

1. Q) Find f_{ave} , where $f(x) = 3x^2 + 4x$, on $[-1, 3]$

$$A) \int f(x)dx = \int (3x^2 + 4x)dx$$

$$= x^3 + 2x^2 + C$$

$$\implies f_{ave} = \frac{[x^3+2x^2]_{-1}^3}{3+1}$$

$$= \frac{(3^3+2.3^2)-((-1)^3+2(-1)^2)}{4}$$

$$= \frac{(27+18)-(-1+2)}{4}$$

$$= \frac{45-1}{4} = 11$$

2. Q) Find f_{ave} , where $f(x) = 5 \cos(x)$, on $[-\pi/2, \pi/2]$

$$A) \int f(x)dx = \int 5 \cos(x)dx$$

$$= 5 \sin(x) + C$$

$$\implies f_{ave} = \frac{[5 \sin(x)]_{-\pi/2}^{\pi/2}}{-\pi/2 - \pi/2}$$

$$= \frac{(-5) - (5)}{-\pi}$$

$$= \frac{10}{\pi}$$

3. Q) Find f_{ave} , where $f(t) = \frac{t}{\sqrt{3+t^2}}$, on $[1, 3]$

$$A) \int f(t)dt = \int \frac{t}{\sqrt{3+t^2}}dt$$

$$= \int \frac{t}{\sqrt{3+t^2}}dt = \frac{1}{2} \int \frac{1}{\sqrt{u}}du, \text{ for } u = 3 + t^2$$

$$= \sqrt{u} + C = \sqrt{(3+t^2)} + C$$

$$\implies f_{ave} = \frac{[\sqrt{(3+t^2)}]_1^3}{3-1}$$

$$= \frac{(\sqrt{3+9}) - (\sqrt{3+1})}{2} = \frac{\sqrt{12}-2}{2} = \sqrt{3} - 1$$

4. Q) Find f_{ave} , where $f(t) = e^{\sin(t)} \cos(t)$, on $[0, \pi/2]$

$$\text{A) } \int f(t) dt = \int e^{\sin(t)} \cos(t) dt$$

$$= \int e^u du, \text{ for } u = \sin(t)$$

$$= e^u = e^{\sin(t)} + C$$

$$\implies f_{ave} = \frac{[e^{\sin(t)}]_0^{\pi/2}}{\pi/2 - 0}$$

$$= \frac{e^1 - e^0}{\pi/2} = \frac{e-1}{\pi/2} = \frac{2(e-1)}{\pi}$$

5. Q) Find f_{ave} , where $f(x) = 6 \cos^4(x) \sin(x)$, on $[0, \pi]$

$$\text{A) } \int f(x) dx = \int 6 \cos^4(x) \sin(x) dx$$

$$= -6 \int u^4 du, \text{ for } u = \cos(x)$$

$$= -\frac{6}{5} u^5 + C = -\frac{6}{5} \cos^5(x) + C$$

$$\implies f_{ave} = -\frac{6}{5} \frac{[\cos^5(x)]_0^\pi}{\pi - 0}$$

$$= -\frac{6}{5} \frac{\cos^5(\pi) - \cos^5(0)}{\pi}$$

$$= -\frac{6}{5} \frac{-1-1}{\pi} = \frac{12}{5\pi}$$

6. Q) Find f_{ave} , where $f(x) = \frac{\ln(x)}{x}$, on $[1, 3]$

$$\text{A) } \int f(x) dx = \int \frac{\ln(x)}{x} dx = \int u du, \text{ for } u = \ln(x)$$

$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln(x))^2 + C$$

$$\implies f_{ave} = \frac{1}{2} \frac{[(\ln(x))^2]_1^3}{3-1}$$

$$= \frac{1}{2} \frac{(\ln(3))^2 - (\ln(1))^2}{2} = \frac{1}{4} (\ln(3))^2$$