1. Q — $\int e^{8\theta}\sin(9\theta)d\theta$

A — Let $u = e^{8\theta}$ and $v' = \sin(9\theta)$.

$\implies u' = 8e^{8\theta}$ and $v = -\frac{\cos(9\theta)}{9}$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

Therefore $\int e^{8\theta}\sin(9\theta)d\theta$

$= \frac{-e^{8\theta}\cos(9\theta)}{9} + \frac{8}{9}\int e^{8\theta}\cos(9\theta)d\theta$

To calculate $\int e^{8\theta}\cos(9\theta)d\theta$

Let $u = e^{8\theta}$ and $v' = \cos(9\theta)$.

$\implies u' = 8e^{8\theta}$ and $v = \frac{\sin(9\theta)}{9}$

Therefore $\int e^{8\theta}\cos(9\theta)d\theta$

$= \frac{e^{8\theta}\sin(9\theta)}{9} - \frac{8}{9}\int e^{8\theta}\sin(9\theta)d\theta$

Therefore $\int e^{8\theta}\sin(9\theta)d\theta$

$= \frac{-e^{8\theta}\cos(9\theta)}{9} + \frac{8}{9}[\frac{e^{8\theta}\sin(9\theta)}{9} - \frac{8}{9}\int e^{8\theta}\sin(9\theta)d\theta]$

$= \frac{-e^{8\theta}\cos(9\theta)}{9} + \frac{8}{81}e^{8\theta}\sin(9\theta) - \frac{64}{81}\int e^{8\theta}\sin(9\theta)d\theta$

$(1 + \frac{64}{81})\int e^{8\theta}\sin(9\theta)d\theta = \frac{-e^{8\theta}\cos(9\theta)}{9} + \frac{8}{81}e^{8\theta}\sin(9\theta)$

$\frac{145}{81}\int e^{8\theta}\sin(9\theta)d\theta = \frac{-9}{81}e^{8\theta}\cos(9\theta) + \frac{8}{81}e^{8\theta}\sin(9\theta)$

$\int e^{8\theta}\sin(9\theta)d\theta = \frac{-9}{145}e^{8\theta}\cos(9\theta) + \frac{8}{145}e^{8\theta}\sin(9\theta) + C$