1. Q — Sophie's coefficients $G(n,k)$ for $n >= 0$, $k$ arbitrary integer are defined as follows:

$$G(n,k) = \begin{cases} 1, & \text{if } n = k = 0 \\ 0, & \text{if } k < -n \text{ or } k > n \\ G(n-1,k-1) + G(n-1,k) + G(n-1,k+1), & \text{for } n > 0 \text{ and } -n <= k <= n \end{cases}$$

Compute the coefficients $G(4,0)$ and $G(4,1)$.

A — The recursive definition of $G$ is used to calculate the values as follows:

$$\boldsymbol{G(4,0)} = G(3,-1) + G(3,0) + G(3,1) = 6 + 6 + 4 = \mathbf{16}$$

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$$\begin{aligned}
G(3,-1) &= G(2,-2) + G(2,-1) + G(2,0) &&= 1 + 2 + 3 = 6 \\
G(3,0) &= G(2,-1) + G(2,0) + G(2,1) &&= 2 + 3 + 1 = 6 \\
G(3,1) &= G(2,0) + G(2,1) + G(2,2) &&= 3 + 1 + 0 = 4
\end{aligned}$$

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$$\begin{aligned}
G(2,-2) &= G(1,-3) + G(1,-2) + G(1,-1) &&= 1 \\
G(1,-3) & &&= 0 \\
G(1,-2) & &&= 0
\end{aligned}$$

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$$\begin{aligned}
G(1,-1) &= G(0,-2) + G(0,-1) + G(0,0) &&= 1 \\
G(0,-2) & &&= 0 \\
G(0,-1) & &&= 0 \\
G(0,0) & &&= 1
\end{aligned}$$

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$$\begin{aligned}
G(2,-1) &= G(1,-2) + G(1,-1) + G(1,0) &&= 0 + 1 + 1 = 2 \\
G(1,-2) & &&= 0 \\
G(1,-1) & &&= 1 \\
G(1,0) &= G(0,-1) + G(0,0) + G(0,1) &&= 0 + 1 + 0 = 1 \\
G(0,1) & &&= 0
\end{aligned}$$

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$$\begin{aligned}
G(2,0) &= G(1,-1) + G(1,0) + G(1,1) &&= 1 + 1 + 1 = 3 \\
G(1,1) &= G(0,0) + G(0,1) + G(0,2) &&= 1 + 0 + 0 = 1
\end{aligned}$$

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$$\begin{aligned}
G(2,1) &= G(1,0) + G(1,1) + G(1,2) &&= 1 + 0 + 0 = 1 \\
G(1,2) & &&= 0
\end{aligned}$$

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$$\begin{aligned}
G(2,2) &= G(1,1) + G(1,2) + G(1,3) &&= 0 + 0 + 0 = 0 \\
G(1,3) & &&= 0
\end{aligned}$$

$$\boldsymbol{G(4,1)} = G(3,0) + G(3,1) + G(3,2) \quad = 6 + 4 + 1 = \mathbf{11}$$

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$$\begin{aligned} G(3,2) &= G(2,1) + G(2,2) + G(2,3) & &= 1 + 0 + 0 = 1 \\ G(2,3) & & &= 0 \end{aligned}$$