KSBound: Kernel Stick-Breaking Prior Distribution for Spatial Boundary Detection

Statistical Model

$$Y_i | E_i, \lambda_i \overset{\text{ind}}{\sim} \text{Poisson}\left(E_i \lambda_i\right), \ \ln\left(\lambda_i\right) = \mathbf{x}_i^{\text{T}} \boldsymbol{\beta} + \eta_i, \ i = 1, ..., n;$$

$$\eta_i | \boldsymbol{p}_i, \boldsymbol{\theta} \overset{\text{ind}}{\sim} G_i, \ G_i\left(.\right) \overset{d}{=} \sum_{j=1}^{\infty} p_{ij} \delta_{\theta_j}\left(.\right), \ i = 1, ..., n;$$

$$p_{i1} = w_{i1} V_1, \ p_{ij} = w_{ij} V_j \prod_{k=1}^{j-1} \left(1 - w_{ik} V_k\right) \text{ for } j \geq 2;$$

- $V_k | \alpha \overset{\text{iid}}{\sim} \text{Beta}\left(1, \alpha\right)$;
- $\theta_j | \sigma_\theta^2 \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\theta^2\right)$;

$$w_{ij} = 1\left(R_i \in \partial_{R_{\psi_j}}\right), \ \partial_{R_{\psi_j}} = \left\{R_{\psi_j}\right\} \cup \left\{R_k : R_k \text{ and } R_{\psi_j} \text{ are neighbors}\right\},$$

$$\psi_j \overset{\text{iid}}{\sim} \text{Discrete Uniform}\left\{1, n\right\}.$$

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\beta^2\right), \ j = 1, ..., p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma_\beta^2 = 10,000$.

$\sigma_\theta^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\theta^2}, b_{\sigma_\theta^2}\right);$

- Default setting: $a_{\sigma_\theta^2} = 0.01, \ b_{\sigma_\theta^2} = 0.01$.

$\alpha \sim \text{Gamma}\left(a_\alpha, b_\alpha\right);$

- Default setting: $a_\alpha = 0.01, \ b_\alpha = 0.01$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma_\theta^2 = 1.00$;
- $\alpha = 1.00$;
- $V_j = 0.99$ for all j;
- $\psi_j = j$ for all j.