

1. Q) $\int 7 \sin^2(x) \cos^3(x) dx$

A) $I = 7 \int \sin^2(x) \cos^2(x) \cos(x) dx$

Let $u = \sin(x)$; $du = \cos(x) dx$. Also $\cos^2(x) = 1 - \sin^2(x) = 1 - u^2$

$$\implies I = 7 \int u^2(1 - u^2) du$$

$$= 7 \int (u^2 - u^4) du$$

$$= \frac{7u^3}{3} - \frac{7u^5}{5} + C$$

$$= \frac{7 \sin^3(x)}{3} - \frac{7 \sin^5(x)}{5} + C$$

2. Q) $\int 19 \sin^2(x) \cos^3(x) dx$

3. Q) $\int_0^{\pi/2} \sin^5(x) \cos^5(x) dx$

A) Let $I = \int \sin^5(x) \cos^5(x) dx$

Let $u = \sin(x)$; $du = \cos(x) dx$. Also $\cos^2(x) = 1 - \sin^2(x) = 1 - u^2$

$$\implies I = \int u^5(1 - u^2)^2 du$$

$$= \int u^5(1 - 2u^2 + u^4) du$$

$$= \int (u^5 - 2u^7 + u^9) du = u^6/6 - u^8/4 + u^{10}/10 + C$$

$$= \frac{\sin^6(x)}{6} - \frac{\sin^8(x)}{4} + \frac{\sin^{10}(x)}{10} + C$$

$$\implies [I]_0^{\pi/2} = \frac{\sin^6(\pi/2)}{6} - \frac{\sin^8(\pi/2)}{4} + \frac{\sin^{10}(\pi/2)}{10} - 0$$

$$= \frac{1}{6} - \frac{1}{4} + \frac{1}{10} = \frac{10-15+6}{60} = 1/60$$

4. Q) $\int_0^{\pi/2} 3 \cos^2(x) dx$

A) Let $I = \int \cos^2(x) dx$

$$\begin{aligned}
&= \int \frac{1+\cos(2x)}{2} dx \\
&= \int \frac{1}{2} dx + \int \frac{\cos(2x)}{2} dx \\
&= \frac{x}{2} + \frac{\sin(2x)}{4} + C \\
&\implies 3I_0^{\pi/2} = 3\left(\frac{\pi}{4} + 0 - 0\right) = 3\pi/4
\end{aligned}$$

5. Q) $\int_0^\pi 9 \sin^2(x) \cos^4(x) dx$

A) Let $I = \int \sin^2(x) \cos^4(x) dx$

$$\begin{aligned}
&= \int \frac{1-\cos(2x)}{2} \left(\frac{1+\cos(2x)}{2}\right)^2 dx \\
&= \frac{1}{8} \int (1 - \cos(2x))(1 + 2\cos(2x) + \cos^2(2x)) dx \\
&= \frac{1}{8} \int (1 + 2\cos(2x) + \cos^2(2x) - \cos(2x) - 2\cos^2(2x) - \cos^3(2x)) dx \\
&= \frac{1}{8} \int (1 + \cos(2x) - \cos^2(2x) - \cos^3(2x)) dx
\end{aligned}$$

$$\int \cos(2x) dx = \frac{1}{2} \sin(2x) + C_1$$

$$\int \cos^2(2x) dx = \frac{1}{2} \int (1 + \cos(4x)) dx = \frac{1}{2} \left(x + \frac{1}{4} \sin(4x)\right) = \frac{x}{2} + \frac{1}{8} \sin(4x) + C_2$$

$$\int \cos^3(2x) dx = \int \cos^2(2x) \cos(2x) dx = \int \sqrt{1 - \sin^2(2x)} \cos(2x) dx = \frac{1}{2} \int (1 - u^2)^{\frac{1}{2}} du,$$

for $u = \sin(2x)$ and $\frac{1}{2} du = \cos(2x) dx$

$$= -\frac{1}{2} \frac{v^{\frac{3}{2}}}{\frac{3}{2}} + C_3, \text{ for } v = (1 - u^2) \text{ and } -\frac{1}{2} dv = u du$$

$$= -\frac{1}{3} v^{\frac{3}{2}} + C_3 = -\frac{1}{3} (1 - u^2)^{\frac{3}{2}} + C_3 = -\frac{1}{3} (1 - \sin^2(2x))^{\frac{3}{2}} + C_3$$

$$\implies I = \frac{1}{8} \left[x + \frac{1}{2} \sin(2x) - \left(\frac{x}{2} + \frac{1}{8} \sin(4x)\right) - \left(-\frac{1}{3} (1 - \sin^2(2x))^{\frac{3}{2}}\right) \right]$$

$$= \frac{1}{8} \left[\frac{x}{2} + \frac{1}{2} \sin(2x) - \frac{1}{8} \sin(4x) + \frac{1}{3} (1 - \sin^2(2x))^{\frac{3}{2}} \right]$$

$$\implies \int_0^\pi 9 \sin^2(x) \cos^4(x) dx$$

$$= \frac{9}{8} \left[\frac{x}{2} + \frac{1}{2} \sin(2x) - \frac{1}{8} \sin(4x) + \frac{1}{3} (1 - \sin^2(2x))^{\frac{3}{2}} \right]_0^\pi$$

$$= \frac{9}{8} \left[\left(\frac{\pi}{2} + \frac{1}{2} \sin(2\pi) - \frac{1}{8} \sin(4\pi) + \frac{1}{3} (1 - \sin^2(2\pi))^{\frac{3}{2}}\right) - \left(\frac{1}{3}\right) \right]$$

$$= \frac{9}{8} \left[\frac{\pi}{2} + 0 - 0 + \frac{1}{3} (1 - 0)^{\frac{3}{2}} - \frac{1}{3} \right] = \frac{9\pi}{16}$$

$$6. \text{ Q) } \int \tan^6(x) \cos^7(x) dx$$

$$\begin{aligned} \text{A) } &= \int \frac{\sin^6(x)}{\cos^6(x)} \cos^7(x) dx \\ &= \int \sin^6(x) \cos(x) dx \\ &= \int u^6 du, \text{ for } u = \sin(x) \text{ and } du = \cos(x) dx \\ &= \frac{u^7}{7} + C \\ &= \frac{\sin^7(x)}{7} + C \end{aligned}$$

$$7. \text{ Q) } \int 3t \sin^2(t) dt$$

$$\begin{aligned} \text{A) } &= 3 \int uv' dt, \text{ for } u = t, v' = \sin^2(t), \text{ therefore } u' = 1, v = \int \sin^2(t) dt \\ &v = \int \frac{1 - \cos(2t)}{2} dt \\ &= \int \frac{1}{2} dt - \int \frac{\cos(2t)}{2} dt \\ &= \frac{t}{2} - \frac{1}{4} \sin(2t) \\ &\int vu' = \int \left(\frac{t}{2} - \frac{1}{4} \sin(2t) \right) dt \\ &= \frac{t^2}{4} + \frac{1}{8} \cos(2t) \\ &\implies \int 3t \sin^2(t) dt \\ &= 3 \left[t \left(\frac{t}{2} - \frac{1}{4} \sin(2t) \right) - \left(\frac{t^2}{4} + \frac{1}{8} \cos(2t) \right) \right] + C \\ &= 3 \left[\frac{t^2}{2} - \frac{t}{4} \sin(2t) - \frac{t^2}{4} - \frac{1}{8} \cos(2t) \right] + C \\ &= 3 \left[\frac{t^2}{4} - \frac{t}{4} \sin(2t) - \frac{1}{8} \cos(2t) \right] + C \end{aligned}$$

$$8. \text{ Q) } \int 8 \tan(x) \sec^3(x) dx$$

$$\begin{aligned} \text{A) } &= 8 \int \frac{\sin(x)}{\cos(x)} \frac{1}{\cos^3(x)} dx = 8 \int \frac{\sin(x)}{\cos^4(x)} dx = -8 \int \frac{1}{u^4} du, \text{ for } u = \cos(x) \\ &= 8 \frac{1}{3u^3} + C \end{aligned}$$

$$= \frac{8}{3 \cos^3(x)} + C = \frac{8}{3} \sec^3(x) + C$$

$$9. \text{ Q) } \int 2(\tan^2(x) + \tan^4(x))dx$$

$$\text{A) } = \int 2 \tan^2(x)(1 + \tan^2(x))dx$$

$$= \int 2 \tan^2(x) \sec^2(x)dx$$

$$\text{Let } u = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\implies u' = \frac{d}{dx} \frac{\sin(x)}{\cos(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \sec^2(x)$$

$$\implies I = \int 2u^2 du = \frac{2}{3}u^3 + C = \frac{2}{3} \tan^3(x) + C$$