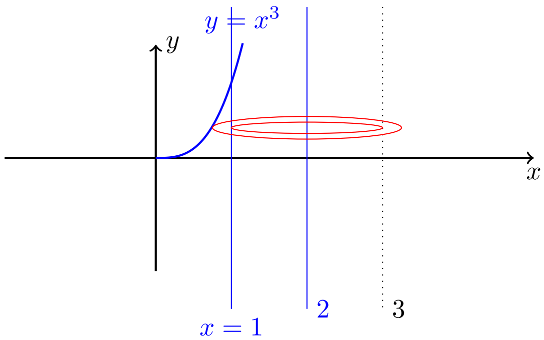
1. Q) Volume of region enclosed by $y = 8x^3, y = 0, x = 1$ about $x = 2$

A)



Since the size of the "ring" is a function of y (think sliding it up and down to change its size), we convert the original function in terms of y. In other words,

$$x = f(y) = \frac{\sqrt[3]{y}}{2} \tag{1}$$

This "ring" has 2 radii: the outer one: R and the inner one: r

We see that $r = 1$ (the distance between $x = 1$ and $x = 2$)

R, however, is the distance between $x = 2$ and $f(y)$. Using (1) above:

$R = 2 - \frac{\sqrt[3]{y}}{2}$

The area of this ring:

$A(y) = \pi R^2 - \pi r^2$

$= \pi(2 - \frac{\sqrt[3]{y}}{2})^2 - \pi(1)^2$

$= \pi \left[(\frac{4 - \sqrt[3]{y}}{2})^2 - 1 \right] = \pi \left[(\frac{16 + y^{2/3} - 8y^{1/3}}{4}) - 1 \right]$

$= \frac{\pi}{4} \left[16 + y^{2/3} - 8y^{1/3} - 4 \right] = \frac{\pi}{4} \left[12 + y^{2/3} - 8y^{1/3} \right]$

The volume of the solid generated by the curve is the integration of this "ring" w.r.t dy, calculated in the range $[0, 8]$ (the extremes of the curve where it intersects with the limiting lines).

$$V = \int_0^8 A(y)dy$$

$$= \int_0^8 \tfrac{\pi}{4}\left[12 + y^{2/3} - 8y^{1/3}\right]dy$$

$$= \tfrac{\pi}{4}\int_0^8 \left[12 + y^{2/3} - 8y^{1/3}\right]dy$$

$$= \tfrac{\pi}{4}\left[12y + \tfrac{y^{5/3}}{5/3} - \tfrac{8y^{4/3}}{4/3}\right]_0^8$$

$$= \tfrac{\pi}{4}\left[12(8) + \tfrac{(8)^{5/3}}{5/3} - \tfrac{8(8)^{4/3}}{4/3}\right]$$

$$= \tfrac{\pi}{4}\left[12(8) + \tfrac{32}{5/3} - \tfrac{8(16)}{4/3}\right]$$

$$= \tfrac{\pi}{4}\left[96 + \tfrac{32}{5/3} - \tfrac{128}{4/3}\right]$$

$$= \tfrac{\pi}{4}\left[96 + \tfrac{96}{5} - 96\right]$$

$$= \tfrac{\pi}{4}(\tfrac{96}{5}) = \boxed{\frac{24\pi}{5}}$$