

$$1. \text{ Q — } \int_0^{4\pi} t^2 \sin(2t) dt$$

$$\text{A — Let } u = t^2 \text{ and } v' = \sin(2t) \implies u' = 2t \text{ and } v = \frac{-1}{2} \cos(2t)$$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

$$\text{Therefore } \int t^2 \sin(2t) dt = \frac{-t^2}{2} \cos(2t) + \int t \cos(2t) dt$$

To calculate $\int t \cos(2t) dt$

$$\text{Let } u = t \text{ and } v' = \cos(2t) \implies u' = 1 \text{ and } v = \frac{1}{2} \sin(2t)$$

$$\text{Therefore } \int t \cos(2t) dt = \frac{t}{2} \sin(2t) - \frac{1}{2} \int \sin(2t) dt = \frac{t}{2} \sin(2t) + \frac{1}{4} \cos(2t)$$

$$\text{Therefore } \int t^2 \sin(2t) dt = \frac{-t^2}{2} \cos(2t) + \frac{t}{2} \sin(2t) + \frac{1}{4} \cos(2t) + C$$

$$\text{Therefore } \int_0^{4\pi} t^2 \sin(2t) dt$$

$$= \left[\frac{-t^2}{2} \cos(2t) + \frac{t}{2} \sin(2t) + \frac{1}{4} \cos(2t) \right]_0^{4\pi}$$

$$= \left(\frac{-16\pi^2}{2} \cos(8\pi) + 2\pi \sin(8\pi) + \frac{1}{4} \cos(8\pi) \right) - \left(0 + 0 + \frac{1}{4} \cos(0) \right)$$

$$= \left(\frac{-16\pi^2}{2} + \frac{1}{4} \right) - \left(\frac{1}{4} \right)$$

$$= -8\pi^2$$