

## Math 501 Homework (§4.2 Limit Theorems)

**Problem 1.** Show that  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1$ .

**Solution.** We start with observing that  $\sin x < x < \tan x$  for  $0 < x < \pi/2$ .  
Dividing all parts by  $\sin x$ ,

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$
$$\cos x < \frac{\sin x}{x} < 1$$

Since  $\lim_{x \rightarrow 0} (1) = \lim_{x \rightarrow 0} (\cos x) = 1$ , by **Squeeze theorem** we have that

$$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1$$

□