1. Q — Find an explicit formula for the recurrence:

$a_0 = 5, a_{n+1} = 2a_n - 2, n \geq 1$

A — The recurrence can be re-written as a **nonhomogenous advancement operator equation**:

$p(A)a = (A - 2)a = -2$

$p(A)$ only has one root, $A = 2$. A general solution to a homogenous equation of this form is:

$a_1(n) = c2^n$

Let's try $a_0(n) = d(-2)^n$ from the R.H.S of the original equation.

$$\begin{aligned}(A-2)a_0(n) &= (A-2)d(-2)^n \\ &= d(-2)^{n+1} - 2d(-2)^n \\ &= d(-2)^{n+1} + d(-2)^{n+1} \\ &= 2d(-2)^{n+1}.\end{aligned}$$

If this were a solution to the nonhomogenous equation,

$(A - 2)a_0(n) = 2d(-2)^{n+1} = -2$

$\therefore d = -(-2)^{-n-1}$

Hence the general solution to the equation is:

$a(n) = a_1(n) + a_0(n) = -(-2)^{-n-1} + c2^n$

Since $a(0) = 5 = -(-2)^{-1} + c$,

$c = 5 + (-2)^{-1} = 5 - 1/2 = 9/2$

Therefore the exact formula for $a(n)$ is:

$$
\begin{aligned}
a(n) &= \frac{9}{2}2^n - (-2)^{-n-1} \\
&= \frac{9}{2^{n+1}}2^{2n} - \frac{(-1)^{n+1}}{2^{n+1}} \\
&= \boxed{\frac{9.2^{2n} + (-1)^n}{2^{n+1}}}
\end{aligned}
$$