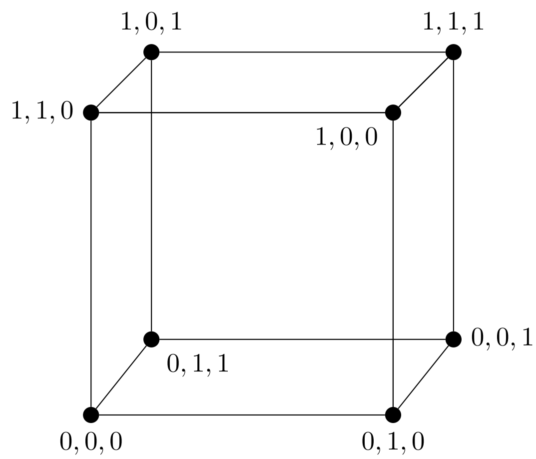
1. Q — Let V be the set of all 0–1 sequences of length d. The graph on V in which two such sequences form an edge if and only if they differ in exactly one position is called the *d-dimensional cube*. Determine the average degree, number of edges, and the length of a smallest cycle in this graph.

A — Let's visualize the graph $G = (V, E)$ for *dimension*, $d = 3$.



<u>**Features (for 3 dimensions):**</u>

Dimensions, $d = 3$

Number of vertices, $[V] = 2^d = 8$

Number of edges, $[E] = 12(= \frac{8\times 3}{2})$

Length of smallest cycle, $= 4$

Average degree, $deg_G(v) = 3$

<u>**features (For d dimensions):**</u>

Number of vertices, $[V] = 2^d$

Number of edges, $[E] = \frac{[V]\times d}{2}$ [$\because$ for a sequence of length d, there are exactly d ways to change exactly 1 bit (0 or 1) and derive d neigbors for each vertex. We divide the result by 2 because each vertex would be counted twice.]

Length of smallest cycle, $= d + 1$

Average degree, $deg_G(v) = d$, since every vertex is connected to exactly d other vertices.