1. Q — Explain why there is no generating function $\frac{1}{x}$. In general, try to figure out exactly what the rules are for dividing two generating functions.

A — Trying to understand the claim that $\frac{1}{1-x}$ is the Generating function $F(x)$ for which $(1-x)F(x) = 1$.

Let $G(x) = 1 - x = 1 + (-1)x + 0x^2 + \dots$

and $F(x) = b_0 + b_1 x + b_2 x^2 + \dots$

Then according to Proposition 8.3, $F(x)G(x)$ represents the Generating Function on a sequence whose n^{th} term is:

$c_n = (1)b_n + (-1)b_{n-1} + (0)b_{n-2} + \dots$

Therefore $c_1 = (b_1 - b_0)$, $c_2 = (b_2 - b_1)$, $c_3 = (b_3 - b_2) \dots$

The Generating Function $F(x)G(x)$ must be:

$c_0 + (b_1 - b_0)x + (b_2 - b_1)x^2 + \dots$

It's not clear to me how this = 1.

This is how I attempted answering the presented question:

Let's assume there was a generating function $F(x) = \frac{1}{x}$. By simple arithmeric, it would follow that

$xF(x) = 1$

By definition of Generating Functions,

$F(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$

Therefore by the previous equality,

$x\left[a_0 + a_1 x + a_2 x^2 + \dots\right] = 1$

The two Generating Functions in play here are:

$1 = F(x) = (1) + (0)x + (0)x^2 + (0)x^3 + \dots$

and $x = G(x) = (0) + (1)x + (0)x^2 + (0)x^3 + \ldots$

Therefore according to Proposition 8.3, the n^{th} term of the multplication, $F(x)G(x)$ is:

$a_0b_n + a_1b_{n-1} + a_2b_{n-2} + \ldots$

$= (1)b_n + (0)b_{n-1} + (0)b_{n-2} + \ldots$

And I got lost :)
