Estimating Utility Functions

# install.packages("tidyverse")  
library(tidyverse)  
library(purrr)  
library(tidyr)

# Data

The data for estimating the utility function of player B is from Charness and Rabin (2002: 829), Table 1, Two-person dictator games.

# Spalten der Tabelle als Vektoren erstellen  
Game <- c("Berk29", "Barc2", "Berk17", "Berk23", "Barc8", "Berk15", "Berk26")  
LeftPayoffA <- c(400, 400, 400, 800, 300, 200, 0)  
LeftPayoffB <- c(400, 400, 400, 200, 600, 700, 800)  
RightPayoffA <- c(750, 750, 750, 0, 700, 600, 400)   
RightPayoffB <- c(400, 375, 375, 0, 500, 600, 400)  
ObsProbRight <- c(.69, .48, .50, 0, .33, .73, .22)  
  
# Vektoren in eine Tabelle packen  
dataRaw <- data.frame(Game,   
 LeftPayoffA,   
 LeftPayoffB,   
 RightPayoffA,   
 RightPayoffB,   
 ObsProbRight)  
  
# Die Rohdaten ausgeben  
print(dataRaw)

## Game LeftPayoffA LeftPayoffB RightPayoffA RightPayoffB ObsProbRight  
## 1 Berk29 400 400 750 400 0.69  
## 2 Barc2 400 400 750 375 0.48  
## 3 Berk17 400 400 750 375 0.50  
## 4 Berk23 800 200 0 0 0.00  
## 5 Barc8 300 600 700 500 0.33  
## 6 Berk15 200 700 600 600 0.73  
## 7 Berk26 0 800 400 400 0.22

# Utility function of player B

The model is a simplified version from Charness and Rabin (2002: 822).

# Nutzenfunktion definieren  
Utility <- function(ownPayoff, # Input1  
 weightOtherPayoff,# Input2  
 otherPayoff) { # Input3  
 weightOwnPayoff <- 1 - weightOtherPayoff # Calculation  
 return(weightOtherPayoff \* otherPayoff + weightOwnPayoff \* ownPayoff) # Output  
}  
  
# Anwendung der Nutzenfunktion  
Utility(ownPayoff = 5,  
 weightOtherPayoff = 0.5,  
 otherPayoff = 40)

## [1] 22.5

We assume a fixed value for the parameter weightPayoffA and calculate the utility of B for our data. Then we predict which option will be chosen and calculate the sum of squared differences.

# Das Gewicht, das Spieler B auf die Auszahlung von Spieler A legt  
parameter = .2  
  
# Berechnung der Nutzen von Spieler B für die Alternativen Left und Right  
# Berechnung der vorhergesagten Wahlwahrscheinlichkeit für die Alternative Right  
# Berechung der quadrierten Abweichungen zwischen vorhergesagten Wahlwahrscheinlichkeiten und der beobachteten Wahlwahrscheinlichkeiten bzw. Frequenzen  
dataFix <- dataRaw %>%   
 mutate(LeftUtilityB = Utility(ownPayoff = LeftPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = LeftPayoffA),  
 RightUtilityB = Utility(ownPayoff = RightPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = RightPayoffA)) %>%   
 mutate(PredProbRight = ifelse(test = RightUtilityB > LeftUtilityB, yes = 1,   
 ifelse(test = RightUtilityB == LeftUtilityB, yes = .5, no = 0)),  
 SquaredDiff = (ObsProbRight - PredProbRight)^2)  
  
# Summe der quadrierten Abweichungen  
sum(dataFix$SquaredDiff)

## [1] 0.7467

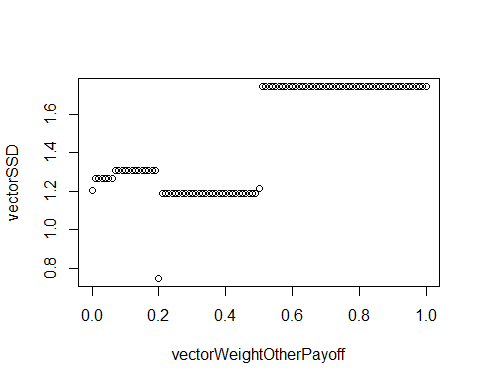
## Model with one free parameter

We search for the parameter weightPayoffA that maximizes the fit to the data.

vectorWeightOtherPayoff <- 0:100 / 100  
#vectorWeightOtherPayoff  
  
vectorSSD = rep(NA, length(vectorWeightOtherPayoff))  
count <- 1  
for (parameter in vectorWeightOtherPayoff) {  
dataEst <- dataRaw %>%   
 mutate(LeftUtilityB = Utility(ownPayoff = LeftPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = LeftPayoffA),  
 RightUtilityB = Utility(ownPayoff = RightPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = RightPayoffA)) %>%   
 mutate(PredProbRight = ifelse(test = RightUtilityB > LeftUtilityB, yes = 1,   
 ifelse(test = RightUtilityB == LeftUtilityB, yes = .5, no = 0)),  
 SquaredDiff = (ObsProbRight - PredProbRight)^2)  
  
SSD = sum(dataEst$SquaredDiff) # Sum of squared differences  
  
vectorSSD[count] = SSD  
  
count <- count + 1  
}  
  
dataEstimation <- data.frame(vectorWeightOtherPayoff, vectorSSD)

We plot the optimization search results

plot(dataEstimation)



We print the table of the optimization results

dataEstimation

## vectorWeightOtherPayoff vectorSSD  
## 1 0.00 1.2067  
## 2 0.01 1.2667  
## 3 0.02 1.2667  
## 4 0.03 1.2667  
## 5 0.04 1.2667  
## 6 0.05 1.2667  
## 7 0.06 1.2667  
## 8 0.07 1.3067  
## 9 0.08 1.3067  
## 10 0.09 1.3067  
## 11 0.10 1.3067  
## 12 0.11 1.3067  
## 13 0.12 1.3067  
## 14 0.13 1.3067  
## 15 0.14 1.3067  
## 16 0.15 1.3067  
## 17 0.16 1.3067  
## 18 0.17 1.3067  
## 19 0.18 1.3067  
## 20 0.19 1.3067  
## 21 0.20 0.7467  
## 22 0.21 1.1867  
## 23 0.22 1.1867  
## 24 0.23 1.1867  
## 25 0.24 1.1867  
## 26 0.25 1.1867  
## 27 0.26 1.1867  
## 28 0.27 1.1867  
## 29 0.28 1.1867  
## 30 0.29 1.1867  
## 31 0.30 1.1867  
## 32 0.31 1.1867  
## 33 0.32 1.1867  
## 34 0.33 1.1867  
## 35 0.34 1.1867  
## 36 0.35 1.1867  
## 37 0.36 1.1867  
## 38 0.37 1.1867  
## 39 0.38 1.1867  
## 40 0.39 1.1867  
## 41 0.40 1.1867  
## 42 0.41 1.1867  
## 43 0.42 1.1867  
## 44 0.43 1.1867  
## 45 0.44 1.1867  
## 46 0.45 1.1867  
## 47 0.46 1.1867  
## 48 0.47 1.1867  
## 49 0.48 1.1867  
## 50 0.49 1.1867  
## 51 0.50 1.2167  
## 52 0.51 1.7467  
## 53 0.52 1.7467  
## 54 0.53 1.7467  
## 55 0.54 1.7467  
## 56 0.55 1.7467  
## 57 0.56 1.7467  
## 58 0.57 1.7467  
## 59 0.58 1.7467  
## 60 0.59 1.7467  
## 61 0.60 1.7467  
## 62 0.61 1.7467  
## 63 0.62 1.7467  
## 64 0.63 1.7467  
## 65 0.64 1.7467  
## 66 0.65 1.7467  
## 67 0.66 1.7467  
## 68 0.67 1.7467  
## 69 0.68 1.7467  
## 70 0.69 1.7467  
## 71 0.70 1.7467  
## 72 0.71 1.7467  
## 73 0.72 1.7467  
## 74 0.73 1.7467  
## 75 0.74 1.7467  
## 76 0.75 1.7467  
## 77 0.76 1.7467  
## 78 0.77 1.7467  
## 79 0.78 1.7467  
## 80 0.79 1.7467  
## 81 0.80 1.7467  
## 82 0.81 1.7467  
## 83 0.82 1.7467  
## 84 0.83 1.7467  
## 85 0.84 1.7467  
## 86 0.85 1.7467  
## 87 0.86 1.7467  
## 88 0.87 1.7467  
## 89 0.88 1.7467  
## 90 0.89 1.7467  
## 91 0.90 1.7467  
## 92 0.91 1.7467  
## 93 0.92 1.7467  
## 94 0.93 1.7467  
## 95 0.94 1.7467  
## 96 0.95 1.7467  
## 97 0.96 1.7467  
## 98 0.97 1.7467  
## 99 0.98 1.7467  
## 100 0.99 1.7467  
## 101 1.00 1.7467

# Why is a weight of .2 optimal?

parameter = .2 # change to .2 and change to .21 - What do you observe?  
  
dataEst <- dataRaw %>%   
 mutate(LeftUtilityB = Utility(ownPayoff = LeftPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = LeftPayoffA),  
 RightUtilityB = Utility(ownPayoff = RightPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = RightPayoffA)) %>%   
 mutate(PredProbRight = ifelse(test = RightUtilityB > LeftUtilityB, yes = 1,   
 ifelse(test = RightUtilityB == LeftUtilityB, yes = .5, no = 0)),  
 SquaredDiff = (ObsProbRight - PredProbRight)^2)  
  
dataEst

## Game LeftPayoffA LeftPayoffB RightPayoffA RightPayoffB ObsProbRight  
## 1 Berk29 400 400 750 400 0.69  
## 2 Barc2 400 400 750 375 0.48  
## 3 Berk17 400 400 750 375 0.50  
## 4 Berk23 800 200 0 0 0.00  
## 5 Barc8 300 600 700 500 0.33  
## 6 Berk15 200 700 600 600 0.73  
## 7 Berk26 0 800 400 400 0.22  
## LeftUtilityB RightUtilityB PredProbRight SquaredDiff  
## 1 400 470 1.0 0.0961  
## 2 400 450 1.0 0.2704  
## 3 400 450 1.0 0.2500  
## 4 320 0 0.0 0.0000  
## 5 540 540 0.5 0.0289  
## 6 600 600 0.5 0.0529  
## 7 640 400 0.0 0.0484

The model predicts that the player will choose Left if weight < 0.2 or Right if weight > 0.2. Both predictions are suboptimal to weight == 0, which predicts that player B is indifferent between Left and Right in both games Barc8 and Berk15.

## Model with two free parameters

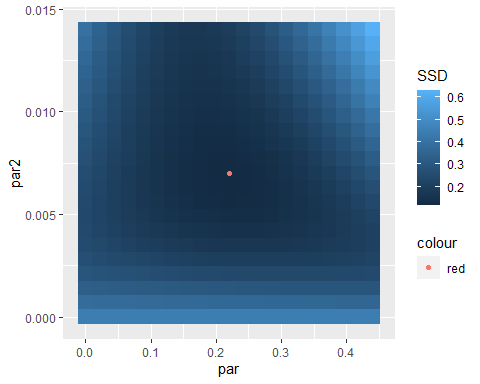
We introduce a second parameter gamma (see Charness and Rabin 2002: 839). The precision parameter gamma measures the sensitivity of player B to differences in utility.

sumSqDiff <- function(parameter, parameter2){  
  
dataEst <- dataRaw %>%   
 mutate(LeftUtilityB = Utility(ownPayoff = LeftPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = LeftPayoffA),  
 RightUtilityB = Utility(ownPayoff = RightPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = RightPayoffA)) %>%   
 mutate(PredProbRight = exp(RightUtilityB \* parameter2) /   
 (exp(LeftUtilityB \* parameter2) + exp(RightUtilityB \* parameter2)),  
 SquaredDiff = (ObsProbRight - PredProbRight)^2)  
  
SSD = sum(dataEst$SquaredDiff) # Sum of squared differences is returned by function  
}  
  
print(sumSqDiff(parameter = .1, parameter2 = 0))

## [1] 0.4467

### Heatmap  
myheatmap <- function(parameter, parameter2, dev){  
# Grid  
comparison\_grid <- expand.grid(par = seq(parameter \* (1 - dev),  
 parameter \* (1 + dev),   
 length.out = 21),  
 par2 = seq(parameter2 \* (1 - dev),  
 parameter2 \* (1 + dev),   
 length.out = 21)) %>%  
 group\_by(par, par2) %>%  
 nest()  
  
# Results  
grid\_results <-  
 comparison\_grid %>%  
 mutate(SSD = map2(par, par2, ~sumSqDiff(parameter = .x, parameter2 = .y))) %>%   
 unnest(cols = SSD) %>%   
 arrange(SSD)  
  
# Best fitting paramters  
opt\_par <- grid\_results$par[1]  
opt\_par2 <- grid\_results$par2[1]  
  
# Heatmap  
ggplot(data = grid\_results) +   
 geom\_tile(aes(x = par, y = par2, fill = SSD)) +  
 geom\_point(aes(x = opt\_par, y = opt\_par2, color = "red"))  
}

myheatmap(parameter = 0.22, # Gewicht auf Auszahlung von A  
 parameter2 = .007, # Gamma\*  
 dev = 1)# Intervall der Paramtervariation



# \* Wie stark richte ich mich nach meinen Präferenzen?   
# 0 = Random Choice   
# hohes Gamma = Nutzenmaximierung

The red dot shows the best fitting parameter combination

#help(optim)  
  
sumSqDiff2 <- function(par){  
 parameter <- par[1]  
 parameter2 <- par[2]  
  
dataEst <- dataRaw %>%   
 mutate(LeftUtilityB = Utility(ownPayoff = LeftPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = LeftPayoffA),  
 RightUtilityB = Utility(ownPayoff = RightPayoffB,   
 weightOtherPayoff = parameter,  
 otherPayoff = RightPayoffA)) %>%   
 mutate(PredProbRight = exp(RightUtilityB \* parameter2) /   
 (exp(LeftUtilityB \* parameter2) + exp(RightUtilityB \* parameter2)),  
 SquaredDiff = (ObsProbRight - PredProbRight)^2)  
  
SSD = sum(dataEst$SquaredDiff) # Sum of squared differences is returned by function  
}  
  
optim(par = c(0, .1), # Startwerte für Parameter weightOtherPayoff, gamma  
 fn = sumSqDiff2) # zu minimierende funktion

## $par  
## [1] 0.218409321 0.006681957  
##   
## $value  
## [1] 0.1176641  
##   
## $counts  
## function gradient   
## 83 NA   
##   
## $convergence  
## [1] 0  
##   
## $message  
## NULL

# Model with three free parameters

We estimate a noisy utility function with different weights on other payoff depending on whether player B is better off or worse off.

# Nutzenfunktion definieren  
Utility2 <- function(ownPayoff, # Input1  
 weightOtherPayoffBetterOff,# Input2  
 weightOtherPayoffWorseOff,# Input3  
 otherPayoff) { # Input4  
   
 weightOwnPayoff <- 1 - weightOtherPayoffBetterOff # Calculation  
   
 if (otherPayoff > ownPayoff) {  
 weightOwnPayoff <- 1 - weightOtherPayoffWorseOff # Calculation  
 }  
   
 return( (1 - weightOwnPayoff) \* otherPayoff + weightOwnPayoff \* ownPayoff) # Output  
}  
  
sumSqDiff3 <- function(par){  
 parameter <- par[1] # gewicht anderer payoff, wenn besser gestellt  
 parameter2 <- par[2] # gewicht anderer payoff, wenn schlechter gestellt  
 parameter3 <- par[3]# gamma  
  
dataEst <- dataRaw %>%   
 mutate(LeftUtilityB = Utility2(ownPayoff = LeftPayoffB,   
 weightOtherPayoffBetterOff = parameter,  
 weightOtherPayoffWorseOff = parameter2,  
 otherPayoff = LeftPayoffA),  
 RightUtilityB = Utility2(ownPayoff = RightPayoffB,  
 weightOtherPayoffBetterOff = parameter,  
 weightOtherPayoffWorseOff = parameter2,  
 otherPayoff = RightPayoffA)) %>%   
 mutate(PredProbRight = exp(RightUtilityB \* parameter3) /   
 (exp(LeftUtilityB \* parameter3) + exp(RightUtilityB \* parameter3)),  
 SquaredDiff = (ObsProbRight - PredProbRight)^2)  
  
SSD = sum(dataEst$SquaredDiff) # Sum of squared differences is returned by function  
}  
  
optim(par = c(0, 0, 0), # Startwerte für Parameter = \*  
 fn = sumSqDiff3)

## $par  
## [1] 0.24385967 0.06086561 0.04251490  
##   
## $value  
## [1] 0.04987446  
##   
## $counts  
## function gradient   
## 126 NA   
##   
## $convergence  
## [1] 0  
##   
## $message  
## NULL

# \* weightOtherPayoffBetterOff, weightOtherPayoffWorseOff, gamma

* Player B puts a higher weight on the other payoff when he is better off
* The model fits the data better than a model that does not take the direction of inequality into account
* In comparison to Charness and Rabin we used less games to estimate the utility function and a different criterion: minimize sum squared differences instead of maximize log likelihood