

Math 501 Homework (§6.2 MVT)

Problem 1. Prove that the function that has a strictly positive derivative on an interval is strictly increasing.

Solution. Let $f : I \rightarrow \mathbb{R}$ have a derivative in I , $f'(x) > 0$. Let $x_1, x_2 \in I$ satisfy $x_1 < x_2$. Applying the Mean Value Theorem to f on $[x_1, x_2]$ we find $c \in (x_1, x_2)$ such that

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1).$$

Since $f'(c) > 0$ by hypothesis, and $x_2 - x_1 > 0$, we have that

$$f(x_2) - f(x_1) > 0.$$

In other words, $x_2 > x_1 \implies f(x_2) > f(x_1)$, or that f is strictly increasing in $[x_1, x_2]$. Since x_1, x_2 are arbitrary, f is strictly increasing in I . \square