

1. Q) $\int \cos^{-1}(x)dx$

A) Let $y = \cos^{-1}(x)$

$$\implies x = \cos(y)$$

$$\implies \frac{dx}{dy} = -\sin(y)$$

$$\implies dx = -\sin(y)dy$$

$$\text{Therefore } \int \cos^{-1}(x)dx = \int y(-\sin(y)dy)$$

$$= -\int y \sin(y)dy$$

$$\text{Let } u = y; v' = \sin(y). \text{ Therefore } v = -\cos(y)$$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

$$\text{Therefore } -\int y \sin(y)dy$$

$$= -(y(-\cos(y)) - \int -\cos(y)dy)$$

$$= y \cos(y) - \int \cos(y)dy$$

$$= y \cos(y) - \sin(y) + C$$

$$= \cos^{-1}(x)x - \sin(\cos^{-1}(x)) + C, \text{ (substituting } y = \cos^{-1}(x))$$

$$= x \cos^{-1}(x) - \sqrt{1-x^2} + C, \text{ (substituting } \sin(\cos^{-1}(x)) = \sqrt{1-x^2})$$