Q.4 Find the volume of a solid obtained by rotating the region bounded by $y = x^3, y = 8, x = 0$ about $x = 5$.

A) The curves are:
$y = x^3$ and
$y = 8$

Intersection points:
$x = 2, y = 8$ and
$x = 5, y = 125$

The curcumference of a cylinder $= 2\pi(5 - x)$

The height of a cylinder $= 8 - x^3$

Volume of a cylinder $= 2\pi(5 - x)(8 - x^3)dx$

Total volume:

$$V = \int_0^2 2\pi(5 - x)(8 - x^3)dx \tag{1}$$

$$= 2\pi \int_0^2 (40 - 5x^3 - 8x + x^4)dx \tag{2}$$

$$= 2\pi \left[40x - \frac{5x^4}{4} - 4x^2 + \frac{x^5}{5}\right]_0^2 \tag{3}$$

$$= 2\pi \left[80 - \frac{(5)2^4}{4} - (4)2^2 + \frac{2^5}{5}\right] \tag{4}$$

$$= 2\pi \left[44 + \frac{32}{5}\right] \tag{5}$$

$$= \frac{504\pi}{5} \tag{6}$$

$$\tag{7}$$

The volume of the lower curve,

$$V_2 = 72\pi \tag{8}$$

$$\tag{9}$$

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