

KDExp: Kernel Density Estimation Prior Distribution for Exposure Uncertainty Propagation

Statistical Model

$$Y_i | \mu_i, \zeta \stackrel{\text{ind}}{\sim} f(y | \mu_i, \zeta), \quad i = 1, \dots, n,$$

$$g(\mu_i) = O_i + \mathbf{x}_i^T \boldsymbol{\beta} + z_i \theta$$

Likelihood Options

- Gaussian likelihood with identity link function: $Y_i | \boldsymbol{\beta}, \theta, \sigma_\epsilon^2 \stackrel{\text{ind}}{\sim} N(\mathbf{x}_i^T \boldsymbol{\beta} + z_i \theta, \sigma_\epsilon^2)$
- Bernoulli likelihood with logit link function: $Y_i | \boldsymbol{\beta}, \theta \stackrel{\text{ind}}{\sim} \text{Bernoulli}(p_i)$; $\text{logit}(p_i) = \mathbf{x}_i^T \boldsymbol{\beta} + z_i \theta$
- Negative binomial likelihood with logit link function: $Y_i | \boldsymbol{\beta}, \theta \stackrel{\text{ind}}{\sim} \text{Negative Binomial}(r, p_i)$; $\text{logit}(p_i) = O_i + \mathbf{x}_i^T \boldsymbol{\beta} + z_i \theta$

Kernel Density Estimation Prior Distributions

- UKDE:

$$f(z_i) = \frac{1}{m} \sum_{j=1}^m \frac{1}{\sqrt{2\pi h_i^2}} \exp \left\{ -\frac{1}{2h_i^2} (z_i - z_{ij}^*)^2 \right\}, \quad i = 1, \dots, n$$

- MKDE:

$$f(\mathbf{z}) = \frac{1}{m} \sum_{j=1}^m \frac{1}{\sqrt{(2\pi)^n |H|}} \exp \left\{ -\frac{1}{2} (\mathbf{z} - \mathbf{z}_{\cdot j}^*)^T H^{-1} (\mathbf{z} - \mathbf{z}_{\cdot j}^*) \right\}$$

Prior Information

$$\beta_j, \theta \stackrel{\text{iid}}{\sim} N(0, \sigma_r^2), \quad j = 1, \dots, p;$$

- p : Length of \mathbf{x}_i vector (same for all i);
- Default setting: $\sigma_r^2 = 10,000$.

$$\sigma_\epsilon^2 \sim \text{Inverse Gamma}(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2});$$

- Default setting: $a_{\sigma_\epsilon^2} = 0.01, b_{\sigma_\epsilon^2} = 0.01$.

$$r \sim \text{Discrete Uniform}[a_r, b_r];$$

- Default setting: $a_r = 1, b_r = 100$.

Default Initial Values

- $\beta_j = \theta = 0$ for all j ;
- $\sigma_\epsilon^2 = \text{variance}(\mathbf{Y})$;
- $r = 100$.

Likelihood Indicator

- `likelihood_indicator = 0`: Bernoulli;
- `likelihood_indicator = 1`: Gaussian;
- `likelihood_indicator = 2`: Negative binomial.