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Math 501 Homework (§5.4 Uniform Continuity)

Problem 1. Let $g(x) = \sqrt{x}$ on $I = [0, 2]$

Solution. 1. For a given $\epsilon > 0$, we pick $\delta = \epsilon\sqrt{2}$ and $x, y \in I$ for which $|x - y| < \delta$. We now have that

$$
\begin{aligned}
|g(x) - g(y)| &= |\sqrt{x} - \sqrt{y}| \\
&= |\frac{x - y}{\sqrt{x} + \sqrt{y}}| \\
&= \frac{|x - y|}{\sqrt{x} + \sqrt{y}} \\
&= \frac{\delta}{2\sqrt{2}} = \epsilon
\end{aligned}
$$

2. For all $x \in I$ there is no number $K > 0$ such that $|g(x)| = \sqrt{x} \le K|x|$. This is because as $x \to 0$ the slope $\frac{1}{\sqrt{x}}$ gets asymptotically larger and larger and cannot be contained by any $K > 0$.

$\square$