1. Q) $\int \frac{\sqrt{x^2-81}}{x}dx$

A) Let $x = 9\sec(\theta) \therefore \sec(\theta) = \frac{x}{9}$

Also $dx = 9\sec(\theta)\tan(\theta)d\theta$

$\sqrt{x^2 - 9^2}$

$= \sqrt{9^2\sec^2(\theta) - 9^2}$

$= 9\sqrt{\sec^2(\theta) - 1}$

$= 9\tan(\theta)$

$\therefore \int \frac{\sqrt{x^2-9^2}}{x}dx$

$\int \frac{9\tan(\theta)}{9\sec(\theta)} 9\sec(\theta)\tan(\theta)d\theta$

$9\int tan^2(\theta)d\theta$

$= -8\int \frac{1-u^2}{u^4}du$, for $u = \cos(\theta)$, $du = -\sin(\theta)d\theta$

$= 8\int \frac{u^2-1}{u^4}du$

$= 8[\int u^{-2}du - \int u^{-4}du]$

$= 8[-\frac{1}{u} + \frac{1}{3u^3} + C]$

$= 8[\frac{1}{3u^3} - \frac{1}{u} + C]$

$= \frac{8}{3u^3} - \frac{8}{u} + C$

$= \frac{8}{3\cos^3(\theta)} - \frac{8}{\cos(\theta)} + C$

$= \frac{8}{3}(\frac{\sqrt{x^2+4}}{2})^3 - 8(\frac{\sqrt{x^2+4}}{2}) + C$

$= \frac{1}{3}(\sqrt{x^2+4})^3 - 4\sqrt{x^2+4} + C$