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# Math 501 Homework (limits of sequences)

**Problem 1.** Prove that

$$\lim_{n\to\infty} (\frac{\sin n}{n})^n = 0$$

**Solution.** Let $X = (x_n)$ be the sequence of all

$$x_n = (\frac{\sin n}{n})^n$$

Since $|\sin n| \leq 1$ $|\sin n|^n \leq 1$. Or

$$|(\sin n)^n| \leq 1$$

Since $n > 0$, $n^n = |n^n| > 0$. Dividing the inequality above by $|n^n|$, we get that

$$\frac{|(\sin n)^n|}{|n^n|} = \left|(\frac{\sin n}{n})^n\right| \leq \frac{1}{n^n} \leq \frac{1}{n}$$

This implies that

$$|x_n - 0| \leq \frac{1}{n}$$

Let $(a_n) = (\frac{1}{n})$. We know that $\lim(a_n) = 0$ and since for all $n$,

$$|x_n - 0| \leq a_n$$

it follows that

$$\lim(x_n) = 0$$

$\square$