

Math 501 Homework (sets)

Problem 1. Consider an infinite sequence of sets A_1, A_2, \dots such that for all $i \geq 1$, we have that $A_{i+1} \subseteq A_i$, and $A_i \neq \phi$. Is it possible that the intersection is empty?

Solution. Since $A_2 \subseteq A_1$, $A_2 \cap A_1 = A_2$.
It's easy to see that in general $A_{i+1} \cap A_i = A_{i+1}$. Hence all these sets intersected in succession will be $I = A_i$ as $i \rightarrow \infty$. Since $A_i \neq \phi$, I is not empty. \square