

1. Q) Find the exact length of the curve $x = \frac{y^4}{8} + \frac{1}{4y^2}, 1 \leq x \leq 2$

A) $L = \int_1^2 \sqrt{1 + [f'(y)]^2} dy$

$$f'(y) = \frac{y^3}{2} - \frac{1}{2y^3}$$

$$L = \int_1^2 \sqrt{1 + \left[\frac{y^3}{2} - \frac{1}{2y^3}\right]^2} dy$$

$$= \int_1^2 \sqrt{1 + \left[\frac{y^6-1}{2y^3}\right]^2} dy$$

$$= \int_1^2 \sqrt{1 + \frac{y^{12}+1-2y^6}{4y^6}} dy = \int_1^2 \sqrt{\frac{y^{12}+1+2y^6}{4y^6}} dy$$

$$= \int_1^2 \frac{y^6+1}{2y^3} dy = \int_1^2 \frac{y^3}{2} + \frac{1}{2y^3} dy$$

$$= \left[\frac{y^4}{8} - \frac{1}{4y^2} \right]_1^2 = \left[\frac{16}{8} - \frac{1}{16} - \left(\frac{1}{8} - \frac{1}{4} \right) \right]$$

$$= \frac{15}{8} + \frac{3}{16} = \frac{33}{16}$$