SpGPCW: Spatially Varying Gaussian Process Model for Critical Window Estimation

Statistical Model

$$y_{ij}|\boldsymbol{\beta}, \boldsymbol{\theta}_i \overset{\text{ind}}{\sim} \text{Bernoulli}\left\{p_{ij}\left(\boldsymbol{\beta}, \boldsymbol{\theta}_i\right)\right\},\ i = 1, ..., s;\ j = 1, ..., n_i;$$

$$\log\left\{\frac{p_{ij}\left(\boldsymbol{\beta}, \boldsymbol{\theta}_i\right)}{1 - p_{ij}\left(\boldsymbol{\beta}, \boldsymbol{\theta}_i\right)}\right\} = \mathbf{x}_{ij}^{\text{T}}\boldsymbol{\beta} + \sum_{k=1}^{m_{ij}} \text{z}_{ijk}\theta_i\left(k\right);$$

$$\boldsymbol{\theta} = \left(\boldsymbol{\theta}_1^{\text{T}}, ..., \boldsymbol{\theta}_s^{\text{T}}\right)^{\text{T}};\ \boldsymbol{\theta}_i = \left\{\theta_i\left(1\right), ..., \theta_i\left(m\right)\right\}^{\text{T}};$$

$$\boldsymbol{\theta}_i|\boldsymbol{\theta}_{-i}, \rho, \phi, \sigma_{\theta}^2, \boldsymbol{\eta}, \overset{\text{ind}}{\sim} \text{MVN}\left\{\frac{\rho\sum_{j=1}^{s} w_{ij}\boldsymbol{\theta}_j + \left(1-\rho\right)\boldsymbol{\eta}}{\rho\sum_{j=1}^{s} w_{ij} + \left(1-\rho\right)}, \frac{\sigma_{\theta}^2\Sigma\left(\phi\right)}{\rho\sum_{j=1}^{s} w_{ij} + \left(1-\rho\right)}\right\},\ i = 1, ..., s;$$

$$\boldsymbol{\eta} = \left\{\eta\left(1\right), ..., \eta\left(m\right)\right\}^{\text{T}}|\sigma_{\eta}^2, \phi \sim \text{MVN}\left\{\mathbf{0}_m, \sigma_{\eta}^2\Sigma\left(\phi\right)\right\};$$

$$\Sigma\left(\phi\right)_{ij} = \exp\left\{-\phi|i-j|\right\},\ \phi > 0;$$

- $m = \max\left\{m_{ij} : i = 1, ..., s;\ j = 1, ..., n_i\right\};$
- s: Number of unique spatial locations;
- n_i: Number of observations with spatial location i;
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_{\beta}^2\right),\ j = 1, ..., p;$

- p: Length of $\mathbf{x}_{ij}$ vector (same for all i, j);
- Default setting: $\sigma_{\beta}^2 = 10,000$.

$\sigma_{\theta}^2 \sim \text{Inverse Gamma}\left(a_{\sigma_{\theta}^2}, b_{\sigma_{\theta}^2}\right);$

- Default setting: $a_{\sigma_{\theta}^2} = 3$, $b_{\sigma_{\theta}^2} = 2$.

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$\rho \sim \text{Uniform}\left(a_{\phi}, b_{\phi}\right);$

- Default setting: $a_{\phi} = 0,\ b_{\phi} = 1$.

$\phi \sim \text{Uniform}\left(a_{\phi}, b_{\phi}\right);$

- Default setting: $a_{\phi} = \log(0.9999)/(-(m-1))$, $b_{\phi} = \log(0.0001)/(-1)$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\eta_j = 0$ for all j;
- $\sigma^2_\theta = 1.00$;
- $\sigma^2_\eta = 1.00$;
- $\rho = 0.50 * (b_\phi - a_\phi)$;
- $\phi = 0.01\,(b_\phi - a_\phi)$.

Special Notes

- Do not center the exposures. Instead, we recommend that they are only scaled at each outcome time period (e.g., by interquartile range).