

1. Q) Surface area of revolution:

$$y = \sqrt{6-x}, 4 \leq x \leq 6$$

A)

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)^2]} dx$$

$$f = (6-x)^{1/2}$$

$$f' = -\frac{1}{2\sqrt{6-x}}$$

$$S = \int_4^6 2\pi(6-x)^{1/2} \sqrt{1 + \left(-\frac{1}{2\sqrt{6-x}}\right)^2} dx$$

$$= 2\pi \int_4^6 (6-x)^{1/2} \sqrt{1 + \frac{1}{4(6-x)}} dx$$

$$= 2\pi \int_4^6 (6-x)^{1/2} \sqrt{\frac{25-4x}{4(6-x)}} dx$$

$$= \pi \int_4^6 \sqrt{25-4x} dx$$

$$\text{Let } I = \int \sqrt{25-4x} dx$$

$$= \frac{-1}{4} \int \sqrt{u} du$$

(for $u = 25 - 4x$ and $du = -4dx$)

$$= \frac{-1}{6} u^{3/2}$$

$$= \frac{-1}{6} (25-4x)^{3/2}$$

$$S = \frac{-\pi}{6} \left[(25-4x)^{3/2} \right]_4^6$$

$$= \frac{-\pi}{6} \left[(25-24)^{3/2} - (25-16)^{3/2} \right]$$

$$= \frac{-\pi}{6} \left[1 - (9)^{3/2} \right]$$

$$= \frac{\pi}{6} [27 - 1]$$

$$= \boxed{\frac{13\pi}{3}}$$