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Math 501 Homework (§3.2)

Problem 15. To prove: If $0 < a < b, z_n := (a^n + b^n)^{1/n} \to b$

Solution. We first claim that $(2^{1/n})$ converges to 1. We can prove this by observing that

$$\log_2(2^{1/n}) = \frac{1}{n}\log_2(2) = \frac{1}{n}$$
$$\therefore 2^{1/n} \to 2^0 = 1$$

since $1/n$ converges to 0. Now since

$$\begin{aligned} 0 < a < b \\ 0 < a^n < b^n \\ b^n < a^n + b^n < 2b^n \\ (b^n)^{1/n} < (a^n + b^n)^{1/n} < (2b^n)^{1/n} \\ b < z_n < 2^{1/n}b \\ b \leq z_n \leq 2^{1/n}b \\ \text{(by laxing the inequality)} \end{aligned}$$

Since we know that $(2^{1/n}) \to 1$ and $(b) \to b$, using Squeeze Theorem, we have that $z_n \to b$. $\square$