1. Q — Let S be the set of strings on the alphabet $\{0, 1, 2, 3\}$ that do not contain 12 or 20 as a substring. Give a recursion for the number $h(n)$ of strings in S of length n.

A —

$h(0) = 1, \{\text{“”}\}$

$h(1) = 4, \{0, 1, 2, 3\}$

$h(2) = 14, \{00, 01, 02, 03, 10, 11, 13, 21, 22, 23, 30, 31, 32, 33\}$

A string could either end in 0, 1, 2, or 3

h_2 :
$- - - - -2$ (good)
$- - - - 12$ (bad)
$= h(n-1) - h(n-2)$

h_0 :
$- - - - -0$ (good)
$- - - - 20$ (bad)
$= h(n-1) - h_2$
$= h(n-1) - [h(n-2) - h(n-3)]$

h_3 :
$- - - - -3$ (good)
$= h(n-1)$

h_1 :
$- - - - -1$ (good)
$= h(n-1)$

$\boxed{\text{h(n) = 4h(n-1) - 2h(n-2) + h(n-3)}}$