1. Q) Find the exact length of the curve $x = \frac{\sqrt{y}(y-3)}{3}, 1 \leq y \leq 4$

A) $L = \int_1^4 \sqrt{1 + [f'(y)]^2} dy$

$f'(y) = \frac{1}{3}\frac{d}{dy}(y^{\frac{3}{2}} - 3y^{\frac{1}{2}}) = \frac{1}{3}(\frac{3}{2}y^{\frac{1}{2}} - \frac{3}{2}y^{\frac{-1}{2}}) = \frac{1}{2}(y^{\frac{1}{2}} - y^{\frac{-1}{2}})$

$[f'(y)]^2 = \frac{1}{4}(y + \frac{1}{y} - 2) = \frac{(y-1)^2}{4y}$

$L = \int_1^4 \sqrt{1 + \frac{(y-1)^2}{4y}} dy = \int_1^4 \frac{y+1}{2\sqrt{y}} dy = \frac{1}{2}\int_1^4 (y^{\frac{1}{2}} + y^{-\frac{1}{2}}) dy$

$= \frac{1}{2}\left[\frac{2}{3}y^{\frac{3}{2}} + 2y^{\frac{1}{2}}\right]_1^4$

$= \frac{1}{2}\left[\frac{2}{3}4^{\frac{3}{2}} + 2.4^{\frac{1}{2}} - (\frac{2}{3} + 2)\right] = \frac{1}{2}(\frac{14}{3} + 2) = \frac{10}{3}$