1. Q — Prove, without resorting to Kuratowski's Theorem, that the Petersen's Graph is non-planar.

A — The Petersen's Graph has the following parameters:

Number of vertices, $n = 10$

Number of edges, $m = 15$

Also, the faces that exist in the graph are, f_5, f_6, f_7, f_8, f_9, where f_x is a face formed by an x-cycle.

If we start pairing (e, F) for every edge, e that touches a face, F, the total number of such pairs,

$p = 5f_5 + 6f_6 + 7f_7 + 8f_8 + 9f_9$

According to Euler's Theorem, if this graph were to be planar, the following will hold: $n + f - m = 2$ where f is the total number of faces. Given the parameters it follows that:

$f = 7$

Also, $f = f_5 + f_6 + f_7 + f_8 + f_9$, the total number of faces.

$\therefore f_5 + f_6 + f_7 + f_8 + f_9 = 7$

By a conservative estimate, $f_5 = 3, f_6 = f_7 = f_8 = f_9 = 1$

$\therefore p \geq 45$

HELP!!