

1. Q) Find the exact length of the curve $y = \frac{x^2}{4} - \frac{\ln x}{2}, 1 \leq x \leq 2$

A) $L = \int_1^2 \sqrt{1 + [f'(x)]^2} dx$

$$f'(x) = \frac{x}{2} - \frac{1}{2x}$$

$$L = \int_1^2 \sqrt{1 + \left[\frac{x}{2} - \frac{1}{2x}\right]^2} dx = \int_1^2 \sqrt{1 + \left[\frac{x^2-1}{2x}\right]^2} dx = \int_1^2 \sqrt{\frac{(x^2+1)^2}{4x^2}} dx$$

$$= \int_1^2 \frac{x^2+1}{2x} dx = \int_1^2 \left(\frac{x}{2} + \frac{1}{2x}\right) dx$$

$$= \left[\frac{x^2}{4} + \frac{\ln x}{2} \right]_1^2$$

$$= \left[\frac{2^2}{4} + \frac{\ln 2}{2} - \left(\frac{1^2}{4} + \frac{\ln 1}{2} \right) \right]$$

$$= \frac{\ln 2}{2} + \frac{3}{4}$$