EMP191 Rocket Lab 7

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Abstract

In this lab, students used Mathematica to make a model of and plot the trajectory of the rocket launch, based on differential equations from the force equations.

1 Introduction

The goal of this lab was to be able to make an accurate model of the trajectory using Newton's laws, as well as become more well versed in Mathematica.

2 Measurement Procedure

The procedure was followed through as written.

3 Analysis Results

For the first part of the procedure, Oleg and I derived a series of differential equations with which we could make plots. Part A:

$$F = ma \rightarrow \quad PA_n - mg = ma \rightarrow \quad a = \frac{PA_n}{m_f} - g \rightarrow \quad \frac{dv}{dt} = \frac{PA_n}{m_f} - g \rightarrow \tag{1}$$

where m_f is the mass of the fully filled rocket.

$$\frac{dv}{dt} = \frac{4.40 * 10^5 * (4.75 * 10^{-3})^2 * \pi}{.1889} - 9.8 = 155m/s^2 \tag{2}$$

$$\Delta x = v0 + at^2 \rightarrow \quad 0.22 = 0 + 155t^2 \rightarrow \quad t = 0.0377s \tag{3}$$

Part B: Using Bernoulli's Equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \rightarrow \quad P_1 = P_2 + \frac{1}{2}\rho v_2^2 \rightarrow \quad v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \tag{4}$$

where v_2 is the exit velocity of the water from the rocket and A_n is the area of the bottle nozzle.

$$v_2 = \sqrt{\frac{2(4.40 * 10^5 - 10^5)}{1000}} = 26.07m/s \tag{5}$$

here is the derivation of $\frac{dM}{dt}$ for 0.27 seconds until depletion.

$$\frac{dM}{dt} = Area * v_2 * 1000 = 1.84kg/s \tag{6}$$

Using the Rocket Equation and Newton's laws one can derive the differential equation for $\frac{dv}{dt}$.

$$\frac{dv}{dt} = -u * ln(\frac{M_i - \frac{dM}{dt}t}{M_i}) - gt \tag{7}$$

The drag force equation is derived below: where A_b is the area of the bottle cross section and m_e is the mass of the empty rocket.

$$mg \pm kv^2 = ma \rightarrow \quad g \pm \frac{kv^2}{m_e} = \frac{dv}{dt} \rightarrow \quad \frac{dv}{dt} = g \pm \frac{\rho_a A_b C_d v^2}{2m_e} \tag{8}$$

For Launch Tube Time : Solving for time with the equation above (and acceleration = $155 m/s^2$) we get 0.0377s which is relatively close to my time of 0.0234s obtained in the previous lab. I used the times from the previous lab for the phases 2-4, getting t2 = 5.175, t3 = 7.9 seconds

4 Conclusion

Note that I was unable to obtain the graphs for to make the model. If I were, I would have been able to observe some small differences between the graph made from data from the accelerometer and that of data from the equations because of small things not taken account for (like launch angle in the simulation).