PS1, Question 3, part a) i)

Derivation of matrix equations:

For a pair of vanishing points, $\mathbf{v}_i$ and $\mathbf{v}_j$, corresponding to 2 sets of parallel lines such that lines from the set meeting at vanishing point $\mathbf{v}_i$ are orthogonal to lines from the set vanishing at $\mathbf{v}_j$, the following is true:

$$0 = \mathbf{v}_i^T \boldsymbol{\omega} \mathbf{v}_j$$

Using the notation v_i^x to mean the x component of vanishing point index *i* and v_j^y for the y component of vanishing point index *j* and so forth, this can be written:

$$0 = \begin{pmatrix} \mathrm{v}_i^x \\ \mathrm{v}_i^y \\ 1 \end{pmatrix}^T \begin{pmatrix} \omega_1 & 0 & \omega_2 \\ 0 & \omega_1 & \omega_3 \\ \omega_2 & \omega_3 & \omega_4 \end{pmatrix} \begin{pmatrix} \mathrm{v}_j^x \\ \mathrm{v}_j^y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathrm{v}_i^x \\ \mathrm{v}_i^y \\ 1 \end{pmatrix}^T \begin{pmatrix} \omega_1 \mathrm{v}_j^x + \omega_2 \\ \omega_1 \mathrm{v}_j^y + \omega_3 \\ \omega_2 \mathrm{v}_j^x + \omega_3 \mathrm{v}_j^y + \omega_4 \end{pmatrix}$$

$$= \begin{pmatrix} \mathrm{v}_i^x \\ \mathrm{v}_i^y \\ 1 \end{pmatrix}^T \begin{pmatrix} \mathrm{v}_j^x & 1 & 0 & 0 \\ \mathrm{v}_j^y & 0 & 1 & 0 \\ 0 & \mathrm{v}_j^x & \mathrm{v}_j^y & 1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix}$$

$$= \begin{pmatrix} \mathrm{v}_i^x \mathrm{v}_j^x + \mathrm{v}_i^y \mathrm{v}_j^y & | & \mathrm{v}_i^x + \mathrm{v}_j^x & | & \mathrm{v}_i^y + \mathrm{v}_j^y & | & 1 \end{pmatrix} \boldsymbol{w}, \qquad \text{where } \boldsymbol{w} \equiv \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix}$$

Using the 3 possible pairings of indices 1,2,3 = (1,2), (1,3), (2,3), and stacking them together, gives:

$$\begin{pmatrix} \mathrm{v}_1^x \mathrm{v}_2^x + \mathrm{v}_1^y \mathrm{v}_2^y & | & \mathrm{v}_1^x + \mathrm{v}_2^x & | & \mathrm{v}_1^y + \mathrm{v}_2^y & | & 1 \\ \mathrm{v}_1^x \mathrm{v}_3^x + \mathrm{v}_1^y \mathrm{v}_3^y & | & \mathrm{v}_1^x + \mathrm{v}_3^x & | & \mathrm{v}_1^y + \mathrm{v}_3^y & | & 1 \\ \mathrm{v}_2^x \mathrm{v}_3^x + \mathrm{v}_2^y \mathrm{v}_3^y & | & \mathrm{v}_2^x + \mathrm{v}_3^x & | & \mathrm{v}_2^y + \mathrm{v}_3^y & | & 1 \end{pmatrix} \boldsymbol{w} = \mathbf{0}$$

or in other words, $\mathbf{A}\boldsymbol{w} = 0$, which can be solved for $\boldsymbol{w}$ by taking the SVD of **A**.