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Math 501 Homework (§5.2 Combinations of Continuous Functions)

Problem 1. Examples showing that in order for the composition $g \circ f$ to be continuous, it is necessary that both f and g are continuous on their respective domains.

Solution.

1. Let $f(x) := \text{sgn}(x), g(x) := \sin(x)$. Clearly, the range of f, $\{-1, 1\} \in \mathbb{R}$ and g is continuous in its domain. But since f is not continuous at $c = 0$ the composition, $\sin(\text{sgn}(x))$ is not continuous at $c = 0$.

2. The inverse, $f \circ g$ is also discontinuous at $c = 0$, since f is discontinuous.

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