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Math 501 Homework (§3.4 Subsequences)

> **Problem 1.** Prove if $\lim a_n = L_1 \neq L_2 = \lim b_n$, then their 'merge', (c_n) diverges and converges if $L_1 = L_2$.

Solution. Let's assume c_n converges with $L_1 \neq L_2$. W.L.O.G., let $L_2 > L_1$. By definition of limits, $a_n \nrightarrow L_2$. Since a_n is a subsequence of c_n, by theorem 3.4.2, $c_n \nrightarrow L_2$. Also by the same theorem since $b_n \to L_2$ it cannot be a subsequence of c_n which is against the premise. Hence we must conclude that c_n diverges.

If, on the contrary, $L_1 = L_2$, by definition of the 'merge', c_n is bounded by L_1, and every subsequence of c_n converges to L_1. Hence by theorem 3.4.9, $c_n \to L_1$. □