

Harmonic Progression

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Arithmetic Progression

- There must be a fixed interval between any two digits in a series.
- There must not be any other reason for the numbers to come in that order.
- The numbers either increase or decrease.

Example of AP?

8, 13, 18, 23, 28, 33

Here we have an AP, because there is a fixed gap and the series is increasing in a series we have 6 terms:

a = first term = 8

d = difference between any two terms = 5

n = number of terms

Basic formula of AP?

In order to find sum of some terms, you can use any of these formula:

1. $\frac{n}{2} (2a + (n - 1) d)$
2. $\frac{n}{2} (\text{first term} + \text{last term})$

Example of the formula:

What will be the sum of first 100 terms of this series:

8, 13, 18, 23, 28,

$$n/2 (2a + (n - 1) d)$$

$$n= 100, a=8, d = 5$$

$$100/2 (2* 8 + (100 - 1) * 5)$$

$$=50 (16 + 495)$$

$$=25550$$

How to identify a term in an AP?

In order to identify any term, use this formula:

$$T_n = a + (n - 1) d$$

What is 30th term in this series:

$$8, 13, 18, 23, 28, \dots$$

$$= 8 + (30 - 1) 5$$

$$= 8 + 145$$

$$= 153$$

What will be 17th term in the series: 103, 123, 143, 163, ..?

$$T_n = a + (n - 1) d$$

Where $a = 103$, $d = 20$, $n = \text{number of term}$

$$= 103 + (17 - 1) * 20$$

$$= 103 + 320 = 423$$

Another problem

There are 31 terms in an AP. The first term is 100340 and the last term is 100580.
What is the sum of that series?

Formula: $n/2 * (\text{first term} + \text{last term})$

$n = 31$, first term = 100340, last term = 100580

$= 31/2 * (100340 + 100580)$

$= 3114260$

What is harmonic progression?

A harmonic progression is a sets of values that, once reciprocated, results to an Arithmetic progression. To check, the reciprocated values must possess a rational common difference. Once this has been identified, we may say that the sequence is a Harmonic Progression.

Harmonic Mean

Harmonic Means are the terms found in between two terms of a harmonic progression.

What is harmonic progression?

It is just reverse of AP

For example: 2, 4, 6, 8 in AP

$$\text{HP} = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$$

Harmonic progression will convert into AP when you reverse it.

Find HM between 20 and 30?

$$AM = (20 + 30) / 2 = 25$$

$$HM = \text{reverse of } \frac{1}{2} \left(\frac{1}{20} + \frac{1}{30} \right) = 24$$

Problems

Determine which of the following are Harmonic Progressions:

1) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

2) $1, \frac{1}{4}, \frac{1}{5}, \frac{1}{7}, \dots$

Solutions

Step 1: Find the reciprocals

The reciprocals are: 1, 2, 3, 4, ...

Step 2: Identify whether the reciprocated sequence is an Arithmetic Progression by checking if a common difference exists in the terms.

Answer

So,

- 1) It is a Harmonic Progression
- 2) It is not a Harmonic Progression

Problems

Determine the next three terms of each of the following HP.

1) 24, 12, 8, 6, ,,,

Solution:

$$24, 12, 8, 6, \dots$$

$$= 1/24, 1/12, 1/8, 1/6, \dots$$

To find the common difference:

$$1/12 - 1/24$$

$$= 2/24 - 1/24$$

$$= 1/24$$

Note:

You can subtract the second term to the first term, the third term to the second term, the fourth to the third term, and so on and so forth.

So now:

To get the next three terms:

$$\text{5th Term} = 1/6 + 1/24$$

$$= 4/24 + 1/24$$

$$= 5/24$$

- Reciprocate

$$= 24/5$$

$$\text{6th Term} = 5/24 + 1/24$$

$$= 6/24$$

- Reciprocate

$$= 4$$

$$\begin{aligned}\text{7th Term} &= 1/4 + 1/24 \\ &= 6/24 + 1/24 \\ &= 7/24\end{aligned}$$

- Reciprocate
 $= 24/7$

So the three terms are: $5/24$, 4 and $24/7$

Problem

Find the Harmonic Mean between the following terms:

1) 12 and 8

Solutions:

The reciprocals are: $\frac{1}{12}$ and $\frac{1}{8}$

Step 2: Arrange the given terms as follows:

$\frac{1}{12}$	Harmonic Mean	$\frac{1}{8}$
1st Term	2nd Term	3rd Term

Formula

$$T_n = a + (n - 1) d$$

Now we may find T_3

So

$$T_3 = 1/12 + (3 - 1) d$$

$$1/8 - 1/12 = 2d$$

$$(3 - 2) / 24 = 2d$$

$$(3 - 2) = 48d$$

$$1 = 48d$$

$$d = 1/48$$

- After getting the common difference, add it to the first term to get the Harmonic Mean between the two terms.

$$T_2 = a + d$$

$$= 1/12 + 1/48$$

$$= (4 + 1) / 48$$

$$= 5/48$$

- Reciprocate

$$= 48/5$$

Problem

Insert three Harmonic Means between the following terms:

1) 36 and $\frac{36}{5}$

Solution:

- The reciprocals are: $\frac{1}{36}$ and $\frac{5}{36}$
- Arrange the given terms as follows:

Formula

$$T_n = a + (n - 1) d$$

- Same as before, at first find d

$$T_5 = 1/36 + (5 - 1) d$$

$$5/36 - 1/36 = 4d$$

$$(5 - 1) / 36 = 4d$$

$$4/36 = 4d$$

$$d = 1/36$$

- Now, after getting the Common Difference, add it to the first term, then add it to the second term, and then add it to the third term to get the Harmonic Mean between the two terms.

So,

$$T_2 = t_1 + d$$

$$= 1/36 + 1/36$$

$$= 1/18$$

- Reciprocate

$$= 18$$

$$T_3 = t_2 + d$$

$$= 2/36 + 1/36$$

$$= 3/36$$

$$= 1/12$$

- Reciprocate

$$= 12$$

$$T_4 = t_3 + d$$

$$= 3/36 + 1/36$$

$$= 4/36$$

$$= 1/9$$

- Reciprocate

$$= 9$$

So there terms between 36 and $36/5$ are 18, 12 and 9

Homework

- Determine if the following are harmonic progressions or not:

1) $1/12, 1/24, 1/36$

2) $2, 5, 7, 8$

3) $1/5, 1/10, 1/15$

- Find the next three terms in the following harmonic progressions

1) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$

2) $19, 17, 15, 13, \dots$

3) $12, 6, 4, 3, \dots$

- Find the harmonic mean between

1) $\frac{1}{2}$ and $\frac{1}{5}$

2) 1 and $\frac{1}{9}$

- Insert three harmonic means between:

1) $\frac{1}{2}$ and $\frac{1}{8}$

2) 1 and $\frac{1}{10}$

Geometric Progression

When a series is rising at an increasing fast speed, it is geometric progression.

Example:

2, 4, 8, 16, 32, 64, Here $a = 2$ (starting term)

Ratio = $16/8 = 2$ (you may pick up any two terms, the ration will be the same)

Essential components of GP?

Take any GP: Let us say: 3, 9, 27, 81, 243

There are 3 terms:

1. Starting term $a = 3$
2. Growth ratio of two terms: $r = 27/9 = 3$
3. $n = \text{number of terms} = n = 5$

Can ratio be less than 1?

The answer is Yes.

In that case the series will be a decreasing series

Example:

81, 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$,

What is the sum of this series for 5 terms:

$1^2, 2^2, 3^2, \dots$

Formula: $(n(n+1)(2n+1)) / 6$

$= (5(6)(11)) / 6$

$= 55$

What is the sum of this series for 5 Terms:

$1^3, 2^3, 3^3, \dots$

Formula: $n (n (n + 1) / 2)^2$

$= (5 (5 + 1) / 2)^2$

$= 225$

What is the sum of this infinite series: 512, 256, 128, 64...for first 10 terms?

Solution:

Formula = $a / (1 - r)$

$$\begin{aligned} a &= 512, r = 256/512 = \frac{1}{2} = .5 \\ &= 512 / (1 - .5) \\ &= 1024 \end{aligned}$$

What is the sum of this series:

2, 4, 8, 16 for first 30 terms???

$$S_n = a (r^n - 1) / (r - 1)$$

$$a = 2, r = \text{ratio}, n = 30$$

$$= 2 (2^{30} - 1) / (2 - 1)$$

$$= 2 * 1073741823$$

$$= 2147483646$$

What is the sum of first 8 terms of this series:

8, 64, 512

$$a (r^n - 1) / (r - 1)$$

$$a = 8, r = 8, n = 8$$

$$8 (8^8 - 1) / (8 - 1)$$

$$= 19173960$$

What is the sum of first 5 terms of this series:

81, 27, 9

$$a (1 - r^n) / (1 - r)$$

$$= 81 (1 - (\frac{1}{3})^5) / (1 - (\frac{1}{3}))$$

$$= 121$$

Which of this is geometric progression?

3, 5, 88, 122

3, 5, 7, 9, 11,

4, 16, 64, 256,

In order to study at Hyderabad, Maynak starts saving money. First day he saves Rs. 200000. Next day he saves half of this money. This process continues. Now this becomes his habit. If Maynak continues saving like this, how much money can he save in his lifetime?

Formula:

$$a / (1 - r)$$

Thank You!!