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Math 560 Homework (#4)

Problem 1. The distribution of moth counts is discrete and skewed with the mean number of moths trapped $\mu = 0.5$ and the standard deviation of the number of moths trapped $\sigma = 0.8$. A random sample of $n = 100$ traps will be selected. Use the Central Limit Theorem to approximate the probability that the average number of moths in the 100 traps will be greater than 0.6.

Solution. Since the distribution is Normal, the mean of the average number of moths, $\mu_{\bar{x}} = \mu = 0.5$. Similarly the standard deviation of the average number of moths, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.8}{10} = 0.08$

Using the Central Limit Theorem, $\bar{x}$ follows the Normal distribution, $N(0.5, 0.08)$.

Since the required probability, $P(\bar{x} > 0.6)$ is the compliment of the probability $P(\bar{x} \leq 0.6)$, "standardizing" both sized we have that

$$P(\bar{x} \leq 0.6) = P(\frac{\bar{x} - 0.5}{0.08} \leq \frac{0.6 - 0.5}{0.08}) = P(z \leq 1.25)$$

For $z = \frac{\bar{x}-0.5}{0.08}$. Using the table *Standard Normal Cumulative Probabilities* we have $P(z \leq 1.25) = 0.8944$

Hence the required probability, $P(\bar{x} > 0.6) = 1 - 0.8944$

$$= \boxed{0.1056}$$

□

Problem 2. Consider a population which has a standard deviation $\sigma = 10$. A random sample with replacement of size n will be selected and the mean of the sample will be computed. What is the smallest sample size n such that the standard deviation of the sampling distribution for the sample mean will be less than 1?

Solution. According to CLT the standard deviation of the sample mean, $\sigma_{\bar{x}}$ is given by $\frac{\sigma}{\sqrt{n}}$.

For $\frac{\sigma}{\sqrt{n}} < 1$ we obtain $\sigma^2 < n$. Or $n > 10^2 = 100$ (since $\sigma = 10$).

Hence we see that the smallest sample size for the given condition to hold is

$$\boxed{n = 101}$$

□

> **Problem 3.** Suppose that a person's glucose level one hour after ingesting a sugary drink varies according to a Normal distribution with mean $\mu = 130$ mg/dl and standard deviation $\sigma = 10$ mg/dl. One hour after injesting a sugary drink, four readings ($n = 4$) are taken of the person's glucose level and the sample mean of these four readings is computed. (It can be assumed that the readings are independent observations from a $N(\mu, \sigma)$ distribution.) Find the value L such that the probability that the sample mean based on the person's four readings falls above L is 0.05.

Solution. The mean of the sample mean, $\mu_{\bar{x}} = \mu = 130$. The standard deviation of the sample mean, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{2} = 5$

The probability that the sample mean, $\bar{x}$ falls above L can be given by $P(\bar{x} > L)$. On "standardizing" the distribution:

$$P(\frac{\bar{x} - 130}{5} > \frac{L - 130}{5})$$

Let's call it $P(z > \frac{L-130}{5})$ which is given to equal 0.05. Using the *Standard Normal Cumulative Probabilities* table we can look for z such that $P(z \leq \frac{L-130}{5}) = 1 - 0.05 = 0.95$. This gets us $z \approx \frac{1.64+1.65}{2} = 1.645$.

Therefore we see that $1.645 = \frac{L-130}{5}$. Or

$$L = 5(1.645) + 130 = \boxed{138.225}$$

□

Problem 4. The probability distribution of a random variable X is given, and X_1 and X_2 are random samples of size 2. Assuming X_1 and X_2 are independent:

Solution. a) Since the variables are independent, the 9 possible pairs (X_1, X_2) and their probabilities are:

1. $P(0,0) = P(X_1 = 0 \wedge X_2 = 0) = P(X_1 = 0)P(X_2 = 0) = 0.6^2 = 0.36$
2. Similarly $P(0,1) = 0.6 * 0.3 = 0.18$
3. $P(0,2) = 0.6 * 0.1 = 0.06$
4. $P(1,0) = P(0,1) = 0.18$
5. $P(1,1) = 0.3^2 = 0.09$
6. $P(1,2) = 0.3 * 0.1 = 0.03$
7. $P(2,0) = P(0,2) = 0.06$
8. $P(2,1) = P(1,2) = 0.03$, and
9. $P(2,2) = 0.1^2 = 0.01$

b) As for the random variable $Y = \frac{X_1+X_2}{2}$ the different possible values are:

Y	0	1	2
0	0	0.5	1
1	0.5	1	1.5
2	1	1.5	2

Therefore the distribution for Y is:

Y	0	0.5	1	1.5	2
$P(Y)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

$\square$