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Math 501 Homework (§6.4 Taylor's Theorem)

Problem 1. Show where Newton's method of finding roots fails.

Solution. Let $f(x) := \sin x$. If we pick $I := [\pi/2, 3\pi/2]$, since $f'(x) = \cos x$, it is possible to pick a subinterval $I^* \subset I$ such that $|f'(x)| > m = 1/2$.
Similarly since $f''(x) = -\sin x$ and $|f''(x)| \leq M = 1$, we can set $K = \frac{M}{2m} = 1$.
If we define a sequence x_n such that:

$$x_{n+1} := x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \tan x_n,$$

we see that since the value of $\tan x$ goes to ∞ in I x_n does not converge. $\square$