

SpGPCW: Spatially Varying Gaussian Process Model for Critical Window Estimation

Statistical Model

$$Y_{ij}|\boldsymbol{\beta}, \boldsymbol{\theta}_i \stackrel{\text{ind}}{\sim} \text{Bernoulli} \{p_{ij}(\boldsymbol{\beta}, \boldsymbol{\theta}_i)\}, \quad i = 1, \dots, s; \quad j = 1, \dots, n_i;$$

$$\log \left\{ \frac{p_{ij}(\boldsymbol{\beta}, \boldsymbol{\theta}_i)}{1 - p_{ij}(\boldsymbol{\beta}, \boldsymbol{\theta}_i)} \right\} = \mathbf{x}_{ij}^T \boldsymbol{\beta} + \sum_{k=1}^{m_{ij}} z_{ijk} \theta_i(k);$$

$$\boldsymbol{\theta} = \left(\boldsymbol{\theta}_1^T, \dots, \boldsymbol{\theta}_s^T \right)^T; \quad \boldsymbol{\theta}_i = \{\theta_i(1), \dots, \theta_i(m)\}^T;$$

$$\boldsymbol{\theta}_i | \boldsymbol{\theta}_{-i}, \rho, \phi, \sigma_\theta^2, \boldsymbol{\eta}, \stackrel{\text{ind}}{\sim} \text{MVN} \left\{ \frac{\rho \sum_{j=1}^s w_{ij} \boldsymbol{\theta}_j + (1 - \rho) \boldsymbol{\eta}}{\rho \sum_{j=1}^s w_{ij} + (1 - \rho)}, \frac{\sigma_\theta^2 \Sigma(\phi)}{\rho \sum_{j=1}^s w_{ij} + (1 - \rho)} \right\}, \quad i = 1, \dots, s;$$

$$\boldsymbol{\eta} = \{\eta(1), \dots, \eta(m)\}^T | \sigma_\eta^2, \phi \sim \text{MVN} \{ \mathbf{0}_m, \sigma_\eta^2 \Sigma(\phi) \};$$

$$\Sigma(\phi)_{ij} = \exp \{ -\phi |i - j| \}, \quad \phi > 0;$$

- $m = \max \{m_{ij} : i = 1, \dots, s; \quad j = 1, \dots, n_i\};$
- s : Number of unique spatial locations;
- n_i : Number of observations with spatial location i ;
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$$\beta_j \stackrel{\text{iid}}{\sim} \text{N} \left(0, \sigma_\beta^2 \right), \quad j = 1, \dots, p;$$

- p : Length of \mathbf{x}_{ij} vector (same for all i, j);
- Default setting: $\sigma_\beta^2 = 10,000$.

$$\sigma_\theta^2 \sim \text{Inverse Gamma} \left(a_{\sigma_\theta^2}, b_{\sigma_\theta^2} \right);$$

- Default setting: $a_{\sigma_\theta^2} = 3, b_{\sigma_\theta^2} = 2$.

$$\sigma_\eta^2 \sim \text{Inverse Gamma} \left(a_{\sigma_\eta^2}, b_{\sigma_\eta^2} \right);$$

- Default setting: $a_{\sigma_\eta^2} = 3, b_{\sigma_\eta^2} = 2$.

$$\rho \sim \text{Uniform} (a_\phi, b_\phi);$$

- Default setting: $a_\phi = 0, b_\phi = 1$.

$$\phi \sim \text{Uniform} (a_\phi, b_\phi);$$

- Default setting: $a_\phi = \log(0.9999)/(-(m-1)), b_\phi = \log(0.0001)/(-1)$.

Default Initial Values

- $\beta_j = 0$ for all j ;
- $\theta_j = 0$ for all j ;
- $\eta_j = 0$ for all j ;
- $\sigma_\theta^2 = 1.00$;
- $\sigma_\eta^2 = 1.00$;
- $\rho = 0.50 * (b_\phi - a_\phi)$;
- $\phi = 0.01 (b_\phi - a_\phi)$.

Alternate Likelihood: Gaussian

$Y_{ij} | \beta, \theta_i, \sigma_\epsilon^2 \stackrel{\text{ind}}{\sim} \text{Normal}(\mathbf{x}_{ij}^T \beta + \sum_{k=1}^{m_{ij}} z_{ijk} \theta_i(k), \sigma_\epsilon^2)$, $i = 1, \dots, s$; $j = 1, \dots, n_i$;

- $\sigma_\epsilon^2 \sim \text{Inverse Gamma}(a_{\sigma_\epsilon^2}, b_{\sigma_\epsilon^2})$;
- Default setting: $a_{\sigma_\epsilon^2} = 0.01$, $b_{\sigma_\epsilon^2} = 0.01$;
- Default initial value: $\sigma_\epsilon^2 = 1.00$;

Alternate Likelihood: Negative Binomial

$Y_{ij} | \beta, \theta_i, r \stackrel{\text{ind}}{\sim} \text{Negative Binomial}\{r, \lambda_{ij}(\beta, \theta_i)\}$, $i = 1, \dots, s$; $j = 1, \dots, n_i$;

$$\ln \left\{ \frac{\lambda_{ij}(\beta, \theta_i)}{1 - \lambda_{ij}(\beta, \theta_i)} \right\} = \mathbf{O}_{ij} + \mathbf{x}_{ij}^T \beta + \sum_{k=1}^{m_{ij}} z_{ijk} \theta_i(k).$$

- $r \sim \text{Discrete Uniform}[a_r, b_r]$;
- Default setting: $a_r = 1$, $b_r = 50$;
- Default initial value: $r = a_r$;

Likelihood Indicator

- likelihood_indicator = 0: Bernoulli;
- likelihood_indicator = 1: Gaussian;
- likelihood_indicator = 2: Negative binomial.

Special Notes

- Do not center the exposures. Instead, we recommend that they are only scaled at each outcome time period (e.g., by interquartile range).