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Math 501 Homework (§4.3 Limits at Infinity)

Problem 1. Let $f : \mathbb{R}^+ \to \mathbb{R}$. Let $f_n = f(n)$ for $n = 1, 2, \ldots$ a sequence corresponding to the function f. Which of the following statements are true?

1. If $\lim_{x \to \infty} f(x) = L$, then $\lim(f_n) = L$.

2. If $\lim(f_n) = L$, then $\lim_{x \to \infty} f(x) = L$

Solution.   1. Let $\lim_{x \to \infty} f(x) = L$. By definition this implies that for every sequence (x_n) in a subset (a, ∞) of $\mathbb{R}^+$, such that $(x_n) \to \infty$, $(f(x_n)) \to L$. We can use this property to pick $(x_n) = \mathbb{N}$ which we know tends to ∞, hence $(f(x_n)) \to L$. **TRUE**.

2. Conversely, let $(f(x_n)) \to L$. Again by definition every convergent sequence f_n converges to the same limit as the function itself, hence $f(x)$ must converge to L. **TRUE**

$\square$