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Math 501 Homework (§2.2)

Problem 1. Let A be ϵ-neighborhood of a, and let B be the ϵ-neighborhood of b. Show that if A and B are not disjoint, then their intersection is the ϵ-neighborhood of a real number.

Solution. From definition, $A = \{x \in \mathbb{R} : |x - a| < \epsilon\}$ and $B = \{x \in \mathbb{R} : |x - b| < \epsilon\}, \epsilon > 0$. Their intersection, $C = A \cap B$ is the set of $x \in \mathbb{R}$ such that:

$$|x - a| < \epsilon, |x - b| < \epsilon \tag{1.1}$$

Now take $|(x + x) - (a + b)| = |(x - a) + (x - b)|$ which according to the Triangular Inequality

$$\begin{aligned} &\leq |x - a| + |x - b| \\ &< 2\epsilon && \text{(using 1.1)} \\ \therefore |2x - (a + b)| &< 2\epsilon \\ |x - \frac{a + b}{2}| &< \epsilon && \text{(dividing both sides by 2)} \end{aligned}$$

$C = \{x \in \mathbb{R} : |x - \frac{a+b}{2}| < \epsilon\}$,

This is the ϵ-neighborhood of $\frac{a+b}{2}$, a real number. □