

1. Q — Number of binary strings of length n with no isolated ones.

A — Let $S(n)$ be the desired number.

$$S(0) = 1, \{\}''$$

$$S(1) = 1, \{0\}$$

$$S(2) = 2, \{00, 11\}$$

$$S(3) = 4, \{000, 011, 110, 111\}$$

$$S(4) = 7, \{0000, 0011, 0110, 1100, 0111, 1110, 1111\}$$

$$S(5) = 12, \{00000, 00011, 00110, 01100, 11000, 00111, 01110, 11100, \mathbf{01111}, \mathbf{11110}, \mathbf{11011}, 11111\}$$

The strings in bold are courtesy of Cristian!. And we can establish that:

$$\boxed{S(n) = S(n-1) + S(n-2) + S(n-4)}$$