

# Lab 3: Inertial Measurement Unit (Allan Variance)

## Abstract

In this lab, an inertial measurement unit (IMU) was used to collect two datasets of 5 min and 5 hours. Allan variance and deviation analysis was done on the collected datasets and noise parameters were calculated. The data's time series and frequency plots are also plotted and analysed.

## Methodology

The Allan variance (AVAR) Method is a method of representing root mean square (RMS) random drift error as a function of average time and can be used to determine the character of the underlying random processes that give rise to the data noise.<sup>[1]</sup>

The Allan variance analysis of a time domain signal  $\Omega(t)$  consists of computing its root Allan variance or Allan deviation as a function of different averaging times  $\tau$  and then analyzing the characteristic regions and log-log scale slopes of the Allan deviation curves to identify the different noise modes.<sup>[2]</sup>

Different types of random processes cause slopes with different gradients to appear on the Allan deviation plot. These different processes can be identified from their presence in different parts of the deviation plot; the numerical values can then be directly identified from the plot. For our IMU, we look at the following:

- Bias instability(BI): Can be identified from the flat region of the Allan deviation plot around the minimum. The numerical value is the minimum value on the Allan deviation plot. For a gyroscope, the bias stability measures how the bias of the gyroscope changes over a specified period of time at a constant temperature. Usually expressed in degrees per hour ( $^{\circ}/h$ ) or radian per second (rad/s) for gyros, and in meters per second squared or in units of acceleration due to gravity ( $m/s^2$  or mg) for accelerometers.
- Random walk (RW): The random walk noise parameter describes the average deviation that can occur when the signal might be integrated. This error will increase with the integration time and hence, limits any angle/velocity measurement based on the integration of velocity/acceleration. It presents itself as a straight line of slope negative 0.5 along the Allan deviation(ADEV) plot. The numerical value is the ADEV value at  $\tau = 1$ . Usually expressed in degrees per sec ( $^{\circ}/s/\sqrt{Hz}$ ) or radian per second ( $rad/s/\sqrt{Hz}$ ) for gyros, and in meters per second squared ( $m/s^2/\sqrt{Hz}$ ) or ( $mg/\sqrt{Hz}$ ) for accelerometers.

## Procedure

This lab was executed with a VN-100 IMU unit and an Ubuntu system with a ROS driver to read and parse a VYNMR string to publish a custom-built message to a topic /imu. The data was collected at two spots:

- One dataset was collected from the basement of an apartment complex away from pipes and other possible sources of major disturbances like washing machines and trains. Data was recorded for a duration of 5 hours in a ROSbag.
- A second shorter dataset of 5 min was collected in the basement of the Forsyth building.

The AVAR and ADEV plots were plotted using MATLAB and the rest were analysed using python.

## Allan Variance analysis for the 5-hour data

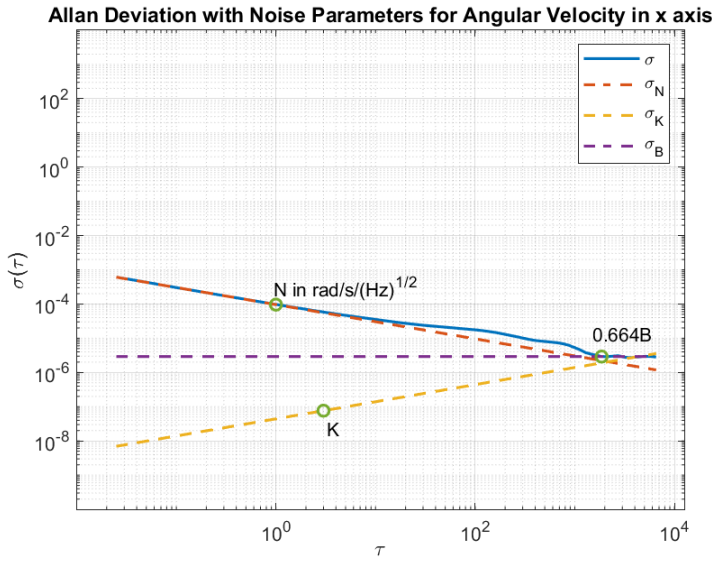


Fig 1.1 ADEV plot for angular velocity in x

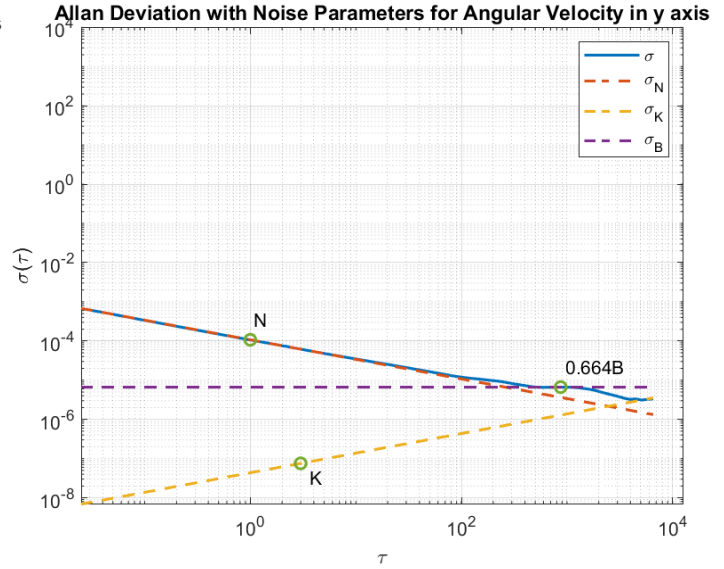


Fig 1.2 ADEV plot for angular velocity in y

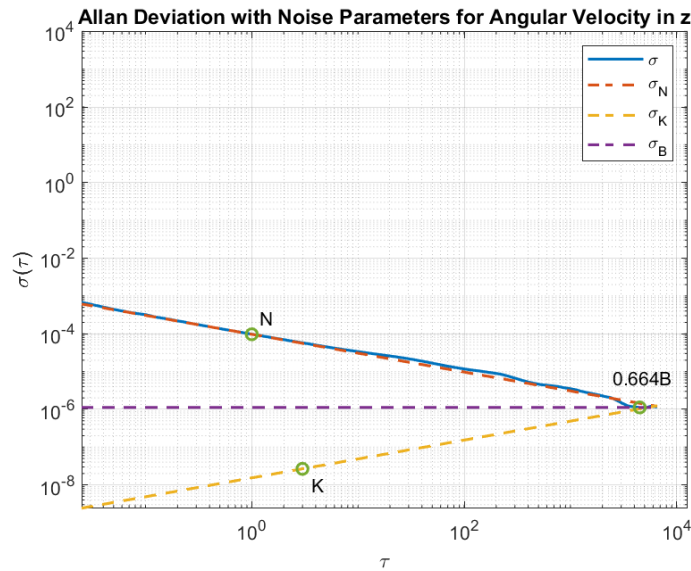


Fig 1.3 ADEV plot for angular velocity in z

In fig. 1.1, fig. 1.2 and fig. 1.3,  $\sigma_N$  represents a line of slope -0.5 to find N(angle random walk) and  $\sigma_B$  represents a line of slope 0 to find B(bias instability). Similarly,  $\sigma_K$  represents a line of slope 0.5 to find K(rate random walk). The parameters noted are  $B \cdot 3600 \cdot 180/\pi$  and  $N \cdot 180/\pi$ . These calculations are for comparison with the datasheet. K is the rate random walk obtained by  $K \cdot 180/\pi$ .

Noise parameters for the plots were as follows:

Angular velocity around Axis	Bias instability ( $^{\circ}/h$ )	Angle Random Walk ( $^{\circ}/s/\sqrt{Hz}$ )	Rate Random Walk ( $^{\circ}/s/\sqrt{Hz}$ )
x	0.9046	0.0055	$4.3817 \cdot 10^{-6}$
y	2.0666	0.0061	$4.3214 \cdot 10^{-6}$
z	0.3491	0.0056	$1.5271 \cdot 10^{-6}$

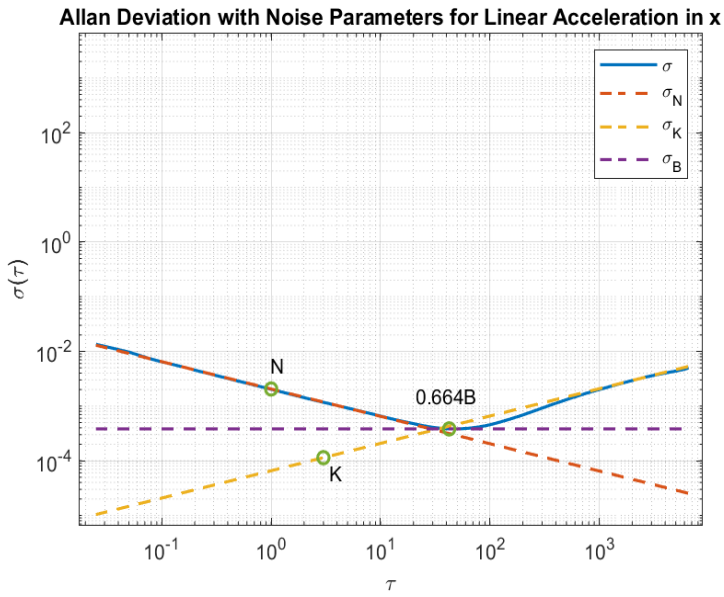


Fig 1.4 ADEV plot for linear velocity in x

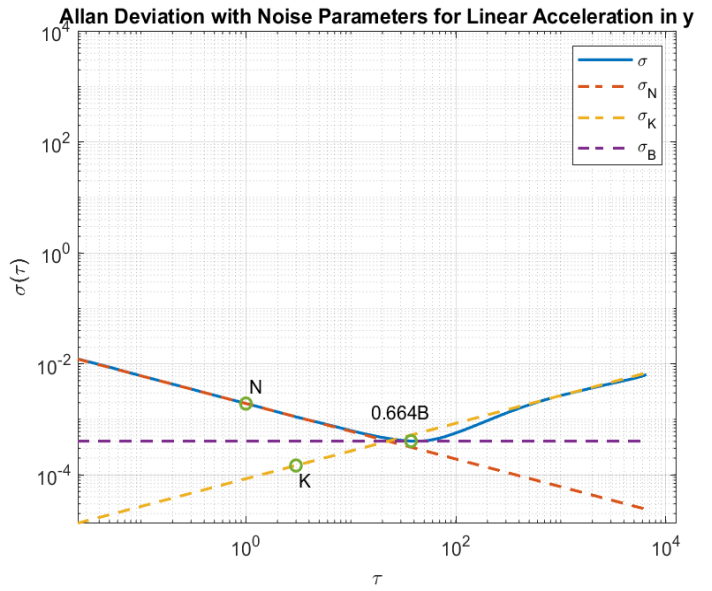


Fig 1.5 ADEV plot for linear velocity in y

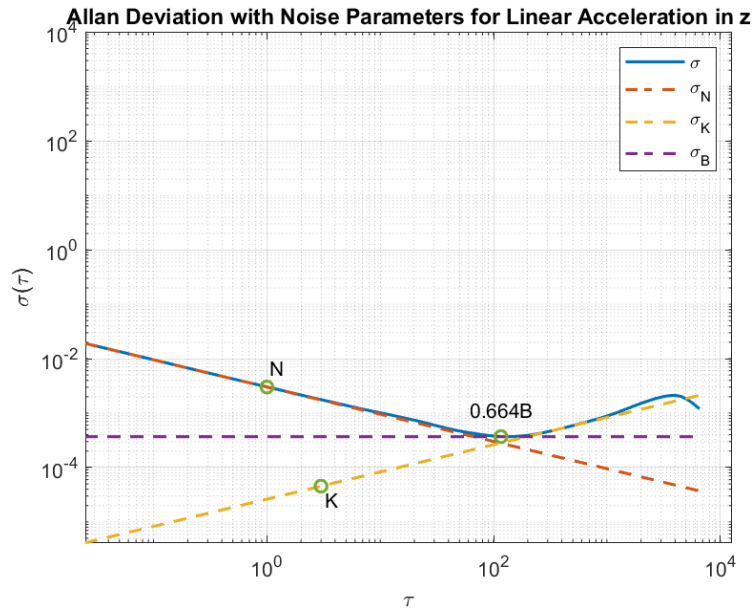


Fig 1.6 ADEV plot for linear velocity in z

In fig. 1.4, fig. 1.5 and fig. 1.6,  $\sigma_N$  represents a line of slope -0.5 to find N(velocity random walk) and  $\sigma_B$  represents a line of slope 0 to find B(bias instability). Similarly,  $\sigma_K$  represents a line of slope 0.5 to find K(rate random walk). The parameters obtained are  $N \cdot 101.97$  for VRW and  $B \cdot 101.97$ . The rate random walk is  $K \cdot 101.97$ .

Noise parameters for the plots were obtained as follows:

Linear velocity around Axis	Bias instability (mg)	Velocity Random Walk ( $\text{mg}/\sqrt{\text{Hz}}$ )	Rate Random Walk ( $\text{m/s}^2/\sqrt{\text{Hz}}$ )
x	0.0587	0.2089	0.0116
y	0.0622	0.1965	0.0150
z	0.0566	0.3070	0.0046

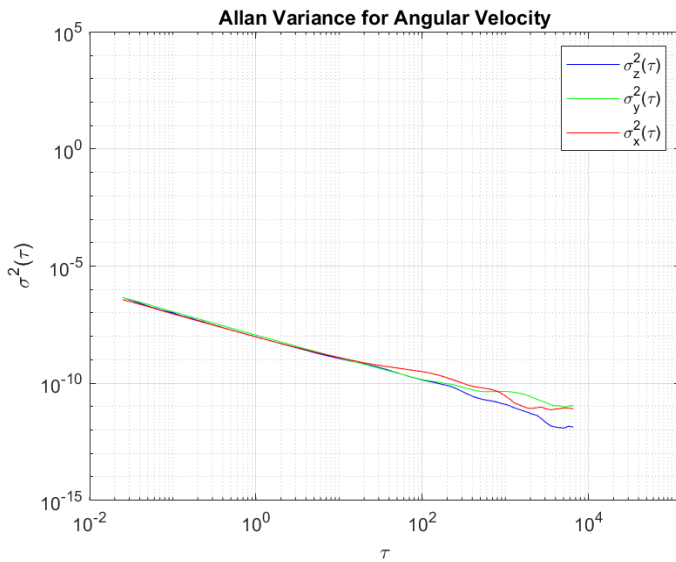


Fig 1.7 Allan Variance in angular velocity

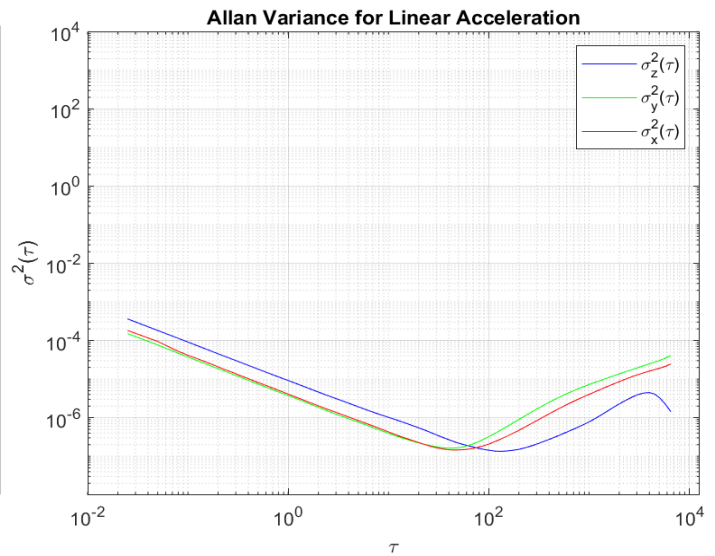


Fig 1.8 Allan Variance in linear acceleration

## Inferences

The following inferences were made about the plots:

- Bias instability and Random walk have been described by the VectorNav manual for consumer and industrial grade as the follows:

IMU Specifications	ACCELEROMETER	GYROSCOPE
Range	$\pm 16$ g	$\pm 2,000^\circ/\text{s}$
In-Run Bias Stability (Allan Variance)	$< 0.04$ mg	$< 10^\circ/\text{hr}$ (5-7 $^\circ/\text{hr}$ typ.)
Noise Density	0.14 mg/ $\sqrt{\text{Hz}}$	0.0035 $^\circ/\text{s} / \sqrt{\text{Hz}}$

- The obtained noise parameters indicate that the IMU used has noise parameters slightly higher than the values described in the datasheet provided.
- The in-run bias stability, often called bias instability, is a measure of how the bias will drift during operation over time at a constant temperature. It is present as a flat line (slope = 0) around a minima of the curve. Its value has been reduced by a factor of 0.664 (approximated constant to convert data from log scale).
- The units were then converted to the required dimensions as mentioned above the tables.  $1 \text{ m/s}^2 = 101.97 \text{ mg}$  was used for the accelerometer noise parameter conversions.
- Bias instability is a measure of how the bias will drift during operation over time at a constant temperature and so is an important value in the device specifications.<sup>[2]</sup>
- Random walk can be visualised as the random deviations that occur from one step to the next and these minor deviations can become very apparent on integration. This random walk is due to gaussian white noise in sensor readings over time.
- Random walk for both accelerometers and gyroscopes are obtained by fitting a line of slope = - 0.5 along the ADEV plot and marking the spot where  $\tau = 1$  on the fitted line. This is the value of random walk. Random walk has also been attributed to red noise.

## Analysis of the 5-minute data

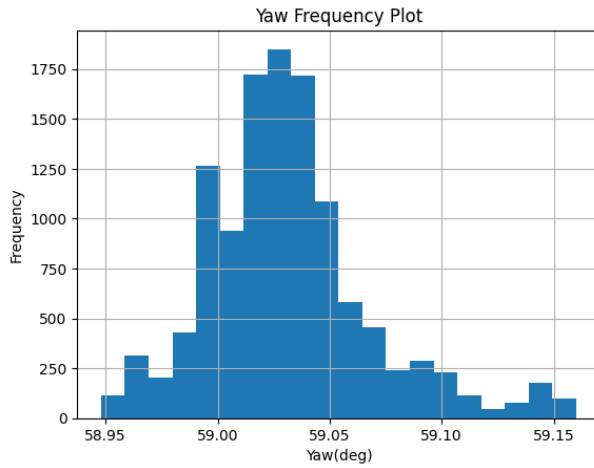


Fig 2.1 Yaw histogram

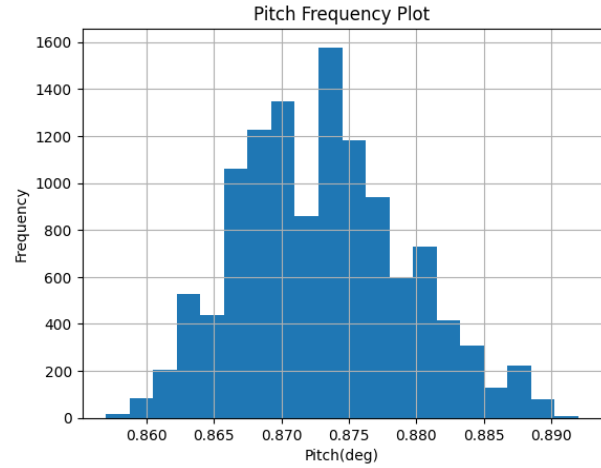


Fig 2.2 Pitch histogram

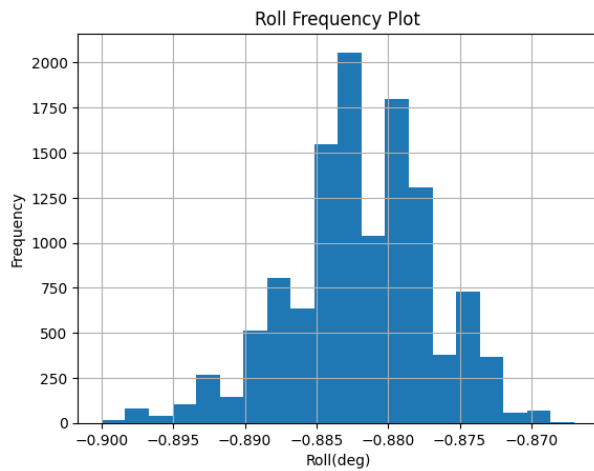


Fig 2.3 Roll histogram

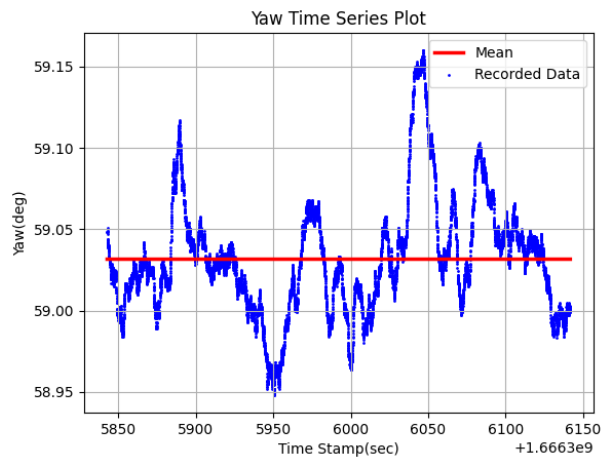


Fig 2.4 Yaw against Time

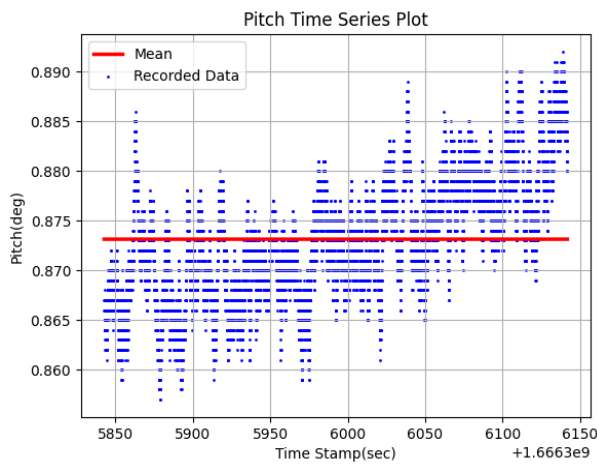


Fig 2.5 Pitch against Time

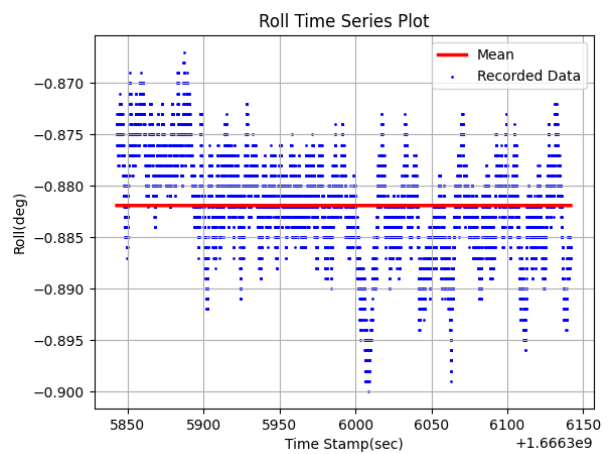


Fig 2.6 Roll against Time

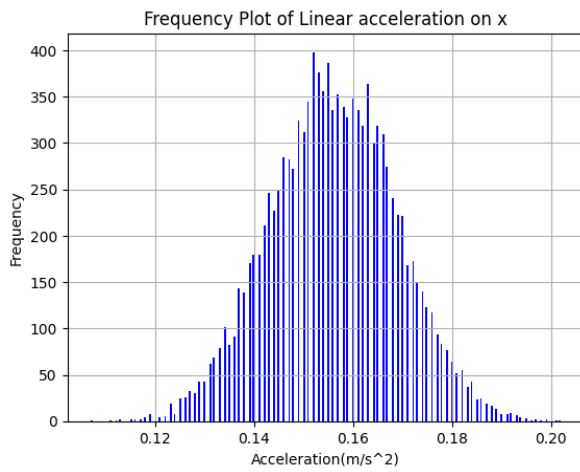


Fig 2.7 Histogram of Lin. Acceleration on x

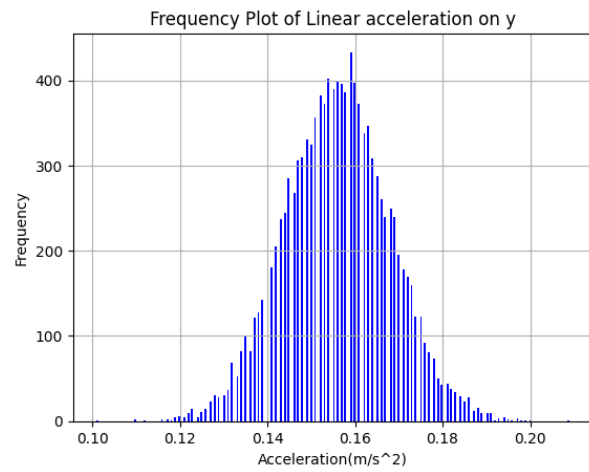


Fig 2.8 Histogram of Lin. Acceleration on y

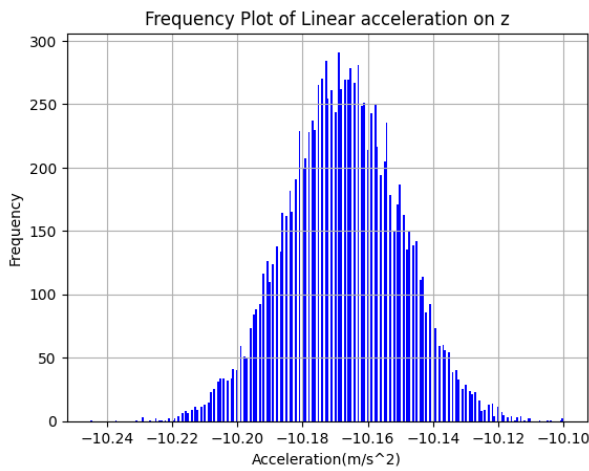


Fig 2.9 Histogram of Lin. Acceleration on z

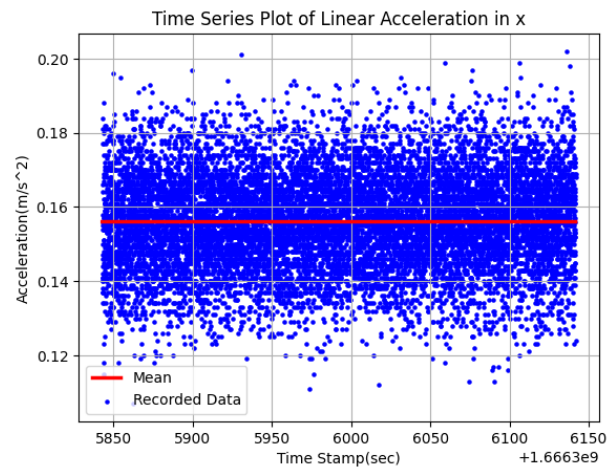


Fig 2.10 Lin. Acceleration on x against time

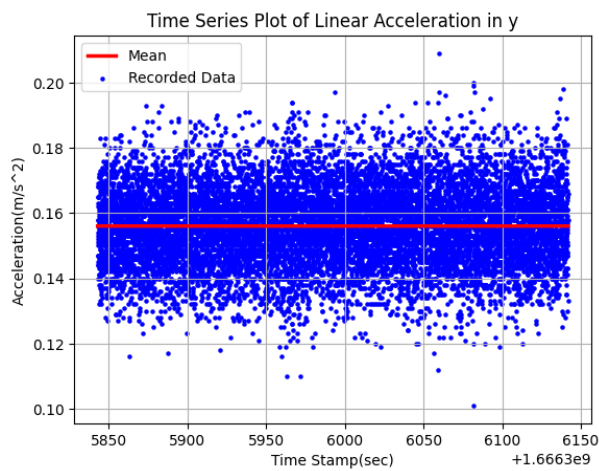


Fig 2.11 Lin. Acceleration on y against time

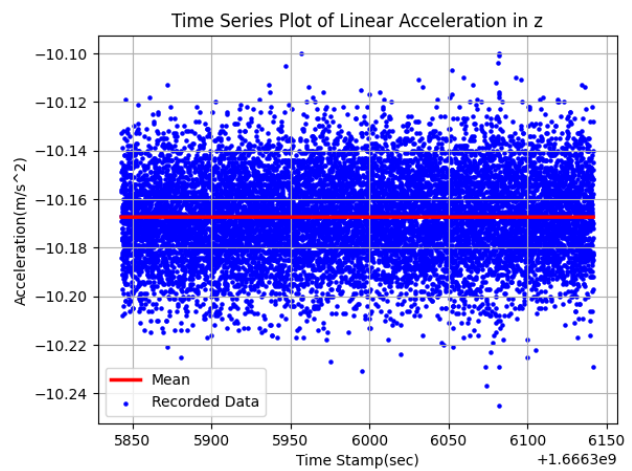


Fig 2.12 Lin. Acceleration on z against time



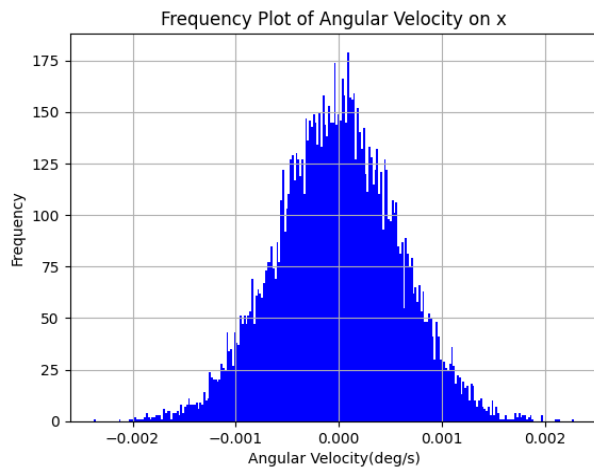


Fig 2.13 Histogram of Ang. Velocity on x

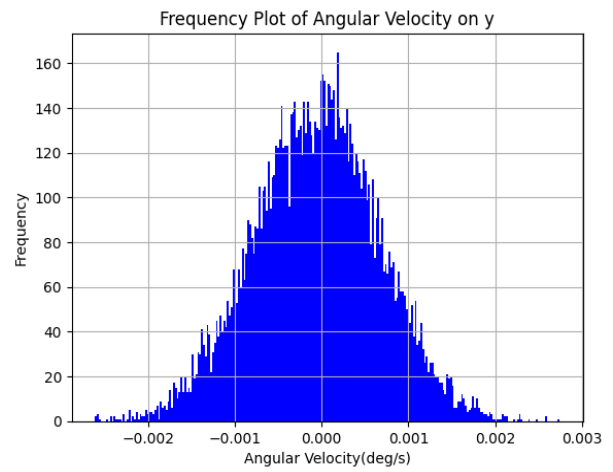


Fig 2.14 Histogram of Ang. Velocity on y

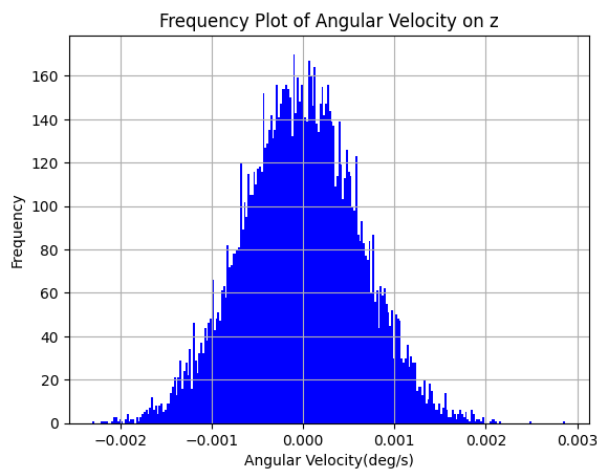


Fig 2.15 Histogram of Ang. Velocity on z

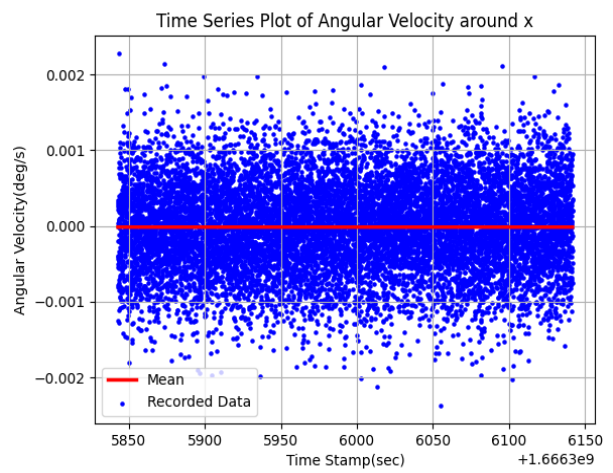


Fig 2.16 Ang. Velocity on x against time

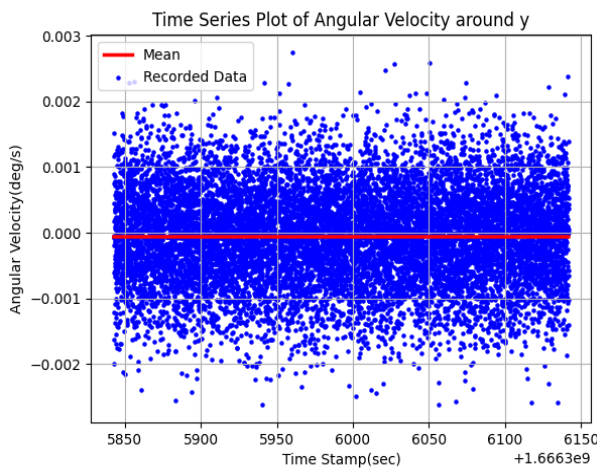


Fig 2.17 Ang. Velocity on y against time

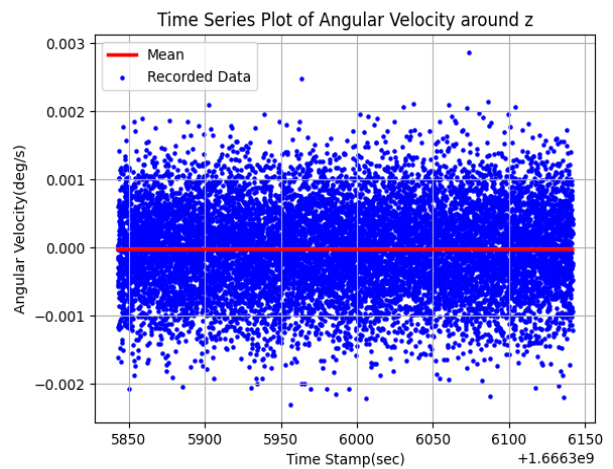


Fig 2.18 Ang. Velocity on z against time

Error characteristics for the data were as follows:

Quantity	Mean	Standard Deviation
Yaw	59.0315°	0.0368°
Pitch	0.8731°	0.0062°
Roll	-0.8819°	0.0051°
Linear Acceleration in x	0.1560 m/s <sup>2</sup>	0.0128 m/s <sup>2</sup>
Linear Acceleration in y	0.1560 m/s <sup>2</sup>	0.0120 m/s <sup>2</sup>
Linear Acceleration in z	-10.1674 m/s <sup>2</sup>	0.0176 m/s <sup>2</sup>
Angular Velocity in x	-1.6020*10 <sup>-5</sup> °/s	5.889*10 <sup>-4</sup> °/s
Angular Velocity in y	-6.245*10 <sup>-5</sup> °/s	7.218*10 <sup>-4</sup> °/s
Angular Velocity in z	-2.415*10 <sup>-5</sup> °/s	6.381*10 <sup>-4</sup> °/s

## Inferences

The following inferences were made from the plots:

- The quaternions were converted back to Euler angle in degrees for better understanding.
- Linear acceleration in z is seen to have a high negative value. This is because of the acceleration due to gravity, which is around  $g = 9.8 \text{ m/s}^2$ .
- Yaw has a higher value, despite keeping the IMU stationary. This is caused by the Earth's rotation.
- Pitch, roll, linear acceleration(in the x and y axis) and angular velocities have mean values close to zero. The bar widths have been adjusted to show their nature as gaussian distributions.
- These bell-shaped plots imply that the noise these sensor measurements suffer is purely gaussian and hence, it can be concluded that the errors mostly originate from random walk errors.
- Since the IMU was kept stationary during the 5 minutes, we can assume that these values are not exactly zero because of drift errors that are characterised by the bias instability and random walk noises obtained from the Allan deviation plots. These can be adjusted with periodic calibrations.
- Some errors could also be attributed to the wood sway of the building and vibrations caused by the nearby railway stations. These could have caused minor vibrations on the table where the IMU was placed. For better readings, it can be concluded that the IMU should be placed on the ground, preferably on a cushioned surface.



## Results

Deviations in the stationary data of the IMU were explored and attributed to different reasons. An Allan deviation analysis of the IMU was also carried out and its plots were obtained along with the noise characteristics slightly higher than those described in the datasheet. These sensor measurements give gaussian distributions around their estimated or validated true values and their errors can be attributed to the noise factors explored in the Allan deviation plots.

## Conclusion

Allan variance method was used on stationary IMU data of 5 hours and an analysis was also done of a 5-minute bag of IMU recordings to explore error causes and values that relate to the Allan deviation plots.

## References

- [1] Barreda Pupo, Leslie. *Characterization of errors and noises in MEMS inertial sensors using Allan variance method*. MS thesis. Universitat Politècnica de Catalunya, 2016.
- [2] El-Sheimy, Naser, Haiying Hou, and Xiaoji Niu. "Analysis and modelling of inertial sensors using Allan variance." *IEEE Transactions on instrumentation and measurement* 57.1 (2007): 140-149.
- [3] M. Matejček and M. Šostronek, "New experience with Allan variance: Noise analysis of accelerometers," 2017 Communication and Information Technologies (KIT), 2017, pp. 1-4, doi: 10.23919/KIT.2017.8109457.