Rohit Wason
Spring 2021

# Math 501 Homework (§1.3)

§1.3

**Problem 1.** Let $A$ be the set of positive integers. Let $A_i$ be the set of $i$-element subsets of $A$ ($i = 0, 1, ...$). Then let $B$ be the union of the sets $A_0, A_1, A_2, ...$ Which of the sets $A_0, A_1, A_2, ...$ and $B$ are countable?

**Solution.** $A_0 = \{\phi\}$, and has 1 element. It is therefore *countable*.
$A_1 = \{\{1\}, \{2\}, \ldots\}$. There seems to be a surjection from $\mathbb{N}$ onto $A_1$. Since $\mathbb{N}$ is *countable*, so is $A_1$.
$A_2$ is nothing but $\mathbb{N} \times \mathbb{N}$ and can be shown as *countable* with the "diagonal procedure".
The higher sets $A_3, A_4, \ldots$ can be shown as cross products of the previous set with $A_1$ e.g., $A_3 = A_2 \times A_1$, $A_4 = A_3 \times A1$ and so on. Since the cross product of 2 countable sets is also countable, all higher $A_i$ are *countable*.

In the end, $B = \bigcup_{i=0}^{\infty} A_i$ being a union of all countable sets, is also *countable* because intuitively it is just an additive process of counting. $\square$