Proof Examples

Peter Cappello
`cappello@cs.ucsb.edu`

Introduction

My hope in producing these proofs is to enable students to learn by example what I would like to see, at least in form. If you think any proof below is incorrect, or could be simpler, clearer, or better in any other way, please email me.

I also am producing the latex used to produce these proofs as a way of showing by example that latex is a labor-saving device, even for mathematical work that is as ephemeral as homework.

Style

I am encouraging a mathematical style that I think is appropriate for a student taking a em first course where mathematical proofs are the coin of the realm: As the student matures, some elements can be abbreviated or omitted. For now, err on the side of giving an explanations where none is needed. Similarly, strive for a form that is clear to another first-year student. Indeed, asking another student to read your proof is a good source of feedback.

One source of thoughts about mathematical writing style is a document by Goss [**?**]. Another is by Gillman [**?**].

Below, I produce a few style notes, certainly not a comprehensive list. Also, some of these notes run against the grain of convention. Please use your own judgment.

1. It generally is considered bad form to mix mathematical symbols with English words (e.g., $\forall n \in N, n^2$ is greater than 0.) I disagree. Please use mathematical symbols as much as possible; This results in fewer characters, enabling the eye to see more content at once. I also disagree, and for the same reasons, with the rule that numbers less than 10 should be written out. In my opinion, this is like saying, for the number 7, "Why use a single universally understood symbol, when we can use a sequence of five letters that only English speakers understand?"

Mathematical Induction

The following proofs are of exercises in Rosen [**?**], chapter 5: Mathematical Induction.

Exercise 62

Show that n lines separate the plane into $(n^2+n+2)/2$ regions, if no 2 of these lines are parallel and no 3 pass through a common point.

Theorem. $\forall n \in \mathbf{N}$, *n lines in the plane, where no lines are parallel and no 3 lines intersect at the same point, partition the plane into $(n^2+n+2)/2$ regions.*

Proof. By induction on n, the number of lines.
Basis: $n = 0$: If the plane is partitioned by 0 lines, there is $1 = (1^2+1+2)/2$ region.
Induction hypothesis: Assume n lines in the plane, where no lines are parallel and no 3 lines intersect at the same point, partition the plane into $(n^2+n+2)/2$ regions.
Induction: Let there be n lines in the plane, where no lines are parallel and no 3 lines intersect at the same point. Without loss of generality, remove any 1 line. By the inductive hypothesis, the remaining n lines partition the plane into $(n^2+n+2)/2$ regions. When line $n+1$ is placed so that it is not parallel to any of the other n lines and it intersects each line l at a point where no other lines intersect line l, then n such intersections are formed. Order these intersection points from left to right and bottom to top among points that have the same y coordinate. The semi-infinite region to the left and below the 1^{st} intersection point, is divided into 2 regions by line $n+1$. Similarly, when this line passes into the region to the right of an intersetion point, it subdivides this regtion into 2 regions. Since there are n intersection points, this increases the number of regions by n, plus 1 more for the subdivided semi-infinite region, totalling $n+1$ additional regions:

$$r(n+1) = r(n) + n + 1 \tag{1}$$

$$= \frac{n^2+n+2}{2} + n + 1 \tag{2}$$

$$= \frac{(n+1)^2 + (n+1) + 2}{2} \tag{3}$$

Above, the inductive hypothesis is used to go from Eqn. (1) to (2).

□