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Math 501 Homework (§5.6 Monotone & Inverse Functions)

Problem 1. (excellent problem & diagram!)

Solution. (a) To see if $f : [0, 1] \to \mathbb{R}$ is monotone we take $x_1, x_2 \in [0, 1]$. And consider the "removed" sets that these elements may belong to. Two cases arise: If $x_1, x_2 \in R_i$. The value of f depends on how many 1's exist after taking said steps (esp. converting the 2's to 1). The existence of 2's in x will depend on if it goes on the "right" partition, while constructing the set C. I.e., larger the value of x, larger $f(x)$ gets. Hence $x_1 \geq x_2 \implies f(x_1) \geq f(x_2)$. This is a sufficient condition that f is monotone.
In the other case, when $x_1 \in R_i$ and $x_2 \in R_{i+m}$ things get interesting. x_2 being in a "later" removed set than R_i, may have more zeros than x_1 in which case, $f(x_2) < f(x_1)$, and f is not monotone.
Hence we see that f is **not monotone**. □