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Math 501 Homework (ternary representation)

Problem 1. Define $[0,1] \supseteq C := \{x : x = (.c_1c_2 \dots c_n \dots)_3, c_i \in \{0,2\}, \forall i \in \mathbb{N}\}$.

Solution. a) To see if C is countable let's look at another set, $C' := \{x : x = (.c_1c_2 \dots c_n \dots)_3, c_i = 1, \forall i \in \mathbb{N}\}$. All the digits in x are 1's. Since there's only one such number $\frac{1}{2} = (.1111111 \dots)_3$, C' is *countable*. Since $C \cup C' = [0,1]$ is uncountable, C is **uncountable**?

b) Assuming there is an interval (a,b) with length $l > 0$ such that $(a,b) \subseteq C$. By definition, the subinterval $(a + \frac{l}{3}, a + \frac{2l}{3}) \not\subset C$. Therefore such (a,b) cannot exist. $\square$