EMP191 Rocket Lab 7

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Abstract

In this lab, students used Mathematica to make a model of and plot the trajectory of the rocket launch, based on differential equations from the force equations.

1 Introduction

The goal of this lab was to be able to make an accurate model of the trajectory using Newton's laws, as well as become more well versed in Mathematica.

2 Measurement Procedure

The procedure was followed through as written.

3 Analysis Results

For the first part of the procedure, Oleg and I derived a series of differential equations with which we could make plots. Part A:

$$F = ma \rightarrow \quad PA_n - mg = ma \rightarrow \quad a = \frac{PA_n}{m_f} - g \rightarrow \quad \frac{dv}{dt} = \frac{PA_n}{m_f} - g \rightarrow \tag{1}$$

where m_f is the mass of the fully filled rocket.

$$\frac{dv}{dt} = \frac{4.40 * 10^5 * (4.75 * 10^{-3})^2 * \pi}{.1889} - 9.8 = 155m/s^2 \tag{2}$$

$$\Delta x = v0 + at^2 \rightarrow \quad 0.22 = 0 + 155t^2 \rightarrow \quad t = 0.0377s \tag{3}$$

Part B: Using Bernoulli's Equation

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2 \rightarrow \quad P_1 = P_2 + \frac{1}{2}\rho v_2^2 \rightarrow \quad v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \tag{4}$$

where v_2 is the exit velocity of the water from the rocket and A_n is the area of the bottle nozzle.

$$v_e = \sqrt{\frac{2(4.40 * 10^5 - 10^5)}{1000}} = 26.07m/s \tag{5}$$

here is the derivation of $\frac{dM}{dt}$ for 0.27 seconds until depletion.

$$\frac{dM}{dt} = Area_{nozzle} * v_e * 1000 = 1.84kg/s \tag{6}$$

Using the Rocket Equation and Newton's laws one can derive the differential equation for $\frac{dv}{dt}$.

$$\frac{dv}{dt} = \frac{2A(P_{bottle} - P_{atm})}{M_i - \rho v_e A_n t} - g \tag{7}$$

The drag force equation is derived below: where A_b is the area of the bottle cross section and m_e is the mass of the empty rocket.

$$-mg \mp kv^2 = ma \rightarrow \quad g \pm \frac{kv^2}{m_e} = \frac{dv}{dt} \rightarrow \quad \frac{dv}{dt} = g \pm \frac{\rho_a A_b C_d v_b^2}{2m_e} \tag{8}$$

Where v_b is the velocity of the bottle and A_b,ρ_a,C_d are the Area of the bottle, the density of air, and the C constant of drag respectively.

For Launch Tube Time : Solving for time with the equation above (and acceleration $= 155m/s^2$) I got 0.1138s. I used Mathematica to solve analytically for the other times.

4 Conclusion

I was able to observe some small differences between the graph made from data from the accelerometer and that of data from the equations because of small things not taken account for (like launch angle in the simulation). I also was able to only approximate the times using simplified equations so I doubt that it is very accurate.