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Math 501 Homework (§3.3 Monotone Convergence Theorem)

Problem 1. If (b_n) is a bounded sequence, and $lim(a_n) = 0$, then $lim(a_n b_n) = 0$.

Solution. The argument $|m||a_n| = |ma_n| \leq |a_n b_n| \leq |Ma_n| = |M||a_n|$ is valid and using **Squeeze Theorem** leads to the conclusion $lim|a_n b_n| = 0$.

However this does not imply that $lim(a_n b_n) = 0$ (only the converse is known to be a theorem).

By definition for every $\epsilon > 0$ there exists $K \in \mathbb{N}$ such that when $n \geq K$, $||a_n b_n| - 0| < \epsilon$. Considering both cases:

$$\begin{aligned}
\text{Case I} &: a_n b_n \geq 0 \implies |a_n b_n| = a_n b_n, \text{ and} \\
&\quad lim(a_n b_n) = 0 \text{ follows from above.} \\
\text{Case II} &: a_n b_n < 0 \implies |a_n b_n| = -a_n b_n, \text{ therefore} \\
&\quad ||a_n b_n| - 0| = |-a_n b_n - 0| < \epsilon \\
&\quad ???
\end{aligned}$$

$\square$