1. Q) $\int \frac{x^3}{\sqrt{x^2+4}} dx$

A) $\int \frac{x^3}{\sqrt{x^2+4}} dx$

Let $x = 2\tan(\theta) \therefore \tan(\theta) = \frac{x}{2}, \cos(\theta) = \frac{2}{\sqrt{x^2+4}}, \sec(\theta) = \frac{\sqrt{x^2+4}}{2}$

Also $dx = 2\sec^2(\theta)d\theta$

$\sqrt{x^2+2^2} = \sqrt{2^2\tan^2(\theta)+2^2} = 2\sqrt{\tan^2(\theta)+1} = 2\sec(\theta)$

$\therefore \int \frac{x^3}{\sqrt{x^2+2^2}} dx$

$= \int \frac{8\tan^3(\theta)}{2\sec(\theta)} 2\sec^2(\theta)d\theta$

$= 8\int \tan^3(\theta)\sec(\theta)d\theta$

$= 8\int \frac{\sin^3(\theta)}{\cos^4(\theta)} d\theta$

$= 8\int \frac{\sin^2(\theta)\sin(\theta)}{\cos^4(\theta)} d\theta$

$= 8\int \frac{(1-\cos^2(\theta))\sin(\theta)}{\cos^4(\theta)} d\theta$

$= -8\int \frac{1-u^2}{u^4} du$, for $u = \cos(\theta)$, $du = -\sin(\theta)d\theta$

$= 8\int \frac{u^2-1}{u^4} du$

$= 8[\int u^{-2}du - \int u^{-4}du]$

$= 8[-\frac{1}{u} + \frac{1}{3u^3} + C]$

$= 8[\frac{1}{3u^3} - \frac{1}{u} + C]$

$= \frac{8}{3u^3} - \frac{8}{u} + C$

$= \frac{8}{3\cos^3(\theta)} - \frac{8}{\cos(\theta)} + C$

$= \frac{8}{3}(\frac{\sqrt{x^2+4}}{2})^3 - 8(\frac{\sqrt{x^2+4}}{2}) + C$

$= \frac{1}{3}(\sqrt{x^2+4})^3 - 4\sqrt{x^2+4} + C$