

# SpMeta: Spatial Meta Analysis/Regression Modeling

## Statistical Model

$$\hat{\theta}_{ij}|\theta_{ij} \stackrel{\text{ind}}{\sim} N\left(\theta_{ij}, \hat{\delta}_{ij}^2\right), \quad i = 1, \dots, n; \quad j = 1, \dots, m_i;$$

- $n$ : Number of geographically separated spatial regions (e.g., states);
- $m_i$ : Number of contiguous spatial units within spatial region  $i$  (e.g., counties within a state);
- $\hat{\theta}_{ij}$ : Point estimate from first stage modeling;
- $\hat{\delta}_{ij}$ : Standard error of the point estimate.

$$\theta_{ij} = \mathbf{x}_{ij}^T \boldsymbol{\beta} + \phi_{ij} + \epsilon_{ij};$$

$$\theta_{ij}|\boldsymbol{\theta}_{i,-j}, \rho_i, \tau_i^2 \stackrel{\text{ind}}{\sim} N\left(\frac{\rho_i \sum_{k=1}^{m_i} w_{jk}^{(i)} \theta_{ik}}{\rho_i \sum_{j=k}^{m_i} w_{jk}^{(i)} + 1 - \rho}, \frac{\tau_i^2}{\rho_i \sum_{k=1}^{m_i} w_{jk}^{(i)} + 1 - \rho_i}\right), \quad j = 1, \dots, m_i;$$

- Independence between parameters across the different geographically separated regions (i.e.,  $\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_n$ );
- $\boldsymbol{\theta}_{i,-j}^T = (\theta_{i1}, \dots, \theta_{i,j-1}, \theta_{i,j+1}, \dots, \theta_{i,m_i})$ ;
- $w_{jk}^{(i)}$ : Equal to one if areal units  $j$  and  $k$  are neighbors, zero otherwise.  $w_{jj}^{(i)} = 0$  by definition.

$$\phi_{ij}|\sigma_i^2 \stackrel{\text{ind}}{\sim} N(0, \sigma_i^2).$$

## Prior Information

$$\beta_k \stackrel{\text{iid}}{\sim} N(0, \sigma_\beta^2), \quad k = 1, \dots, p;$$

- $p$ : Length of  $\mathbf{x}_{ij}$  vector (same for all  $i, j$ );
- Default setting:  $\sigma_\beta^2 = 10,000$ .

$$\sigma_i^2 \stackrel{\text{iid}}{\sim} \text{Inverse Gamma}(a_{\sigma^2}, b_{\sigma^2});$$

- Default setting:  $a_{\sigma^2} = 0.01, b_{\sigma^2} = 0.01$ .

$$\tau_i^2 \stackrel{\text{iid}}{\sim} \text{Inverse Gamma}(a_{\tau^2}, b_{\tau^2});$$

- Default setting:  $a_{\tau^2} = 0.01, b_{\tau^2} = 0.01$ .

$$\rho_i \stackrel{\text{iid}}{\sim} \text{Uniform}(a_\rho, b_\rho);$$

- Default setting:  $a_\rho = 0.00, b_\rho = 1.00$ .

## Default Initial Values

- $\theta_{ij} = \hat{\theta}_{ij}$  for all  $i, j$
- $\beta_k = 0$  for all  $k$ ;
- $\phi_{ij} = 0$  for all  $i, j$ ;
- $\sigma_i^2 = 1.00$  for all  $i$ ;
- $\tau_i^2 = 1.00$  for all  $i$ ;
- $\rho_i = 0.50$  for all  $i$ .

## Model Indicator

- `model_indicator = 0`: Non-spatial:
  - $\phi_{ij} \equiv 0$  for all  $i, j$ ;
- `model_indicator = 1`: Spatial Option 1:
  - $\epsilon_{ij} \equiv 0$  for all  $i, j$ ;
- `model_indicator = 2`: Spatial Option 2:
  - Both  $\phi_{ij}$  and  $\epsilon_{ij}$  included in the model.