SpMeta: Spatial Meta-Analysis/Regression Modeling

Statistical Model

$$\hat{\theta}_{ij}|\theta_{ij} \overset{\text{ind}}{\sim} \text{N}\left(\theta_{ij}, \hat{\delta}_{ij}^2\right),\ i = 1, ..., n;\ j = 1, ..., m_i;$$

- n: Number of geographically separated spatial regions (e.g., states);
- m_i: Number of contiguous spatial units within spatial region i (e.g., counties within a state);
- $\hat{\theta}_{ij}$: Point estimate from first stage modeling;
- $\hat{\delta}_{ij}$: Standard error of the point estimate.

$$\theta_{ij} = \mathbf{x}_{ij}^{\text{T}}\boldsymbol{\beta} + \phi_{ij} + \epsilon_{ij};$$

$$\theta_{ij}|\boldsymbol{\theta}_{i,-j}, \rho_i, \tau_i^2 \overset{\text{ind}}{\sim} \text{N}\left(\frac{\rho_i \sum_{k=1}^{m_i} w_{jk}^{(i)}\theta_{ik}}{\rho_i \sum_{j=k}^{m_i} w_{jk}^{(i)} + 1 - \rho}, \frac{\tau_i^2}{\rho_i \sum_{k=1}^{m_i} w_{jk}^{(i)} + 1 - \rho_i}\right),\ j = 1, ..., m_i;$$

- Independence between parameters across the different geographically separated regions (i.e., $\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_n$);
- $\boldsymbol{\theta}_{i,-j}^{\text{T}} = (\theta_{i1}, ..., \theta_{i,j-1}, \theta_{i,j+1}, ..., \theta_{i,m_i})$;
- $w_{jk}^{(i)}$: Equal to one if areal units j and k are neighbors, zero otherwise. $w_{jj}^{(i)} = 0$ by definition.

$$\phi_{ij}|\sigma_i^2 \overset{\text{ind}}{\sim} \text{N}\left(0, \sigma_i^2\right).$$

Prior Information

$\beta_k \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\beta^2\right),\ k = 1, ..., p;$

- p: Length of $\mathbf{x}_{ij}$ vector (same for all i, j);
- Default setting: $\sigma_\beta^2 = 10,000$.

$\sigma_i^2 \overset{\text{iid}}{\sim} \text{Inverse Gamma}\left(a_{\sigma^2}, b_{\sigma^2}\right)$;

- Default setting: $a_{\sigma^2} = 0.01$, $b_{\sigma^2} = 0.01$.

$\tau_i^2 \overset{\text{iid}}{\sim} \text{Inverse Gamma}\left(a_{\tau^2}, b_{\tau^2}\right)$;

- Default setting: $a_{\tau^2} = 0.01$, $b_{\tau^2} = 0.01$.

$\rho_i \overset{\text{iid}}{\sim} \text{Uniform}\left(a_\rho, b_\rho\right)$;

- Default setting: $a_\rho = 0.00$, $b_\rho = 1.00$.

Default Initial Values

- $\theta_{ij} = \hat{\theta}_{ij}$ for all i, j
- $\beta_k = 0$ for all k;
- $\phi_{ij} = 0$ for all i, j;
- $\sigma_i^2 = 1.00$ for all i;
- $\tau_i^2 = 1.00$ for all i;
- $\rho_i = 0.50$ for all i.

Model Indicator

- model_indicator = 0: Non-spatial:
  - $\phi_{ij} \equiv 0$ for all i, j;
- model_indicator = 1: Spatial Option 1:
  - $\epsilon_{ij} \equiv 0$ for all i, j;
- model_indicator = 2: Spatial Option 2:
  - Both ϕ_{ij} and ϵ_{ij} included in the model.