1. Q — Find the number of ways to distribute n different objects to five different boxes, if an even number of objects are distributed to box 5.

A — Let's examine the exponential generating functions for finding the # of ways n distinguishable objects can be distributed among 5 different places:

The Generating Function for the respective boxes are:

$a_1, a_2, a_3, a_4 : \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$

$a_5 : 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \frac{e^x + e^{-x}}{2}$

The # of solutions to the above system of problems, N is the coefficient of the $\frac{x^n}{n!}$ term in the expansion of

$e^{4x} \cdot \frac{e^x + e^{-x}}{2} = \frac{e^{5x} + e^{3x}}{2} = \frac{1}{2} \left[\sum_{n=0}^{\infty} 5^n \cdot \frac{x^n}{n!} + \sum_{n=0}^{\infty} 3^n \cdot \frac{x^n}{n!} \right]$

Hence we see that the # of ways to distribute the objects with the given restrictions is:

$$\boxed{\frac{5^n + 3^n}{2}}$$