Rohit Wason
Spring 2021

Math 501 Homework (§3.7 Comparison Tests)

Problem 1. Show that $\sum \frac{\cos n}{n^2}$ converges.

Solution. We start with separating the sequence $x_n = \frac{\cos n}{n^2}$ into x_{pos} and x_{neg}, the subsequences containing positive and negative values of x_n.
We see that $0 < x_{pos} \leq 1/n^2$ for all n and since $\sum 1/n^2$ converges so does $\sum x_{pos}$.

Similarly $0 < -x_{neg} < 1/n^2$. This implies that the summation $\sum -x_{neg}$ converges (let's say to x). Taking the negative sign out of the summation, $\sum -x_{neg}$ must converge to $-x$.

Since $\sum x_n = \sum x_{pos} + \sum x_{neg}$ it follows that $\sum x_n$ converges. □