

$$1. \text{ Q — } \int \cos^{-1}(x) dx$$

$$\text{A — Let } y = \cos^{-1}(x)$$

$$\implies x = \cos(y)$$

$$\implies \frac{dx}{dy} = -\sin(y)$$

$$\implies dx = -\sin(y) dy$$

$$\text{Therefore } \int \cos^{-1}(x) dx = \int y(-\sin(y) dy)$$

$$= -\int y \sin(y) dy$$

$$\text{Let } u = y; v' = \sin(y). \text{ Therefore } v = -\cos(y)$$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

$$\text{Therefore } -\int y \sin(y) dy$$

$$= y(-\cos(y)) - \int -\cos(y) dy$$

$$= -y \cos(y) + \int \cos(y) dy$$

$$= -y \cos(y) + \sin(y) + C$$

$$= -\cos^{-1}(x)x + \sin(\cos^{-1}(x)) + C, \text{ (substituting } y = \cos^{-1}(x))$$

$$= -x \cos^{-1}(x) + \sqrt{1-x^2} + C, \text{ (substituting } \sin(\cos^{-1}(x)) = \sqrt{1-x^2})$$