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Math 501 Homework (§2.4)

Problem 1. M-function: The domain D is the set of positive reals. It is defined as follows: if x is irrational, then $M(x) = 0$. If x is rational, write $x = p/q$, where p, q are relatively prime integers. Then $M(x) = 1/q$. The questions is is M bounded? What is $sup\ M(D)$, what is $inf\ M(D)$?

Solution. Every element in M is either 0 or $1/q$. Since $0 < 1/q \leq 1$ for all $q \in \mathbb{Z}^+$, every element $m \in M(D)$ is bound by 0 on the left and 1 to the right with both $0, 1 \in M(D)$. So

$$inf\ M(D) = 0$$

$$sup\ M(D) = 1$$

$\square$