1. Q — Determine the explicit formula for t_n, the fewest moves in which you can solve the problem of **Towers of Hanoi**.

A — Let p_1, p_2, p_3 be the three pegs and t_n be the number of minimum moves required to move n disks $(d_1, d_2, \ldots, d_n)$ from peg p_1 to p_3, using p_2 as an intermediary.

To accomplish this, we first move $n-1$ disks from $p1$ to p_2, using p_3 as an intermediary (t_{n-1} moves); move the largest disk (d_n) from p_1 to p_3 (1 move); and finally move the $n-1$ disks from p_2 to p_3, using p_1 as an intermediary (t_{n-1} moves). Hence we see that $t_n = 2t_{n-1}+1, t_1 = 1$. On simplifying:

$$t_n - 2t_{n-1} = 1 \tag{1}$$

This is a non-homogeneous equation which can be written as an *advancement operator equation* $(A-2)f = 1, f(1) = 1$. For the homogenous part $(A-2)f = 0$ or $Af = 2f$, the solution would be a function whose terms double with each progression, we can say that $f_1(n) = c_1 2^n$ is a solution.

For the non-homogeneous part let c_2 be a *particular solution* to the equation. On trying it out in the equation, $c_2 - 2c_2 = 1$ we get $c_2 = -1$. Therefore the solution to the original equation is: $t(n) = c_1 2^n - 1$. However using the initial condition, $t(1) = 1$, we have $2c_1 - 1 = 1$ or $c_1 = 1$.

Hence the explicit formula that we desire is $\boxed{t(n) = 2^n - 1}$.