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Math 501 Homework (§5.5 Gauges)

Problem 1. Let $\delta(x) = e^x$ be a gauge on $I = [-2, -1]$. Find a δ-fine partition on I, or show it doesn't exist.

Solution. Such a partition definetely exists, according to **Theorem 5.5.5**, so let's find one!

We define 10 non-overlapping subintervals of I as

$$\begin{aligned} I_1 &:= [-2, -1.8], \\ I_2 &:= [-1.8, -1.7], \dots, \\ I_{10} &:= [-1.1, -1] \end{aligned}$$

and a tagged partition $\dot{P} := \{(I_1, -2), (I_2, -1.8), \dots\}$.

We also observe that

$$\begin{aligned} -2 \in I_1 &\subseteq [-2 - e^{-2}, -2 + e^{-2}] \\ -1.8 \in I_2 &\subseteq [-1.8 - e^{-1.8}, -1.8 + e^{-1.8}], \dots, \\ -1.1 \in I_{10} &\subseteq [-1.1 - e^{-1.1}, -1.1 + e^{-1.1}] \end{aligned}$$

Hence $\dot{P}$ is one such partition on I. □