1. Q — $\int 5z^3 e^z dz$

A — Let $u = 5z^3$ and $v' = e^z \implies u' = 15z^2$ and $v = e^z$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

Therefore $\int 5z^3 e^z dz = 5z^3 e^z - 15 \int z^2 e^z dz$

To calculate $\int z^2 e^z dz$

Let $u = z^2$ and $v' = e^z \implies u' = 2z$ and $v = e^z$

Therefore $\int z^2 e^z dz = z^2 e^z - 2 \int z e^z dz$

To calculate $\int z e^z dz$

Let $u = z$ and $v' = e^z \implies u' = 1$ and $v = e^z$

Therefore $\int z e^z dz = z e^z - \int e^z dz = (z-1)e^z$

Therefore $\int z^2 e^z dz = z^2 e^z - 2(z-1)e^z = (z^2 - 2z + 2)e^z$

Therefore $\int 5z^3 e^z dz = 5z^3 e^z - 15(z^2 - 2z + 2)e^z = (5z^3 - 15z^2 + 30z - 30)e^z + C$