

Spillover: Spatial Change Point Estimation Due to Spillover from a Point Source

Statistical Model

$$y_i(\mathbf{s}_i) | p_i(\mathbf{s}_i) \stackrel{\text{ind}}{\sim} \text{Bernoulli}(p_i(\mathbf{s}_i)), \quad i = 1, \dots, n;$$

$$\log \left(\frac{p_i(\mathbf{s}_i)}{1 - p_i(\mathbf{s}_i)} \right) = \mathbf{x}_i^T \boldsymbol{\beta} + \lambda g(\|\mathbf{s}_i - \mathbf{s}_p\|; \theta) + w(\mathbf{s}_i);$$

$$\mathbf{w} = (w(\mathbf{s}_1^*), \dots, w(\mathbf{s}_m^*))^T | \sigma_w^2, \phi \sim \text{MVN}(\mathbf{0}_m, \sigma_w^2 \Sigma(\phi));$$

$$\Sigma(\phi)_{ij} = \begin{cases} 1 - 1.5\phi \|\mathbf{s}_i^* - \mathbf{s}_j^*\| + 0.5(\phi \|\mathbf{s}_i^* - \mathbf{s}_j^*\|)^3, & \text{if } 0 \leq \|\mathbf{s}_i^* - \mathbf{s}_j^*\| \leq 1/\phi; \\ 0, & \text{if } \|\mathbf{s}_i^* - \mathbf{s}_j^*\| \geq 1/\phi. \end{cases}$$

- \mathbf{s}_p : Location of the point source;
- $\mathbf{s}_i \in \{\mathbf{s}_1^*, \dots, \mathbf{s}_m^*\}$ for all i , where $m < n$ is the number of unique spatial locations;
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

$g(\|\mathbf{s}_i - \mathbf{s}_p\|; \theta)$ Options

- Change Point: $I(\|\mathbf{s}_i - \mathbf{s}_p\| \leq \theta)$;
- Exponential: $I(\|\mathbf{s}_i - \mathbf{s}_p\| \leq \theta) \exp\{-\|\mathbf{s}_i - \mathbf{s}_p\|\}$;
- Gaussian: $I(\|\mathbf{s}_i - \mathbf{s}_p\| \leq \theta) \exp\left\{-\left(\|\mathbf{s}_i - \mathbf{s}_p\|\right)^2\right\}$.

Prior Information

$$\beta_j, \lambda \stackrel{\text{iid}}{\sim} \text{N}(0, \sigma_\delta^2), \quad j = 1, \dots, p;$$

- p : Length of \mathbf{x}_i vector (same for all i);
- Default setting: $\sigma_\delta^2 = 10,000$.

$$\theta \sim \text{Uniform}(a_\theta, b_\theta);$$

- Default settings: $a_\theta = \min\{\|\mathbf{s}_i - \mathbf{s}_p\| : i = 1, \dots, n\}$, $b_\theta = \max\{\|\mathbf{s}_i - \mathbf{s}_p\| : i = 1, \dots, n\}$.

$$\sigma_w^2 \sim \text{Inverse Gamma}(a_{\sigma_w^2}, b_{\sigma_w^2});$$

- Default setting: $a_{\sigma_w^2} = 3$, $b_{\sigma_w^2} = 2$.

$$\phi \sim \text{Gamma}(\alpha_\phi, \beta_\phi);$$

- Default setting: $\alpha_\phi = 1$, $\beta_\phi = 1$.

Default Initial Values

- $\beta_j = \lambda = 0$ for all j ;
- $\theta = (b_\theta - a_\theta)/2$;
- $w(\mathbf{s}_i) = 0$ for all i ;
- $\sigma_w^2 = 1$;
- $\phi = \max\{\|\mathbf{s}_i - \mathbf{s}_p\| : i = 1, \dots, n\}/2$.