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Spring 2021

Math 501 Homework (§5.2 Continuous Functions)

Problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that is continuous on $\mathbb{R}$. Suppose that $c \in \mathbb{R}$ such that $f(c) > 0$. Show that there exists a $\delta > 0$ such that the function f is positive on the δ-neighborhood of c.
What if we only require continuity in c as opposed to $\mathbb{R}$?

Solution. Since f is continuous there exists an $\epsilon > 0$ for every $\delta > 0$ such that $f(x)$ is in the ϵ-neighborhood of $f(c)$. Since $f(c) > 0$, there are infinitely many +ve numbers in every neighborhood of it. And since f **is continuous in all neighborhoods of** c, at least one such neighborhoods contains $f > 0$.

A continuity on *just* c does not guarantee that there exists a neighborhood of $f(c)$ where $f > 0$. □