1. Q) Find f_{ave}, where $f(x) = 3x^2 + 4x$, on $[-1, 3]$

A) $\int f(x)dx = \int (3x^2 + 4x)dx$

$= x^3 + 2x^2 + C$

$\implies f_{ave} = \frac{\left[x^3+2x^2\right]_{-1}^{3}}{3+1}$

$= \frac{(3^3+2.3^2)-((-1)^3+2(-1)^2)}{4}$

$= \frac{(27+18)-(-1+2)}{4}$

$= \frac{45-1}{4} = 11$

2. Q) Find f_{ave}, where $f(x) = 5\cos(x)$, on $[-\pi/2, \pi/2]$

A) $\int f(x)dx = \int 5\cos(x)dx$

$= 5\sin(x) + C$

$\implies f_{ave} = \frac{[5\sin(x)]_{\pi/2}^{-\pi/2}}{-\pi/2-\pi/2}$

$= \frac{(-5)-(5)}{-\pi}$

$= \frac{10}{\pi}$

3. Q) Find f_{ave}, where $f(t) = \frac{t}{\sqrt{3+t^2}}$, on $[1, 3]$

A) $\int f(t)dt = \int \frac{t}{\sqrt{3+t^2}}dt$

$= \int \frac{t}{\sqrt{3+t^2}}dt = \frac{1}{2}\int \frac{1}{\sqrt{u}}du$, for $u = 3 + t^2$

$= \sqrt{u} + C = \sqrt{(3+t^2)} + C$

$\implies f_{ave} = \frac{\left[\sqrt{(3+t^2)}\right]_{1}^{3}}{3-1}$

$= \frac{(\sqrt{3+9})-(\sqrt{3+1})}{2} = \frac{\sqrt{12}-2}{2} = \sqrt{3} - 1$

4. Q) Find f_{ave}, where $f(t) = e^{\sin(t)}\cos(t)$, on $[0, \pi/2]$

A) $\int f(t)dt = \int e^{\sin(t)}\cos(t)dt$

$= \int e^u du$, for $u = \sin(t)$

$= e^u = e^{\sin(t)} + C$

$\implies f_{ave} = \frac{\left[e^{\sin(t)}\right]_0^{\pi/2}}{\pi/2-0}$

$= \frac{e^1-e^0}{\pi/2} = \frac{e-1}{\pi/2} = \frac{2(e-1)}{\pi}$

5. Q) Find f_{ave}, where $f(x) = 6\cos^4(x)\sin(x)$, on $[0, \pi]$

A) $\int f(x)dx = \int 6\cos^4(x)\sin(x)dx$

$= -6\int u^4 du$, for $u = \cos(x)$

$= -\frac{6}{5}u^5 + C = -\frac{6}{5}\cos^5(x) + C$

$\implies f_{ave} = -\frac{6}{5}\frac{\left[\cos^5(x)\right]_0^{\pi}}{\pi-0}$

$= -\frac{6}{5}\frac{\cos^5(\pi)-\cos^5(0)}{\pi}$

$= -\frac{6}{5}\frac{-1-1}{\pi} = \frac{12}{5\pi}$

6. Q) Find f_{ave}, where $f(x) = \frac{\ln(x)}{x}$, on $[1, 3]$

A) $\int f(x)dx = \int \frac{\ln(x)}{x}dx = \int udu$, for $u = \ln(x)$

$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(x))^2 + C$

$\implies f_{ave} = \frac{1}{2}\frac{\left[(\ln(x))^2\right]_1^3}{3-1}$

$= \frac{1}{2}\frac{(\ln(3))^2-(\ln(1))^2}{2} = \frac{1}{4}(\ln(3))^2$

7. Q) Find f_{ave}, where $f(x) = (x-6)^2$, on $[4, 7]$

A) $\int f(x)dx = \int (x-6)^2 dx = \frac{(x-6)^3}{3} + C$

$\Longrightarrow f_{ave} = \frac{1}{3}\frac{\left[(x-6)^3\right]_4^7}{3}$

$= \frac{(1+8)}{9} = 1$

8. Q) Find b, such that f_{ave} for $5 + 10x - 9x^2$, on $[0, b]$ equals 6

A) $\int f(x)dx = \int (5 + 10x - 9x^2)dx = 5x + 5x^2 - 3x^3 + C$

$\Longrightarrow f_{ave} = \frac{\left[5x+5x^2-3x^3\right]_0^b}{b}$

$= \frac{5b+5b^2-3b^3}{b}$ which is given to equal 6

$\Longrightarrow 5 + 5b - 3b^2 = 6$

$\Longrightarrow 3b^2 - 5b + 1 = 0 \Longrightarrow b = \frac{5 \pm \sqrt{25-12}}{6}$

$\Longrightarrow b = \frac{5+\sqrt{13}}{6}, \frac{5-\sqrt{13}}{6}$