3.10.21

CSE 313

Mathematical Analysis for Computer Science

Book: Concrete Mathematics (2nd edition)
· Knuth's book

chapter-1

Recurrent Problem
Tower OF Hanoi

0 সংখ্যক disk এর জন্য $move_0 = 0$

1 " " " " $move_1 = 1$

2 " " " " $move_2 = 3$

3 " " " " $move_3 = 7$

so, $T_n = 2^n - 1$ (closed form)

যদি $(n-1)$ সংখ্যক disk এর জন্য move "
হবে তাহলে n " " " recurrance solve
হবে করি তাহলে একে
বলে। closed form মানে হচ্ছে there
is no more optimal solution.

Recurrent solution of TOH

আমার কাছে n সংখ্যক disk আছে তাদের কে একটা tower থেকে অন্য tower এ নিতে হবে, এর আগে $n-1$ সংখ্যক disk কে অন্য tower এ সরাতে হবে, so $(n-1)$ সংখ্যক disk সরাতে move লাগে T_{n-1} এবং last বড় disk কে সরাতে 1 টা move লাগবে। যেমন বড় ও disk টা উপর বাকিগুলো রাখতে $n-1$ move লাগে।

Upper bound: কারণ এর থেকে কম move ও লাগতে পারে

So Total Move $\leq 2T_{n-1}+1$ for $n>0$

Lower bound: যেহেতু অধিক move হলে তুলে largest disk

$$T_n \geq 2T_{n-1}+1 \rightarrow \text{Lower bound}$$

Two inequalities মিলে যেটা তৈরি হবে।

Trivial solve. (এই solve নিয়ে বেশি দূর আগানো যাবে না)

$$T_0 = 0$$

$$T_n = 2T_{n-1}+1 \text{ ; for } n>0$$

closed form (optimal solⁿ)

Proof by Mathematical Induction

~~Basis~~: Base case

$n=0$, $T_0 = 2^0 - 1 = 0$, so for basis its true

Induction:

Let the closed form is true for $n-1$

i.e $T_{n-1} = 2^{n-1} - 1$

Now we have to prove the closed form for $n=n$.

Hence, $T_n = 2T_{n-1} + 1$

$$= 2(2^{n-1} - 1) + 1$$

$$= 2^{n-1+1} - 2 + 1$$

$$= 2^n - 1$$

closed form is proved for all $n \geq 0$

এই recurrence solve দিয়ে n এর মান অনেক বেশি দিলে time বেশি লাগবে calculate ~~করতে~~, করতে। তাই আমরা closed form বের করি

Double Tower of Hanoi

$T_0 = 0$

$T_1 = 2$

$T_2 = 6$

$T_3 = 14$

[T_n Here n actually represents how many same size of peg is there]

Recursive Solution $T_n = 2T_{n-1} + 2$

closed form $= 2^{n+1} - 2$

Proof

Base Case $n=0$ $2^{0+1} - 2 = 2-2 = 0$

~~$n=1$ $2^{1+1} - 2 = 2^2 - 2 = 4-2 =$~~ 2

Induction: Let's the closed form is true for $n-1$ i.e $T_{n-1} = 2^{n-1+1} - 2$ $= 2^n - 2$

We have to prove for $n=n$

$$T_n = 2T_{n-1} + 2$$

$$= 2(2^{n-1+1} - 2) + 2$$

$$= 2^{n-1+1+1} - 4 + 2$$

$$T_n = 2^{n+1} - 2$$

proved

Upper bound also a lower bound: ?

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Line in a Plane
(Pizza Problem)

n number of line দিয়ে total কয়টা piece করা যাবে।

$n \rightarrow$ number of line

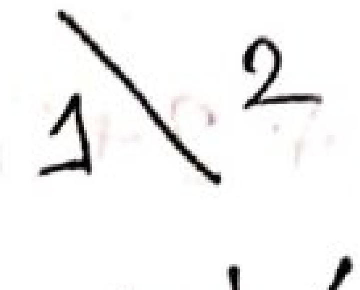
$L_n \rightarrow$ number of region

rules:

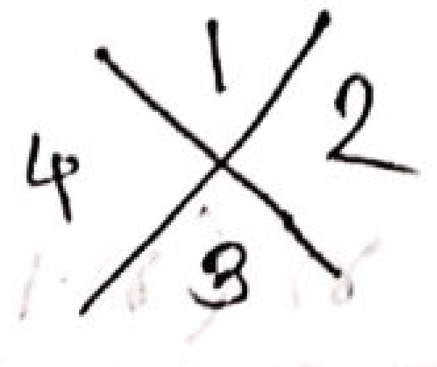
1. newline টা আগের প্রত্যেকটা Line এর সাথে at least একটা জায়গায় intercept করতে হবে।
2. Line গুলো parallel হতে পারবে না।

$L_0 = 1$

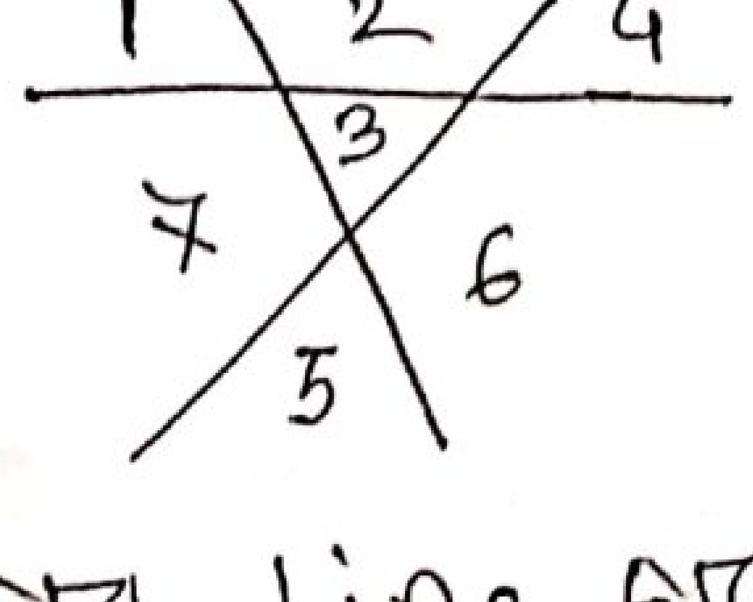
$L_1 = 2$



$L_2 = 4$



$L_3 = 7$



- new line আগের Line গুলোকে আলাদা point এ cut করবে।

$L_n = L_{n-1} + n$

upper bound $L_n \leq L_{n-1} + n$ for $n > 0$

Lower " $L_n \geq L_{n-1} + n$ for $n > 0$

closed form determine

$$L_n = L_{n-1} + n$$

$$= L_{n-2} + n-1 + n$$

$$= L_{n-3} + (n-2) + (n-1) + n$$

$$\vdots$$

$$= L_0 + 1 + 2 + 3 + \dots + (n-3) + (n-2) + (n-1) + n$$

$$= 1 + 1 + 2 + 3 + \dots + (n-3) + (n-2) + (n-1) + n$$

$$= \frac{n(n+1)}{2} + 1$$] closed form

Proof by induction

base case $n = 0$ $\quad L_n = L_{0-1} + 0 = 0$

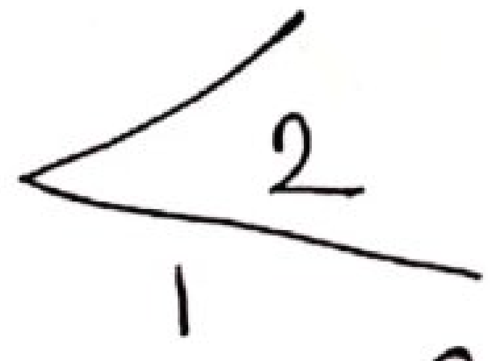
for n $\quad L_n = L_{n-1} + n$

$$= \frac{(n-1)(n-1+1)}{2} + 1 + n$$

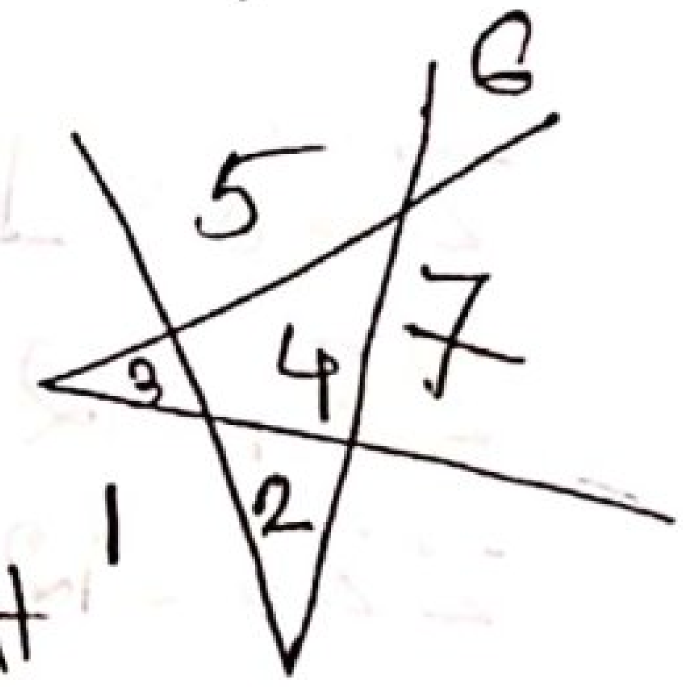
$$= \frac{n(n+1)}{2} + 1$$

Bent Lines in the Plane:

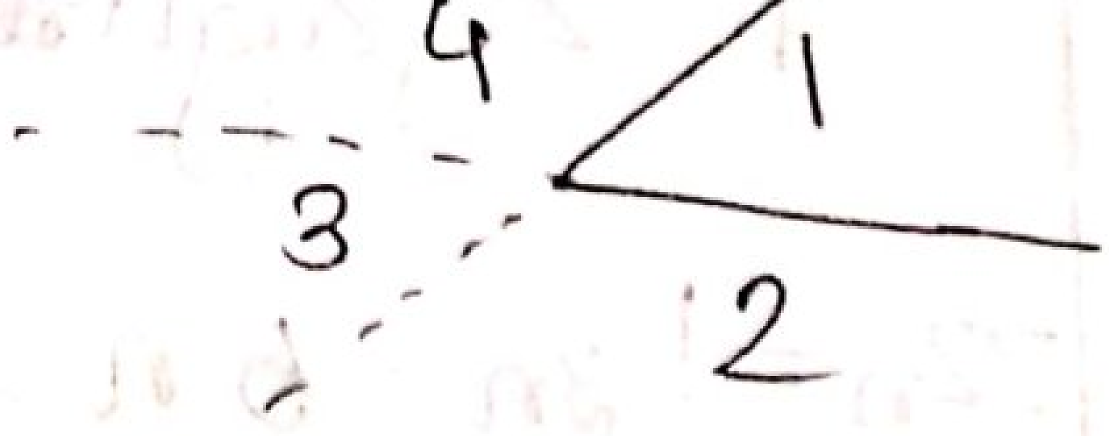
for 1 bent line area = 2



for 2 " " area = 7



একটা bent line = 2 ~~টা~~ straight line



$$Z_n = L_{2n} - 2n$$

$$= \frac{2n(2n+1)}{2} + 1 - 2n$$

$$= 2n^2 - n + 1$$

$$Z_2 = L_{2\times2} - 2\times2 = 11 - 4 = 7$$

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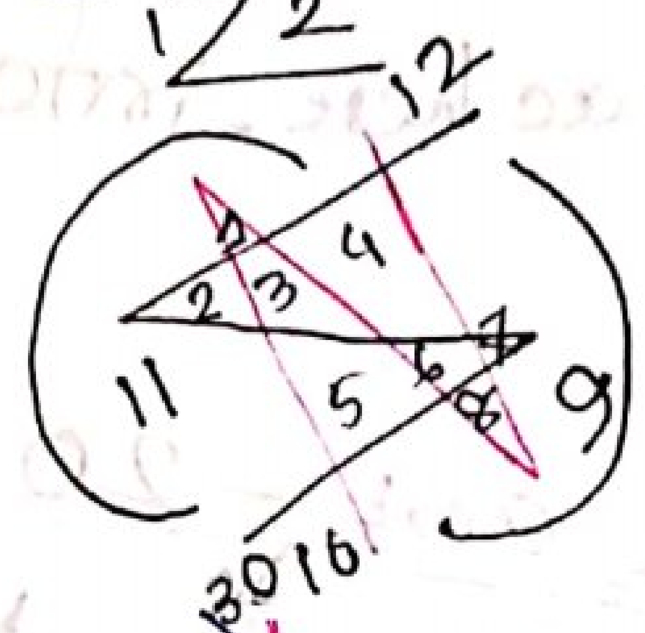
Subject :

Date :

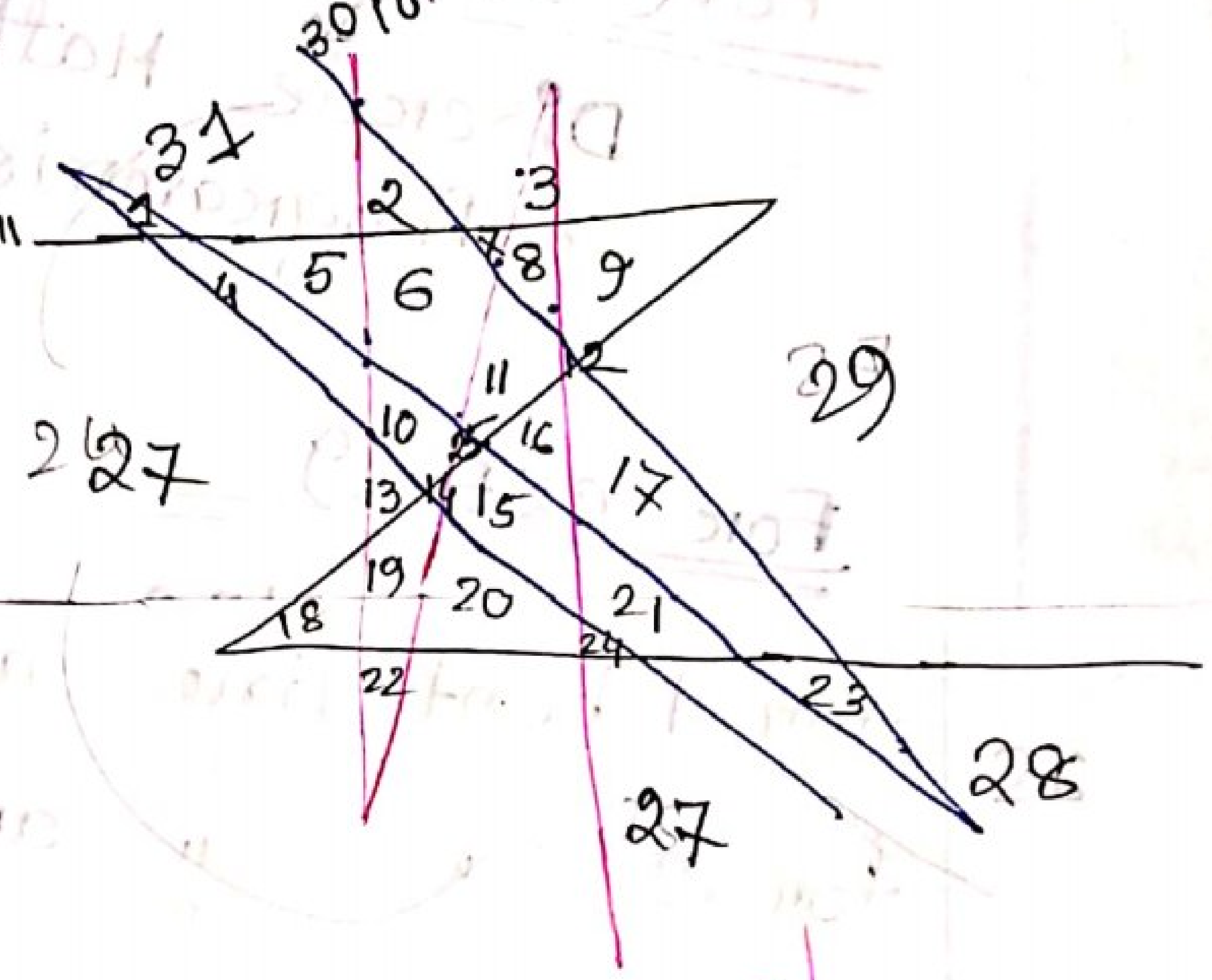
Practise Problem: Zigzagline (Z)

for 1 zigzag Line

for 2 " "



for 3 " "



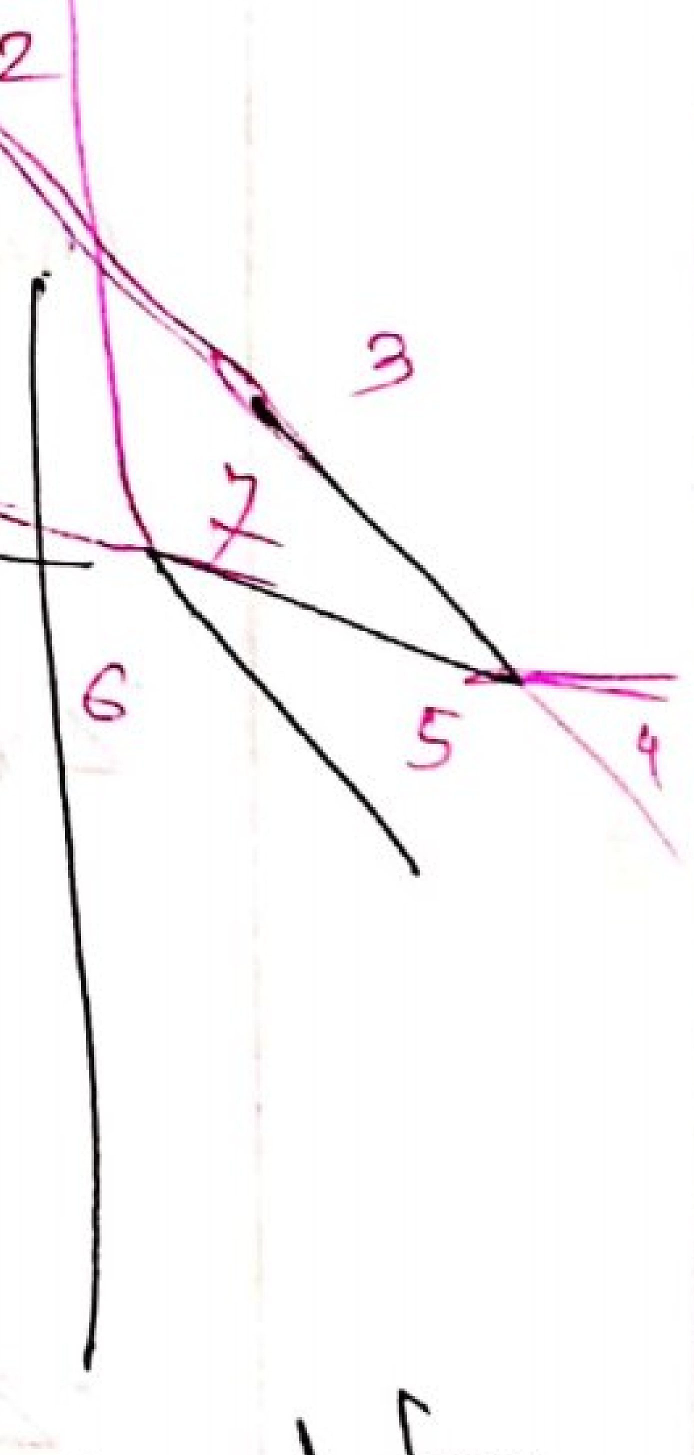
$ZZ_0 = 1$

$ZZ_1 = 2$

$ZZ_2 = 12$

$ZZ_3 = 31$

1 ZigZagLine = 3 straight Line



$$ZZ_n = L_{3n} - 5n$$

$$= \frac{3n(3n+1)}{2} + 1 - 5n$$

$$= \frac{9n^2+3n+2-10n}{2}$$

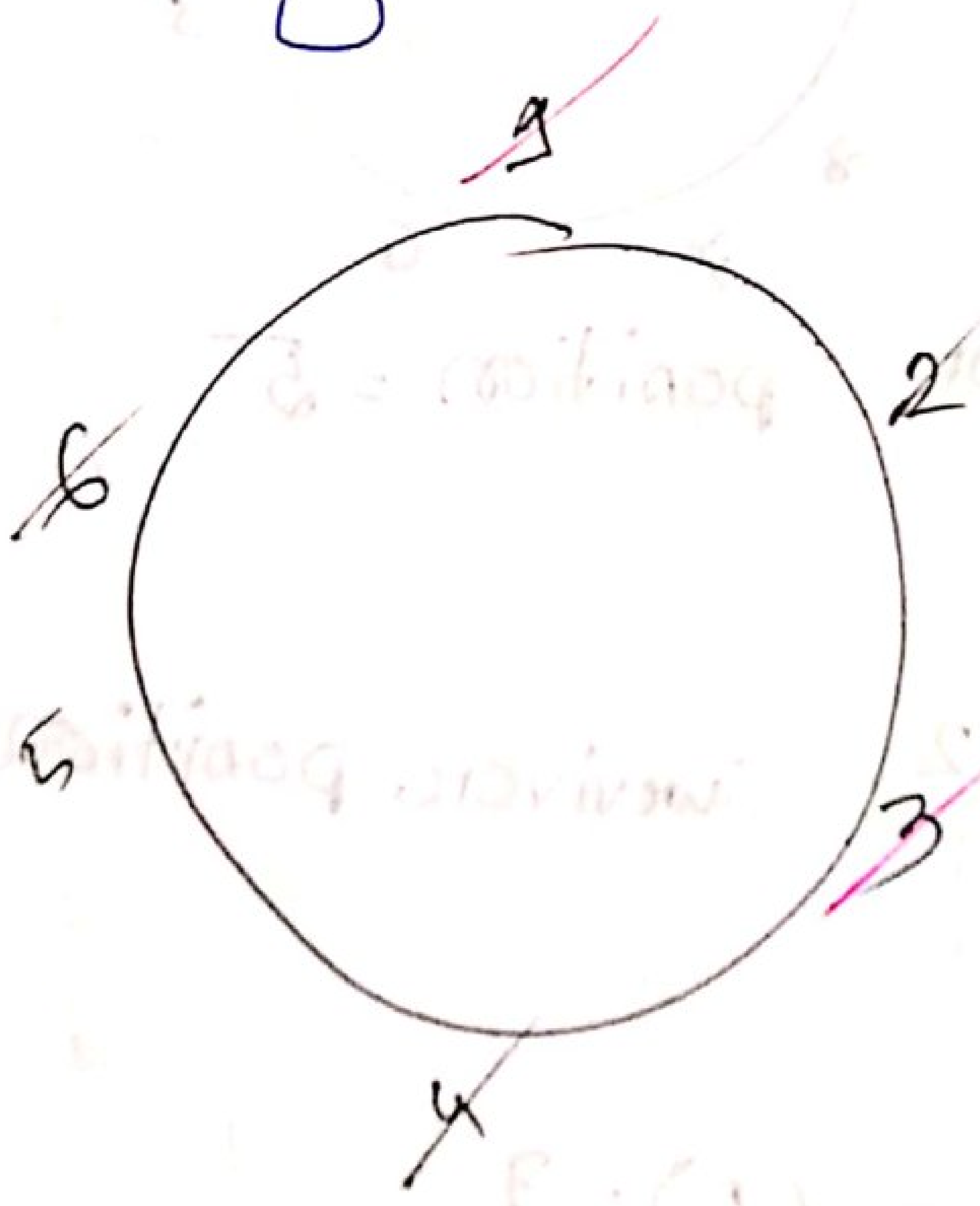
$$= \frac{9n^2-7n+2}{2} = \frac{9n^2-7n}{2} + 1$$ } closed form

$$L_n = \frac{n(n+1)}{2} + 1$$

$$Z_n = 2n^2 - n + 1$$

$L_n = \frac{1}{2}n^2$ so $L_n < Z_n$ [May come On Exam]

Life saving Problem



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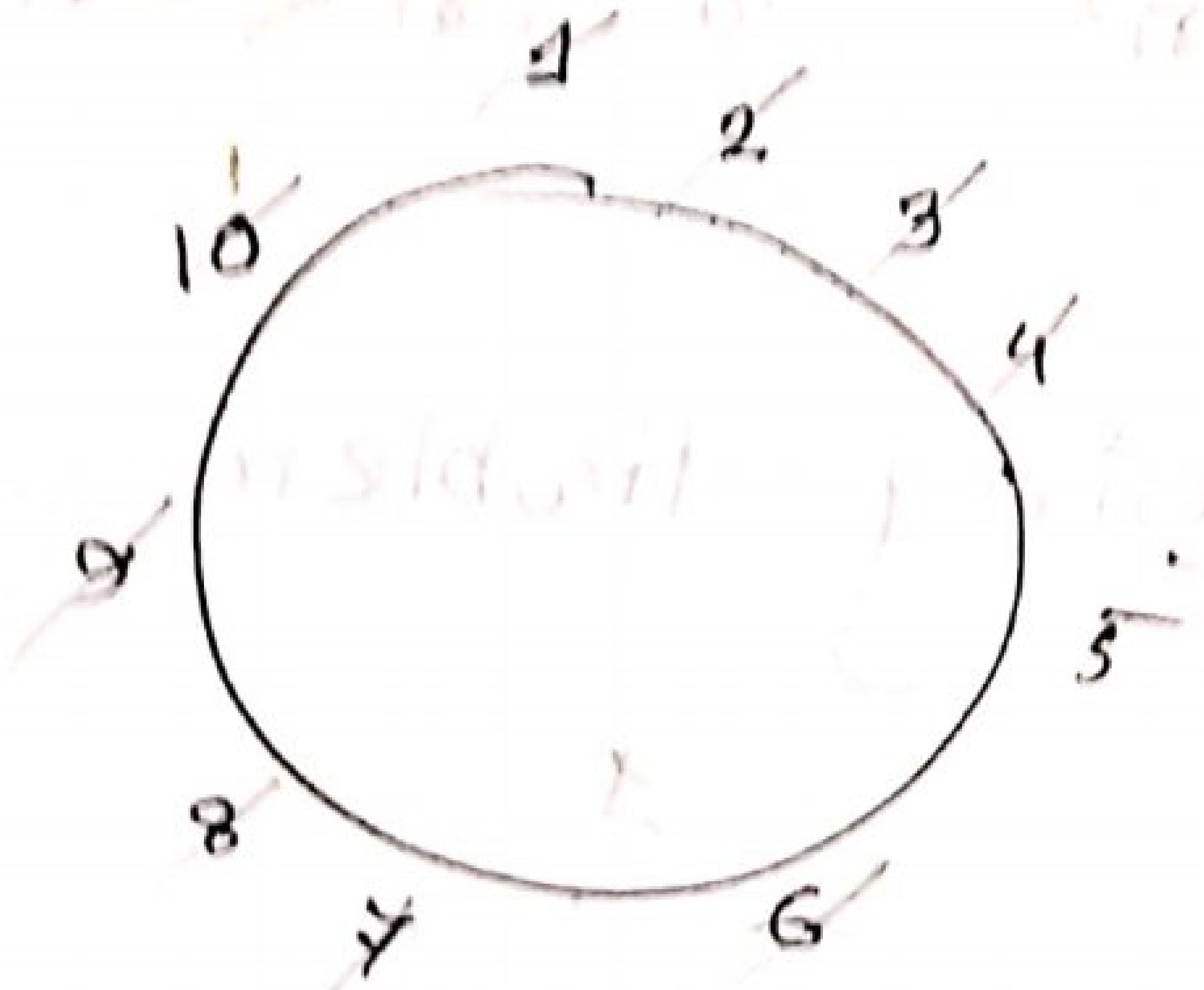
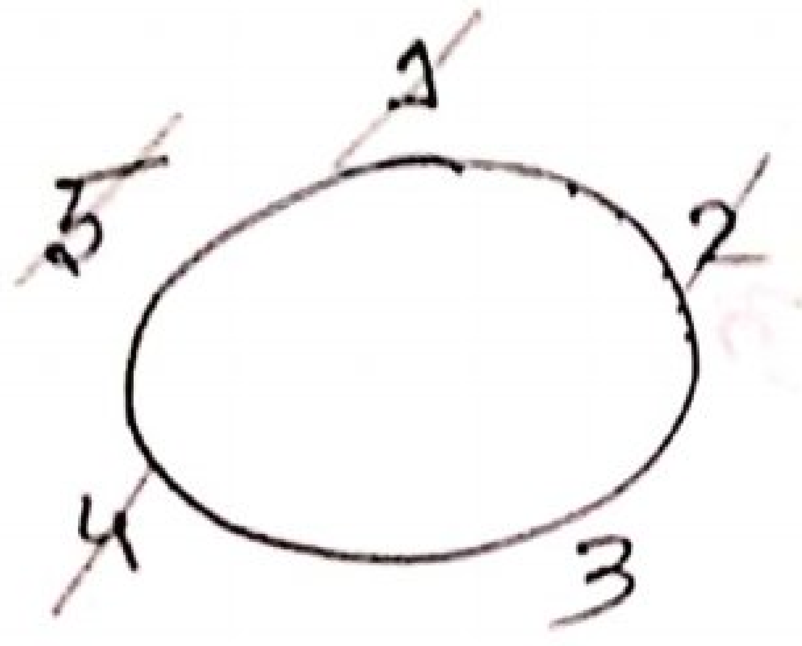
Joshephus Problem :

Let's there are n people.



Survivor position = 5



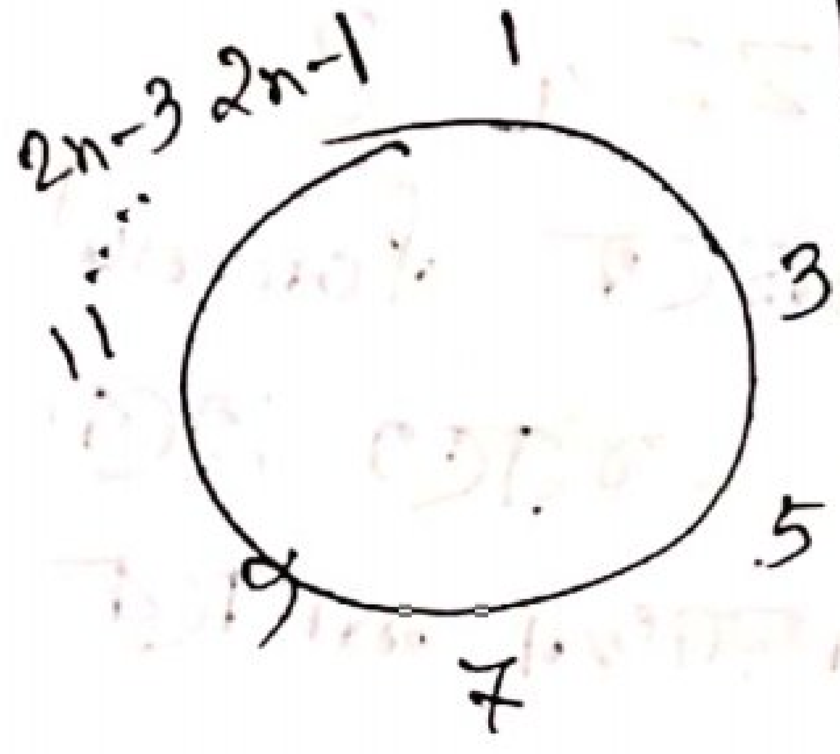
Survivor position = 3

$J(5) = 3$

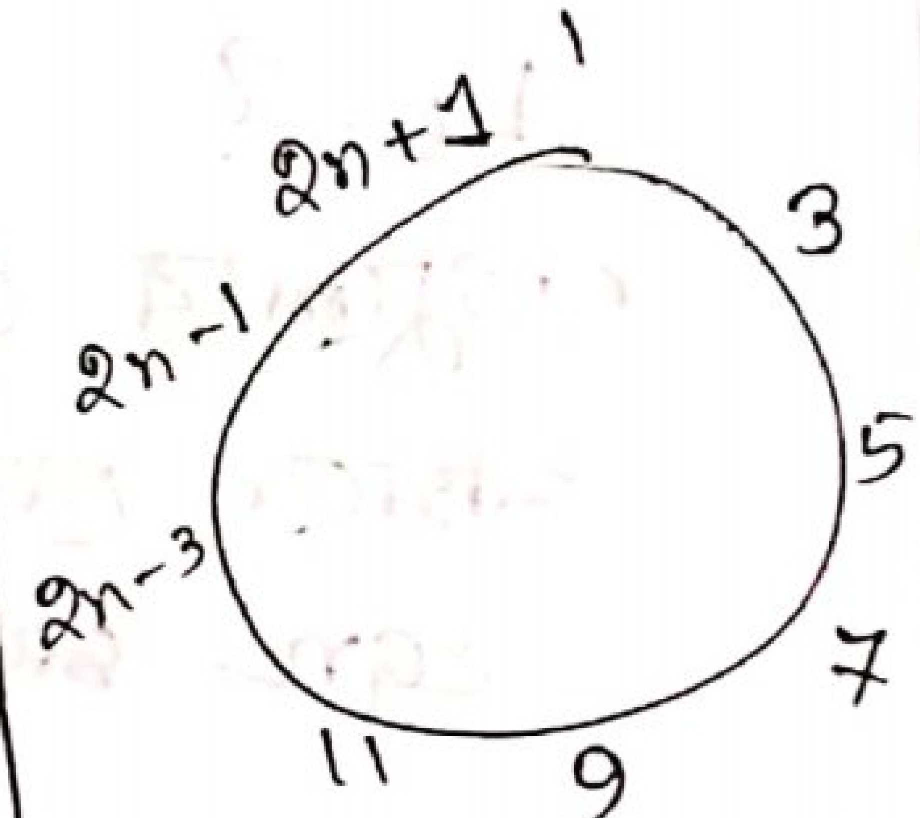
$J(10) = 5$

কখনই even position এর people survive
করবে না তারা first round এ dead হবে,
so Josephus এর solve always ই বেজোড়টা

odd number



if n even



if n odd

Recursive Solve:

even people:

$J(2n) : 2J(n) - 1$

odd people

$J(2n+1) : 2J(n) + 1$

Closed Form

n	J(n)	n	J(n)
2^0 1	1	10	5
2^1 2	1	11	7
3	3	12	9
2^2 4	1	13	11
5	3	14	13
6	5	15	15
7	7	2^4 16	1
2^3 8	1		
9	3		

$$\boxed{J(2^n + h) = 2h + 1}$$

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Proving closed form of Josephus

If $n=0$ then

$$J(2^0+0) = J(1) = 2\times 0+1 = 1$$

for basis it's true

Now, $2^n+h = 2n$ when

$$J(2^n+h) = 2\,J\left(\frac{2^n+h}{2}\right) - 1 \quad [J(2n) = 2J(n)-1]$$

$$= 2\,J\left(2^{n-1}+\frac{h}{2}\right)-1$$

$$= 2\left(\cancel{2}\times\frac{h}{\cancel{2}}+1\right)-1 \quad [J(2^n+h) = 2h+1]$$

$$= 2h+2-1 = 2h+1$$

If $2^n+h = 2n+1$

$$J(2^n+h) = 2\,J\left(\frac{2^n+h-1}{2}\right)+1$$

$$= 2\,J\left(2^{n-1}+\frac{h-1}{2}\right)+1$$

$$= 2\left(\cancel{2}\,\frac{h-1}{\cancel{2}}+1\right)+1$$

$$= 2h-\cancel{2}+\cancel{2}+1$$

$$= 2h+1$$

Question Pattern

$L_5 = ?$ $ZZ_4 = ?$

এগুলোর ক্ষেত্রে formula তে মান বসিয়ে বের করতে হবে।
eqn জেনে রাখা লাগবে

Chapter-2

Sums

2.1 Sequences:

Repertoire Method:

Recurrence eqn থেকে closed form বের করার way. পুরো problem কে ছোট ছোট part করে প্রত্যেকটা part হতে solve বের করে পুরো problem টার solve বের করা। এই method এ 3 টা coefficient use হয় α, β, γ

Example 1:

$$a_n = a + bn$$

Recurrent eqn for sum: $S_n = a_0 + a_1 + a_2 + \dots + a_n$

$$S_0 = a$$
$$S_n = S_{n-1} + (a + bn)$$

$$R_0 = \alpha$$
$$R_n = R_{n-1} + (\beta + \gamma n)$$

Now,

$$R_1 = R_0 + \beta + \gamma \times 1 = \alpha + \beta + \gamma$$
$$R_2 = R_1 + \beta + \gamma \times 2 = \alpha + \beta + \gamma + \beta + 2\gamma$$
$$= \alpha + 2\beta + 3\gamma$$
$$R_3 = R_2 + \beta + 3\gamma = \alpha + 3\beta + 6\gamma$$
$$\vdots$$
$$R_n = A(n)\alpha + B(n)\beta + C(n)\gamma$$

$R_n = 1$ for all n

$R_0 = 1 = \alpha$

$R_n = R_{n-1} + (\beta + \gamma n)$

$R_1 = R_0 + (\beta + \gamma \times 1)$

$1 = 1 + \beta + \gamma$

তাহলে eqn true হবে যদি

$\beta = 0$ & $\gamma = 0$

$\beta = -\gamma$ or $\gamma = -\beta$

when $\beta = 0$ & $\gamma = 0$, $\alpha = 1$

then $r = 1$ justified

$R_n = A(n) \times 1 + B(n) \times 0 + C(n) \times 0$

$1 = A(n)$

$A(n) = 1$

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Now for B_n

$R_n = n$ for all n

$R_0 = 0 = \alpha$

$\cancel{R_n = R_{n-1} + (\beta + \gamma n)}$

$\cancel{n = n-1 + (\beta + \gamma n)}$ [~~এটা true হতে~~ $\beta = 0$ $\gamma = 0$]

$n = n-1 + \beta + \gamma n$

$1 = \beta + \gamma n$

$\beta = 1 - \gamma n$

এখানে $\gamma = 0$ দিলেই শুধু n যুক্ত term eliminate হয়েছে, otherwise n যুক্ত term থেকে যায়।

$\beta = 1$

$n = 0 + B(n) + 0$

$B(n) = n$

Now for $C(n)$

$R_n = n^2$ for all n, $R_0 = 0 = \alpha$

$R_n = R_{n-1} + (\beta + \gamma n)$

$n^2 = (n-1)^2 + \beta + \gamma n$

$n^2 = n^2 - 2n + 1 + \beta + \gamma n$

$2n = 1 + \beta + \gamma n$

$2n = 1 - 1 + \gamma n$

$2n = \gamma n$

$\gamma = 2$

Now,

$R_n = A(n)\alpha + B(n)\beta + C(n)\cdot\gamma$

$n^2 = 0 - B(n) + 2C(n)$

$n^2 = 2C(n) - n$

$C(n) = \frac{n^2 + n}{2}$

$A(n) = 1$

$B(n) = n$

$C(n) = \frac{n^2 + n}{2}$

$$R_n = \alpha + n\beta + \frac{n^2+n}{2}\gamma$$

$$\cancel{2R_n = 2\alpha + 2n\beta + (n^2+n)\gamma}$$

Now, $\alpha = \beta = a$, $\gamma = b$

$$\cancel{2R_n = 2a + 2na + (n^2+n)b}$$

$$S_n = a + na + \frac{n^2+n}{2} b$$

$$S_n = a(n+1) + \left(\frac{n^2+n}{2}\right) b$$

Question: $S_n = \sum_{k=0}^{n} (a+bk)$

3rd Class

Reduction of Recurrences to Sums

$T_0 = 0$

$T_n = 2T_{n-1} + 1$; for $n > 0$

if we divide both side by 2^n:

$T_0 = 2^0 = 0$;

$$\frac{T_n}{2^n} = \frac{T_{n-1}}{2^{n-1}} + \frac{1}{2^n}$$

$$S_n = S_{n-1} + 2^{-n}$$

$$S_n = \sum_{k=1}^{n} 2^{-k}$$

Our actual eqn $T_n = 2T_{n-1} + 1$

Now we make

$a_n T_n = b_n T_{n-1} + C_n$ - (1)

we can tell $a_n = 1$, $b_n = 2$, $C_n = 1$

multiply by S_n (1)

$S_n a_n T_n = S_n b_n T_{n-1} + S_n C_n$

Let, $S_n b_n = S_{n-1} a_{n-1}$

Now

$$S_n a_n b_n = S_{n-1} a_{n-1} T_{n-1} + S_n c_n$$

$$S_n = S_{n-1} + S_n c_n$$

Now compare with $S_n = S_{n-1} + 2^{-n}$.

we get

$$S_n = S_{n-1} + S_n c_n$$

$$= S_0 a_0 T_0 + \sum_{k=1}^{n} S_k c_k$$

$$S_n = S_1 b_1 T_0 + \sum_{k=1}^{n} S_k c_k$$

$$S_n a_n T_n = S_1 b_1 T_0 + \sum_{k=1}^{n} S_k c_k$$

$$T_n = \frac{1}{S_n a_n}\left(S_1 b_1 T_0 + \sum_{k=1}^{n} S_k c_k\right)$$

$$S_n c_n = 2^{-n} \Rightarrow S_n = \frac{2^{-n}}{c_n}$$

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Question: 1. Tower of Hanoi এর sum এর value derive কর
2. eqn তুলে দিয়ে prove করতে বলতেপারে
3. value বের করতে দেয়া

Quick Sort

Recurrence Solve

$$C_0 = 0$$

$$C_n = n+1 + \frac{2}{n} \sum_{k=0}^{n-1} C_k \qquad n > 0$$

multiply by n to both sides

$$nC_n = n^2 + n + 2\sum_{k=0}^{n-1} C_k \quad \ldots (1)$$

replace n by $(n-1)$

$$(n-1)C_{n-1} = (n-1)^2 + (n-1) + 2\sum_{k=0}^{n-2} C_k \quad \ldots (11)$$

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Quick Sort

Recursive Solve to Summation

$c_0 = 0$

$$c_n = n+1 + \frac{2}{n} \sum_{K=0}^{n-1} c_K \,,\; n > 0$$

multiply by n to both sides

$$n C_n = n^2 + n + 2 \sum_{K=0}^{n-1} c_K \quad n > 0 \quad \text{-- (1)}$$

replace n by $(n-1)$

$$(n-1) C_{n-1} = (n-1)^2 + (n-1) + 2 \sum_{K=0}^{n-2} c_K \quad \text{-- (2)}$$

(1) - (2)

$$n C_n - (n-1) C_{n-1} = n^2 + n - (n-1)^2 - (n-1) + 2 C_{n-1}$$

?

$$n C_n - (n-1) C_{n-1} = 2n + 2C_{n-1} \quad \text{-- (3)}$$

$$\Rightarrow n C_n = n C_{n-1} - C_{n-1} + 2n + 2 C_{n-1}$$

$$n C_n = (n+1) C_{n-1} + 2n$$

compare this with Tower of Hanoi

eqn $a_n T_n = b_n T_{n-1} + c_n$ if $T_n = C_n$

$a_n = n$, $b_n = n+1$, $c_n = 2n$

Put this values on Tower of Hanoi's summation eqn

$$s_n b_n = s_{n-1} a_{n-1}$$

$$s_n = \frac{s_{n-1} a_{n-1}}{b_n}$$

$$=$$

[s_{n-1} কে 1 ধরে নিলে]

$$= \frac{a_{n-1} a_{n-2} \cdots a_1}{b_n \cdot b_{n-1} \cdots b_2}$$

last page [$\because a_n = n$, $b_n = n+1$]

$$= \frac{(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}{(n+1)(n)(n-1)(n-2) \cdots 3}$$

$$s_n = \frac{2}{n(n+1)}$$

We know

$$T_n = \frac{1}{s_n a_n} \left(s_1 b_1 T_0 + \sum_{k=1}^{n} s_k c_k \right)$$

$$c_n = \frac{1}{s_n a_n} \left(s_1 b_1 c_0 + \sum_{k=1}^{n} s_k c_k \right.$$

$$S_1 = \frac{2}{1(1+1)} = \frac{2}{2}$$

$b_n = 1+1 = 2$

$c_0 = 2 \times 0 = 0$

$$= \frac{1}{\frac{2}{n(n+1)} \times n} \left(1 \cdot 2 \cdot 0 + \sum_{k=1}^{n} a_k c_k\right)$$

$$= \frac{(n+1)}{2} \left(0 + \sum_{k=1}^{n} \frac{2}{\cancel{k}(k+1)} \times 2\cancel{k}\right)$$

$$\cancel{= \frac{2}{(n+1)} (}$$

$$= \frac{(n+1)}{2} \left(4 \times \sum_{k=1}^{n} \frac{1}{k+1}\right)$$

$$= \frac{(n+1)}{2} \times 4 \left(\sum_{k=1}^{n} \frac{1}{k+1}\right)$$

$$C_n = 2(n+1) \sum_{k=1}^{n} \frac{1}{k+1}$$

chapters 6. 2
harmonium number
জিনিসটাকে ভালোকরে
elaborate করতে
হবে

Perturbation Method:

linear term গুলো সরিয়ে non-linear term এ আনে

$$\boxed{\text{Geometric seq}^n \quad a_n = ax^n}$$

$$S_0 = a$$

$$S_n = S_{n-1} + ax^n = a_0 + a_1 + \dots + a_{n-1} + a_n$$

$$S_n + a_{n+1} = a_0 + a_1 + \dots + a_{n-1} + a_n + a_{n+1}$$ [adding a_{n+1} on both side]

$$S_n + a_{n+1} = a_0 + \sum_{1 \le k+1 \le n+1} a_{k+1}$$

$$= a_0 + \sum_{1 \le k+1 \le n+1} ax^{(k+1)}$$

$$= a_0 + x \sum_{1 \le k \le n} ax^k$$

$$= a_0 + x \times (ax + ax^2 + ax^3 + \dots + ax^n)$$

$$= a_0 + x \cdot a\cancel{x}\underbrace{(x + x^2 + x^3 + \dots x^n)}$$

$$= a_0 + x S_n$$

$$S_5 = S_4 + ax^5$$
$$= S_3 + ax^4 + ax^5$$
$$= S_2 + ax^3 + ax^4 + ax^5$$
$$= S_1 + ax^2 + ax^3 + ax^4 + ax^5$$
$$= S_0 + ax + ax^2 + ax^3 + ax^4 + ax^5$$
$$= a + a(x + x^2 + x^3 + x^4 + x^5)$$

$$S_n + ax^{n+1} = a + xS_n$$

$$\Rightarrow S_n - xS_n = a - ax^{n+1}$$

$$\Rightarrow S_n(1-x) = a(1-x^{n+1})$$

$$S_n = \frac{a(1-x^{n+1})}{1-x}$$

Not sure

$$S_n = \sum_{0 \le k \le n} k 2^k$$

$$S_n + (n+1)2^{n+1} = \sum_{0 \le k \le n} (k+1) 2^{k+1}$$

$$= \sum_{0 \le k \le n} k 2^{k+1} + \sum_{0 \le k \le n} 2^{k+1}$$

$$= 2\left(\sum_{0 \le k \le n} k 2^k + \sum_{0 \le k \le n} 2^k\right)$$

$$\cancel{= 2(S_n + 2^{n+1} - 2)}$$

$$= 2\left(S_n + \frac{1-2^{n+1}}{1-2}\right) \left[\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}\right]$$

$$= 2S_n + \frac{2-2^{n+2}}{1-2}$$

If we replace ~~2~~ by x

$$S_n + (n+1)x^{n+1} = xS_n + \frac{x-x^{n+2}}{1-x}$$

$$S_n - xS_n = -(n+1)x^{n+1} + \frac{x-x^{n+2}}{1-x}$$

$$S_n(1-x) = \frac{(1-x)\{-(n+1)x^{n+1}\} + x - x^{n+2}}{1-x}$$

$$S_n = \frac{-(n+1)x^{n+1} + (n+1)x^{n+2} + x - x^{n+2}}{(1-x)^2}$$

$$S_n = \frac{x-(n+1)x^{n+1} + nx^{n+2} + \cancel{x^{n+2}} - \cancel{x^{n+2}}}{(1-x)^2}$$

$$S_n = \frac{x-(n+1)x^{n+1} + nx^{n+2}}{(1-x)^2}$$

for $x \neq 1$

General Methods

$$\square_n = \sum_{0 \le k \le n} n^2$$

$n =$	0	1	2	3	4	5	6	7	8	9	10	11	12
$n^2 =$	0	1	4	9	16	25	36	49	64	81	100	121	144
$\square_n =$	0	1	5	14	30	55							6650

We know $\square_n = \dfrac{n(n+1)(2n+1)}{6}$

By Induction Method

$$\square_n = \square_{n-1} + n^2$$

$$= \frac{(n-1)n(2n-2+1)}{6} + n^2$$

$$= \frac{n(n-1)(2n-1)}{6} + n^2$$

$$= \frac{(n^2-n)(2n-1)}{6} + n^2$$

$$= \frac{2n^3 - 2n^2 - n^2 + n + 6n^2}{6}$$

$$= \frac{2n^3 + 3n^2 + n}{6}$$

$$= \frac{2n^3 + 2n^2 + n^2 + n}{6}$$

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$$= \frac{2n^2(n+1)+n(n+1)}{6}$$

$$= \frac{(n+1)(2n^2+n)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Perturbation:

$$\Box_n = \sum_{0 \le k \le n} n^2$$

$$\Rightarrow \Box_n + (n+1)^2 = \sum_{0 \le k \le n} (k+1)^2$$

$$= \sum_{0 \le k \le n} k^2+2k+1)$$

যদি solve হয়
হবে না cont
→ $\Box_n$ term
চলে যাবে
হয়

$$= \sum_{0 \le k \le n} k^2 + \sum_{0 \le k \le n} 2k + \sum_{0 \le k \le n} 1$$

$$= \sum k^2 + \sum 2k + n$$

$$= \sum n^2 + \sum 2k + n$$

$$\Box_n + (n+1)^2 = \Box_n + \sum 2k + n$$

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$\blacksquare_n + (n+1)^3 = \sum_{0 \le k \le n} (k+1)^3$

$= \sum_{0 \le k \le n} k^3 + 3k^2 + 3k + 1$

$= \sum k^3 + \sum 3k^2 + \sum 3k + \sum_{0 \le k \le n} 1$

$= \sum k^3 + \sum 3k^2 + \sum 3k + \underline{n+1}$ k এর range 0 থেকে n হওয়ায় 1 বেশি

$= \blacksquare_n + 3\square_n + \cancel{3n} \; \frac{3n(n+1)}{2} + n + 1$

$\cancel{3\square_n = (n+1)^3 - 4n}$

$3\square_n = (n+1)^3 - 3\frac{n(n+1)}{2} - n - 1$

$= n^3 + 3n^2 + 3n + 1 - \frac{3n^2 + 3n}{2} - n - 1$

$= \frac{2n^3 + 6n^2 + 6n + 2 - 3n^2 - 3n - 2n - 2}{2}$

$= \frac{2n^3 + 3n^2 + n + 2 - 2}{2}$

$= \cancel{n(2n+1) + 1}$

$= \frac{2n^3 + 2n^2 + n^2 + n + \cancel{2}}{2}$

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$$= \frac{2n^2(n+1) + n^2(n+1)}{2}$$

$$\square_n = \frac{(n+1)(2n^2+n)}{2}$$

$$\square_n = \frac{n(n+1)(2n+1)}{6}$$

By Repertoire

$R_0 = 0$

$R_n = R_{n-1} + \alpha + \beta n + \gamma n^2$

$R_n = A(n)\alpha + B(n)\beta + C(n)\gamma$

At first let $R_n = n$

$n = n-1 + \alpha + \beta n + \gamma n^2$

$1 = \alpha + \beta n + \gamma n^2$

~~So compare with $R_n = A(n)\alpha + B(n)\beta + C(n)\gamma$~~

$\alpha = 1$ ধরলে $\beta = 0$ & $\gamma = 0$ হতে হবে

$n = A(n) \cdot 1 + 0 + 0$

$A(n) = n$

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Now Let $R_n = n^2$

$$n^2 = (n-1)^2 + \alpha + \beta n + \gamma n^2$$

$$\Rightarrow n^2 = n^2 - 2n + 1 + \alpha + \beta n + \gamma n^2$$

$$\Rightarrow 0 = n(\beta - 2) + (\alpha + 1) + \gamma n^2$$

For Getting β $\alpha = -1$, $\gamma = 0$

$$0 = n(\beta - 2) + 0 + 0$$

$$\beta - 2 = 0$$

$$\beta = \cancel{0}2$$

$$n^2 = A(n)(-1) + B(n) \times 2 + 0$$

$$n^2 = -A(n) + 2B(n)$$

$$n^2 = -n + 2B(n)$$

$$B(n) = \frac{n^2 + n}{2}$$

Now, Let $R_n = n^3$

$$n^3 = (n-1)^3 + \alpha + \beta n + \gamma n^2$$

$$n^3 = n^3 - 3n^2 + 3n - 1 + \alpha + \beta n + \gamma n^2$$

$$0 = n^2(\gamma - 3) + n(\beta + 3) + (\alpha - 1)$$

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For getting $\beta = -3$, $\alpha = 1$

$\gamma = 3$

Now,

$$n^3 = A(n)\cdot 1 + B(n)(-3) + 3C(n)$$

$$n^3 = n - \frac{3(n^2+n)}{2} + 3C(n)$$

$$n^3 = \frac{2n - 3n^2 - 3n + 6C(n)}{2}$$

$$2n^3 = -3n^2 - n + 6C(n)$$

$$6C(n) = 2n^3 + 3n^2 + n$$

$$C(n) = \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{n(2n^2+2n+n+1)}{6}$$

$$= \frac{n\{2n(n+1)+1(n+1)\}}{6}$$

$$C(n) = \frac{n(n+1)(2n+1)}{6}$$

we have

$$R_n = R_{n-1} + \alpha + \beta n + \gamma n^2$$

if $\alpha = 0, \beta = 0, \gamma > 0$

$$R_n = R_{n-1} + \gamma n^2$$

According to recurrence relation

$$R_{n-1} + n^2 \Rightarrow \sum n^2$$

So, $R_n = \square_n$

So, $\square_n = \frac{n(n+1)(2n+1)}{6}$

Practise Problem

$$n^3 = \left(\frac{n(n+1)}{2} \right)^2$$

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Chapter-6 (follow book for this Chapter)
Special Numbers

Stirling Numbers:

James stirling এই number গুলোর উদ্ভাবক।

Stirling Second Kind / Stirling Subset Number

representation: $\left\{ {n \atop K} \right\}$

এটাকে "n subset K" বলে।

n হচ্ছে number of elements

K " কয়টি subset

$\left\{ {n \atop K} \right\}$ n সংখ্যক element দিয়ে K ভাগে কয়টি partition করা যাবে তাকেই n subset K বলে।

$$\left\{ {4 \atop 2} \right\} = \left\{ {1\ 2\ 3\ 4 \atop 2} \right\} = \{1,2,3\}\{4\},$$

$$\{1,3,4\}\{2\},$$

$$\{1,2,4\}\{3\},$$

$$\{2,3,4\}\{1\},$$

$$\{1,2\}\{3,4\}, \{1,3\}\{2,4\},$$

$$\{1,4\}\{2,3\}$$

$lcm(m,n) = \min\{K \mid K > 0$

4, 16
এর gcd 4
Now
$4 \times 2 + 16 \times 2$
$= 40$
$40/4 = 0$

Now Proof by GCD

$m = ka$

$n = kb$

if $k \mid m$ & $k \mid n$ then it can also divide $m'm + n'n$

$gcd(m,n) = m'm + n'n$

$= m'(ka) + n'(kb)$

$= m'ka + n'kb$

$= k(m'a + n'b)$

m' & n' any integer

As k, $(m'a + n'b)$ এর সাথে multiple আকারে আছে তাই বলা যায় K $(m'a + n'b)$ দ্বারা divisible. আর ~~right~~ left side এ $gcd(m,n) = k(m'a + n'b)$ তাই বলা যায় K হচ্ছে $gcd(m,n)$

LCM দ্বারা prove practise

Primes Proof by LCM

Must ত Exam

Prime

$$n = p_1 \dots p_m = \prod_{k=1}^{m} p_k$$

if $m=0$, $n=1$ কারণ কোন production নাই তাই empty production.

First proof production হলো prime number এর production.

if $n>1$

suppose $n = n_1 \cdot n_2$

~~এখন~~ এখন আমরা prove করতে try করব n_1 & n_2 prime number এর productio. so

$n_1 \cdot n_2$ " " " "

$36 = 4 \times 9 = \underbrace{2 \times 2}_{} \times \underbrace{3 \times 3}_{}$

$n_1 = 2 \times 2$

$n_2 = 3 \times 3$

proved

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2nd proof production is unique number production

Suppose $n = pq$ [যদি n এর দুইটি product p & q]

$p = p_1 \ldots p_m$, $p_1 \le \ldots \le p_m$

$q = q_1 \ldots q_k$ $q_1 \le \ldots \le q_k$

Now,

$n = p_1 \ldots p_m = q_1 \ldots q_k$

$\boxed{36 = 2 \times 2 \times 3 \times 3}$

Let $p_1 < q_1$

$ap_1 + bq_1 = \underset{\text{As gcd 1}}{1}$ .. ① if $\gcd(p_1, q_1) = 1$ [As $p_1 \ne q_1$ & both are prime]

multiply by $q_2 q_3 \ldots q_k$

$ap_1 (q_2 q_3 \ldots q_k) + b q_1 (q_2 q_3 \ldots q_k) = q_2 q_3 \ldots q_k$

$ap_1 (q_2 \ldots q_k) + bn = q_2 \ldots q_k$

left side p_1 দ্বারা divisible but right side divisible না,

right side তে ~~divible~~ divisible করতে q_1 যুক্ত দিতে হবে। কিন্তু ~~এখ~~ আমরা বলেছি $p_1 < q_1$. so q_1 লিখলেও আমরা $q_1 \cdots q_k = n = p_1 \cdots p_m$ লিখতে পারতেছি না।

তাই $p_1 < q_1$ হয় না। $p_1 = q_1$ হলেই আমরা $q_1 \cdots q_k = n = p_1 \cdots p_m$ লিখতে পারব right side এ। আর তখন right side ও p_1 দ্বারা divisible হবে।

আর তখন $p = q$ বলতে পারব।

Factorial

Prove that $n!$ increases exponentially / $n!$ is a big number

By Gauss trick prove $n!$ is a big number

n	0	1	2	3	4	5	6	7	8	9	10
$n!$	1	1	2	6	24						

Gauss trick

$S_n = 1 + 2 + 3 + \cdots + 100$

$S_n = 100 + \underset{(n-1)}{99} + 98 + \cdots + 1$

$(n+1) + (n+1) + (n+1) + \cdots + (n+1)$

$2S_n = n(n+1)$

$S_n = \dfrac{n(n+1)}{2}$

$n! = (n-1)!\, n = \prod_{K=1}^{n} K$

Now, $(n!)^2 = \prod_{K=1}^{n} \cdot \prod_{K=1}^{n} K$

$= \prod_{K=1}^{n} (1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-2) \cdot (n-1) \cdot n)$

$= \prod_{K=1}^{n} \{1(n-0) \cdot 2 \cdot (n-1) \cdot 3 \cdot (n-2) \cdots\}$

$= \prod_{K=1}^{n} K(n+1-K)$

$1(n-0) = 1(n+1-1)$

$2(n-1) = 2(n+1-2)$

$3(n-2) = 3(n+1-3)$

Now $\underbrace{K}_{a}\underbrace{(n+1-K)}_{b}$

$$= \frac{(K+n+1-K)^2-(K-n-1+K)^2}{4} \quad \left[ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2\right]$$

$$= \frac{(n+1)^2}{4} - \frac{(2K-(n+1))^2}{4}$$

$$= \frac{1}{4}(n+1)^2 - \frac{4K^2-4K(n+1)+(n+1)^2}{4}$$

$$= \frac{1}{4}(n+1)^2 -$$

$$= \frac{(n+1)^2}{4} - \left(\frac{2K-(n+1)}{2}\right)^2$$

$$= \frac{(n+1)^2}{4} - \left(K-\frac{1}{2}(n+1)\right)^2$$

যেহেতু K এর smallest value = 1

K " largest " $= \frac{1}{2}(n+1)$

যেহেতু ও aside এ 0 হবে যদি

সুতরাং $\frac{(n+1)^2}{4}$

Now, $k = 1$

$$k(n+1-k) = \frac{1}{4}(n+1)^2 - \left(1 - \frac{1}{2}(n+1)\right)^2$$

$$= \frac{1}{4}(n+1)^2 - \left(1 - (n+1) + \frac{1}{4}(n+1)^2\right)$$

$$= \frac{1}{4}(n+1)^2 - \left(\cancel{1} - n - \cancel{1} + \frac{1}{4}(n+1)^2\right)$$

$$= \cancel{\frac{1}{4}(n+1)^2} + n - \cancel{\frac{1}{4}(n+1)^2}$$

$$= n$$

$$\cancel{= \frac{(n+1)^2 - (1-n)^2}{4}}$$

$$\cancel{= \frac{n^2+2n+1-1+2n-n^2}{4}}$$

$$\cancel{= \frac{4n}{4} = n}$$

Now, $k = \frac{1}{2}(n+1)$

$$k(n+1) - k = \frac{(n+1)^2}{4} - 0$$

$$\prod_{k=1}^{n} n \le n!^2 \le \prod_{k=1}^{n} \frac{(n+1)^2}{4}$$

$$n!^2 = \prod_{k=1}^{n} k(n+1-k)$$

$$n^n \le n!^2 \le \frac{(n+1)^{2n}}{4^n}$$

$$n^{n/2} \le n! \le \frac{(n+1)^n}{2^n}$$

So $n!$ exponential আকারে বৃদ্ধি পায়।

CT

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Chapter-6 (follow book for this Chapter)
Special Number

Stirling Numbers:

James Stirling এর number গুলোর উদ্ভাবক।

Stirling Second kind / Stirling Subset Number

representation: $\left\{ {n \atop k} \right\}$

একে "n subset k" বলে।

n হচ্ছে number of elements

k " কতটি subset

$\left\{ {n \atop k} \right\}$ n সংখ্যক element দিয়ে k উপাদানে কতটি partition করা যাবে তাকেই n subset k বলে।

$\left\{ {4 \atop 2} \right\} = \{1\ 2\ 3\ 4\} = \{1,2,3\}\{4\},$
$\{1,3,4\}\{2\},$
$\{1,2,4\}\{3\},$
$\{2,3,4\}\{1\},$
$\{1,2\}\{3,4\}, \{1,3\}\{2,4\},$
$\{1,4\}\{2,3\}$

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$\left\{ {1,2,3,4 \atop 4} \right\} = \cancel{\{1,2,3,4\}}\ \{1\}\{2\}\{3\}\{4\}$

$\left\{ {1,2,3,4 \atop 1} \right\} = \{1,2,3,4\}$

$\left\{ {n \atop n} \right\} = 1 \qquad \left\{ {n \atop 1} \right\} = 1 \qquad \left\{ {n \atop 0} \right\} = 0$

$\left\{ {0 \atop 1} \right\} = 0 \qquad \left\{ {0 \atop 0} \right\} = 1$ [empty set কে empty হিসাবে ধরা করছি]

$\left\{ {n \atop 2} \right\}$

$\left\{ {0 \atop 2} \right\} = 0$

$\left\{ {n \atop 2} \right\} = 2^{n-1} - 1 = \frac{2^n}{2} - 1$

Reason of this formula

আমরা subset তৈরি করতে পারি 2^n. ~~যার মধ্যে~~

Example : $\{1,2,3\} = \{1\},\{2\},\{3\},\{1,2\},\{2,3\}$
$\{3,4\}, \{1,2,3\}, \phi$

এখানে প্রকৃত উপাদানটি consider করলে $\{1,2,3\}$ বাদ, এই formula তে -1 করেছি। যেমন এ উপাদানটি এ একটা করে element নিয়েও term আছে। আমাদের কে 2 টা করে নিতে হবে।

$\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}$

So single element গুলো pair এ convert করলে 1/2 ~~element~~ হয়ে যায়। তাই 2 দ্বারা divide করা হয়েছে।

$\left\{ {n \atop 2} \right\} \left\{ {n \atop 3} \right\}$

5 15 25

6 90

এখানে $90 = 25 \times 3 + 15$

$$\left\{ {n \atop k} \right\} = \left\{ {n-1 \atop k} \right\} \times k + \left\{ {n-1 \atop k-1} \right\}$$ Recurrence

Question: $\left\{ {n \atop 2} \right\}$ বের করো

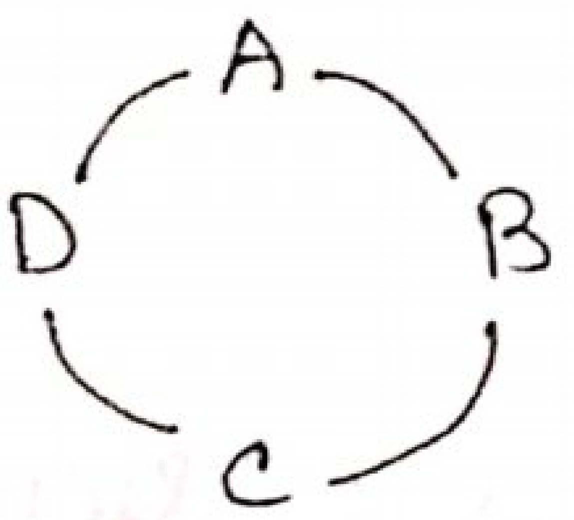
$\left\{ {n \atop 0} \right\}$ হলে 0

$\left\{ {0 \atop 0} \right\} = 1$ হলে

recurrence relation টা বের করো

Stirling Number First Kind: Stirling Cycle

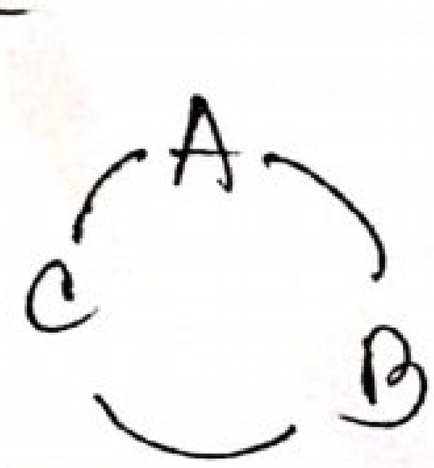
$\left[{n \atop k} \right]$. n cycle k

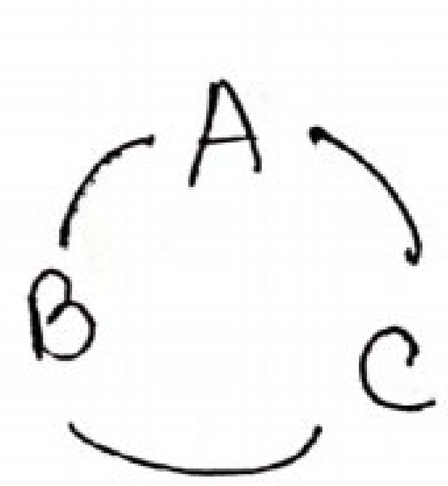


$\left\{ {1,2,3,4 \atop 2} \right\}$

$\left[{A,B,C,D \atop 2} \right]$ = [A,B,C] [D], [A,C,D] [B],
[A,B,D] [C], [A,B] [C,D],
[A,C] [B,D], [A,D] [B,C]
[B,C,D] [A]

যদি [A,B,C] & [A,C,B] ~~Cycle~~ subset হিসেবে same
কিন্তু cycle হিসেবে দেখলে same না





Now

$$\begin{bmatrix} A\ B\ C\ D \\ 2 \end{bmatrix} = [A,B,C][D],\ [A,C,D][B],\ [A,B,D][C],\ [B,C,D][A]$$

$$\cancel{[A,B][C,D],\ [A\ C][B,D]}$$

$$[A,D,B][C],\ [B,D,C][A],\ [A,C,B][D],\ [A,D,C][B];\ [A,D][B,C]$$

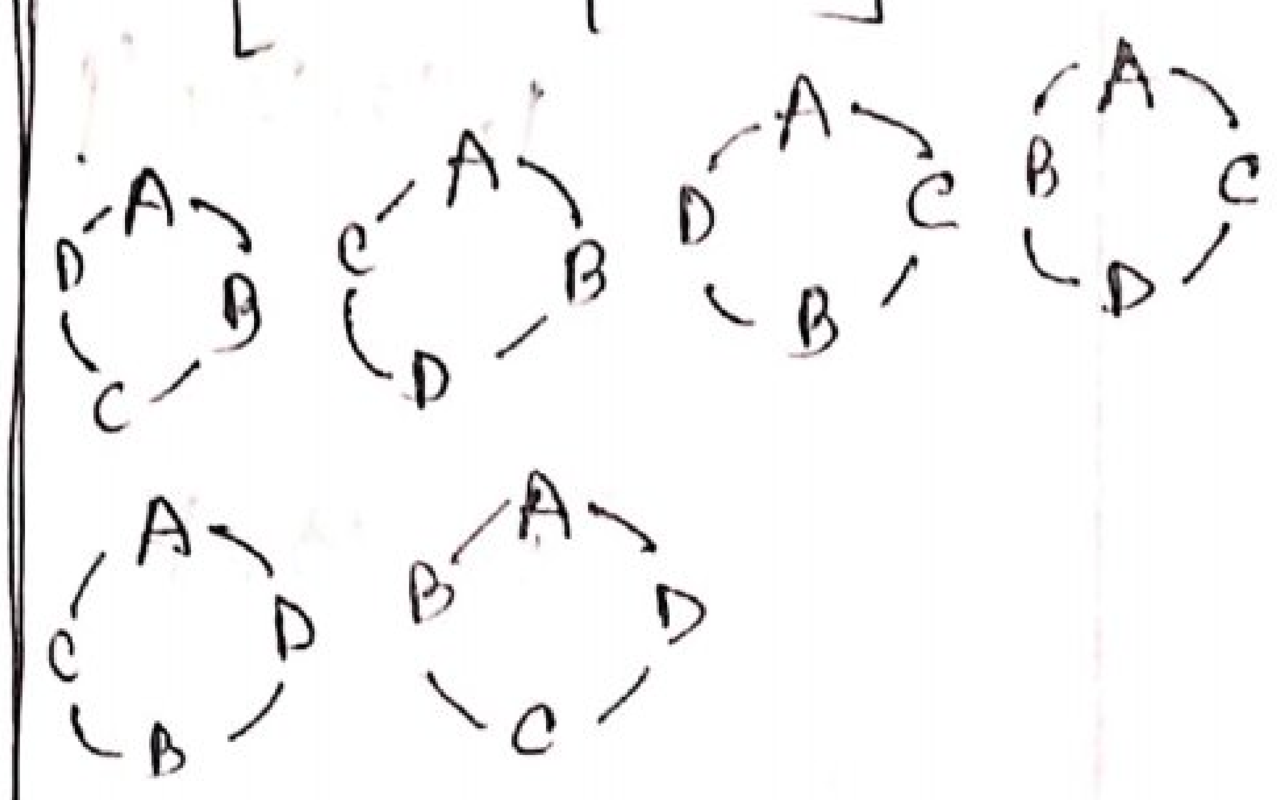
$$[A,B],\ [C,D],\ [A,C][B,D];$$

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix} = 11$$

$$\begin{bmatrix} n \\ 0 \end{bmatrix} = 0 = \left\{ \begin{matrix} n \\ 0 \end{matrix} \right\} \qquad \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 1 = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$\begin{bmatrix} A\ B\ C\ D \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} n \\ 1 \end{bmatrix} = (n-1)! \longrightarrow$$



$$\left\{ \begin{matrix} n \\ n-1 \end{matrix} \right\} \qquad \begin{bmatrix} n \\ k \end{bmatrix} \geq \left\{ \begin{matrix} n \\ k \end{matrix} \right\}$$

So, Recurrence

$$\begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$$

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} \qquad \left\{ \begin{matrix} 6 \\ 4 \end{matrix} \right\} = 85$$

$$\left\{ \begin{matrix} n-1 \\ k \end{matrix} \right\} \qquad \left\{ \begin{matrix} 5 \\ 4 \end{matrix} \right\} = 10 \times 5 + 35 = 80$$

$$\begin{bmatrix} n-1 \\ k-1 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 3 \end{bmatrix} = 35$$

এগুলোর recurrence ~~শিখতে~~ হবে না

Eulerian Number ঃ [এগুলোর recurrsion ~~মনে রাখতে~~ করতে হবে না]

$\left\langle {n \atop k} \right\rangle =$

n সংখ্যক number এর subset থেকে ~~k সংখ্যক আলাদা~~ ঐ সকল subset যাদের k সংখ্যক place এ $\pi_i < \pi_{i+1}$

$\left\langle {4 \atop 2} \right\rangle = \left\langle {1,2,3,4 \atop 2} \right\rangle \Rightarrow$ এখানে 3 টা place এ $\pi_i < \pi_{i+1}$ হয়ে যাচ্ছে।

$1,3,2,4 \Rightarrow \pi_1 < \pi_2$
$\pi_3 < \pi_4$

$2,3,1,4 \Rightarrow \pi_1 < \pi_2$
$\pi_3 < \pi_4$

এভাবে totall ১১ ~~position টা~~ combination possible following the condition.

Q $\left\langle {n \atop n} \right\rangle = 0$ কেন?

$\left\langle {1,2,3 \atop 3} \right\rangle$ এখানে আমার 3 টা place ই নাই যে compare করব, সব

$\left\langle {n \atop n} \right\rangle$ এর ক্ষেত্রে place always $(n-1)$ টা

তাই 0

Q. $\left\langle {n \atop 0} \right\rangle = 1$ কেন?

~~উদাহরণ~~ $\left\langle {5\;4\;3\;2\;1 \atop 0} \right\rangle$

এর মানে হচ্ছে 0 টা place এ $\pi_i < \pi_{i+1}$ permutation অনুসারে হতে হবে, এমন ~~comp~~ permutation একটাই কেবল possible

$$\left\langle {n\;\; n-1\;\; n-2\;\; \cdots\;\; 1 \atop 0} \right\rangle$$

তাই $\left\langle {n \atop 0} \right\rangle = 1$

Harmonic Number:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n} = \sum_{K=1}^{n} \frac{1}{K}, \text{ integer } n \geq 0$$

Card

per card length = 2 unit

condition:

Table এর উপরে Card এর centre of gravity table এর edge বরাবর

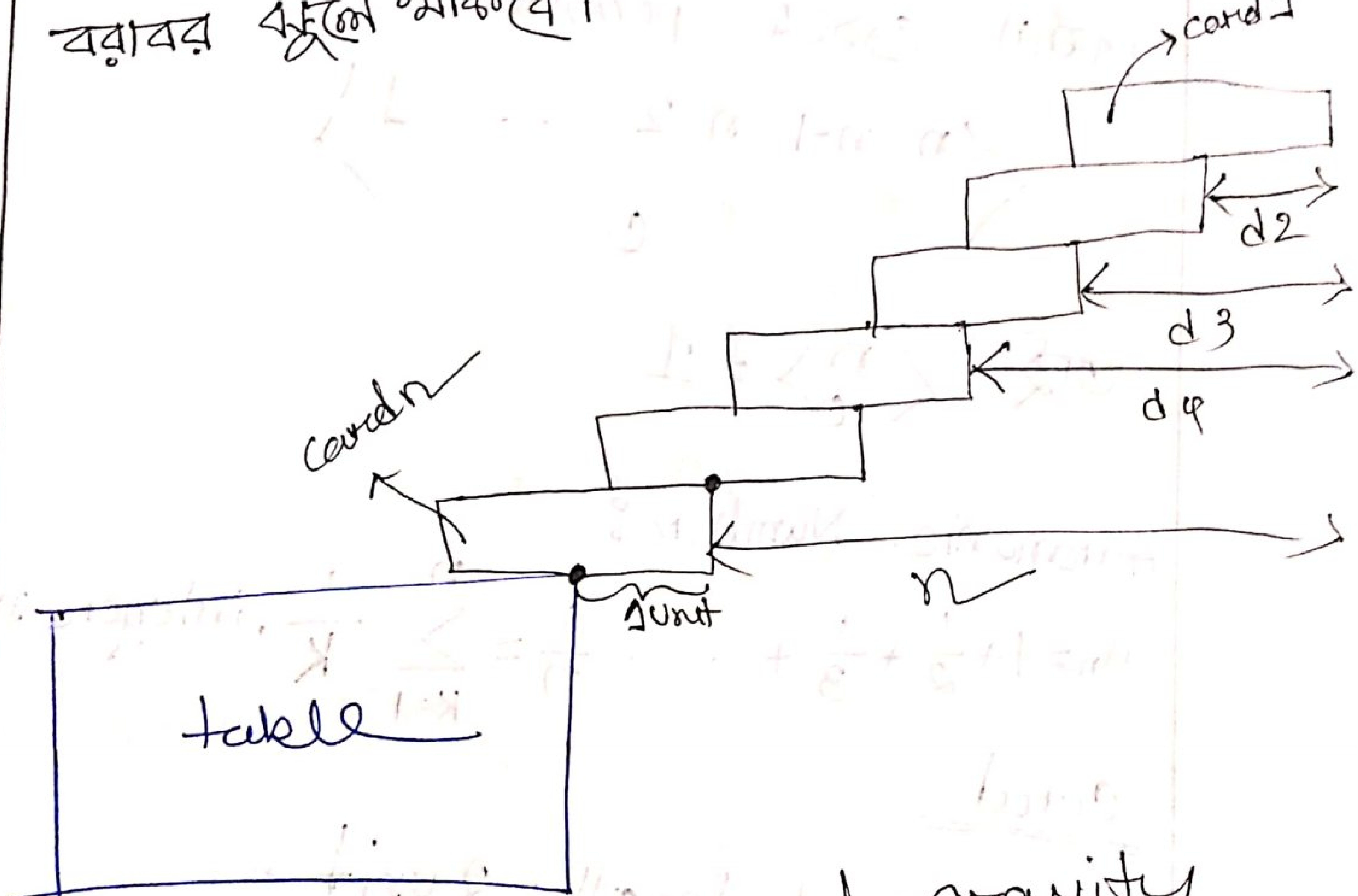
আসলে Card এর edge বরাবর

2. প্রত্যেক card কে ~~৪~~ অন্যদের সাথে parallel হতে হবে

3\.

n সংখ্যক card আছে।

Question: ~~n সংখ্যক card~~ কত সংখ্যক card নিলে cardগুলো table এর edge বরাবর ঝুলে থাকবে।



যদি ~~first~~ card এর centre of gravity centre বরাবর, Here if we consider table it plays role as ~~nth~~ (n+1) th card

d_K distance - top card edge থেকে তার
পরের card, ~~edge~~ এর corresponding distance

$d_1 = 0$ $\qquad 1 \le k \le n$

$d_{K+1} = d_1 + d_2 + d_3 + \dots + d_K$

$= \dfrac{\cancel{d_1 + d_2 + d_3 + \dots + d_K}}{}$

$= \dfrac{d_1+1+d_2+1+d_3+1 \dots d_K+1}{K}$

$K d_{K+1} = K + d_1 + d_2 + \dots + d_K$, $K \geq 0$ ... (1)

1 টা K কমে গেলে

$(K-1) d_K = K - 1 + d_1 + d_2 + \dots + d_{K-1}$, $K \geq 1$ ... (2)

(1) − (11)

$K d_{K+1} - K d_K + d_K = 1 + d_K$

~~$d_{K+1} = d_K$~~

$K d_{K+1} = 1 + \cancel{d_K} - d_K + K d_K$

$d_{K+1} = \dfrac{1}{K} + d_K$

$d_{K+1} = d_K + \dfrac{1}{K}$

$= H_K$ [From harmonic number]

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$d_{n+1} = H_n$

যেকোনো card highest তার length এর সমানুপাতিক ভাবে ঝুলে থাকতে পারে।

n	.
H_n	

Question: Card length change করে H_n এর value কত হলে overhand হবে

2nd Problem of Harmonic Number

Worm on the rubber band

1. Worm rubber band দিয়ে 1 min এ 1cm যায়। K streches, the rubber by 1cm band when worm cross 1 cm of rubber band.

each minute $\frac{1}{100}$ th rubber band

$$\frac{1}{100} + \frac{1}{200} + \frac{1}{300} + \frac{1}{400} + \dots + \frac{1}{n \times 100}$$

$$= \frac{1}{100}\left(1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n}\right) = \frac{H_n}{100}$$

when H_n become greater than 100 then worm can cross rubber band.

Question: 1. when $H_n \geq 100$ for that we have to find n

2. Worm কবে rubber band cross করবে কত min লাগবে।

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NOT from any chapter Totally some distinct Topic

7.11.21

Harmonic Progression (new ppt)

Arithmatic Progression:

1. first find interval of ~~each~~ two digits in a series
2. অন্য কোনো number মাঝে আসবে না
3. হবে increasing / decreasing

8, 13, 18, 23, 28, 33, ...

a = first term = 8

d = diff^n bet^n any two number = 5

n = number of terms.

Basic formula for sum

$\frac{n}{2}\left(2a+(n-1)d\right)$

what will be sum of first 100 ~~numbers~~ terms of series 8, 13, 18, 23, 28, 33, ...

$a = 8,\ d = 5,\ n = 100$

$S = \frac{100}{2}\left(2\times 8 + (100-1)5\right)$

$= 25550$

Formula to identify any term

$T_n = a + (n-1)d$

30th element of series 8, 13, 18, 23

$T_{30} = 8 + (30-1)5 = 8 + 145 = 153$

Another problem

there are 31 terms. first term

Last term

Harmonic progression is reciprocal of arithmatic progression.

Harmonic progression এ আমাকে যে series দিবে তা reciprocate করতে হবে then সেটা arithmetic ~~কে~~ series হবে then formula দিয়ে result বের করে result কে আবার reciprocate করবো,

Harmonic Means: দুইটি term এর মাঝের ~~[illegible]~~ যা যা value থাকবে তার সবই harmonic means

$\frac{1}{5}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{10}$

$\frac{1}{5}$ & $\frac{1}{8}$ এর মধ্যকার H.M কত? Ans: $\frac{1}{2}, \frac{1}{4}$

যদি এমন বলে $\frac{1}{4}, \frac{1}{6}$ এর মধ্যকার H.M কত?।

এক্ষেত্রে $\frac{1}{4}$ & $\frac{1}{6}$ এর interval বের করতে হবে, then সেটা থেকে মাঝের term গুলো বের হবে।

Arithmatic progression: 2, 4, 6, 8

Harmonic " : $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$

Problems: (Slide 13)

~~Determine~~

1) $1, 1/2, 1/3, 1/4, \ldots$

reciprocate series: 1, 2, 3, 4, ...

It is harmonic progression

2) $1, 1/4, 1/5, \frac{1}{7}$

reciprocate series: 1, 4, 5, 7

It is not harmonic progression

Determine next 3terms of follow HP

24, 12, 8, 6, ...

Solve: $\frac{1}{24}, \frac{1}{12}, \frac{1}{8}, \frac{1}{6}$

$diff^n = \frac{1}{12} - \frac{1}{24} = \frac{1}{24}$

5th term $= \frac{1}{6} + \frac{1}{24} = \frac{5}{24} = \frac{24}{5}$

6th term $= \frac{5}{24} + \frac{1}{24} = \frac{1}{4} = 4$

6th term $= \frac{1}{4} + \frac{1}{24}$

$= \frac{7}{24}$

$= \frac{24}{7}$

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Find Harmonic Mean betn following terms:

1) 12 and 8 [এমন problem এ always 2nd number টাকে 3rd term ধরে নিবো

reciprocate : $\frac{1}{12}$ and $\frac{1}{8}$

$\frac{1}{12}$ Harmonic Mean $\frac{1}{8}$:

1st term Harmonic Mean last term

এখানে 1st term ধরাটা assumption. এখানে
$\frac{1}{8}$ কোনোদিন second term হবে না কারণ
Harmonic Mean ই বের হবে না,
তাহলে
so $\frac{1}{8}$ কে atleast 3rd term হতে হবে,

$$T_3 = a + (n-1)d$$

$$\frac{1}{8} = \frac{1}{12} + (3-1)d$$

$$\frac{1}{24} = 2d$$

$$d = \frac{1}{48}$$

~~এখন first term~~

$$2nd\ term = \frac{1}{12} + \frac{1}{48} = \frac{5}{48} = \frac{48}{5}$$

Insered three H. M betn following term

36 & $\frac{36}{5}$

Reciprocate: $\frac{1}{36}$ & $\frac{5}{36}$

$T_5 = a + (n-1)d$

$\frac{5}{36} = \frac{1}{36} + (5-1)d$

$\frac{1}{9} = 4d$

$d = \frac{1}{36}$

2nd term $= \frac{1}{36} + \frac{1}{36} = \frac{1}{18} = 18$

3rd " $= \frac{1}{18} + \frac{1}{36} = \frac{1}{12} = 12$

4th " $= \frac{1}{12} + \frac{1}{36} = \frac{1}{9} = 9$

6, 4, 1

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

এই harmonic progression এর 10th term

Reciprocal : 2, 4, 6, 8

$$T_{10} = 2 + (10-1)2$$
$$= 20$$

Reciprocate $= \frac{1}{20}$

যদি ~~Reciproc:~~ Series reciprocate করে
পেলে term গুলোর interval সমান না
তাহলে এর reason টা mention করে
বলবো এটার n'তম term বের করা
possible না

Slide 33

3) 12, 6, 4, 3 → এটা increasing order

Reciprocate: $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$

$\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$

~~24~~ Slide 34

2) 1 and $\frac{1}{9}$

$T_3 = a \cdot 1 + (3-1)d$

$\frac{1}{9} = 1 + 2d$

$-\frac{8}{9} = 2d$

$d = -\frac{4}{9}$

1 and $\frac{1}{9}$

Reciprocate: 1 and 9

$T_3 = 1 + (3-1)d$

$9 - 1 = 2d$

$8 = 2d$

$d = 4$

2nd term $= 1 + 4 = 5 = \frac{1}{5}$

So harmonic mean $= \frac{1}{5}$

Geometric Progression

3, 9, 27, 81, 243

starting term $a = 3$

ratio $r = \frac{9}{3} = 3$

n = number of term $= 5$

what is the sum of this series for 5 terms

$1^2, 2^2, 3^2, \ldots$

$\cancel{\frac{1}{2^2}}$

Formula: $\frac{n(n+1)(2n+1)}{6}$

এসব ক্ষেত্রে কোন fixed formula নেই

$= \frac{5(5+1)(2\times5+1)}{6}$

$= 55$

$1^3, 2^3, 3^3, \ldots$

Formula: $\left(\frac{n(n+1)}{2}\right)^2$

$= \left(\frac{5(5+1)}{2}\right)^2$

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Q. Sum of infinite series : $512, 256, 128, 64, \ldots$ for first 10 terms

$a = 512$, $r = \frac{256}{512} = \frac{1}{2} = 0.5$

As series is in decreasing ~~or~~ order

Formula $= \frac{a}{1-r}$ [if $r < 1$]

if $(r > 1)$

formula $= \frac{a}{r-1}$

সিরের করে use না করে

nth term এর sum বের করার formula

$a * (r^n - 1) / (r-1)$ if [$r > 1$] [series 2, 4, 8, 16 ...]

$a * (1 - r^n) / (1-r)$ if [$r < 1$] [16, 8, 4, 2 ...]

হবে বের করে use করে

8, 64, 512

$r = 8$

$a = 8$

$$Sum = \frac{a*(r^n - 1)}{r - 1}$$

81, 27, 9

$r = \frac{1}{3}$

$a = 81$

$$Sum = \frac{a*(1 - r^n)}{1 - r}$$

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512 256 12 64

$r = 0.5$

$a(1 - r^n) / 1 - r$

$= 512 \times (1 - (0.5)^{10}) / 1 - 0.5$

$= 1023$

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200000, 100000, 50,000, 25,000, 12500,

$$S_n = \frac{a}{1 - r} \qquad \left[r = \frac{100000}{200000} = \frac{1}{2} \right.$$

$$= \frac{200000}{1 - \frac{1}{2}}$$

$$= 400000$$

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$$S_n = \frac{a*(1-r^n)}{1-r}$$

$$400000 = \frac{200000(1-(\frac{1}{2})^n)}{1/2}$$

$$\cancel{100000 = 1-\left(\frac{1}{2}\right)^n}$$

$$\cancel{99999 = \left(\frac{1}{2}\right)^n}$$

$$1 = 1 - \frac{1}{2^n}$$

$$\frac{1}{2^n} = 0$$

$$2^{-n} = 0$$

বের করাটা tough
so n বের করছি না।

14.11.21

Generating Functions

dice এর জন্য

$1 \rightarrow$ আসবার probability $P_1 = \frac{1}{6}$

$2 \rightarrow$ " " $P_2 = \frac{1}{6}$

$3 \rightarrow$ " " $P_3 = \frac{1}{6}$

$4 \rightarrow$ " " $P_4 = \frac{1}{6}$

$5 \rightarrow$ " " $P_5 = \frac{1}{6}$

$6 \rightarrow$ " " $P_6 = \frac{1}{6}$

G(x) এর normal formula

$$G(x) = \sum_{k=0}^{\infty} a_k x^k$$

For dice

$$G(x) = P_1 x + P_2 x^2 + P_3 x^3 + P_4 x^4 + P_5 x^5 + P_6 x^6$$

differenciate

$$G'(x) = P_1 + 2P_2 x + 3P_3 x^2 + 4P_4 x^3 + 5P_5 x^4 + 6P_6 x^5$$

$$G'(0) = P_1 + 0 + 0 + 0 + 0 + 0$$

Like this we can find P_2, P_3 . . . . . .

$$G''(0) = 2P_2$$

$$G'''(0) = 6P_3$$

so, $P_1 = G'(0)$

$P_2 = \frac{G''(0)}{2}$

$P_3 = \frac{G'''(0)}{6}$

$P_4 = \frac{G^{IV}(0)}{24}$

$P_5 = \frac{G^{V}(0)}{120}$

$$P(x) = \frac{1}{K!} \frac{d^K G(x)}{dx}$$

1 টা dice হলে

~~max~~ min probability $G_1(x) = 2$

max " $G_2(x) = 12$

2 টা dice একসাথে হলে

$$G^2(x) = (P_1x + P_2x^2 + P_3x^3 + P_4x^4 + P_5x^5 + P_6x^6) \cdot (P_1x + P_2x^2 + P_3x^3 + P_4x^4 + P_5x^5 + P_6x^6)$$

$$= P_1P_1x^2 + P_1P_2x^3 + \ldots\ldots\ldots + P_6P_6x^{12}$$

2 আসার	~~probability~~ way	1 টা	P_1P_1
3 "	"	2 "	P_1P_2, P_2P_1
4 "	"	3 "	P_1P_3, P_2P_2, P_3P_1

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6 বেডার way 5 টা P_1P_5 P_1P_5

P_2P_4 P_5P_1

P_3P_3 P_3P_3

P_4P_2 P_2P_4

P_5P_1 P_4P_2

Math

conditions

even number of red candies

$x^0+x^2+x^4+x^6+x^8+x^{16}$

more than 6 blue candies

$x^7+x^8+x^9+x^{16}$

less than 3 green candies

$x^0+x^1+x^2$

so for choosing 10 candies maintaining constraints:

$x^0(x^7+x^8+x^9+x^{10}+x^0+x^1+x^2)$

$x^2($ " $)$

$x^4($ " $)$

$x^6($ " $)$

$x^8($ " $)$

$x^{10}($ " $)$

$x^0 x^{10}$

$x^2 x^8$

$\boxed{x^4 x^6 \quad x^6.}$ কারণ এইগুলো মাত্র x^{10} make করতে পারে না

$x^8 x^2$

$x^{10} x^0$

Problems: A girl can choose 2 items from the basket. basket containing 1 apple, 1 pair, 1 orange, 1 banana and 1 papaya. How many ways she can do this?

For apple $\rightarrow x^0 + x^1 = 1+x$

For pair $\rightarrow x^0 + x^1 = 1+x$

" orange $\rightarrow x^0 + x^1 = 1+x$

" banana $\rightarrow x^0 + x^1 = 1+x$

" papaya $\rightarrow x^0 + x^1 = 1+x$

[আমাদের যেখানে x^2 আছে তাদের co-eff যোগ করব]

For ~~total~~ 2 item chose $= (1+x)^5$

$= 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

Here coeff of $x^2 = 10$

So 10 way we can select 2 items

যদি previous problem এ basket এ ~~1টা~~ ~~appl~~ 2টা apple থাকে তাহলে কিভাবে 2টা items select করবে

For apple $\rightarrow 1+x+x^2$

" pair $\rightarrow 1+x$

" orange $\rightarrow 1+x$

" banana $\rightarrow 1+x$

" papaya $\rightarrow 1+x$

For selective 2 items $=(1+x+x^2)(1+x)^4$

$=(1+x+x^2)(1+4x+6x^2+4x^3+x^4)$

only for x^2

$=4x^2+x^2+6x^2$

$=11x^2$

Problem 2: In a bakery there are 3 cheese, 2 cherry, 4 raspberry pastries. How many ways one can choose 7 pastries.

For cheese pastries $= 1+x+x^2+x^3$ $\cancel{+x^3}$

For cherry " $= 1+x+x^2$

For Rasberry " $= 1+x+x^2+x^3+x^4$

Now, For selecting 7 pastries

$= (1+x+x^2+x^3)(1+x+x^2)(1+x+x^2+x^3+x^4)$

$= (1+x+x^2+x+x^2+x^3+x^2+x^3+x^4+x^3+x^4+x^5)$
$(1+x+x^2+x^3+x^4)$

$= (1+2x+3x^2+3x^3+2x^4+x^5)(1+x+x^2+x^3+x^4)$

only for x^7

$= 3x^7+2x^7+x^7$

$= 6x^7$

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modification of problem 2

rasberry pastries ~~can~~ come in box of 2 (even numbers)

Now solve ~~$\$$~~

For cheese pastries $= 1+x+x^2+x^3$

" cherry " $= 1+x+x^2$

" rasberry " $= 1+x^2+x^4$

Now, For selecting 7 pastries

$= (1+x+x^2+x^3)(1+x+x^2)(1+x^2+x^4)$

~~$=(1+x+x^2) + x + x^2 +$~~

$= (1+2x+3x^2+3x^3+2x^4+x^5)(1+x^2+x^4)$

$= 3x^7 + x^7$

$= 4x^7$

so 4 ways to find 7 pastries

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Miss Revolving Number

142857

7 1 4
5 ×1
8 2

4 2 8
×2
1 5
7

1 4
7 ×3 2
5 8

6 পর্যন্ত গুণ দিলে এই seq maintain করে।

$\underline{\underline{720}} = 1\times2\times3\times4\times5\times6$

$= 8\times9\times10$

[1–10 পর্যন্ত সব digit আছে except 720

Easy trick for square

$11 \times 11 = 121$

$12 \times 12 = 144$

$13 \times 13 = 169$ [~~last dig x last dig~~ to (first dig × first dig) (last dig + last dig) (last dig × last dig)

$14 \times 14 = 196$

$15 \times 15 = 225$

$16 \times 16 = 256$

$17 \times 17 = 289$

$18 \times 18 = 324$

$19 \times 19 = 361$

$22 \times 22 =$ 4

23

$24 \times 24 =$ 576

$27 \times 27 = 729$ [(f.d × f.d) (f.d × f.d × 2nd dig) (last d × last d)

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