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Math 501 Homework (§3.3 Monotone Convergence Theorem)

Problem 1. Examine whether (x_n) converges, where $x_1 = a > 0$ and $x_{n+1} = x_n + \frac{1}{x_n}$.

Solution. Since $x_n > 0$, $\frac{1}{x_n} > 0$. It is easy to see that

$$x_1 < x_2 < \cdots < x_n, n \in \mathbb{N}$$

Assume the sequence had a limit, x. Since (x_{n+1}) is a 1-tail of (x_n), they should both converge to the same limit. In order words

$$lim(x_{n+1}) = lim(x_n) = x$$

Since the two sequences are linearly related their limits would also follow the same relation,

$$x = x + \frac{1}{x}$$

Since this leads to an absurdity $\frac{1}{x} = 0$, our assumption that (x_n) converges must be **incorrect**. □