Complexity certification of Mixed-Integer Quadratic Programming

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Motivation & Research Goals

In hybrid model predictive control (MPC), a non-convex optimization problem has to be solved at each time step, which in real-time applications makes it important to solve these efficiently and to have good upper bounds on worst-case solution time. For linear hybrid MPC problems, the optimization problem is often a multi-parametric mixed-integer quadratic program (mp-MIQP) that depends on parameters such as system states and reference signals. The aim of the research is to certify the complexity of MIQPs by computing which sequence of subproblems are required to solve in the branch and bound (B&B) method for every parameter of interest. These sequences can be used to compute the worst-case bounds on how many iterations, floating-point operations and, ultimately, the maximum solution time, the B&B algorithm would require to converge online.

Methods

- **Problem Formulation**
  - mp-MIQP

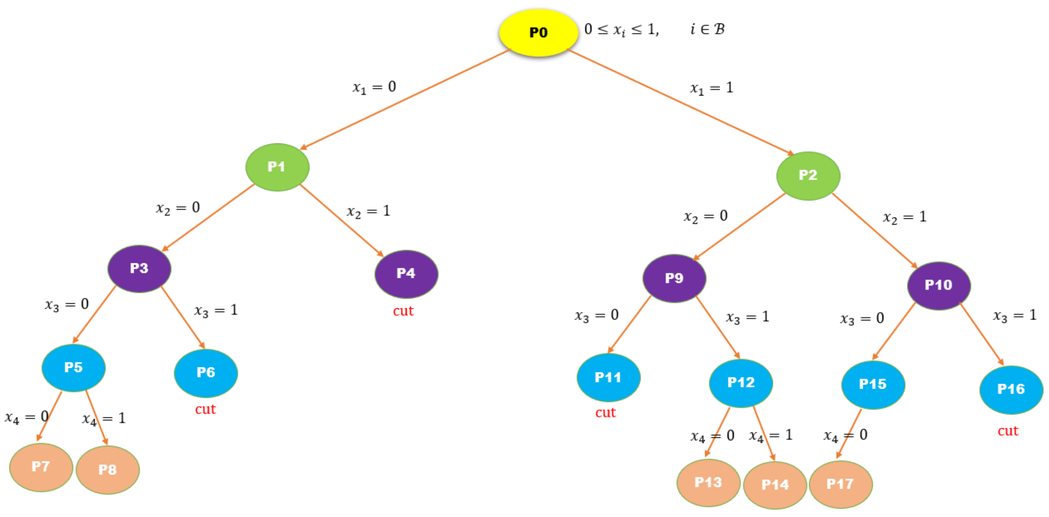
$$\begin{aligned} \min_{x} \quad & \frac{1}{2}x^T H x + f^T x + \theta^T f_\theta^T x, \\ \text{s.t.} \quad & Ax \leq b + W\theta, \\ & x_i \in \{0,1\}, \quad \forall i \in \mathcal{B} \end{aligned}$$

  * $x = [x_c^T, x_b^T]^T \in R^{n_c} \times \{0,1\}^{n_b}$: state vector
  * $\theta \in \Theta_0 \subset R^{n_\theta}$: parameter vector

- **B&B method**

Solving a sequence of relaxed convex mp-QP problems by fixing a binary variable to 0 and 1, forming nodes in the B&B search tree and cut a node if the solution of a relaxation is

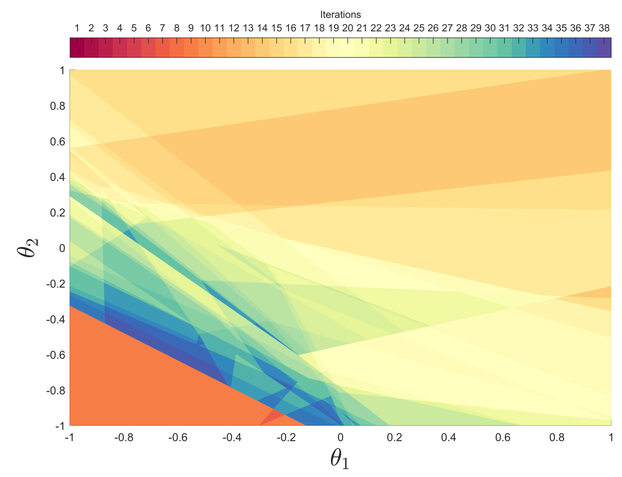
- *infeasible*
- *does not provide better solution*
- *integer feasible*



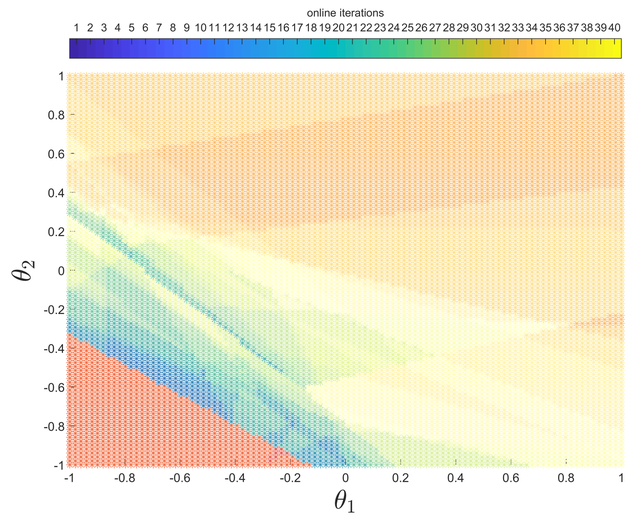
Contribution:
An algorithm for computing a useful upper bound of the worst-case computational complexity for solving any possible MIQP that can arise from a specific parameter in a polyhedral parameter ser

Results

Partitioning the parameter space based on the total number of QP iterations, i.e., the total number of linear system of equations solved, for a random example with with $n_c = 2$, $n_b = 4$, $n_\theta = 2$, from the proposed certification algorithm. Points with the same color share the same number of complexity measure.



The total QP iteration number for 10000 samples specified by * in the parameter space derived by applying online B&B to the same example.



The complexity certification result coincides with the online algorithm in all sample points, despite that the conservative upper bound is used in the certification method, using depth-first search strategy

Ongoing & Future works:

- Exact complexity certification of mixed-integer linear programming (MILP) (*ongoing*)
- Complexity certify the B&B method for different node selection strategies such as best-first strategy
- Certification of the warm-started algorithm to decrease the computational complexity

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