1. Q) $\int \cos^{-1}(x)dx$

A) Let $y = \cos^{-1}(x)$

$\implies x = cos(y)$

$\implies \frac{dx}{dy} = -\sin(y)$

$\implies dx = -\sin(y)dy$

Therefore $\int \cos^{-1}(x)dx = \int y(-\sin(y)dy)$

$= -\int y\sin(y)dy$

Let $u = y; v' = \sin(y)$. Therefore $v = -\cos(y)$

According to integration by parts:

$$\int uv' = uv - \int vu'$$

Therefore $-\int y\sin(y)dy$

$= -(y(-\cos(y)) - \int -\cos(y)dy)$

$= y\cos(y) - \int \cos(y)dy$

$= y\cos(y) - \sin(y) + C$

$= \cos^{-1}(x)x - \sin(cos^{-1}(x)) + C$, (substituting $y = cos^{-1}(x)$)

$= x\cos^{-1}(x) - \sqrt{1-x^2} + C$, (substituting $\sin(cos^{-1}(x)) = sqrt1 - x^2$)