1. Q — $\int e^{5\theta}\sin(6\theta)d\theta$

A — Let $u = e^{5\theta}$ and $v' = \sin(6\theta)$.

$\implies u' = 5e^{5\theta}$ and $v = -\frac{\cos(6\theta)}{6}$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

Therefore $\int e^{5\theta}\sin(6\theta)d\theta$

$= \frac{-e^{5\theta}\cos(6\theta)}{6} + \frac{5}{6}\int e^{5\theta}\cos(6\theta)d\theta$

To calculate $\int e^{5\theta}\cos(6\theta)d\theta$

Let $u = e^{5\theta}$ and $v' = \cos(6\theta)$.

$\implies u' = 5e^{5\theta}$ and $v = \frac{\sin(6\theta)}{6}$

Therefore $\int e^{5\theta}\cos(6\theta)d\theta$

$= \frac{e^{5\theta}\sin(6\theta)}{6} - \frac{5}{6}\int e^{5\theta}\sin(6\theta)d\theta$

Therefore $\int e^{5\theta}\sin(6\theta)d\theta$

$= \frac{-e^{5\theta}\cos(6\theta)}{6} + \frac{5}{6}[\frac{e^{5\theta}\sin(6\theta)}{6} - \frac{5}{6}\int e^{5\theta}\sin(6\theta)d\theta]$

$= \frac{-e^{5\theta}\cos(6\theta)}{6} + \frac{5}{36}e^{5\theta}\sin(6\theta) - \frac{25}{36}\int e^{5\theta}\sin(6\theta)d\theta$

$(1 + \frac{25}{36})\int e^{5\theta}\sin(6\theta)d\theta = \frac{-e^{5\theta}\cos(6\theta)}{6} + \frac{5}{36}e^{5\theta}\sin(6\theta)$

$\frac{61}{36}\int e^{5\theta}\sin(6\theta)d\theta = \frac{-6}{36}e^{5\theta}\cos(6\theta) + \frac{5}{36}e^{5\theta}\sin(6\theta)$

$\int e^{5\theta}\sin(6\theta)d\theta = \frac{5}{61}e^{5\theta}\sin(6\theta) - \frac{6}{61}e^{5\theta}\cos(6\theta) + C$