# Proof By Induction

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**Reading Exercise.** *Suppose that in a gossiping village of $n$ people, every resident knows exactly one rumor. Any resident can call another one over the phone, at which point they can share every rumor they know. Show that if $n$ is at least 4, then $2n-4$ phone calls is sufficient to achieve that every resident knows every rumor.*

*Proof.* By induction on $n$, the number of people.
**Basis:** Let $C_n$ denote the number of calls required to achieve the goal. For $n = 4$, the 4 people $(p_1, p_2, p_3, p_4)$ will need to make the following calls (in this order):
$p_{1,2}$ results in $p_1 = (1, 2)$ and $p_2 = (1, 2)$
$p_{3,4}$ results in $p_3 = (3, 4)$ and $p_4 = (3, 4)$
$p_{2,3}$ results in $p_2 = (1, 2, 3, 4)$ and $p_3 = (1, 2, 3, 4)$
$p_{1,4}$ results in $p_1 = (1, 2, 3, 4)$ and $p_4 = (1, 2, 3, 4)$
Therefore, $C_4 = 4$
**Induction hypothesis:** Assume For $n$ people, the number of calls require to achieve the goal:

$$C_n = 2n - 4 \tag{1}$$

**Induction:** For $n + 1$ people, one person, say $p_x$ calls another, say $p_y$ to tell their rumor. This is one call:

$$C_{x,y} = 1 \tag{2}$$

The remaining $n$ people (excluding $p_x$) can learn all their rumor and that of $p_x$'s in $2n - 4$ calls (induction hypothesis):

$$C_n = 2n - 4 \tag{3}$$

Now $p_x$ gets "caught up" with the everybody else by calling $p_y$ again. This will be one more call:

$$C_{x,y} = 1 \tag{4}$$

Adding the equations (2), (3) and (4), the total calls for $n + 1$ people:

$$C_{n+1} = 2n - 4 + 1 + 1 = 2n + 2 - 4 = 2(n + 1) - 4 \tag{5}$$

The truth of the proposition for a specific $n$ implies truth for $n + 1$. Hence the proposition must be true for all $n > 4$. □