

1. Q —  $\int e^{5\theta} \sin(6\theta) d\theta$

A — Let  $u = e^{5\theta}$  and  $v' = \sin(6\theta)$ .

$$\implies u' = 5e^{5\theta} \text{ and } v = -\frac{\cos(6\theta)}{6}$$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

Therefore  $\int e^{5\theta} \sin(6\theta) d\theta$

$$= \frac{-e^{5\theta} \cos(6\theta)}{6} + \frac{5}{6} \int e^{5\theta} \cos(6\theta) d\theta$$

To calculate  $\int e^{5\theta} \cos(6\theta) d\theta$

Let  $u = e^{5\theta}$  and  $v' = \cos(6\theta)$ .

$$\implies u' = 5e^{5\theta} \text{ and } v = \frac{\sin(6\theta)}{6}$$

Therefore  $\int e^{5\theta} \cos(6\theta) d\theta$

$$= \frac{e^{5\theta} \sin(6\theta)}{6} - \frac{5}{6} \int e^{5\theta} \sin(6\theta) d\theta$$

Therefore  $\int e^{5\theta} \sin(6\theta) d\theta$

$$= \frac{-e^{5\theta} \cos(6\theta)}{6} + \frac{5}{6} \left[ \frac{e^{5\theta} \sin(6\theta)}{6} - \frac{5}{6} \int e^{5\theta} \sin(6\theta) d\theta \right]$$

$$= \frac{-e^{5\theta} \cos(6\theta)}{6} + \frac{5}{36} e^{5\theta} \sin(6\theta) - \frac{25}{36} \int e^{5\theta} \sin(6\theta) d\theta$$

$$\left(1 + \frac{25}{36}\right) \int e^{5\theta} \sin(6\theta) d\theta = \frac{-e^{5\theta} \cos(6\theta)}{6} + \frac{5}{36} e^{5\theta} \sin(6\theta)$$

$$\frac{61}{36} \int e^{5\theta} \sin(6\theta) d\theta = \frac{-6}{36} e^{5\theta} \cos(6\theta) + \frac{5}{36} e^{5\theta} \sin(6\theta)$$

$$\int e^{5\theta} \sin(6\theta) d\theta = \frac{5}{61} e^{5\theta} \sin(6\theta) - \frac{6}{61} e^{5\theta} \cos(6\theta) + C$$