

Math 501 Homework (§3.4 Limit superior & inferior)

Problem 1. Let (a_n) and (b_n) be two sequences. Show that $\liminf(a_n + b_n) \leq \liminf a_n + \limsup b_n$ (assuming all these limits exist). Also show an example in which case equality happens, and one, in which case strict inequality happens.

Solution. Since all 3 limits exist, $(a_n), (b_n), (a_n + b_n)$ are all bounded. Hence \limsup and \liminf are defined for all.

(strict equality) If $\limsup(a_n) = \liminf(a_n)$, according to theorem 3.4.12, (a_n) converges. Let $(a_n) \rightarrow a$. Similarly let $(b_n) \rightarrow b$. According to the rules of limits, $(a_n + b_n) \rightarrow a + b$. If we can show that in this case $\limsup(a_n) = \liminf(a_n) = a$, and similarly for b_n , the equality is proved.

□