1. Q) Find the exact length of the curve $f(x) = \frac{x^2}{2}, P = (-3, \frac{9}{2}), Q = (3, \frac{9}{2})$

A) $L = \int_{-3}^{3} \sqrt{1 + [f'(x)]^2} dx$

$f'(x) = \frac{2x}{2} = x$

$L = \int_{-3}^{3} \sqrt{1 + x^2} dx$

$\int \sqrt{1 + x^2} dx = \int \sqrt{1 + (\tan\theta)^2} (\sec\theta)^2 d\theta$

for $x = \tan\theta, dx = (\sec\theta)^2 d\theta$

$\int \sqrt{1 + x^2} dx = \int \sqrt{1 + (\tan\theta)^2} (\sec\theta)^2 d\theta = \int (sec\theta)^3 d\theta$

$= \int \frac{1}{(\cos\theta)^2} \frac{1}{\cos\theta} d\theta$

(applying integration by parts $\int u.dv = u.v - \int v.du$),

$= \sec\theta \tan\theta - \int \sec\theta (\tan\theta)^2 d\theta$

(for $u = \frac{1}{\cos\theta}, dv = \frac{1}{(\cos\theta)^2}$ and $du = \sec\theta \tan\theta d\theta, v = \tan\theta$)

$\implies \int (\sec\theta)^3 d\theta = \sec\theta \tan\theta - \int (\sec\theta)^3 (\sin\theta)^2 d\theta$

$= \sec\theta \tan\theta - \int (\sec\theta)^3 (1 - \cos\theta)^2 d\theta$

$= \sec\theta \tan\theta - \int (\sec\theta)^3 d\theta + \int \sec\theta d\theta$

$= \sec\theta \tan\theta - \int (\sec\theta)^3 d\theta + \ln|\sec\theta + \tan\theta|$

$\implies 2 \int (\sec\theta)^3 d\theta = \sec\theta \tan\theta + \ln|\sec\theta + \tan\theta|$

$\implies \int (\sec\theta)^3 d\theta = \frac{1}{2} [\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta|]$

$= \frac{1}{2} [\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta|]$

$\implies L = \frac{1}{2} \left[\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta|\right]_{x=-3}^{x=3}$

For $x = \pm 3$, $\tan\theta = \pm 3, \sec\theta = \sqrt{10}$

$\implies L = \frac{1}{2} \left[3\sqrt{10} + \ln|\sqrt{10} + 3| - (-3\sqrt{10} + \ln|\sqrt{10} - 3|)\right]$

$= 3\sqrt{10} + \frac{1}{2} \ln(\frac{\sqrt{10}+3}{\sqrt{10}-3})$