1. Q) Find the exact length of the curve $f(x) = \frac{x^3}{3} + \frac{1}{4x}, 1 \leq x \leq 2$

A) $L = \int_1^2 \sqrt{1 + [f'(x)]^2} dx$

$f'(x) = x^2 - \frac{1}{4x^2}$

$[f'(x)]^2 = [x^2 - \frac{1}{4x^2}]^2$

$= x^4 + \frac{1}{16x^4} - \frac{1}{2}$

$L = \int_1^2 \sqrt{1 + x^4 + \frac{1}{16x^4} - \frac{1}{2}} dx$

$= \int_1^2 \sqrt{x^4 + \frac{1}{16x^4} + \frac{1}{2}} dx$

$= \int_1^2 \sqrt{\frac{16x^8+1+8x^4}{16x^4}} dx$

$= \int_1^2 \frac{4x^4+1}{4x^2} dx = \int_1^2 x^2 + \frac{1}{4x^2} dx = \left[\frac{x^3}{3} - \frac{1}{4x}\right]_1^2$

$= \left[\frac{8}{3} - \frac{1}{8} - (\frac{1}{3} - \frac{1}{4})\right] = \frac{7}{3} + \frac{1}{8}$

$= \frac{59}{24}$