Spillover: Spatial Change Point Estimation Due to Spillover from a Point Source

Statistical Model

$$y_i\left(\mathbf{s}_i\right)|p_i\left(\mathbf{s}_i\right) \overset{\text{ind}}{\sim} \text{Bernoulli}\left(p_i\left(\mathbf{s}_i\right)\right),\ i=1,...,n;$$

$$\log\left(\frac{p_i\left(\mathbf{s}_i\right)}{1-p_i\left(\mathbf{s}_i\right)}\right) = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \lambda g\left(||\mathbf{s}_i - \mathbf{s}_p||;\theta\right) + w\left(\mathbf{s}_i\right);$$

$$\boldsymbol{w} = \left(w\left(\mathbf{s}_1^*\right),...,w\left(\mathbf{s}_m^*\right)\right)^{\text{T}}|\sigma_w^2,\phi \sim \text{MVN}\left(\mathbf{0}_m, \sigma_w^2\Sigma\left(\phi\right)\right);$$

$$\Sigma\left(\phi\right)_{ij} = \begin{cases} 1 - 1.5\phi||\mathbf{s}_i^* - \mathbf{s}_j^*|| + 0.5\left(\phi||\mathbf{s}_i^* - \mathbf{s}_j^*||\right)^3, & \text{if } 0 \leq ||\mathbf{s}_i^* - \mathbf{s}_j^*|| \leq 1/\phi; \\ 0, & \text{if } ||\mathbf{s}_i^* - \mathbf{s}_j^*|| \geq 1/\phi. \end{cases}$$

- $\mathbf{s}_p$: Location of the point source;
- $\mathbf{s}_i \in \{\mathbf{s}_1^*,...,\mathbf{s}_m^*\}$ for all i, where $m < n$ is the number of unique spatial locations;
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

$g\left(||\mathbf{s}_i - \mathbf{s}_p||;\theta\right)$ Options

- Change Point: $I\left(||\mathbf{s}_i - \mathbf{s}_p|| \leq \theta\right)$;
- Exponential: $I\left(||\mathbf{s}_i - \mathbf{s}_p|| \leq \theta\right)\exp\left\{-||\mathbf{s}_i - \mathbf{s}_p||\right\}$;
- Gaussian: $I\left(||\mathbf{s}_i - \mathbf{s}_p|| \leq \theta\right)\exp\left\{-\left(||\mathbf{s}_i - \mathbf{s}_p||\right)^2\right\}$.

Prior Information

$\beta_j, \lambda \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\delta^2\right),\ j=1,...,p;$

- p: Length of $\mathbf{x}_i$ vector (same for all i);
- Default setting: $\sigma_\delta^2 = 10{,}000$.

$\theta \sim \text{Uniform}\left(a_\theta, b_\theta\right)$;

- Default settings: $a_\theta = \min\left\{||\mathbf{s}_i - \mathbf{s}_p|| : i=1,...,n\right\}$, $b_\theta = \max\left\{||\mathbf{s}_i - \mathbf{s}_p|| : i=1,...,n\right\}$.

$\sigma_w^2 \sim \text{Inverse Gamma}\left(a_{\sigma_w^2}, b_{\sigma_w^2}\right);$

- Default setting: $a_{\sigma_w^2} = 3$, $b_{\sigma_w^2} = 2$.

$\phi \sim \text{Gamma}\left(\alpha_\phi, \beta_\phi\right);$

- Default setting: $\alpha_\phi = 1$, $\beta_\phi = 1$.

Default Initial Values

- $\beta_j = \lambda = 0$ for all j;
- $\theta = \left(b_\theta - a_\theta\right)/2$;
- $w\left(\mathbf{s}_i\right) = 0$ for all i;
- $\sigma_w^2 = 1$;
- $\phi = 0.01\max\left\{||\mathbf{s}_i - \mathbf{s}_p|| : i=1,...,n\right\}$.