Q.1 Find the volume of a solid obtained by rotating the region bound by $y = 6\sin(2x^2)$, $x = 0$ and $x = \sqrt{\pi/2}$ about the y-axis.

A) To find the limits, $y' = 24x\cos(2x^2) = 0$ (horizontal slope).

$\implies x = 0$ and $\cos(2x^2) = 0$

$\implies x = 0$ and $2x^2 = \pi/2 \implies x = \sqrt{\pi}/2$

$\implies y = 0$ and $y = 6\sin(\pi/2) = 6$

Using the "shell method", the volume,

$V = \int_0^{\sqrt{\pi/2}} 2\pi x(6\sin(2x^2))dx$

$= 12\pi \int_0^{\sqrt{\pi/2}} x\sin(2x^2)dx$

$= \int x\sin(2x^2)dx = \frac{1}{4}\int \sin(u)du$ for $u = 2x^2$ and $du = 4xdx$

$= -\frac{1}{4}\cos(u) = -\frac{1}{4}\cos(2x^2)$

$\implies V = 12\pi\left[-\frac{1}{4}\cos(2x^2)\right]_0^{\sqrt{\pi/2}}$

$= 12\pi(-\frac{1}{4}\cos(\pi) + \frac{1}{4}\cos(0))$

$= 3\pi(1+1) = 6\pi$

Q.2 Find the volume of a solid obtained by rotating the region bound by $y = 5\sqrt[3]{x}$, $y = 0$ and $x = 1$ about the y-axis.

A) Using the "shell method", the volume,

$V = \int_0^1 2\pi x(5\sqrt[3]{x})dx$

$= 10\pi \int_0^1 x\sqrt[3]{x}dx$

$= 10\pi \int_0^1 x^{\frac{4}{3}}dx$

$= 10\pi\frac{3}{7}\left[x^{\frac{7}{3}}\right]_0^1$

$= \frac{30\pi}{7}$

Q.3

Q.4 Find the volume of a solid obtained by rotating the region bounded by $y = \sqrt{4x}$, $y = \frac{x^2}{4}$ about the y-axis.

A)