Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Development of an RF resonator for a double junction ion trap

Semester Project

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Abstract

In this report I will describe the modeling, designing and testing of an RF helical resonator suitable for delivering RF signal to a double-junction ion trap in a cryogenic environment.

Contents

Chapter 1

Introduction

Quantum computing is an exciting and rapidly evolving field of modern science. One of the most promising implementations of a quantum computer is based on the ability to control and measure systems of trapped ions. Those ions are typically confined in Paul (RF) [14] traps by applying static and oscillating RF signals to the trap electrodes, which on average generates a confining electric potential.

1.1 Why do we need resonators?

One could potentially couple a radio frequency source directly to an ions' trap. However this creates the following challenges:

- noise from a source may contribute to heating of trapped ions [18]
- in order to maintain efficient cryostat cooling amount of generated heat within it should be minimized. In order to stabilize ions in the ion trap confining RF signal must have a reasonably high voltage amplitude. An RF resonator close to the signal consumption area allows to reduce the length of an actively heated (with accordance to the Joule–Lenz law [15]) high voltage wire.
- impedance mismatch between source and trap leads to an additional dissipation of RF power

These issues can be avoided by placing an amplifier close to the Paul trap, which would filter the incoming signal and output it with a voltage suitable for operating the trap. There are two available options: active and passive amplifiers. Core difference between them is that active amplifiers require an additional power source to function while passive amplifiers can be used solely with a source signal itself.

Active amplifiers perform better in terms of a voltage gain at room temperatures. However aim of this project is to create a resonator used within a cryogenic environment. Properties of transistors powering active amplifiers depend heavily on a combination of densities of free electrons and holes. Lowering the temperature tends to reduce [2] these densities significantly, turning semiconductors at room temperatures into practically dielectrics.

It leaves us with passive amplifiers (resonators).

1.2 Context of this project

This semester project aims to be a part of an attempt to create a scalable quantum computing architecture by Chiara Decaroli. It provides the following benefits compared to existing solutions:

- using subtractive laser writing to manufacture wafers eliminates misalignment effects by allowing a "self assembly" of different wafers
- double junction ion trap designed for parallel operations, Decoherence Free Subspace (DFS) ion transport across the junctions, and manipulation of long chains of ions
- potential integrated laser delivery through optical lensed fibers eliminates the need for bulky optics and custom objectives which limit scalability

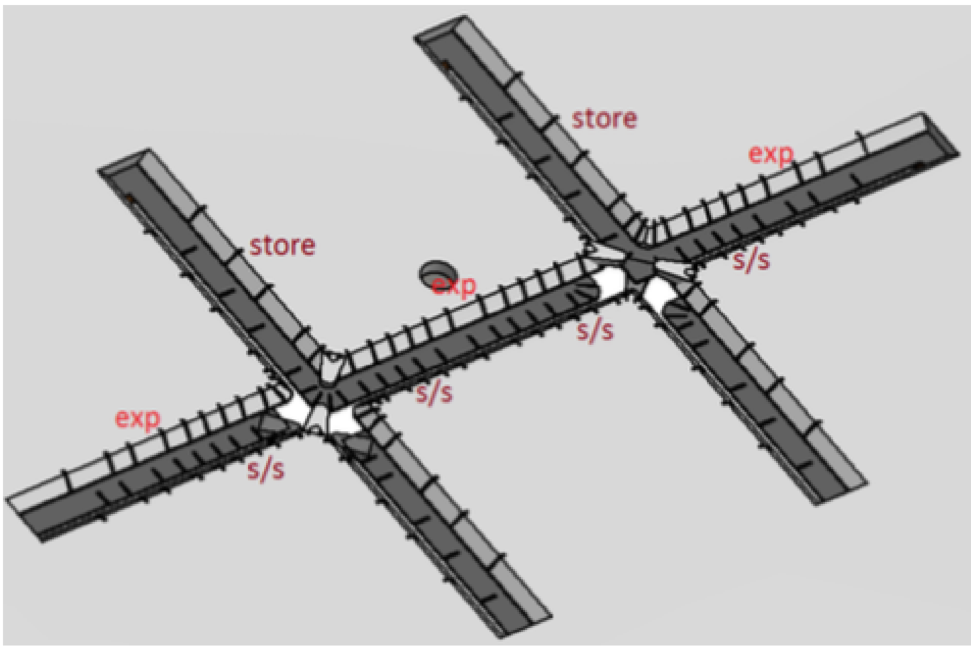

Figure 11: Ion trap overview (should be improved, just filling space now)

1.3 Kinds of resonators

The required frequency of 40 MHz limits our selection to the following types of resonators: helical [6, 7, 19, 10, 9] or, for higher frequencies, coaxial [8], RLC [4, 11, 5], and crystal oscillators. Multiple available solutions require us to do an analysis for a reasoned choice.

1.3.1 Helical

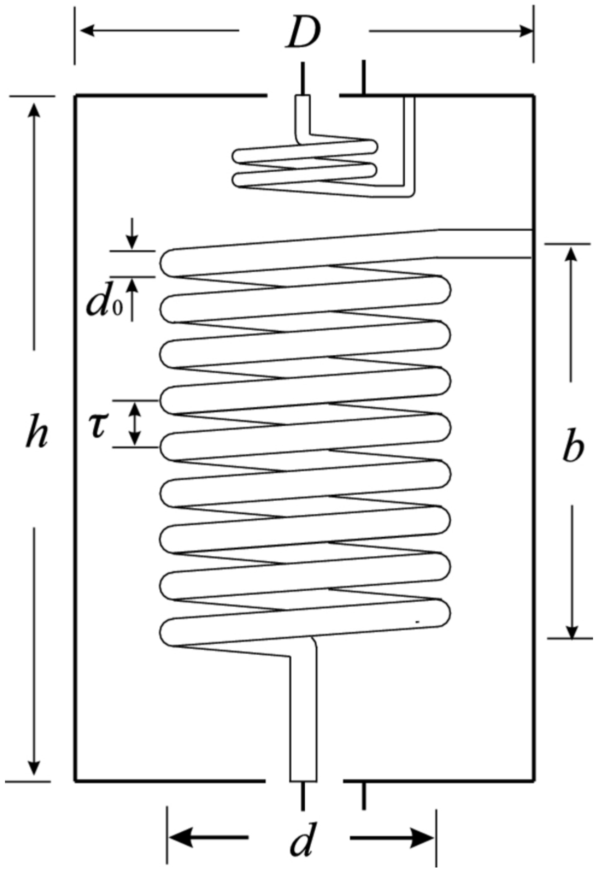

Figure 12: Schematic diagram of a helical resonator indicating shield diameter D, shield height h, coil diameter d, coil height b, winding pitch τ, and coil wire diameter d_0. [1]

Helical resonators are commonly selected to be coupled with ion traps due to their high quality factors and ability to operate on high frequencies. It is a perfect option for ion traps operated at room temperatures, since in absence of space constraints they are able to provide Q values of a couple thousands. However in order to achieve those Q, the fabrication process needs to be quite precise to avoid reflections of traveling waves which negatively influence the overall gain.

1.3.2 RLC

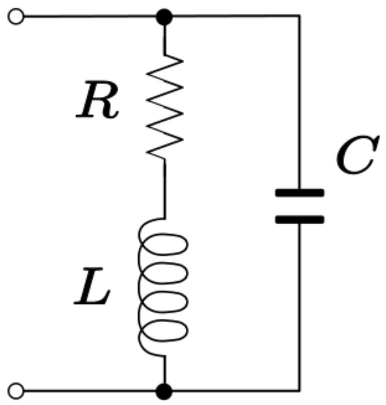

Figure 13: Example of a parallel RLC circuit

RLC amplifiers are a convenient choice for space-bounded environments, such as cryostats. Typical implementation pumps energy between two reactive components — inductor and capacitor. Assembly of RLC circuit is easier than of helical resonator since the physical placement of lumped parts does not influence the resulting quality factor. But it also means that the quality of these components is a major factor for successful creation. Given that units' data sheets rarely provide values for cryogenic setup it takes a lot of trial and error to find the right ones [4].

1.3.3 Crystal

Unlike helical and RLC resonators, the crystal oscillators do not store energy just in the electric field. This type or resonator utilizes piezoelectric effect to transform applied harmonic voltage into surface mechanical modes and vice versa.

Narrow excitation spectrum is provided by physical dimensions imposing hard constraints on vibrational oscillations and could have made such device an ideal filtering solution for ions traps. Unfortunately, there are some major downsides that seriously limit its applicability:

- after fabrication resonant frequency can not be widely tuned
- limited stability of the crystal does not allow high voltages

1.3.4 Choosing the right one

In our setup, the combination of high voltage and frequency values with our constraints on available space makes helical resonator the optimal option. However, difficulties of assembly do not make it a perfect solution in terms of scalability — for a production-grade setup RLC amplifier might be preferred.

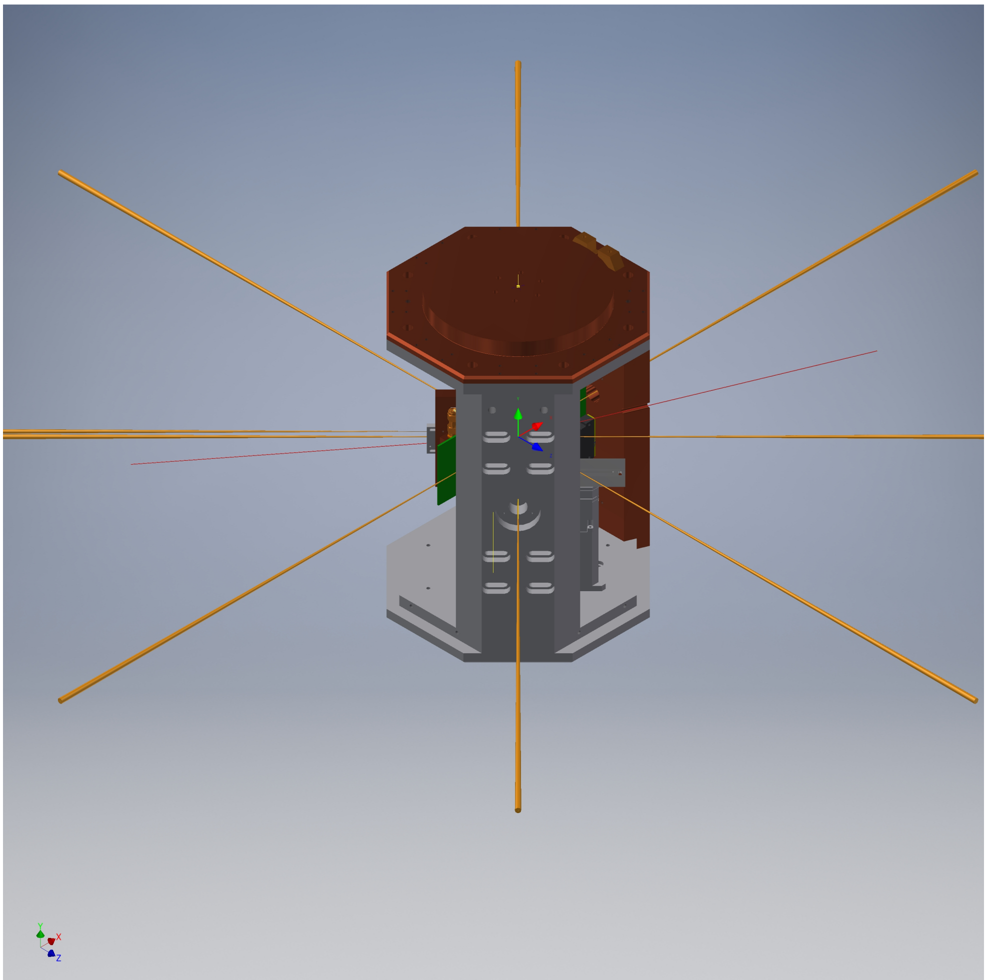

Figure 14: 3D model of a 4K cryostat chamber

Chapter 2

Theory

2.1 Helical resonator models

In order to create a helical resonator satisfying our experimental conditions and limitations we inevitably come to a need for a theoretical model that would be able to predict the essential characteristics of the resulting unit. The following sections aim to provide an overview and comparison between the models.

2.1.1 Macalpine & Schildknecht

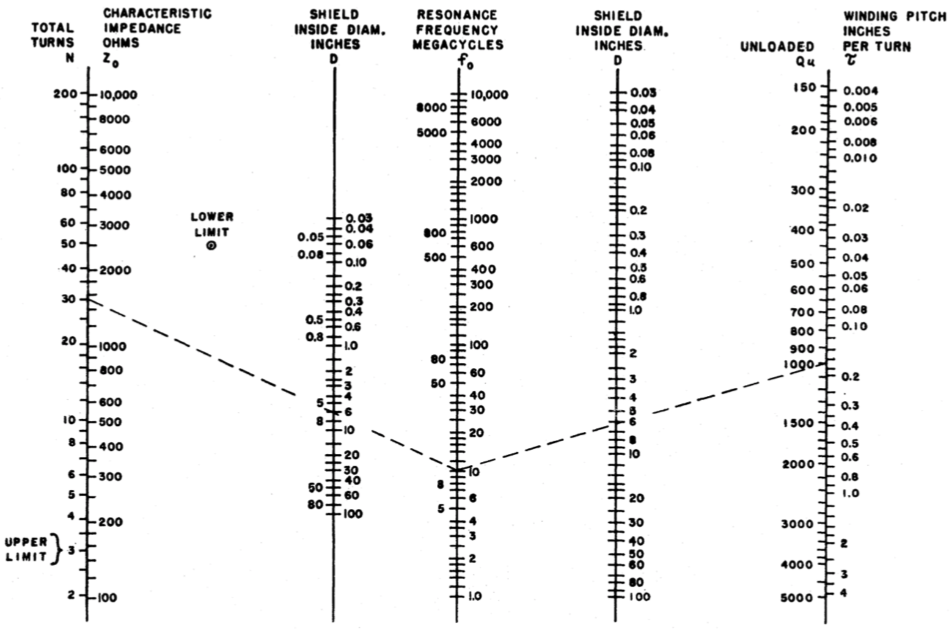

Figure 21: Design chart for quarter-wave helical resonators [13]

A well-known approach [13] for describing helical resonators was introduced in the same year as Richard Feynman's idea [3] to use quantum systems for computations. It was motivated by the possibility to reduce volume compared to TEM-mode coaxial-line resonators (90% volume reduction for the reference case [13]). While skipping a detailed theoretical analysis it nevertheless provides a basis for constructing a resonator: such as regions of usefulness, design considerations and a set of parameters' dependencies maximizing Q.

While describing essential properties of an unloaded helical quarter-wave resonator this paper [13] also predicts the shift of resonant frequency if an external load is connected. In order to define a new frequency one can make use of telegraph equations [16] by effectively treating the ion trap as a capacitor.

Important results of [13] can be found in equations 2.1 with parameters named as in the figure 12.

$$\begin{aligned} d/D &= 0.55 \\ b/d &= 1.5 \\ h &\approx (b + D/2) \end{aligned} \tag{2.1}$$

2.1.2 Siverns et al.

Unfortunately modeling an ion trap as a pure capacitive load is not always accurate. Introducing resistive losses imposes an additional shift of resonant frequency which pushes the deviation from self-resonant frequency even further. It is possible to tune the strength of the inductive coupling between the antenna and the main coil to compensate this shift while losing in efficiency.

These limitations of Macalpine's & Schildknecht's [13] model have been overcome in a newer paper [17] which takes the development of an amplifier, specifically for the needs of quantum computing, one step further. By taking a look at the joint resonator + ion trap system as a whole it aims to predict the effective Q and frequency. This model ensures that it's possible to find optimal parameters for given experimental constraints.

2.2 Comparison

Macalpine's and Schildknecht's [13] model gives insides for designing a helical quarter-wave resonator with a given self-resonant frequency. However major shifts from it can be expected when connecting the ion trap. Siverns' et al. approach [17] investigates connections between various parameters in the total circuit. As a result one could create a resonator which implements a transfer function closer to a desired one. Considering these benefits the Siverns model [17] was selected.

Chapter 3

Design

3.1 On a quest to perfect parameters

3.1.1 First iteration

3.1.2 Precise fit

3.2 Real-world implementation

Chapter 4

Validation

Chapter 5

External circuits & additional features

Appendix A

Mathematica code for Macalpine's model

This is a relevant part of the script supporting calculations in [12].

```
In[1]:= (* Physical Constants and Units *)
μ0 = 4*π*10^-7;
ε0 = 8.854187817*10^-12;
c = 299792458;
AMU = 1.6605402*10^-27;
h = 6.6260755*10^-34;
ħ = h/(2π);
μB = 9.2740154*10^-24;
kB = 1.380658*10^-23;
grav = 9.8;
a0 = 0.5291772108*10^-10;
me = 9.1093826*10^-31;
ee = 1.60217733*10^-19; eV = ee;
Eh = ħ^2/(me a0^2);
m=1; μm=10^-6m; mm=10^-3m; cm=10^-2m; nm=10^-9m; km=10^3m;
in=2.54cm; ft=12in; mi=5280*ft;
K=1; mK=10^-3K; μK=10^-6; nK=10^-9K; pK=10^-12;
T=1; mT=10^-3K; G=10^-4; mG=10^-3G; μG=10^-6;
sec=1; s=1; ms=10.0^-3s; μs=10^-6s; ns=10^-9s;
Ω=1; mΩ=10^-3; kΩ=10^3; MΩ=10^6;
A=1; mA=10^-3A; μA=10^-6; nA=10^-9;
W=1; mW=10^-3; μW=10^-6; nW=10^-9;
Hz=1; kHz=10^3; MHz=10^6; GHz=10^9;
nF=10^-9; pF=10^-12;
nH=10^-9; pH=10^-12;
```

```
In[2]:= (* Calculations for Cu resonator *)
Ds[Bs_]:= 0.75Bs; (*diameter of shield*)
d[Bs_]:= 0.55Ds[Bs]; (*diameter of helix*)
b[Bs_]:= 1.5d[Bs]; (*length of helix*)
```

$$Nt[Bs\_,f0\_] := \frac{48.26*10^{6}}{f0\ Ds[Bs]}; \text{ (*number of turns*)}$$

$$d0[Bs\_,f0\_] := 0.5\frac{b[Bs]}{Nt[Bs,\ f0]}; \text{ (*diameter of wire*)}$$

$$\tau[Bs\_,f0\_] := \frac{b[Bs]}{Nt[Bs,\ f0]}; \text{ (*pitch*)}$$

(*effective inductance*)

$$Leff[Bs\_,f0\_] := 9.84*10^{-7}*\left(\frac{Nt[Bs,\ f0]\ d[Bs]}{b[Bs]}\right)^{2}*\left(1-\left(\frac{d[Bs]}{Ds[Bs]}\right)^{2}\right);$$

$$Ceff[Bs\_] := \frac{2.95*10^{-11}}{Log[10,\frac{Ds[Bs]}{d[Bs]}]}; \text{ (*effective capacitance*)}$$

$$Z0[Bs\_,f0\_] := \sqrt{\frac{Leff[Bs,f0]}{Ceff[Bs]}}; \text{ (*characteristic impedance*)}$$

$$v[Bs\_,f0\_] := \frac{1}{\sqrt{Leff[Bs,f0]\ Ceff[Bs]}}; \text{ (*velocity*)}$$

$$\lambda[Bs\_,f0\_] := \frac{v[Bs,f0]}{f0}; \text{ (*wavelength*)}$$

$$Q[Bs\_,f0\_] := 1.97\ Ds[Bs]\ \sqrt{f0};$$

$$omega[Bs\_,f0\_,Cl\_] := Module[\{w\},$$
$$\quad w = \nu /. FindRoot[\frac{1}{2\pi*Z0[Bs,f0]*Cl*\nu} == Tan[\frac{2\pi\nu\ b[Bs]}{v[Bs,f0]}],\{\nu,f0\}][[1]];$$
$$\quad Return@w;$$
$$]$$

```
In[3]:= Bs = 60mm; (*length of shield*)
(* This frequency needs to be varied
until loaded frequency is close to target frequency*)
f0 = 97.023MHz; (*center frequency*)
Cl = 10pF; (*trap capacitance*)

Print["Bs = ",Bs/mm," mm (length of shield)"]
Print["Ds = ",Ds[Bs]/mm," mm (diameter of shield)"]
Print["d = ",d[Bs]/mm," mm (diameter of helix)"]
Print["b = ",b[Bs]/mm," mm (length of helix)"]
Print["Nt = ",Nt[Bs,f0]," (number of turns)"]
Print["d0 = ",d0[Bs,f0]/mm," mm (diameter of wire)"]
```

```
Print["τ = ",τ[Bs,f0]/mm," mm (pitch of helix)"]
Print["Z0 = ",Z0[Bs,f0]/ω," ω (characteristic impedance)"]
Print["Leff = ",Leff[Bs,f0]/(nH/mm)," nH/mm (effective inductance)"]
Print["Ceff = ",Ceff[Bs]/(pF/mm)," pF/mm (effective capacitance)"]
Print["v = ",v[Bs,f0]/(m/s)," m/s (velocity)"]
Print["λ = ",λ[Bs,f0]/mm," mm (wavelength)"]
Print["f0 = ",f0/MHz," MHz (unloaded frequency)"]
Print["Q = ",Q[Bs,f0]," (quality factor)"]
Print["ν0 = ",omega[Bs,f0,Cl]/MHz," MHz (loaded frequency)"]

Clear[Bs,f0,Cl];
```

```
Bs = 60 mm (length of shield)

Ds = 45. mm (diameter of shield)

d = 24.75 mm (diameter of helix)

b = 37.125 mm (length of helix)

Nt = 11.0535 (number of turns)

d0 = 1.67933 mm (diameter of wire)

τ = 3.35866 mm (pitch of helix)

Z0 = 572.731 Ω (characteristic impedance)

Leff = 37.2698 nH/mm (effective inductance)

Ceff = 0.11362 pF/mm (effective capacitance)

v = 1.53672*10^7 m/s (velocity)

λ = 158.387 mm (wavelength)

f0 = 97.023 MHz (unloaded frequency)

Q = 873.205 (quality factor)

ν0 = 40. MHz (loaded frequency)
```

Appendix B

Mathematica code for Siverns' model

Following calculations are heavily based on a script generously provided by James David Siverns. All variables and references correspond to [17].

```
In[1]:= (* Units and constants *)
        MHz = 10^6;
        pF = 10^-12;
        Ω = 1;
        mm = 10^-3;
        m = 1;
        H = 1;
        μ0 = 4π*10^-7;

In[2]:= (* Trap and wire values *)
        Cw = 0.00001 pF; (* Wire to trap capacitance *)
        Rt = 0.1 Ω; (* Trap resistance *)
        CΣ[Ct_]:= Cw + Ct; (* Sum of above *)

In[3]:= calculateQ[dMillimeters_, γ_]:=Module[{
          (* Arguments naming as in Siverns paper *)
          d0,τ,d,De,α,ρ,eN,b,Cc,KLc,KCs,Cs,LC,ω0,δ,lc,r,Ns,ls,a,
          Rs,Rc,Rj,XLc,XCc,Xct,Xcw,XCs,Zcoil,ZE,Ztot,RealZ,Q,
          maxSize, log, ωRes, Capacitance
        },
          Capacitance = 20pF;
          (* Switch log function for plotting *)
          log = Print;
          log = (#)&;

          (* Resonator parameters *)
          (* Coil wire diameter, we take it from Macalpine *)
          d0 = 1.95mm;
```

```
τ = 2*d0; (* Winding pitch *)
d = dMillimeters*mm; (* Diameter of coil *)
De = d/γ; (* Diameter of a shield *)
α = d/De;
(* Resistivity of resonator material *)
ρ = 1.7*10^-8*Ω*m;
(* Given by the 4K chamber design *)
maxSize = 36mm;
b = 56mm - De/2;
(* Handling case of a too large resonator *)
If[d>maxSize || De>maxSize || b≤0, Return@0];

log["b = ", b/mm, "mm"];
log["d = ", d/mm, "mm"];
log["D = ", De/mm, "mm"];

eN = b/τ;(* Number of turns in the coil *)
log["N = ", eN];
(* Coil self capacitance - equation 25 *)

Cc = ((11.26 b/d)+8+(27/Sqrt[b/d]))d pF;

KLc = 39.37 (0.025 (d)^2 (1-α^2))/(τ)^2 10^-6 H/m;
KCs = 39.37 0.75/Log[10, 1/α] pF/m;

(* Shield-coil capacitance - equation 26 *)
Cs = b KCs;
(* Inductance of coil inside a shield - equation 27 *)
LC = b KLc;
(* Resonant frequency - equation 21 *)
ωRes[Ct_]:= 1/Sqrt[(Cs+Ct+Cw+Cc)LC];

log["ω = ", ωRes[Capacitance]/(2π MHz), "MHz"];

ω0[Ct_]:= 2π*40 MHz; (* This is a target frequency *)
(* Allowing 5% accuracy for the frequency *)
If[
      Abs[ω0[Capacitance] - ωRes[Capacitance]]/ω0[Capacitance] > 0.05,
      Return@0
];
```

```
δ[Ct_]:= Sqrt[2 ρ/(ω0[Ct] μ0 )]; (* Skin depth *)

(* Unwond length of the coil *)
lc = 2π Sqrt[(d/2)^2 +(τ/(2π))^2] b/τ;
r = d/2 (1/α-1);

(* Number of "turns" in the currents path in the shield
- equation 31 *)

Ns = (b lc)/(4π r^2);

(* Distance of current path in the shield - equation 32 *)
ls = Ns Sqrt[π^2 (d/α)^2 +(b/Ns)^2 ];
a[Ct_]:= Ct/(Cs + Cw);
Rs[Ct_]:= (ρ ls)/(b δ[Ct]);
Rc[Ct_]:= (ρ lc)/(d0 π δ[Ct]);

(* Resistance of solder joint as a function of frequency -
the 0.003 is the DC resistance of a typical solder joint
between shield and coil, however this can vary
and is best to measure *)

Rj[Ct_]:= 0.003 Sqrt[ω0[Ct]/(2π 10^5)] Ω;

(* Q calculations *)
XLc[Ct_]:= ω0[Ct ]LC;
XCc[Ct_]:= 1/(ω0[Ct] Cc);
Xct[Ct_]:= 1/(ω0[Ct] Ct);
Xcw[Ct_]:= 1/(ω0[Ct] Cw);
XCs[Ct_]:= 1/(ω0[Ct] Cs);

Zcoil[Ct_]:= (1/((i XLc[Ct]+Rc[Ct])) + 1/(1/i XCc[Ct]))^-1 ;

ZE[Ct_]:= (1/((1/i Xct[Ct]+Rt[Ct])) + 1/(1/i Xcw[Ct]) + 1/(1/i XCs[Ct]))^-1 ;
```

```
  Ztot[Ct_]:= Zcoil[Ct] + ZE[Ct] + Rs[Ct] + Rj[Ct];

  RealZ[Ct_]:= Rc[Ct] XCc[Ct]^2 / (Rc[Ct]^2+(XCc[Ct]-XLc[Ct])^2) +
  Rt XCs[Ct]^2 Xcw[Ct]^2 / (Rt^2(XCs[Ct]+Xcw[Ct])^2+(XCs[Ct](Xct[Ct]+Xcw[Ct])+Xct[Ct]Xcw[Ct])^2)
  + Rs[Ct] + Rj[Ct];

  Q[Ct_]:= LC ω0[Ct] / RealZ[Ct];

  Return@Q[Capacitance];
]
```

In[4]:=
```
SetDirectory[NotebookDirectory[]];
contourData = Table[
  {γ, d, calculateQ[d,γ]},
  {d, 15, 25, 0.01},
  {γ, 0.4, 0.7, 0.01}
] // Flatten[#,1]&;

ListContourPlot[
  contourData,
  PlotLegends → Automatic,
  FrameLabel → {"d/D", "d, mm"}
]
```

In[5]:=
```
(* Final parameters *)
calculateQ[19, 0.55]
```

```
b = 38.7273mm

d = 19mm

D = 34.5455mm

N = 9.93007

ω = 39.7919MHz
```

Out[5]= 362.188

Bibliography

[1] K. Deng, Y. L. Sun, W. H. Yuan, Z. T. Xu, J. Zhang, Z. H. Lu, and J. Luo. A modified model of helical resonator with predictable loaded resonant frequency and Q-factor. *Review of Scientific Instruments*, 85(10):1–8, 2014.

[2] Paul A. M. Dirac. On the Theory of Quantum Mechanics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 112(762):661–677, oct 1926.

[3] Richard Phillips Feynman. The Wonders That Await a Micro-Microscope. *The Saturday Review*, pages 45–47, 1960.

[4] Davide Gandolfi. *Compact RF Amplifier for Scalable Ion-Traps*. PhD thesis, Università degli Studi di Trento and Universität Innsbruck, 2010.

[5] Amy Greene. *Experiments Towards Mitigation of Motional Heating in Trapped Ion Quantum Information Processing*. Master of engineering in electrical engineering and computer science thesis, Massachusetts Institute of Technology, 2016.

[6] S. Gulde. *Experimental realization of quantum gates and the Deutsch-Josza algorithm with trapped ^{40}Ca-ions*. PhD thesis, University of Innsbruck, 2017.

[7] K G Johnson, J. D. Wong-Campos, A Restelli, K A Landsman, B Neyenhuis, J Mizrahi, and C. Monroe. Active stabilization of ion trap radiofrequency potentials. *Review of Scientific Instruments*, 87(5):053110, may 2016.

[8] T. Karin, I. Le Bras, A. Kehlberger, K. Singer, N. Daniilidis, and H. Häffner. Transport of charged particles by adjusting rf voltage amplitudes. *Applied Physics B*, 106(1):117–125, jan 2012.

[9] Ezra Kassa, Hiroki Takahashi, Costas Christoforou, and Matthias Keller. Precise positioning of an ion in an integrated Paul trap-cavity system using radiofrequency signals. *Journal of Modern Optics*, 65(5-6):520–528, mar 2018.

[10] Ezra M. B. Kassa. *Single ion coupled to a high-finesse optical fibre cavity for cQED in the strong coupling regime*. Doctor of philosophy thesis, University of Sussex, 2016.

[11] Muir Kumph. *2D Arrays of Ion Traps for Large Scale Integration of Quantum Information Processors*. PhD thesis, University of Innsbruck, 2015.

[12] Florian Michael Leupold. *Bang-bang Control of a Trapped-Ion Oscillator*. PhD thesis, ETH Zurich, 2015.

[13] W. Macalpine and R. Schildknecht. Coaxial Resonators with Helical Inner Conductor. *Proceedings of the IRE*, 47(12):2099–2105, dec 1959.

[14] Wolfgang Paul. Electromagnetic Traps for Charged and Neutral Particles(Nobel Lecture). *Angewandte Chemie International Edition in English*, 29(7):739–748, jul 1990.

[15] A.M. Prokhorov. Joule–Lenz law. *Great Soviet Encyclopedia*, 1972.

[16] Felix Rohde. *Remote ion traps for quantum networking: Two-photon interference and correlations*. Dissertation, ICFO Barcelona, 2009.

[17] J. D. Siverns, L. R. Simkins, S. Weidt, and W. K. Hensinger. On the application of radio frequency voltages to ion traps via helical resonators. *Applied Physics B*, 107(4):921–934, jun 2012.

[18] Q. A. Turchette, Kielpinski, B. E. King, D. Leibfried, D. M. Meekhof, C. J. Myatt, M. A. Rowe, C. A. Sackett, C. S. Wood, W. M. Itano, C. Monroe, and D. J. Wineland. Heating of trapped ions from the quantum ground state. *Physical Review A*, 61(6):063418, may 2000.

[19] Andre J. S. Van Rynbach. *Microfabricated Surface Trap and Cavity Integration for Trapped Ion Quantum Computing by Abstract Microfabricated Surface Trap and Cavity Integration for Trapped Ion Quantum Computing*. Doctor of philosophy dissertation, Graduate School of Duke University, 2016.

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