Rohit Wason
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Math 501 Homework (§3.2 Limit Theorems)

Problem 1. Assume $X = (x_n) \to x$ and $Z = (z_n) \to z, z \neq 0$.

Solution. Two cases arise:

Case I: There are finitely many $z_i = 0$. This implies there is a $k \in \mathbb{N}$ after which $z_i \neq 0$. In other words, $Z' = (z_n), n > k$ is a tail of Z that does converge to z. Hence the quotient sequence X/Z converges. to x/z.

Case II: There are infinetely many zero- and nonzero-elements in Z. This means no matter how large a $k \in \mathbb{N}$, we can always find a $z_{i>k} = 0$. I.e., z_n cannot converge. In this case the quotient sequence X/Z does not exist.

□