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Math 501 Homework (§5.1 Continuous Functions)

Problem 1. Hypothesis: Let $f : \mathbb{R}^+ \to \mathbb{R}$. For every $\epsilon > 0$ there exists $\delta > 0$ such that $|x - y| < \delta$ and $x, y > 0$ implies $|f(x) - f(y)| < \epsilon$.

Solution. Since $x, y \in \mathbb{R}^+$ are arbitrary, the given hypothesis is true for any point x in, and every corresponding point y in $\mathbb{R}^+$. Hence, by definition, f **is continuous** on the set $\mathbb{R}^+$.

Conversely, let f be continuous on every point of $\mathbb{R}^+$. Let $y \in \mathbb{R}^+$ be one such point. By definition, for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|x - y| < \delta$ implies $|f(x) - f(y)| < \epsilon$. Since $x, y, > 0$ this implication **is** the hypothesis. $\square$