KDExp: Kernel Density Estimation Prior Distribution for Exposure Uncertainty Propagation

Statistical Model

$$Y_i|\mu_i, \boldsymbol{\zeta} \overset{\text{ind}}{\sim} f\left(y|\mu_i, \boldsymbol{\zeta}\right), \ i = 1, \ldots, n,$$
$$g\left(\mu_i\right) = \mathrm{O}_i + \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}_i\theta$$

Likelihood Options

- Gaussian likelihood with identity link function: $Y_i|\boldsymbol{\beta}, \theta, \sigma_\epsilon^2 \overset{\text{ind}}{\sim} \mathrm{N}\left(\mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}_i\theta, \sigma_\epsilon^2\right)$
- Bernoulli likelihood with logit link function: $Y_i|\boldsymbol{\beta}, \theta \overset{\text{ind}}{\sim} \text{Bernoulli}\left(p_i\right); \ \text{logit}\left(p_i\right) = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}_i\theta$
- Negative binomial likelihood with logit link function: $Y_i|\boldsymbol{\beta}, \theta \overset{\text{ind}}{\sim} \text{Negative Binomial}\left(r, p_i\right); \ \text{logit}\left(p_i\right) = \mathrm{O}_i + \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \mathrm{z}_i\theta$

Kernel Density Estimation Prior Distributions

- UKDE:

$$f\left(\mathrm{z}_i\right) = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{\sqrt{2\pi h_i^2}}\exp\left\{-\frac{1}{2h_i^2}\left(\mathrm{z}_i - \mathrm{z}_{ij}^*\right)^2\right\}, \ i = 1, ..., n$$

- MKDE:

$$f\left(\mathbf{z}\right) = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{\sqrt{\left(2\pi\right)^n\left|H\right|}}\exp\left\{-\frac{1}{2}\left(\mathbf{z} - \mathbf{z}_{\cdot j}^*\right)^{\mathrm{T}}H^{-1}\left(\mathbf{z} - \mathbf{z}_{\cdot j}^*\right)\right\}$$

Prior Information

$\beta_j, \theta \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma_r^2\right), \ j = 1, ..., p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma_r^2 = 10,000$.

$\sigma_\epsilon^2 \sim$ Inverse Gamma $\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right)$;

- Default setting: $a_{\sigma_\theta^2} = 0.01$, $b_{\sigma_\theta^2} = 0.01$.

$r \sim$ Discrete Uniform $\left[a_r, b_r\right]$;

- Default setting: $a_r = 1$, $b_r = 100$.

Default Initial Values

- $\beta_j = \theta = 0$ for all j;
- $\sigma_\epsilon^2 = \text{variance}\left(\boldsymbol{Y}\right)$;
- $r = 100$.