GPCW: Gaussian Process Model for Critical Window Estimation

Statistical Model

$$y_i|\boldsymbol{\beta}, \boldsymbol{\theta} \stackrel{\text{ind}}{\sim} \text{Bernoulli}\left(p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)\right),\ i = 1, ..., n;$$

$$\log\left(\frac{p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)}{1 - p_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)}\right) = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\theta\left(j\right);$$

$$\boldsymbol{\theta} = \left(\theta\left(1\right), ..., \theta\left(m\right)\right)^{\text{T}} |\sigma_\theta^2, \phi \sim \text{MVN}\left(\mathbf{0}_m, \sigma_\theta^2 \Sigma\left(\phi\right)\right);$$

$$\Sigma\left(\phi\right)_{ij} = \exp\left\{-\phi|i-j|\right\},\ \phi > 0.$$

- $m = \max\left\{m_i : i = 1, ..., n\right\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$\beta_j \stackrel{\text{iid}}{\sim} \text{N}\left(0, \sigma_\beta^2\right),\ j = 1, ..., p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma_\beta^2 = 10,000$.

$\sigma_\theta^2 \sim \text{Inverse Gamma}\left(a_{\sigma_\theta^2}, b_{\sigma_\theta^2}\right);$

- Default setting: $a_{\sigma_\theta^2} = 3$, $b_{\sigma_\theta^2} = 2$.

$\phi \sim \text{Uniform}\left(a_\phi, b_\phi\right);$

- Default setting: $a_\phi = \log(0.9999)/(-(m-1))$, $b_\phi = \log(0.0001)/(-1)$.

Default Initial Values

- $\beta_j = 0$ for all j;
- $\theta_j = 0$ for all j;
- $\sigma_\theta^2 = 1.00$;
- $\phi = 1.00$.