1. Q) $\int 7\sin^2(x)\cos^3(x)dx$

A) $I = 7\int sin^2(x)cos^2(x)cos(x)dx$

Let $u = \sin(x)$; $du = \cos(x)dx$. Also $\cos^2(x) = 1 - sin^2(x) = 1 - u^2$

$\implies I = 7\int u^2(1-u^2)du$

$= 7\int (u^2 - u^4)du$

$= \frac{7u^3}{3} - \frac{7u^5}{5} + C$

$= \frac{7\sin^3(x)}{3} - \frac{7\sin^5(x)}{5} + C$

2. Q) $\int 19\sin^2(x)\cos^3(x)dx$

3. Q) $\int_0^{\pi/2} \sin^5(x)\cos^5(x)dx$

A) Let $I = \int \sin^5(x)\cos^5(x)dx$

Let $u = \sin(x)$; $du = \cos(x)dx$. Also $\cos^2(x) = 1 - sin^2(x) = 1 - u^2$

$\implies I = \int u^5(1-u^2)^2du$

$= \int u^5(1 - 2u^2 + u^4)du$

$= \int (u^5 - 2u^7 + u^9)du = u^6/6 - u^8/4 + u^{10}/10 + C$

$= \frac{\sin^6(x)}{6} - \frac{\sin^8(x)}{4} + \frac{\sin^{10}(x)}{10} + C$

$\implies [I]_0^{\pi/2} = \frac{\sin^6(\pi/2)}{6} - \frac{\sin^8(\pi/2)}{4} + \frac{\sin^{10}(\pi/2])}{10} - 0$

$= \frac{1}{6} - \frac{1}{4} + \frac{1}{10} = \frac{10-15+6}{60} = 1/60$

4. Q) $\int_0^{\pi/2} 3\cos^2(x)dx$

A) Let $I = \int \cos^2(x)dx$

$= \int \frac{1+\cos(2x)}{2} dx$

$= \int \frac{1}{2} dx + \int \frac{\cos(2x)}{2} dx$

$= \frac{x}{2} + \frac{\sin(2x)}{4} + C$

$\implies 3I_0^{\pi/2} = 3(\frac{\pi}{4} + 0 - 0) = 3\pi/4$