ECE 133A - Homework 3

Wednesday, October 25, 2023 10:35 AM

4.16 Formulate the following problem as a set of linear equations. Find a polynomial

$$p(t) = x_1 + x_2 t + x_3 t^2 + \cdots + x_n t^{n-1}$$

that satisfies the following conditions on the values and the derivatives at m points $t_1, \ldots, t_m$:

$$p(t_1) = y_1, \quad p(t_2) = y_2, \quad \ldots, \quad p(t_m) = y_m, \qquad p'(t_1) = s_1, \quad p'(t_2) = s_2, \quad \ldots, \quad p'(t_m) = s_m.$$

The unknowns in the problem are the coefficients $x_1, \ldots, x_n$. The values of t_i, y_i, s_i are given.

(a) Express the problem as a set of linear equations $Ax = b$. Clearly state how A and b are defined.

$$p'(t) = x_2 + 2x_3 t + \ldots + (n-1) x_n t^{n-2}$$

$$\boxed{\underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 & \cdots & t_n^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & t_m & t_m^2 & \cdots & t_m^{n-1} \\ 0 & 1 & 2t_1 & \cdots & (n-1)t_1^{(n-2)} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 1 & 2t_m & \cdots & (n-1)t_m^{(n-2)} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_m \\ s_1 \\ \vdots \\ s_m \end{bmatrix}}_{b}}$$

(b) Suppose $n = 2m$ and the m points t_i are distinct. Is the matrix A in part (a) nonsingular?

4.12 Consider the matrix $A = I + aJ$, where a is a real scalar and J is the reverser matrix

$$J = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & ⋰ & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

of size $n \times n$. We assume $n > 1$.

(a) For what values of a is A singular?

(b) Assuming A is nonsingular, express its inverse as a linear combination of the matrices I and J.

$A = I + aJ = AA^{-1} + aJ$

4.9 Suppose A is an $m \times p$ matrix with linearly independent columns and B is a $p \times n$ matrix with linearly

independent rows. We are not assuming that A or B are square. Define

$$X = AB, \qquad Y = B^{\dagger}A^{\dagger}$$

where $A^{\dagger}$ and $B^{\dagger}$ are the pseudo-inverses of A and B. Show the following properties.

(a) YX is symmetric.

$YX = (YX)^T$

$$= X^T Y^T$$
$$= (AB)^T (B^+ A^+)^T$$
$$= B^T A^T (A^+)^T (B^+)^T$$
$$= XY$$

(b) XY is symmetric.

$$XY = (XY)^T$$
$$= Y^T X^T$$
$$= (B^+ A^+)^T (AB)^T$$
$$= (A^+)^T (B^+)^T B^T A^T$$
$$= XY$$

(c) $YXY = Y$.

$$YXY = Y AB B^+ A^+$$
$$= Y I I$$
$$= Y$$

(d) $XYX = X$.

$$XYX = X B^{+} A^{+} A B$$
$$= X I I$$
$$= X$$

homework3

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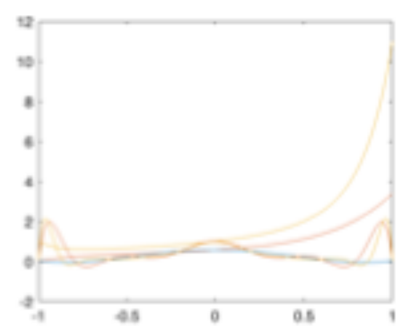
A3.1

```
[x_6, y_6] = interpolation_3_1(6);
[x_11, y_11] = interpolation_3_1(11);
[x_16, y_16] = interpolation_3_1(16);

plot(x_6, y_6);
plot(x_11, y_11);
plot(x_16, y_16);
```



T8.8

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y = [3 5 10 -2 -3];
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f = (c1 + c2*t + c3*t.^2) / (1 + d1*t + d2*t.^2);
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```
function [plot_t, poly] = interpolation_3_1(n)
    t = linspace(-1, 1, n)';
    y = 1 ./ (1 + 25*t.^2);
    A = fliplr(vander(t));
    x = A \ y;

    plot_t = linspace(-1, 1, 400);
    poly = 0;
    for i = 1:n
        poly = poly + x(i)*plot_t.^(i - 1);
    end
end
```