1. Q — Solve the advancement operator equation
$(A^2 + 3A - 10)f = 0$ if $f(0) = 2$ and $f(1) = 10$.

A — The original equation can be written as $(A+5)(A-2)f = 0$.

$(A-2)f = 0$ can also be written as $Af(n) = 2f(n)$, i.e., $f_{n+1} = 2f_n$. A function that, when advanced, gives twice the value as its predecessor is $f_1 = c_1 2^n$. When we try this solution in our original problem we have that $(A+5)(A-2)f_1(n) = (A+5)0 = 0$ hence f_1 is a solution to the original problem.

Similarly $(A+5)f = 0$ has a solution $f_2 = c_2(-5)^n$ which is also a solution to the original problem.

To see if combined, f_1 and f_2 give us all the solutions to the advancement operator equation, we substitute $f(n) = c_1 2^n + c_2(-5)^n$.

$$\begin{aligned}
(A+5)(A-2)f(n) &= (A+5)(A-2)(c_1 2^n + c_2(-5)^n) \\
&= (A+5)(c_1 2^{n+1} + c_2(-5)^{n+1} - 2(c_1 2^n + c_2(-5)^n)) \\
&= (A+5)(c_1 2^{n+1} + c_2(-5)^{n+1} - c_1 2^{n+1} - 2c_2(-5)^n) \\
&= (A+5)(-7c_2(-5)^n) \\
&= -7c_2(A+5)((-5)^n) \\
&= -7c_2(-5(-5)^n + 5(-5)^n) \\
&= 0 \\
\therefore f(n) &= c_1 2^n + c_2(-5)^n \text{ are the solutions.} \\
f(0) &= c_1 + c_2 = 2, \\
f(1) &= 2c_1 - 5c_2 = 10 \text{ (using initial conditions)} \\
c_1 &= \frac{20}{7}, c_2 = -\frac{6}{7} \\
&\boxed{f(n) = \frac{20}{7}2^n - \frac{6}{7}(-5)^n}
\end{aligned}$$