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Math 501 Homework (§2.3)

Problem 1. Is it possible to have two sets of real numbers, A and B, such that $A \subset B$, $sup\ A = sup\ B$ and $inf\ A = inf\ B$?

Solution. We shall see how this is **not possible**. Let $u = sup\ B, w = inf\ B$. Since A is a proper subset of B, there exists at least one $b \in B$ not in A.

It is possible that $b >$ the *upper bound* of A (as we're told $sup\ A$ exists). Then by definition, **upper bound** of A $< b \leq u \implies u \neq sup\ A$.

The only other case is if $b <$ the *lower bound* of A (again since $inf\ A$ exists), $w \geq b >$ **lower bound** of $A \implies w \neq inf\ A$. □