

1. Q — Consider a set X of 10 positive integers, none of which is greater than 100. Show that it has two distinct subsets whose elements have the same sum.

A — The number of ways to choose 10 numbers from $1, 2, 3, \dots, 100$ are $\binom{100}{10}$. Let's call these 'containers'.

From the set of 10 numbers, the number of subsets $= 2^{10} = 1024$. Let's call these 'items'.

According to the Pigeonhole Principle, there exists at least one 'container' with $\left\lceil \frac{\binom{100}{10}}{1024} \right\rceil$ number of items. This number is clearly larger than 2 so there are at least 2 distinct subsets whose elements have the same sum.