1. Q) Surface area of rvolution:

$x = \frac{1}{3}(y^2+2)^{3/2}, 1 \le y \le 3$

A) $3x = (y^2+2)^{3/2}$

$(3x)^{\frac{2}{3}} = y^2 + 2$

$y^2 = (3x)^{\frac{2}{3}} - 2$

$y = ((3x)^{\frac{2}{3}} - 2)^{\frac{1}{2}}, \sqrt{3} \le x \le \frac{11^{3/2}}{3}$

$y' = \frac{1}{2}((3x)^{\frac{2}{3}} - 2)^{-\frac{1}{2}} \frac{2}{3}(3x)^{-\frac{1}{3}} 3$

$= (3x)^{-\frac{1}{3}}((3x)^{\frac{2}{3}} - 2)^{-\frac{1}{2}}$

$[y']^2 = \frac{(3x)^{-\frac{2}{3}}}{(3x)^{\frac{2}{3}} - 2} = \frac{1}{(3x)^{\frac{2}{3}}[(3x)^{\frac{2}{3}} - 2]} = \frac{1}{(3x)^{\frac{4}{3}} - 2(3x)^{\frac{2}{3}}}$

$1 + [y']^2 = 1 + \frac{1}{(3x)^{\frac{4}{3}} - 2(3x)^{\frac{2}{3}}}$

$= \frac{(3x)^{\frac{4}{3}} - 2(3x)^{\frac{2}{3}} + 1}{(3x)^{\frac{4}{3}} - 2(3x)^{\frac{2}{3}}}$

$= \frac{[(3x)^{\frac{2}{3}} - 1]^2}{(3x)^{\frac{2}{3}}[(3x)^{\frac{2}{3}} - 2]}$

$\sqrt{1 + [y']^2} = \frac{(3x)^{\frac{2}{3}} - 1}{\sqrt{(3x)^{\frac{2}{3}}[(3x)^{\frac{2}{3}} - 2]}}$

$= \frac{(3x)^{\frac{2}{3}} - 1}{(3x)^{\frac{1}{3}}\sqrt{(3x)^{\frac{2}{3}} - 2}}$

$$
\begin{aligned}
S &= \int_a^b 2\pi f(x)\sqrt{1+[f'(x)^2]}dx \\
&= \int_{\sqrt{3}}^{\frac{11^{3/2}}{3}} 2\pi\sqrt{(3x)^{\frac{2}{3}}-2)}\frac{(3x)^{\frac{2}{3}}-1}{(3x)^{\frac{1}{3}}\sqrt{(3x)^{\frac{2}{3}}-2}}dx \\
&= 2\pi\int_{\sqrt{3}}^{\frac{11^{3/2}}{3}} \frac{(3x)^{\frac{2}{3}}-1}{(3x)^{\frac{1}{3}}}dx \\
\text{Let } I &= \int \frac{(3x)^{\frac{2}{3}}-1}{(3x)^{\frac{1}{3}}}dx \\
&= \frac{1}{3}\int \frac{u^{\frac{2}{3}}-1}{u^{\frac{1}{3}}}du \qquad (\text{ For } u=3x, dx=\frac{du}{3}\ ) \\
&= \frac{1}{3}\int [u^{\frac{1}{3}}-u^{\frac{-1}{3}}]du \\
&= \frac{1}{3}[\frac{3}{4}u^{\frac{4}{3}}-\frac{3}{2}u^{\frac{2}{3}}] \\
&= \frac{u^{\frac{4}{3}}}{4}-\frac{u^{\frac{2}{3}}}{2} \\
&= \frac{(3x)^{\frac{4}{3}}}{4}-\frac{(3x)^{\frac{2}{3}}}{2} \\
S &= 2\pi\left[\frac{(3x)^{\frac{4}{3}}}{4}-\frac{(3x)^{\frac{2}{3}}}{2}\right]_{\sqrt{3}}^{\frac{11^{3/2}}{3}} \\
&= 2\pi\frac{(11^{3/2})^{\frac{4}{3}}}{4}-\frac{(11^{3/2})^{\frac{2}{3}}}{2}-\frac{(3\sqrt{3})^{\frac{4}{3}}}{4}+\frac{(3\sqrt{3})^{\frac{2}{3}}}{2} \\
&= 2\pi\frac{121}{4}-\frac{11}{2}-\frac{9}{4}+\frac{3}{2} \\
&= 2\pi(28-4) \\
&= \boxed{48\pi}
\end{aligned}
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