1. Use the definition of estimable functions to show that if X has full rank, then every $a'\beta$ is estimable and in particular, every individual parameter β_i is estimable.

Solution. Let $a'\beta$ be some function such that β is from the model $Y = X\beta + \epsilon$. Suppose that X has full rank. Then if β has p rows, then X must have p linearly independent columns. Thus the column space of X is $C(X) = \mathbb{R}^p$. And if follows that a must have p elements. Thus $a \in \mathbb{R}^p$. Thus, $a \in C(X)$. And we know that a function $a'\beta$ is estimable $\iff$ $a \in C(X)$.

$\square$