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Spring 2021

Math 501 Homework (§6.3 L'Hospital's Rules)

Problem 1. Find
$$\lim_{x \to \infty} (1 + \frac{a}{x})^x.$$

Solution.

$$\text{Let } f(x) = (1 + \frac{a}{x})^x$$
$$= e^{x \ln(1 + \frac{a}{x})}$$

$$\text{However, } \lim_{x \to \infty} x \ln(1 + \frac{a}{x}) = \lim_{x \to \infty} \frac{\ln(1 + a/x)}{1/x} \qquad (\frac{0}{0} \text{ form})$$

$$= \lim_{x \to \infty} \frac{(\ln(1 + a/x))'}{(1/x)'} \qquad (\text{L'Hospital Rule})$$

$$= \lim_{x \to \infty} \frac{\frac{1}{1 + a/x}(-a/x^2)}{-1/x^2} \qquad (\text{Chain Rule})$$

$$= \lim_{x \to \infty} \frac{a}{1 + a/x}$$

$$= a$$

$$\therefore \lim_{x \to \infty} f(x) = e^a \qquad (\text{since } e^x \text{ is continuous everywhere}).$$

$\square$