

1. Q) Convergent/Divergent?  $\int_{-\infty}^{\infty} 11xe^{-x^2} dx$

$$\begin{aligned} \text{A) } & 11 \left[ \int_{-\infty}^0 xe^{-x^2} dx + \int_0^{\infty} xe^{-x^2} dx \right] \\ &= 11 \lim_{t \rightarrow \infty^+} \int_t^0 xe^{-x^2} dx + 11 \lim_{t \rightarrow \infty^-} \int_0^t xe^{-x^2} dx \\ & \int xe^{-x^2} dx = -\frac{1}{2} \int e^u du, \text{ for } u = -x^2, \text{ and } xdx = -\frac{1}{2} du \\ &= -\frac{e^u}{2} = -\frac{e^{-x^2}}{2} \\ &\therefore \left[ -\frac{e^{-x^2}}{2} \right]_t^0 = -\frac{1}{2} [e^0 - e^{-t^2}] = -\frac{1}{2} \left[ 1 - \frac{1}{e^{t^2}} \right] \\ &\therefore -\frac{1}{2} \lim_{t \rightarrow \infty^+} \left[ 1 - \frac{1}{e^{t^2}} \right] = -\frac{1}{2} [1 - 0] = -\frac{1}{2} \\ &\text{and } \left[ -\frac{e^{-x^2}}{2} \right]_0^t = -\frac{1}{2} [e^{-t^2} - e^0] = -\frac{1}{2} \left[ \frac{1}{e^{t^2}} - 1 \right] \\ &\text{and } -\frac{1}{2} \lim_{t \rightarrow \infty^-} \left[ \frac{1}{e^{t^2}} - 1 \right] = -\frac{1}{2} [0 - 1] = \frac{1}{2} \\ &\therefore I = 11(0) = 0 \end{aligned}$$