11b. Exhibit a coplanar graph G on 6 vertices such that both G and its complement are connected.

Solution. Observe the graph of G and its complement below.

14a. Let $G = P_3 \vee C_4$. Prove that G is not planar.

Proof. Let $G = P_3 \vee C_4$. Recall Theorem 4.19 that states,

$$\gamma(G) \geq \frac{m}{6} - \frac{n}{2} + 1$$

where $\gamma(G)$ is the genus of G, the minimum number of overpasses required to embed G into a surface. We notice that G has 7 vertices, and P_3 and C_4 have a combined 6 edges before we join them. After we join these graphs to construct G, we have to add 12 edges to end up with 18 edges. Thus,

$$\begin{aligned}\gamma(G) &\geq \frac{18}{6} - 72 + 1 \\ &= 3 - 3.5 + 1 \\ &= .5\end{aligned}$$

And since $\gamma(G) \geq .5$, then we know G cannot be planar since the genus of any planar graph is zero.

□