1. Q) $\int 7\sin^2(x)\cos^3(x)dx$

A) $I = 7\int sin^2(x)cos^2(x)cos(x)dx$

Let $u = \sin(x)$; $du = \cos(x)dx$. Also $\cos^2(x) = 1 - sin^2(x) = 1 - u^2$

$\implies I = 7\int u^2(1-u^2)du$

$= 7\int (u^2 - u^4)du$

$= \frac{7u^3}{3} - \frac{7u^5}{5} + C$

$= \frac{7\sin^3(x)}{3} - \frac{7\sin^5(x)}{5} + C$

2. Q) $\int 19\sin^2(x)\cos^3(x)dx$

3. Q) $\int_0^{\pi/2} \sin^5(x)\cos^5(x)dx$

A) Let $I = \int \sin^5(x)\cos^5(x)dx$

Let $u = \sin(x)$; $du = \cos(x)dx$. Also $\cos^2(x) = 1 - sin^2(x) = 1 - u^2$

$\implies I = \int u^5(1-u^2)^2du$

$= \int u^5(1 - 2u^2 + u^4)du$

$= \int (u^5 - 2u^7 + u^9)du = u^6/6 - u^8/4 + u^{10}/10 + C$

$= \frac{\sin^6(x)}{6} - \frac{\sin^8(x)}{4} + \frac{\sin^{10}(x)}{10} + C$

$\implies [I]_0^{\pi/2} = \frac{\sin^6(\pi/2)}{6} - \frac{\sin^8(\pi/2)}{4} + \frac{\sin^{10}(\pi/2])}{10} - 0$

$= \frac{1}{6} - \frac{1}{4} + \frac{1}{10} = \frac{10-15+6}{60} = 1/60$

4. Q) $\int_0^{\pi/2} 3\cos^2(x)dx$

A) Let $I = \int \cos^2(x)dx$

$= \int \frac{1+\cos(2x)}{2} dx$

$= \int \frac{1}{2} dx + \int \frac{\cos(2x)}{2} dx$

$= \frac{x}{2} + \frac{\sin(2x)}{4} + C$

$\implies 3I_0^{\pi/2} = 3(\frac{\pi}{4} + 0 - 0) = 3\pi/4$

5. Q) $\int_0^{\pi} 9\sin^2(x)\cos^4(x)dx$

A) Let $I = \int \sin^2(x)\cos^4(x)dx$

$= \int \frac{1-\cos(2x)}{2}(\frac{1+\cos(2x)}{2})^2 dx$

$= \frac{1}{8}\int (1-\cos(2x))(1+2\cos(2x)+\cos^2(2x))dx$

$= \frac{1}{8}\int (1+2\cos(2x)+\cos^2(2x)-\cos(2x)-2\cos^2(2x)-\cos^3(2x))dx$

$= \frac{1}{8}\int (1+\cos(2x)-\cos^2(2x)-\cos^3(2x))dx$

$\int \cos(2x)dx = \frac{1}{2}\sin(2x) + C_1$

$\int \cos^2(2x)dx = \frac{1}{2}\int (1+cos(4x))dx = \frac{1}{2}(x+\frac{1}{4}\sin(4x)) = \frac{x}{2} + \frac{1}{8}\sin(4x) + C_2$

$\int \cos^3(2x)dx = \int \cos^2(2x)\cos(2x)dx = \int \sqrt{1-\sin^2(2x)}\cos(2x)dx = \frac{1}{2}\int (1-u^2)^{\frac{1}{2}}du$, for $u = sin(2x)$ and $\frac{1}{2}du = \cos(2x)dx$

$= -\frac{1}{2}\frac{v^{\frac{3}{2}}}{3/2} + C_3$, for $v = (1-u^2)$ and $-\frac{1}{2}dv = udu$

$= -\frac{1}{3}v^{\frac{3}{2}} + C_3 = -\frac{1}{3}(1-u^2)^{\frac{3}{2}} + C_3 = -\frac{1}{3}(1-sin^2(2x))^{\frac{3}{2}} + C_3$

$\implies I = \frac{1}{8}\left[x + \frac{1}{2}\sin(2x) - (\frac{x}{2} + \frac{1}{8}\sin(4x)) - (-\frac{1}{3}(1-sin^2(2x))^{\frac{3}{2}})\right]$

$= \frac{1}{8}\left[\frac{x}{2} + \frac{1}{2}\sin(2x) - \frac{1}{8}\sin(4x) + \frac{1}{3}(1-sin^2(2x))^{\frac{3}{2}}\right]$

$\implies \int_0^{\pi} 9\sin^2(x)\cos^4(x)dx$

$= \frac{9}{8}\left[\frac{x}{2} + \frac{1}{2}\sin(2x) - \frac{1}{8}\sin(4x) + \frac{1}{3}(1-sin^2(2x))^{\frac{3}{2}}\right]_0^{\pi}$

$= \frac{9}{8}\left[(\frac{\pi}{2} + \frac{1}{2}\sin(2\pi) - \frac{1}{8}\sin(4\pi) + \frac{1}{3}(1-sin^2(2\pi))^{\frac{3}{2}}) - (\frac{1}{3})\right]$

$= \frac{9}{8}\left[\frac{\pi}{2} + 0 - 0 + \frac{1}{3}(1-0)^{\frac{3}{2}} - \frac{1}{3}\right] = \frac{9\pi}{16}$

6. Q) $\int \tan^6(x)\cos^7(x)dx$

A) $= \int \frac{\sin^6(x)}{\cos^6(x)}\cos^7(x)dx$

$= \int \sin^6(x)\cos(x)dx$

$= \int u^6 du$, for $u = \sin(x)$ and $du = cos(x)dx$

$= \frac{u^7}{7} + C$

$= \frac{\sin^7(x)}{7} + C$

7. Q) $\int 3t\sin^2(t)dt$

A) $= 3\int uv'dt$, for $u = t, v' = \sin^2(t)$, therefore $u' = 1, v = \int \sin^2(t)dt$

$v = \int \frac{1-\cos(2t)}{2}dt$

$= \int \frac{1}{2}dt - \int \frac{\cos(2t)}{2}dt$

$= \frac{t}{2} - \frac{1}{4}\sin(2t)$

$\int vu' = \int (\frac{t}{2} - \frac{1}{4}\sin(2t))dt$

$= \frac{t^2}{4} + \frac{1}{8}\cos(2t)$

$\implies \int 3t\sin^2(t)dt$

$= 3\left[t(\frac{t}{2} - \frac{1}{4}\sin(2t)) - (\frac{t^2}{4} + \frac{1}{8}\cos(2t))\right] + C$

$= 3\left[\frac{t^2}{2} - \frac{t}{4}\sin(2t) - \frac{t^2}{4} - \frac{1}{8}\cos(2t)\right] + C$

$= 3\left[\frac{t^2}{4} - \frac{t}{4}\sin(2t) - \frac{1}{8}\cos(2t)\right] + C$