KSBound: Kernel Stick-Breaking Prior Distribution for Spatial Boundary Detection

Statistical Model

$$Y_i|\mathrm{E}_i, \lambda_i \overset{\text{ind}}{\sim} \text{Poisson}\left(\mathrm{E}_i\lambda_i\right),\ \ln\left(\lambda_i\right) = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \eta_i,\ i = 1, ..., n;$$

$$\eta_i|\boldsymbol{p}_i, \boldsymbol{\theta} \overset{\text{ind}}{\sim} G_i,\ G_i\left(.\right) \overset{d}{=} \sum_{j=1}^{\infty} p_{ij}\delta_{\theta_j}\left(.\right),\ i = 1, ..., n;$$

$$p_{i1} = w_{i1}V_1,\ p_{ij} = w_{ij}V_j \prod_{k=1}^{j-1}\left(1 - w_{ik}V_k\right) \text{ for } j \geq 2;$$

- $V_k|\alpha \overset{\text{iid}}{\sim} \text{Beta}\left(1, \alpha\right);$
- $\theta_j|\sigma^2_\theta \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma^2_\theta\right);$

$$w_{ij} = 1\left(R_i \in \partial_{R_{\psi_j}}\right),\ \partial_{R_{\psi_j}} = \left\{R_{\psi_j}\right\} \cup \left\{R_k : R_k \text{ and } R_{\psi_j} \text{ are neighbors}\right\},$$
$$\psi_j \overset{\text{iid}}{\sim} \text{Discrete Uniform}\left\{1, n\right\}.$$

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma^2_\beta\right),\ j = 1, ..., p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma^2_\beta = 10,000$.

$\sigma^2_\theta \sim \text{Inverse Gamma}\left(a_{\sigma^2_\theta}, b_{\sigma^2_\theta}\right);$

- Default setting: $a_{\sigma^2_\theta} = 0.01$, $b_{\sigma^2_\theta} = 0.01$.

$\alpha \sim \text{Gamma}\left(a_\alpha, b_\alpha\right);$

- Default setting: $a_\alpha = 0.01$, $b_\alpha = 0.01$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma^2_\theta = 1.00$;
- $\alpha = 1.00$;
- $V_j = 0.99$ for all j;
- $\psi_j = j$ for all j.

Alternate Likelihood: Binomial

$Y_i|\boldsymbol{\beta}, \eta_i, r \overset{\text{ind}}{\sim} \text{Binomial}\left\{c_i, p_i\left(\boldsymbol{\beta}, \eta_i\right)\right\},\ i = 1, ..., n;$

$\ln\left\{\frac{p_i(\boldsymbol{\beta}, \eta_i)}{1 - p_i(\boldsymbol{\beta}, \eta_i)}\right\} = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} + \eta_i.$

Alternate Likelihood: Gaussian

$Y_i|\boldsymbol{\beta}, \eta_i, \sigma_\epsilon^2 \overset{\text{ind}}{\sim} \text{Normal}\left(\mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \eta_i, \sigma_\epsilon^2\right),\ i = 1, ..., n.$

- $\sigma_\epsilon^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2}\right);$
- Default setting: $a_{\sigma_\epsilon^2} = 0.01$, $b_{\sigma_\epsilon^2} = 0.01$;
- Default initial value: $\sigma_\epsilon^2 = \text{variance}\left(\boldsymbol{Y}\right)$.

Alternate Likelihood: Negative Binomial

$Y_i|\boldsymbol{\beta}, \eta_i, r \overset{\text{ind}}{\sim} \text{Negative Binomial}\left\{r, \lambda_i\left(\boldsymbol{\beta}, \eta_i\right)\right\},\ i = 1, ..., n;$

$\ln\left\{\frac{\lambda_i(\boldsymbol{\beta}, \eta_i)}{1-\lambda_i(\boldsymbol{\beta}, \eta_i)}\right\} = \ln\left(\text{E}_i\right) + \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \eta_i.$

- $r \sim \text{Discrete Uniform}\left[a_r, b_r\right];$
- Default setting: $a_r = 1$, $b_r = 100$;
- Default initial value: $r = b_r$.

Likelihood Indicator

- likelihood_indicator = 0: Poisson;
- likelihood_indicator = 1: Binomial;
- likelihood_indicator = 2: Gaussian;
- likelihood_indicator = 3: Negative binomial.