1. Q) Convergent/Divergent? $\int_{-\infty}^{0} \frac{z}{z^4+36} dz$

A) Let

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\begin{aligned}
I &= \int \frac{z}{z^4+36} dz \\
&= \int \frac{z}{(z^2)^2+6^2} dz \\
&= \frac{1}{2} \int \frac{1}{u^2+6^2} du && \text{for } u = z^2, du = 2z dz \\
I_1 &= \int \frac{1}{u^2+6^2} du \\
&= \int \frac{6\sec^2(\theta)}{6^2\tan^2(\theta)+6^2} d\theta && \text{for } u = 6\tan(\theta),\ du = 6\sec^2(\theta) d\theta \\
&= \frac{1}{6} \int \frac{\sec^2(\theta)}{\tan^2(\theta)+1} d\theta \\
&= \frac{1}{6} \int d\theta = \frac{1}{6}\theta + C_1 = \frac{1}{6}\arctan(\frac{u}{6}) + C_2 \\
I &= \frac{1}{12}\arctan(\frac{u}{6}) + C \\
&= \frac{1}{12}\arctan(\frac{z^2}{6}) + C \\
[I]_{-\infty}^{0} &= \frac{1}{12} \lim_{t\to-\infty} \left[\arctan(\frac{z^2}{6})\right]_t^0 \\
&= \frac{1}{12} \lim_{t\to-\infty} \left[0 - \arctan(\frac{t^2}{6})\right] \\
&= \frac{1}{12} \left[0 - (\frac{\pi}{2})\right] && \text{since } \lim_{t\to-\infty} t^2 = \infty \text{ and } \arctan(\infty) = \frac{\pi}{2} \\
&= \boxed{-\frac{\pi}{24}}
\end{aligned}
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