1. Q — $\int_0^1 \frac{r^3}{\sqrt{16+r^2}} dr$

A — Let $u = t^2$ and $v' = sin(2t) \implies u' = 2t$ and $v = \frac{-1}{2}\cos(2t)$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

Therefore $\int t^2 \sin(2t)dt = \frac{-t^2}{2}\cos(2t) + \int t\cos(2t)dt$

To calculate $\int t\cos(2t)dt$

Let $u = t$ and $v' = \cos(2t) \implies u' = 1$ and $v = \frac{1}{2}\sin(2t)$

Therefore $\int t\cos(2t)dt = \frac{t}{2}\sin(2t) - \frac{1}{2}\int \sin(2t)dt = \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t)$

Therefore $\int t^2 \sin(2t)dt = \frac{-t^2}{2}\cos(2t) + \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t) + C$

Therefore $\int_0^{4\pi} t^2 \sin(2t)dt$

$= \left[\frac{-t^2}{2}\cos(2t) + \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t)\right]_0^{4\pi}$

$= (\frac{-16\pi^2}{2}\cos(8\pi) + 2\pi\sin(8\pi) + \frac{1}{4}\cos(8\pi)) - (0 + 0 + \frac{1}{4}\cos(0))$

$= (\frac{-16\pi^2}{2} + \frac{1}{4}) - (\frac{1}{4})$

$= -8\pi^2$