

Math 501 Homework (§6.1 Differentials)

Problem 1. Are there functions f, g (defined on the same interval) such that neither of them is constant, and $(fg)' = f'g'$.

Solution. For the given condition to be true,

$$\begin{aligned}(fg)' &= f'g' \\ fg' + gf' &= f'g' && \text{(Product Rule)} \\ \frac{f}{f'} + \frac{g}{g'} &= 1\end{aligned}$$

Intuitively, we see that f, g could be some exponent functions such that their ratios with their respective derivatives are constants that add up to 1.

In particular, choose $f = e^{2x/3}$ and $g = e^{x/3}$. Using the Chain Rule we see that $f' = \frac{2}{3}f$ and $g' = \frac{1}{3}g$ and hence

$$\frac{f}{f'} + \frac{g}{g'} = 1$$

or the required condition is satisfied. □