Rohit Wason
Spring 2021

Math 501 Homework (§6.2 MVT)

Problem 1. Prove that the sequence of Iterated Cosine is convergent.

Solution. Iterated Cosine: $a_0 = 0; a_{n+1} = \cos a_n, n \geq 1$.
In general $-1 \leq \cos x \leq 1$ but for $n > 0$, $0 < \cos x \leq 1$. Hence for all $n \geq 0$, $0 \leq a_n \leq 1$.

To show that the sequence is contractive, we should prove that there exists a $0 < C < 1$ such that

$$|\frac{a_{n+2} - a_{n+1}}{a_{n+1} - a_n}| \leq C$$

This is true for $n = 0$ since

$$|\frac{a_2 - a_1}{a_1 - a_0}|$$
$$= |\frac{\cos 1 - 1}{1 - 0}|$$
$$= |\cos 1 - 1| \leq \frac{1}{2}$$

We claim that $C = 1/2$ holds true for all a_n and prove it by Induction.

SORRY, my mind isn't open today. But I do want to work on this, and the bonus questions later. $\square$