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Math 501 Homework (§1.2)

§1.2

Problem 1. A pipe maker company produces 3ft and 5ft lengths of pipes. These pieces can be glued together to get longer pipes. Prove that any integer length of at least 8ft can be made out of these pipes.

Solution. ***To prove:*** In other words we need to prove that the equation $3a + 5b = c$ has positive solutions for all $c \geq 8$.

Basis step: Let $P(c)$ be the statement that the last statement is true for a given c. $P(8)$ holds true, since $a = 1, b = 1$ are the positive solutions.

Induction step: Let's assume $P(k)$ holds for an arbitrary $k > 8$. I.e., $\exists a, b : a \geq 0, b \geq 0 : k = 3a + 5b$

Case I Both a and b are greater than zero.

$$\begin{aligned} \therefore k + 1 &= 3a + 5b + 1 \\ &= 3a + 5(b - 1) + 6 \\ &= 3(a + 2) + 5(b - 1) \end{aligned}$$

Case II $a = 0$ and b is greater than zero.

$$\begin{aligned} \therefore k + 1 &= 5b + 1 \\ &= 5(b - 1) + 6 \\ &= 3(2) + 5(b - 1) \end{aligned}$$

Case III a is greater than zero and $b = 0$. But this is only possible if $a \geq 3$ since $k \geq 8$.

$$\begin{aligned} \therefore k + 1 &= 3a + 1 \\ &= 3(a - 3) + 1 \\ &= 3(a - 3) + 10 \\ &= 3(a - 3) + 5(2) \end{aligned}$$

All the possible casese suggest that if $P(k)$ is true, $P(k+1)$ is true $\forall k \geq 8 \in \mathbb{N}$.
I.e., any integer length ≥ 8 can be made out of these pipes. $\square$