1. Q) Surface area of rvolution:

$y = \frac{x^3}{3} + \frac{1}{4x}, \frac{1}{2} \leq x \leq 1$

A)

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)^2]}dx$$

$$f = \frac{x^3}{3} + \frac{1}{4x}$$

$$f' = x^2 - \frac{1}{4x^2} = \frac{4x^4 - 1}{4x^2}$$

$$S = \int_{\frac{1}{2}}^1 2\pi(\frac{x^3}{3} + \frac{1}{4x})\sqrt{1 + (\frac{4x^4 - 1}{4x^2})^2}dx$$

$$= 2\pi \int_{\frac{1}{2}}^1 \frac{4x^4 + 3}{12x} \frac{\sqrt{16x^4 + 16x^8 + 1 - 8x^4}}{4x^2}dx$$

$$= 2\pi \int_{\frac{1}{2}}^1 \frac{4x^4 + 3}{12x} \frac{\sqrt{8x^4 + 16x^8 + 1}}{4x^2}dx$$

$$= 2\pi \int_{\frac{1}{2}}^1 \frac{(4x^4 + 3)(4x^4 + 1)}{48x^3}dx$$

$$= 2\pi \int_{\frac{1}{2}}^1 \frac{(16x^8 + 16x^4 + 3)}{48x^3}dx$$

$$= 2\pi \int_{\frac{1}{2}}^1 (\frac{x^5}{3} + \frac{x}{3} + \frac{1}{16x^3})dx$$

$$= 2\pi \left[\frac{x^6}{18} + \frac{x^2}{6} - \frac{1}{32x^2}\right]_{\frac{1}{2}}^1$$

$$= 2\pi \left[\frac{1}{18} + \frac{1}{6} - \frac{1}{32} - (\frac{1}{18.64} + \frac{1}{24} - \frac{1}{8})\right]$$

$$= 2\pi \left[\frac{16 + 48 - 9}{288} - (\frac{1 + 48 - 144}{1152})\right] = 2\pi \left[\frac{55}{288} + \frac{95}{1152}\right] = 2\pi \left[\frac{55.4 + 95}{1152}\right]$$

$$= 2\pi \left[\frac{315}{1152}\right]$$

$$= \frac{105\pi}{192}$$