

$$\text{Control function: } v = \frac{W1+W2*f}{1+W1+W2*f} = \frac{W1+W2*\frac{(\frac{x}{Ki})^n}{1+(\frac{x}{Ki})^n}}{1+W1+W2*\frac{(\frac{x}{Ki})^n}{1+(\frac{x}{Ki})^n}}$$

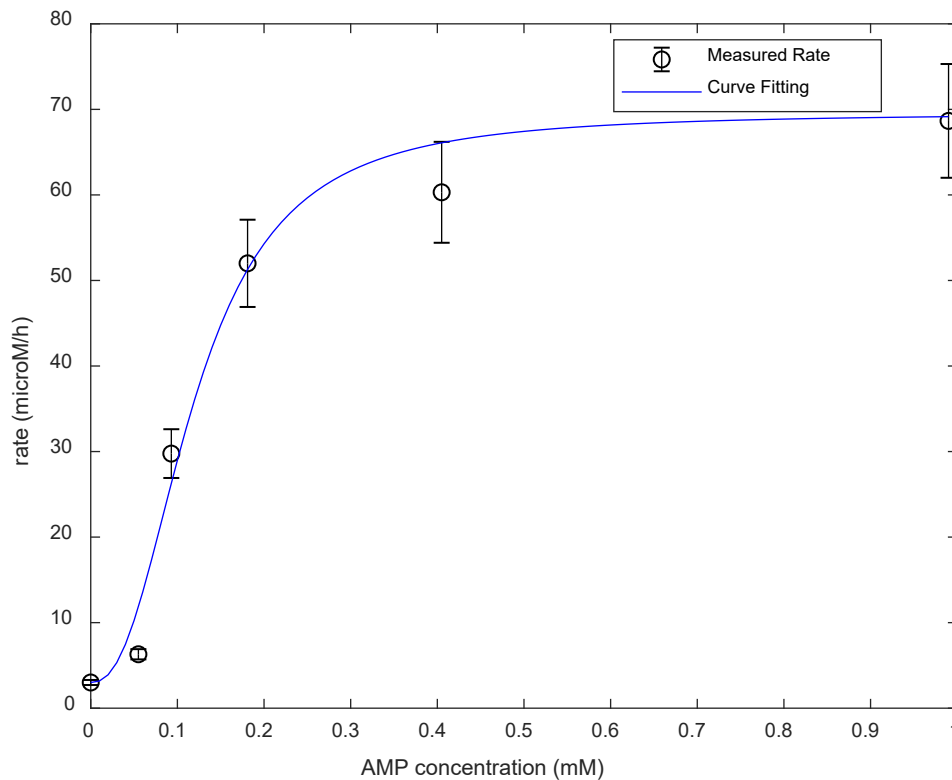
W1=0.0451 (calculated from Matlab); W2=80 (estimate)

R1=69.598 microM/h (saturation rate)

Reasoning: W2 should be significantly larger than W1 because at high concentration of AMP, $f=1$ and the equation would be simplified to $(W1+W2)/(1+W1+W2)$ and the measured rate is almost equal to the saturation rate. Hence, W2 needs to be large enough to render the simplified equation to 1.

Ki=0.7407;

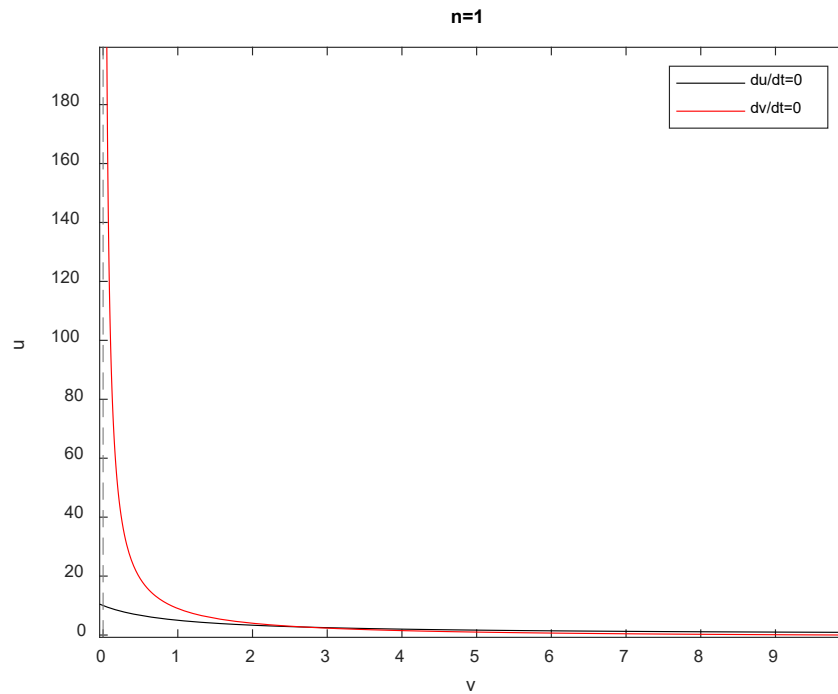
n=2.3894;



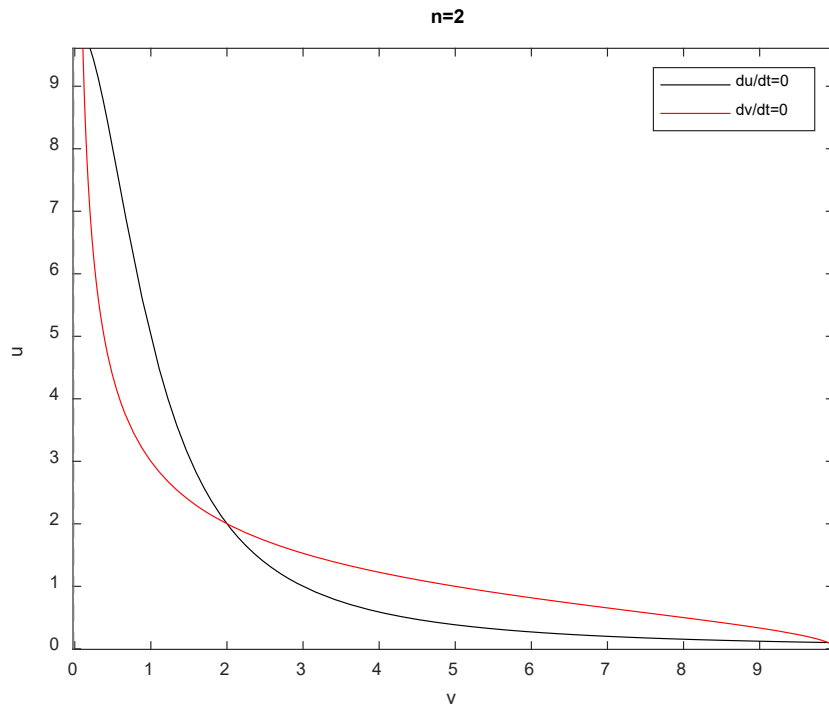
Yes the model describes the data well.

u is [Repressor 1]; v =[Repressor 2]; a_1 = effective rate of synthesis of Repressor 1;
 a_2 = effective rate of synthesis of Repressor 2;
 b is the cooperativity of repression of promoter 2; g is the cooperativity of repression of promoter 1.

Degradation rate constant: 1



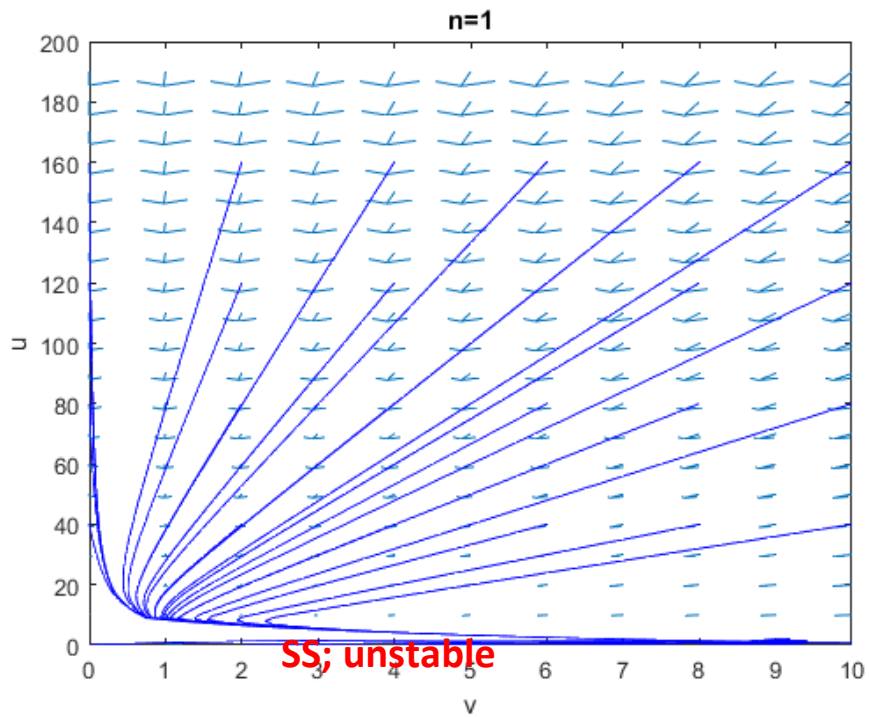
One solution

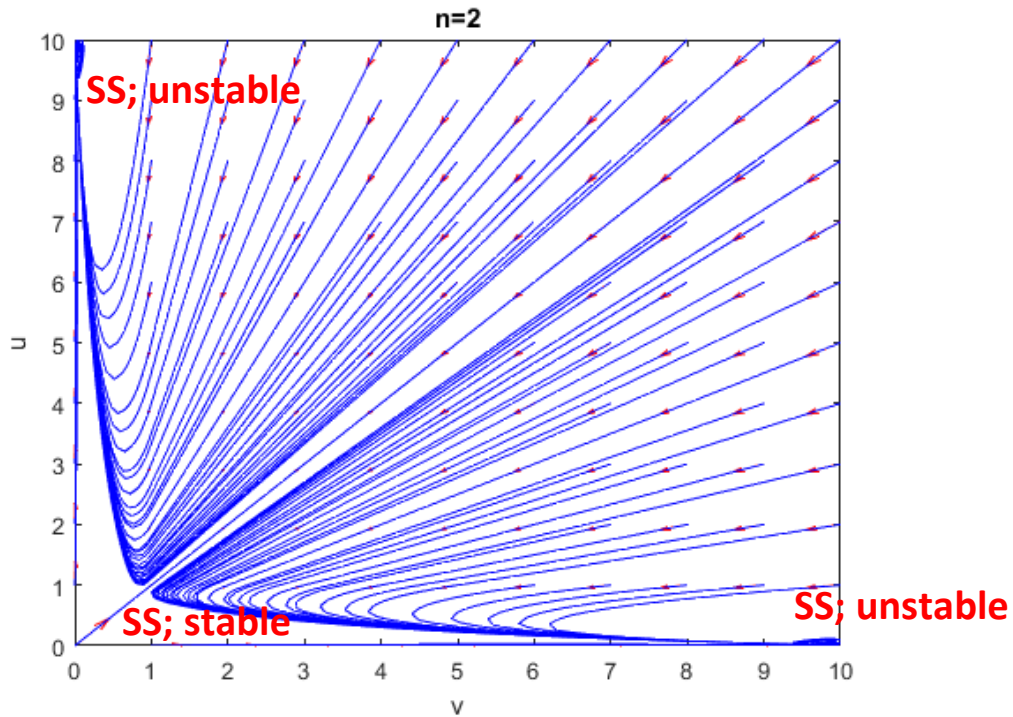


3 solutions

A higher degree of cooperative creates more equilibrium states.

PART C





$N=2$ makes the streamline symmetrical and it also gives us a stable steady state.

PART D

$J =$

$$\left[\frac{-(20*vs)}{(vs^2 + 1)^2}, -1 \right]$$

$$\left[-1, \frac{-(20*us)}{(us^2 + 1)^2} \right]$$

If eigenvalues are smaller than zero, steady state is stable.

PART E

WHEN $n=1$

$(2.7015, 2.7016) \rightarrow$ equilibrium point

$J =$

$$\left[\frac{-10}{(vs + 1)^2}, -1 \right]$$

$$\left[-1, \frac{-10}{(us + 1)^2} \right]$$

Eigenvalue = -1.7298; 0.2702 so it is unstable

WHEN $n=2$

$V_s=0.1010, 2, 9.8990$

$U_s= 9.8990, 2, 0.1010$

There are 3 equilibrium points (0.1010, 9.8990), (2,2) and 99.8990, 0.1010)

$e_1 = -2.3997; 0.4001$ **UNSTABLE**

$e_2 = -2.6000; -0.6000$ **STABLE**

$e_3 = -2.4000; 0.4000$ **UNSTABLE**

An increase in cooperativity not only gives us two more steady states but also gives us a stable states. The steady states are also symmetrical which suggests bistability.

PART F1

$$\frac{dR_i^*}{dt} = k_f L R_i - k_r R_i^* \quad (3)$$

$$\frac{dN_i^*}{dt} = k_f^{ND} N_i D_j - k_r^{ND} N_i^* \quad (4)$$

$$\frac{dD_i}{dt} = k_D R_i^* - \gamma_D D_i \quad (5)$$

$$\frac{dR_i}{dt} = \frac{\beta^n}{K^n + N_i^{*n}} - \gamma_R R_i \quad (6)$$

When $l=1$ and assume steady state for eq (4):

$$k_f^{ND} N_1 D_2 - k_r^{ND} N_i^* = 0$$

So that $N_1^* = \frac{k_f^{ND} N_1 D_2}{k_r^{ND}}$ (7) and now solve for D_2 in term of R_2

When $l=2$ and assume steady state for eq (3) and (5)

$$D_2 = \frac{k_D R_2^*}{g_D}; R_2^* = \frac{k_f L R_2}{k_r} \quad (8)$$

Now combine (6) (7) (8) to yield:

$$\frac{dR_1}{dt} = \frac{b^n}{K^n + (k_c R_2)^n} - g_R R_1; \quad k_c = \frac{k_f^{ND} N_1 k_D k_f L}{k_r^{ND} g_D k_r}$$

Similarly

$$\frac{dR_2}{dt} = \frac{b^n}{K^n + (k_d R_1)^n} - g_R R_2; \quad k_d = \frac{k_f^{ND} N_2 k_D k_f L}{k_r^{ND} g_D k_r}$$

PART F2 (assume n=1)

$$\frac{du}{dt} = \frac{b}{K^2 + k_c v K^2} - \tau u; \quad k_c = \frac{k_f^{ND} N_1 k_D k_f L}{k_r^{ND} g_D k_r}$$

$$\frac{dv}{dt} = \frac{b}{K^2 + k_d u K^2} - \tau v; \quad k_d = \frac{k_f^{ND} N_2 k_D k_f L}{k_r^{ND} g_D k_r}$$

There should be an intermediate concentration where the system is stable; too high or too long of a concentration would lead to instability

One can manipulate the degree of cooperativity as we did previously