$.

Default Initial Values

- $\beta_j, \gamma_k = 0$ for all j, k;
- $\theta_i = 0$ for all i;
- $\eta_i = 0$ for all i;
- $\sigma_\epsilon^2 = \text{variance}\left(\boldsymbol{Y}\right)$;
- $\sigma_\zeta^2 = 1.00$;
- $\tau^2 = 1.00$;
- $\phi = 1.00$.