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Math 501 Homework (limits of sequences)

Problem 1. Let a_0 be a positive real number. Then define the sequence a_i recursively: $a_{i+1} = \sqrt{a_i}, i = 0, 1, 2, \ldots$ This sequence depends on a_0, the number you start with. E.g. if $a_0 = 1$, then every element of the sequence is 1, so it clearly converges. Does it converge for every starting number?

Solution. *claim:* All such sequences converge to 1.
Clearly, such a sequence converges for $a_0 = 1$.
If $a_0 > 1$, $a_{i+1} < a_i$ or for all n, $a_n < a_{n-1} < a_{n-2} \cdots < a_0$. Therefore $|a_n - 1| = a_n - 1 < a_0 - 1$.
For $a_0 < 1$, $a_{i+1} > a_i$ or $\frac{a_{i+1}}{a_i} > 1$ Therefore $\frac{1}{\sqrt{a_i}} > 1$.

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