ELEC 4700

Assignment 2

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Firstly, finite difference method is used in a matrix form to solve the electrostatic potential. The electrostatic potential in a specified rectangular region will be coded and measured. Firstly, a simple case with simple boundary conditions were coded, where top and bottom conditions are not fixed. The code below is shown on how the boundary conditions were set up:

```
for i = 1:nx
    for j = 1:ny
        n = j + (i-1)*ny;
        if i == 1
            G(n,:) = 0;
            G(n,n) = 1;
            F(n) = 1;

        elseif i == nx
            G(n,:) = 0;
            G(n,n) = 1;
            F(n) = 0;

        elseif j == 1
            G(n,:) = 0;
            G(n,n) = -3;
            G(n,n+1) = 1;
            G(n,n-ny) = 1;
            G(n,n+ny) = 1;

        elseif j == ny
            G(n,n) = -3;
            G(n,n-1) = 1;
            G(n,n-ny) = 1;
            G(n,n+ny) = 1;

        else
            G(n,n) = -4;
            G(n,n-1) = 1;
            G(n,n+1) = 1;
            G(n,n-ny) = 1;
            G(n,n+ny) = 1;
        end
    end
end
```

This resulted in plot of V(x) shown below

a)

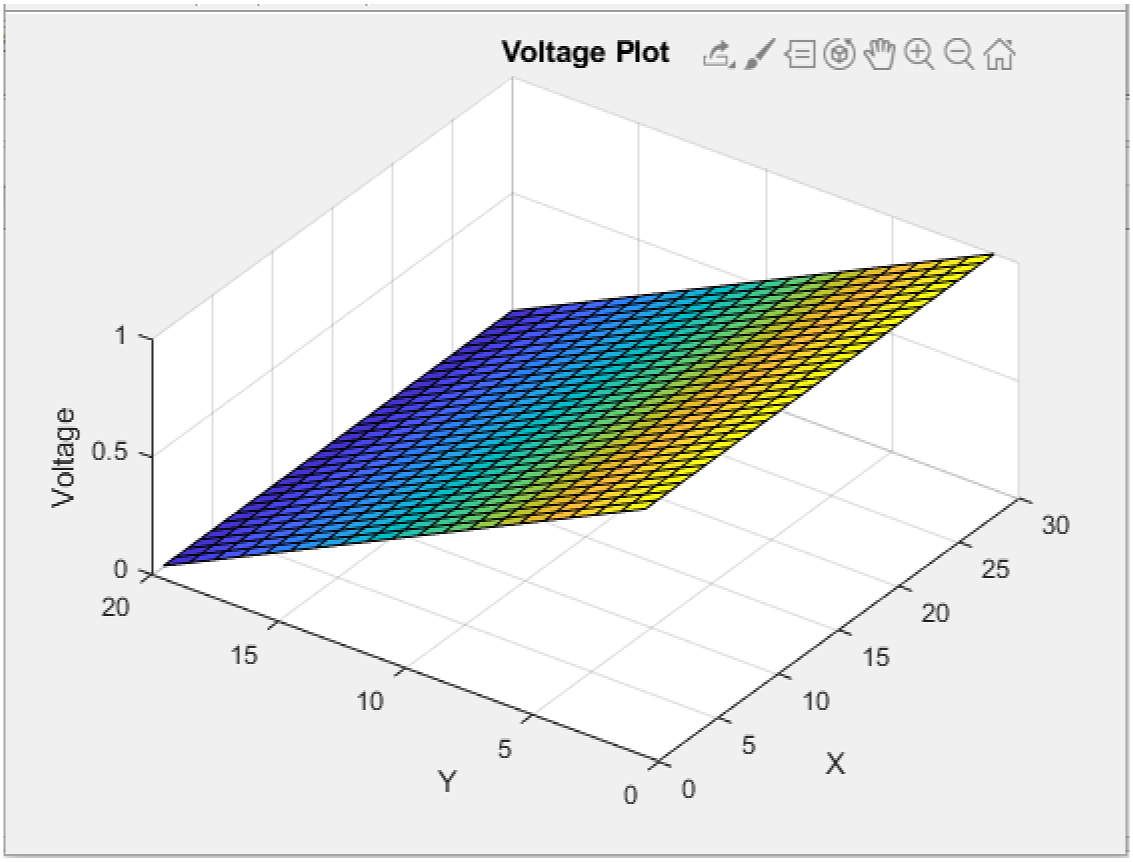

Figure 1: V(x) plot

b)

Secondly, using different boundary conditions are used and the solution of a bunch of mesh siezs and analytics series solutions were coded and plotted. The boundary conditions were set in the code below:

```
for i = 1:nx
    for j = 1:ny
        n = j + (i-1)*ny;
        if i == 1
            G(n,:) = 0;
            G(n,n) = 1;
            F(n) = 1;

        elseif i == nx
            G(n,:) = 0;
            G(n,n) = 1;
            F(n) = 1;

        elseif j == 1
            G(n,:) = 0;
            G(n,n) = 1;

        elseif j == ny
            G(n,:) = 0;
            G(n,n) = 1;
```

```
        else
            G(n,n) = -4;
            G(n,n-1) = 1;
            G(n,n+1) = 1;
            G(n,n-ny) = 1;
            G(n,n+ny) = 1;
        end
    end
end
```

the numerical plot is shown below in figure 2:

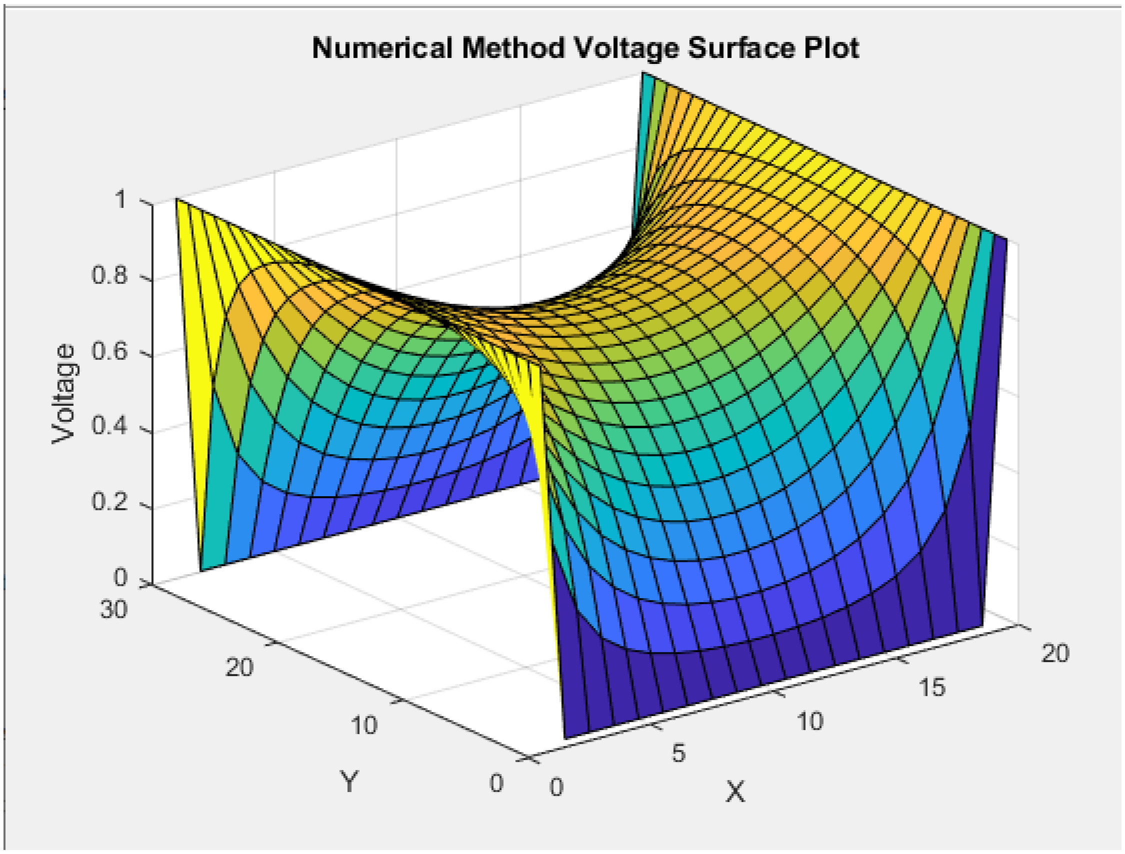

Figure 2: Numerical Voltage surface plot

The figure below shows the analytical method:

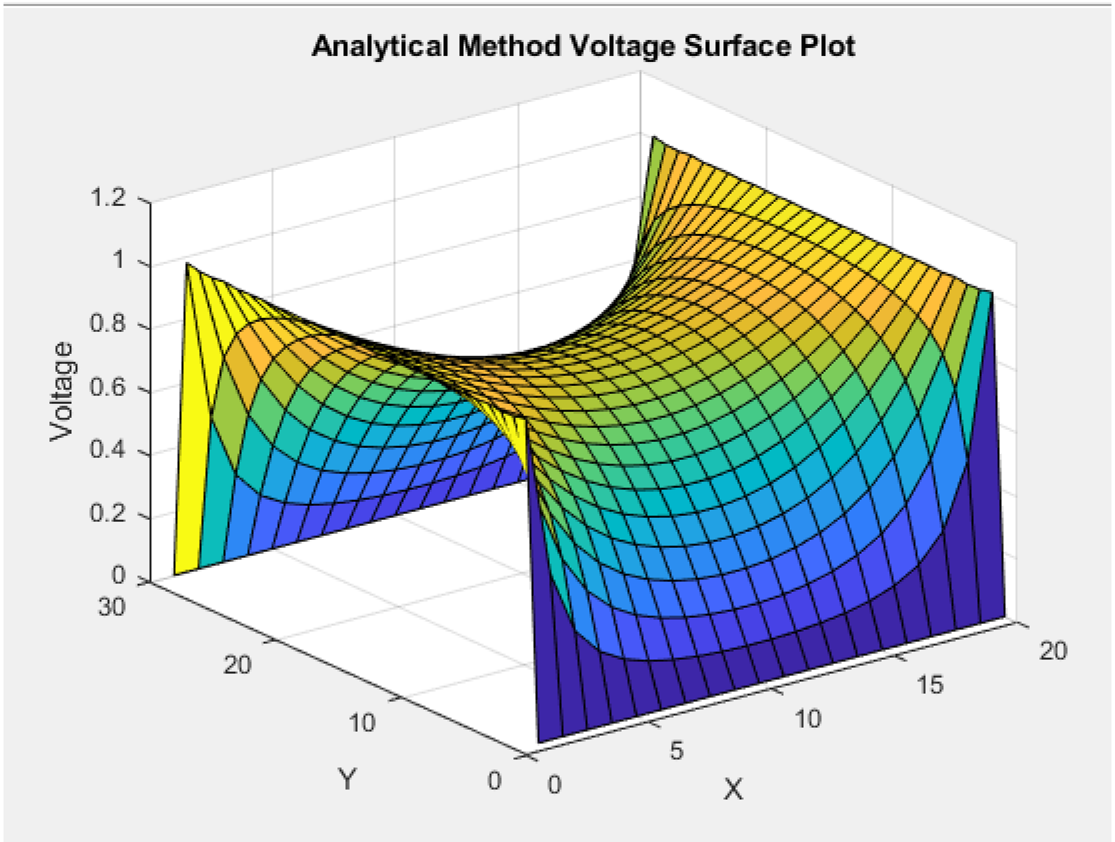

Figure 3: Analytical method voltage surface

When iterating to a higher number, the analytical method could recreate the Finite difference solution. But going too high or too low will not let it be comparable to the true form. The numerical solution is best used when the inputs are not complicated. Using the numerical solution with complex equations will be very difficult and will not work as effectively. For the analytical method, it is much quicker when computing simple equations. When using the analytical method, it is required that the optimal iterations points must be found, or it will not be accurate.

2)

Next, a bottleneck like the one built in the previous assignment is applied to the rectangular system. To model the boxes, a low sigma value is used inside the boxes to acquire high resistivity.

a) Firstly, the current flow at the contacts were calculated and various plots were generated:

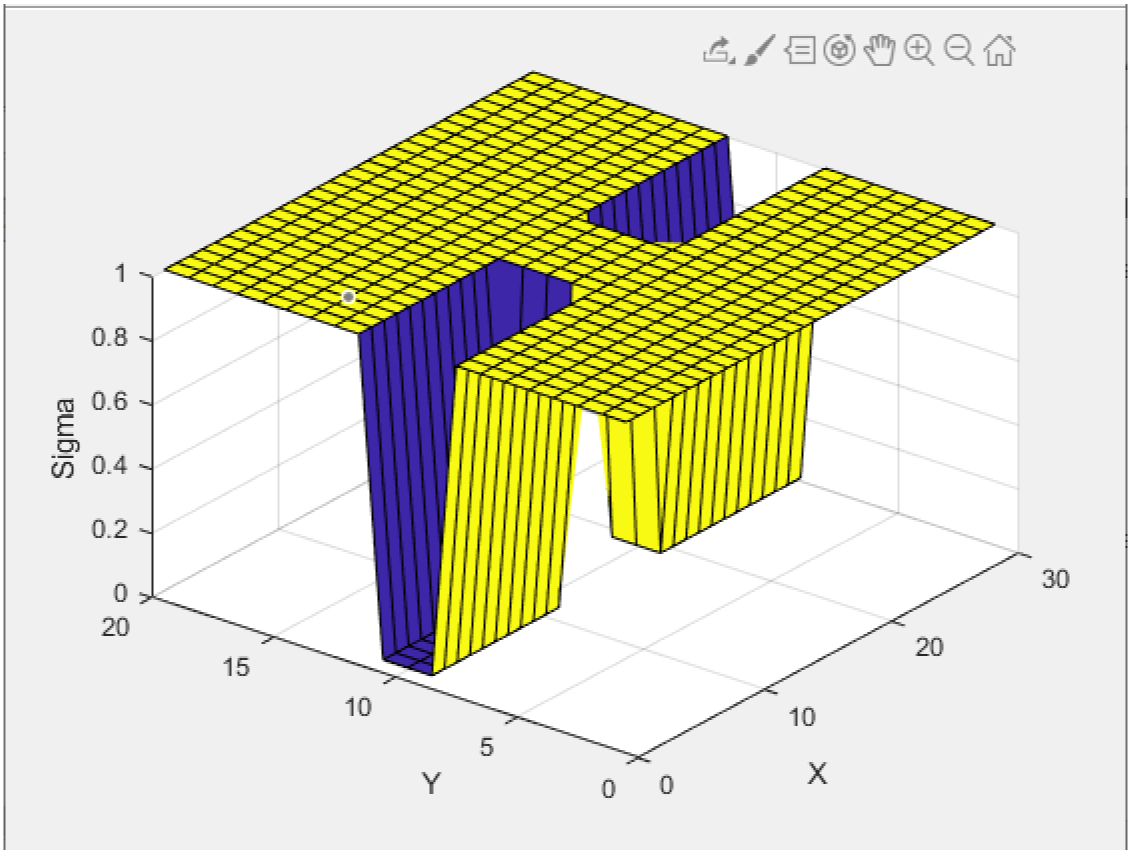

Figure 4: Sigma surface plot

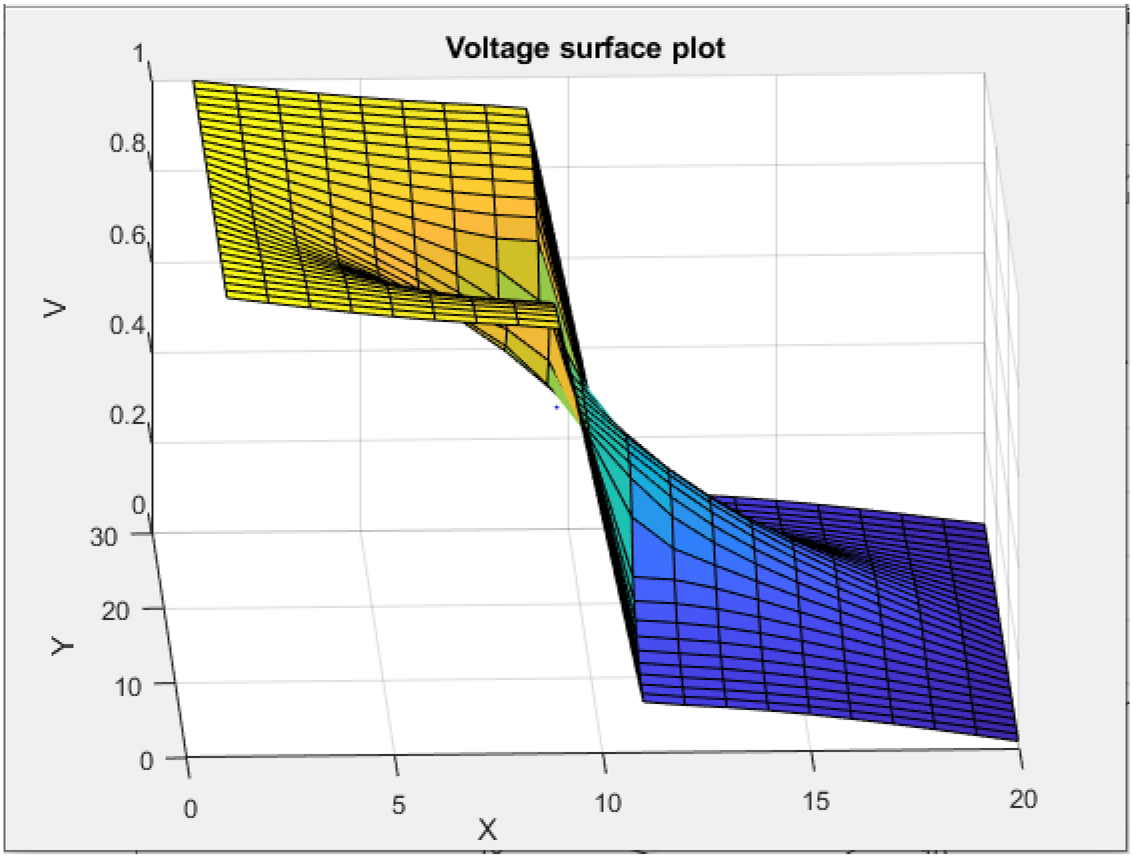

Figure 5: Voltage surface plot

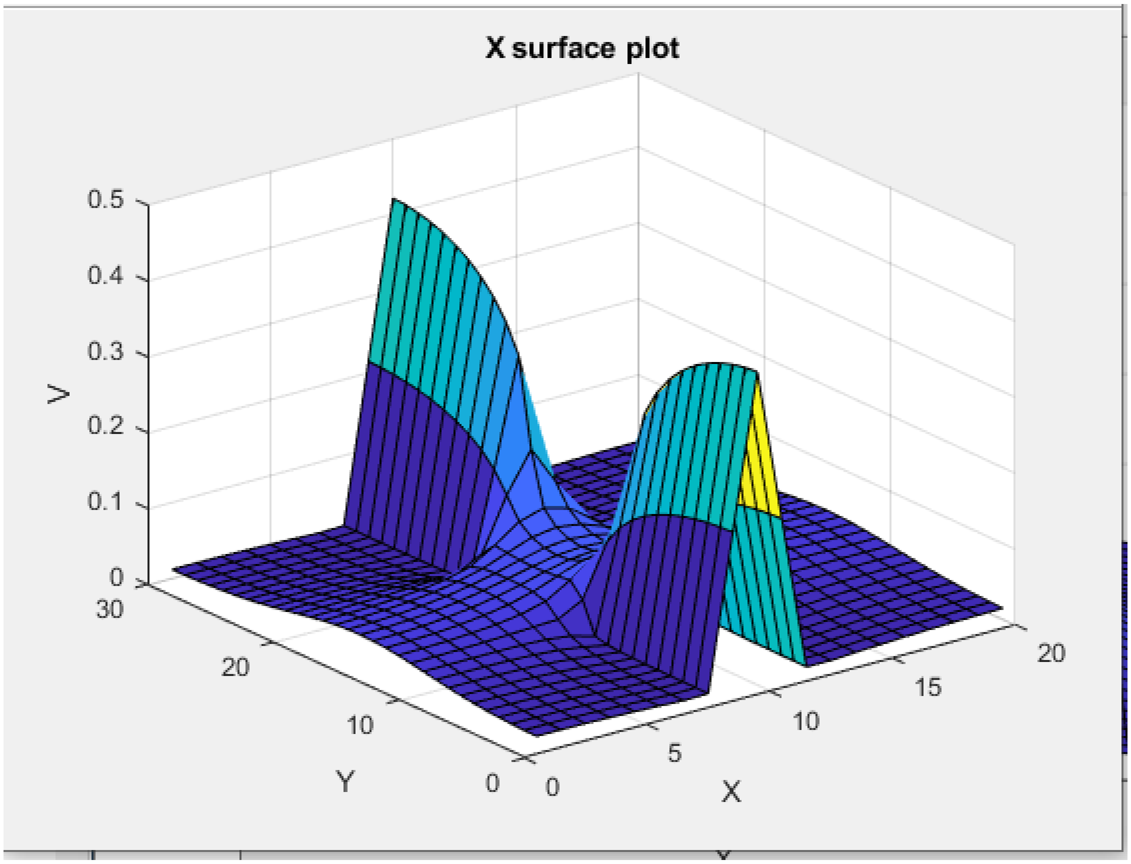

Figure 6: X-component electric field surface plot

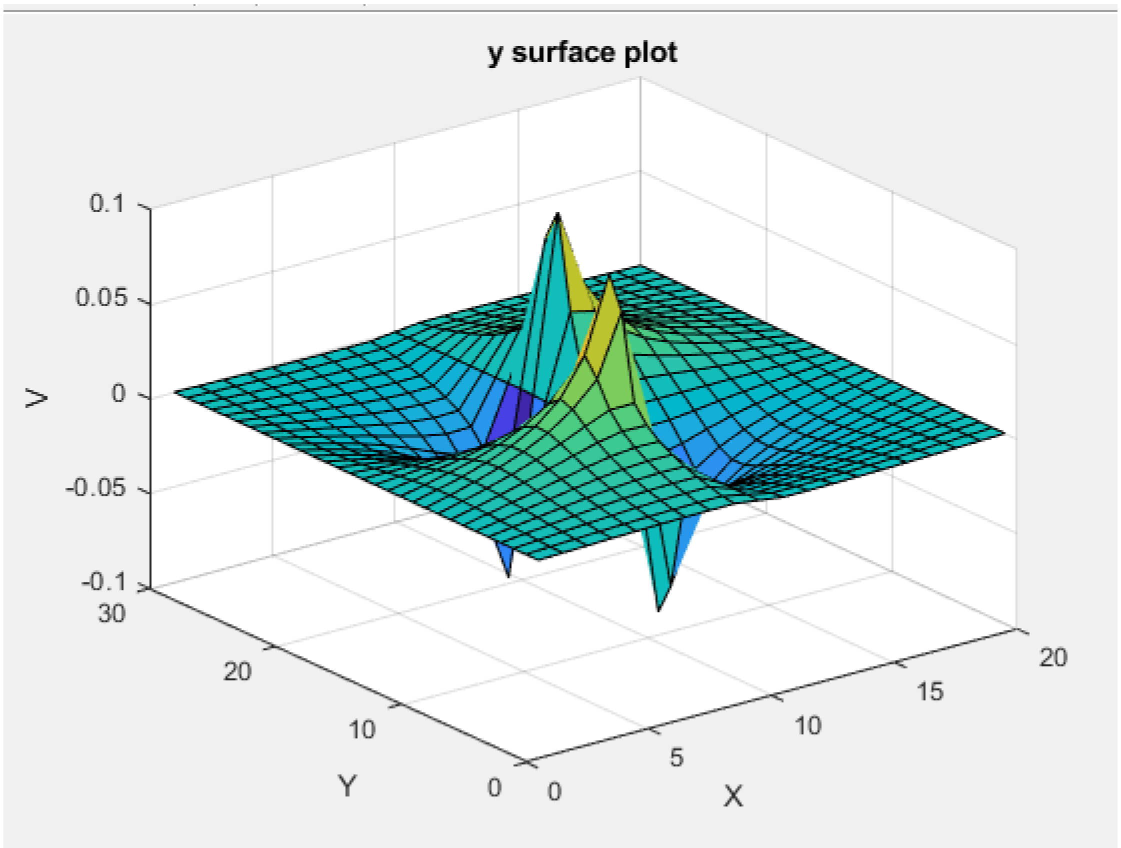

Figure 7: Y-component electric field surface plot

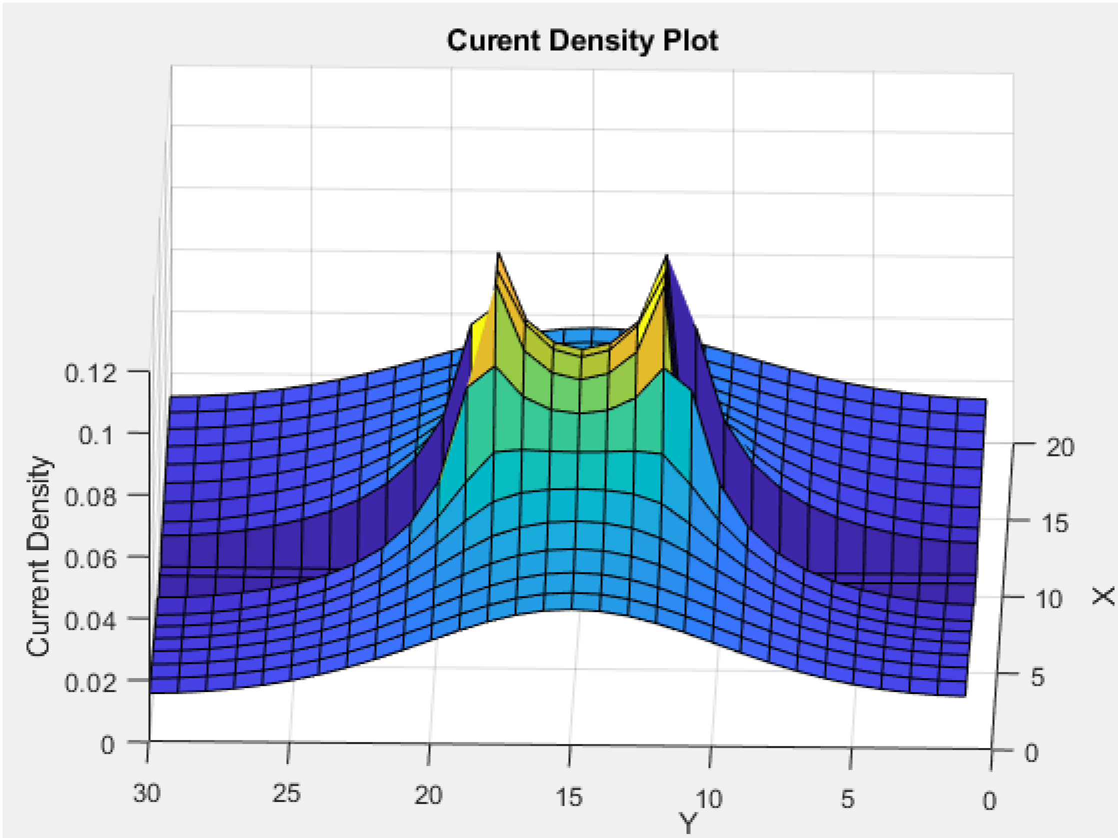

Figure 8: Current density surface plot

The boundary conditions were coded using the code below:

```
for x = 1:nx
    for y = 1:ny
        n = y + (x-1)*ny;
        nxp = y + (x+1-1)*ny;
        nxx = y + (x-1-1)*ny;
        nyp = y + 1 + (x-1)*ny;
        nnegy = y - 1 + (x-1)*ny;
        if x == 1
            G(n, :) = 0;
            G(n, n) = 1;
            F(n) = 1;
        elseif x == nx
            G(n, :) = 0;
            G(n, n) = 1;
            F(n) = 0;
        elseif y == 1
            G(n, nxp) = (sigma(x+1, y) + sigma(x,y))*(1/2);
            G(n, nxx) = (sigma(x-1, y) + sigma(x,y))*(1/2);
            G(n, nyp) = (sigma(x, y+1) + sigma(x,y))*(1/2);
            G(n, n) = -(G(n,nxp)+G(n,nxx)+G(n,nyp));
        elseif y == ny
            G(n, nxp) = (sigma(x+1, y) + sigma(x,y))*(1/2);
            G(n, nxx) = (sigma(x-1, y) + sigma(x,y))*(1/2);
            G(n, nnegy) = (sigma(x, y-1) + sigma(x,y))*(1/2);
            G(n, n) = -(G(n,nxp)+G(n,nxx)+G(n,nnegy));
```

```
        else
            G(n, nyp) = (sigma(x, y+1) + sigma(x,y))*(1/2);
            G(n, nnegy) = (sigma(x, y-1) + sigma(x,y))*(1/2);
            G(n, nxp) = (sigma(x+1, y) + sigma(x,y))*(1/2);
            G(n, nxx) = (sigma(x-1, y) + sigma(x,y))*(1/2);
            G(n, n) = -(G(n,nxp)+G(n,nxx)+G(n,nyp)+G(n,nnegy));
        end
    end
end
```

b) Next, mesh density was investigated to see how the number of meshes will affect current density.
   To plot the mesh vs current density the code below was used:

```
if nx > 30
        oldValue = total;
        total = sum(sum(currDensity, 2));
        plot([nx-30, nx], [oldValue, total])
        xlabel("nx")
        ylabel("Current Density")
    end
    if nx == 30
        total = sum(sum(currDensity, 1));
        oldValue = total;
        plot([nx, nx], [oldValue, total])
    end
end
```

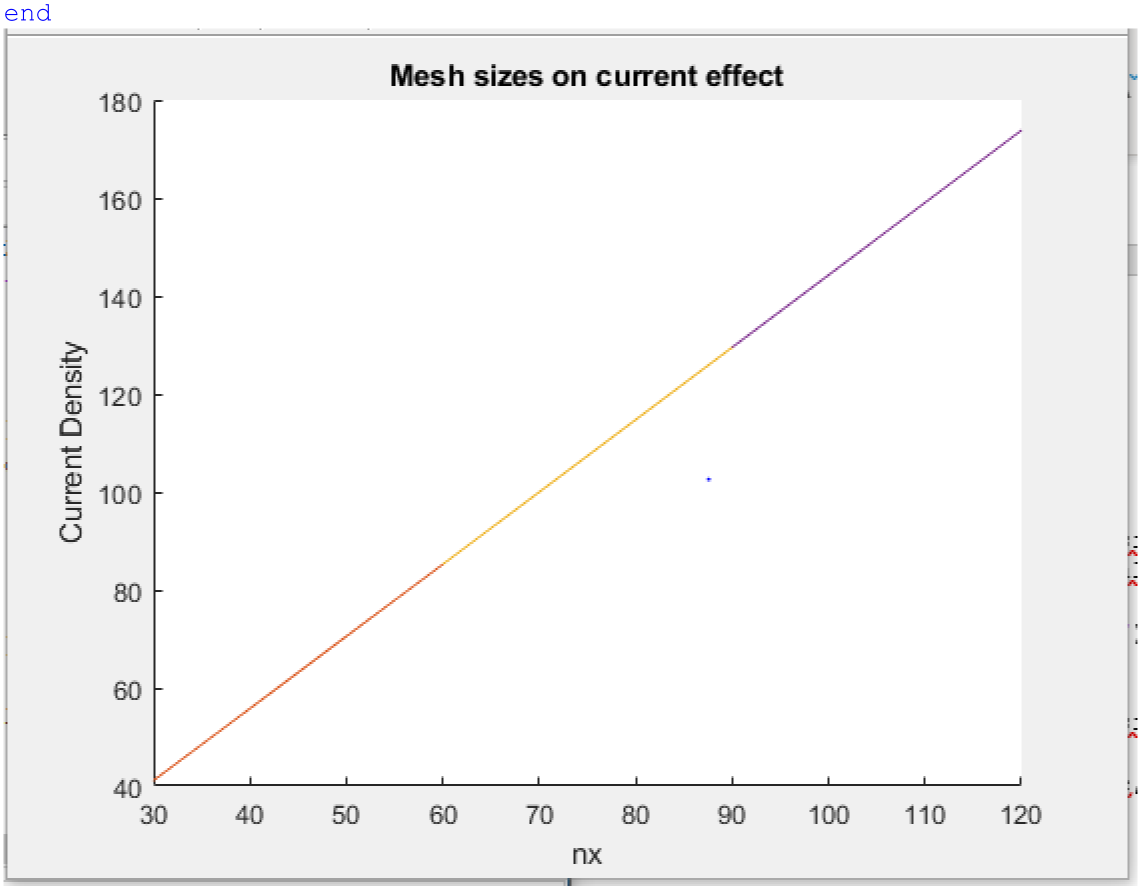

Figure 9: mesh size effect on current density

It is seen above that the size of the mesh is proportional to the current density of the system as expected. The higher the size of mesh, the higher the current density will become.

c) Next, the narrowing of the bottleneck is investigated to see how changing the bottleneck will change the current density.

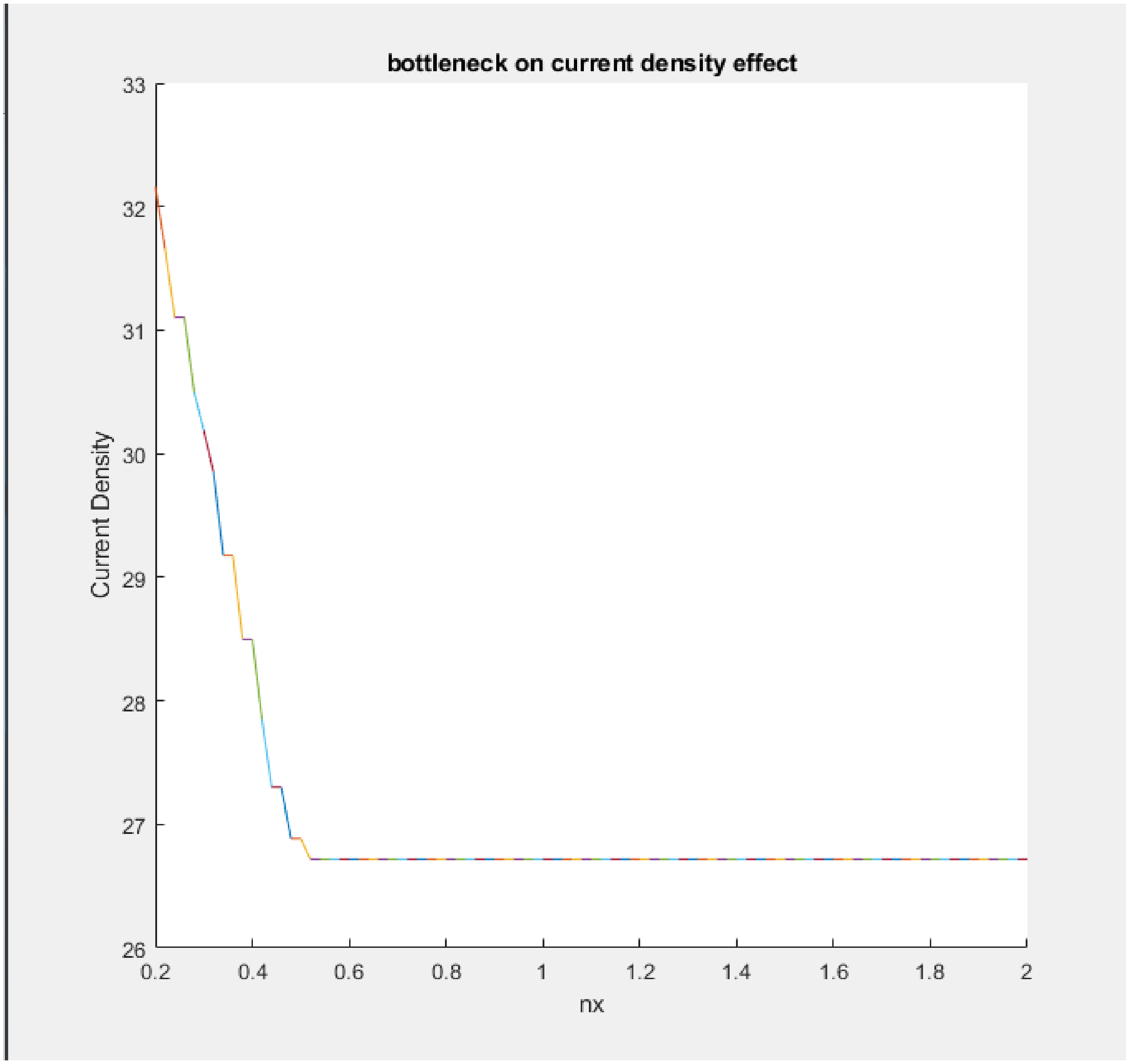

Figure 10: bottleneck effect on current density

The plot above shows how decreasing the bottleneck will directly affect the current density. While increasing the bottleneck, the current density appears to decrease substantially. There comes a point where the bottleneck is wide that the current density does not decrease anymore and just averages out.

d) The last graph shows the current vs sigma

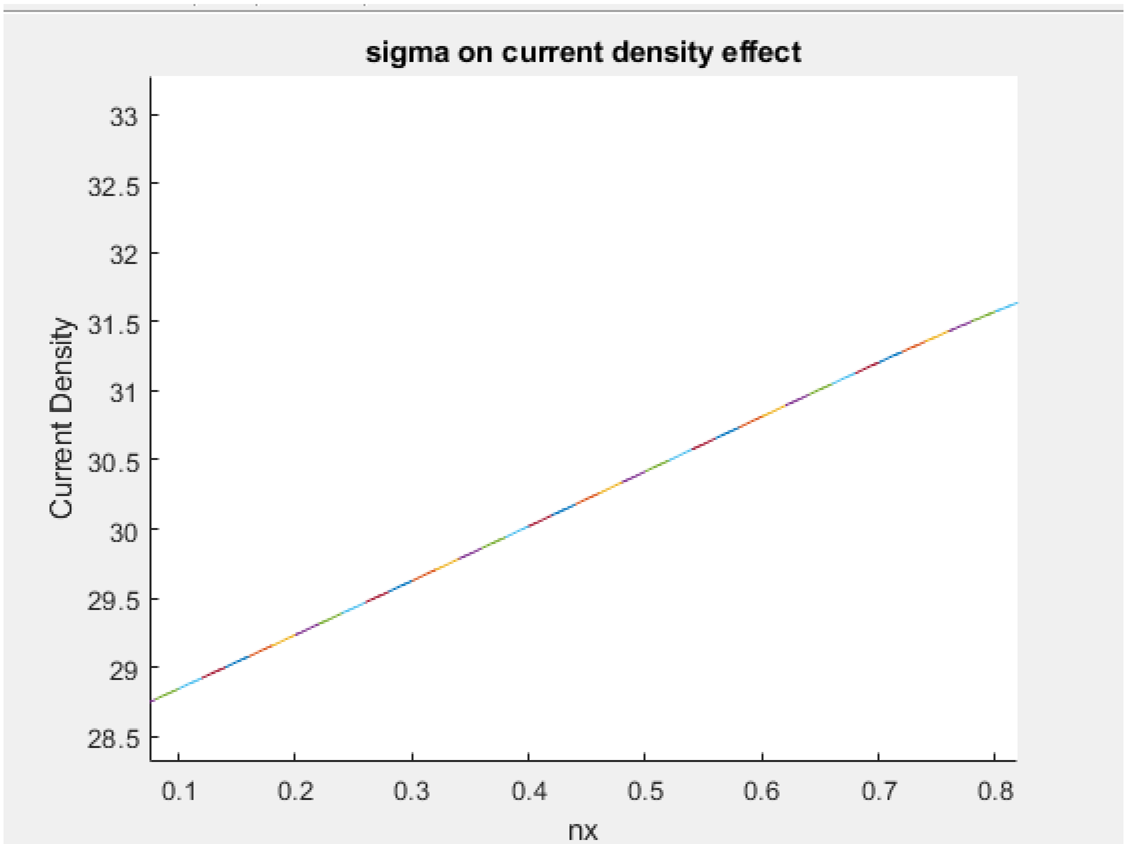

Figure 11: Sigma effect on the current density

The plot above shows a linear relationship between sigma and the current density which is expected since the equation is linear. Increasing sigma will cause the current density to increase proportionally.