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Spring 2021

Math 501 Homework (§6.4 Taylor's Theorem)

Problem 1. Show that $e^x > 1 + x$ for $x > 0$.

Solution. Let $I = (0, \infty]$ and $f(x) = e^x$ defined on $I \to \mathbb{R}$. Since $f^{(k)}(x) = e^x$ are all continuous and diffable on I, Taylor's Theorem states that $f(x) = P_1 x + R_1 x$ (for $n = 1$).

Choose $x_0 = 0$, yielding

$$P_1(x) = e^0 + e^0 x = 1 + x$$
$$R_1(x) = \frac{e^c}{2!} x^2.$$

Since, $x > 0$ by definition, $0 = x_0 < c < x$. Or $e^c > 0$ and $R_1(x) > 0$ impling that $f(x) > 1 + x$. □