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Math 501 Homework (§6.1 Differentials)

Problem 1. Are there functions f, g (defined on the same interval) such that neither of them is constant, and $(fg)' = f'g'$.

Solution. For the given condition to be true,

$$(fg)' = f'g'$$
$$fg' + gf' = f'g' \qquad \text{(Product Rule)}$$
$$\frac{f}{f'} + \frac{g}{g'} = 1$$

Intuitively, we see that f, g could be some exponent functions such that their ratios with their respective derivatives are constants that add up to 1.
In particular, choose $f = e^{3x/2}$ and $g = e^{3x}$. Using the Chain Rule we see that $f' = \frac{3}{2}f$ and $g' = 3g$ and hence

$$\frac{f}{f'} + \frac{g}{g'} = 1$$

or the required condition is satisfied. $\square$