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Math 501 Homework (§2.1)

Problem 1. Page 27, in the proof of Theorem 2.1.8: "then $a^2 = (-a)(-a)$". So why is that?

Solution. We can take the R.H.S.

$$\begin{aligned}
&(-a)(-a) \\
&\qquad = ((-1)\cdot a)\cdot((-1)\cdot a) && \text{(by definition of negative numbers)} \\
&\qquad = (-1\cdot -1)\cdot(a\cdot a) && \text{(by associative property of multiplication)} \\
&\qquad =???
\end{aligned}$$

□

Problem 2. Page 28, in the proof of Theorem 2.1.10: "If $a > 0$, then $1/a > 0$." Why is this true?

Solution. Assuming the contrary, there are 2 cases (Trichotomy Property). In the first case, $1/a = 0$. Hence $a \cdot 1/a = 0$, by *existence of the zero element*. But $a \cdot 1/a = 1$ by *definition of the reciprical*.

In the remaining case, $-(1/a) > 0 \implies -1 \cdot 1/a > 0$. Also by hypothesis $a > 0$. According to the multiplicative closure of $\mathbb{P}$, we have $a \cdot -1 \cdot 1/a > 0$ or $(a \cdot 1/a) \cdot -1 > 0$ (*cummutative prop. of multiplication*). I.e., $1 \cdot -1 = -1 > 0$ which is not true by the definition of -1 rendering our assumption incorrect.

□