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Math 560 Homework (#9, Inference of Two-Way Tables)

Problem 1. A table of two-variables is given:

Coffee	Male	Female	Total
Always	18	15	33
Sometimes	36	36	72
Never	36	9	45
Total	90	60	150

Solution. The expected cell counts are as follows:

Coffee	Male	Female
Always	$\frac{33\times 90}{150} = 19.8$	$\frac{33\times 60}{150} = 13.2$
Sometimes	$\frac{72\times 90}{150} = 43.2$	$\frac{72\times 60}{150} = 28.8$
Never	$\frac{45\times 90}{150} = 27$	$\frac{45\times 60}{150} = 18$

Hypothesis: H_0: There is no association between the row & the column variables vs. H_a: There is an association between them.

The test statistic, χ^2

$$= \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}}$$
$$= \frac{(18-19.8)^2}{19.8} + \frac{(15-13.2)^2}{13.2} + \frac{(36-43.2)^2}{43.2} + \frac{(36-28.8)^2}{28.8} + \frac{(36-27)^2}{27} + \frac{(9-18)^2}{18}$$
$$= 0.1636364 + 0.2454545 + 1.2 + 1.8 + 3 + 4.5$$
$$= 10.90909$$

Looking up the χ^2 *distribution critical values* table, we see that the **Critical value** at $\alpha = 0.01$ and with **Degrees of freedom**, $df = (r-1)(c-1) = 2$ comes out to be, $\chi^{2*} = 4.605$.

Conclusion: Since $\chi^2 > \chi^{2*}$ we **reject** the hypothesis H_0, that there is no association between the coffee consumption & gender of the students. □

Problem 2. Given

Outcome	1	2	3	4	5	6
# of occurances	153	184	160	175	162	166

Solution. We are to perform a Goodness of Fit test at $\alpha = 0.05$ that the die is fair.

Hypothesis: H_0: The outcome from this die follows a fair distribution (where each outcome is $\frac{1}{6}$ likely) vs. H_a: The die is not fair.

The **expected counts** for a fair die, when rolled 1000 times would be ≈ 166.67

The test statistic, χ^2

$$
\begin{aligned}
&= \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \\
&= \frac{1}{166.67}[(153 - 166.67)^2 + (184 - 166.67)^2 + (160 - 166.67)^2 \\
&+ (175 - 166.67)^2 + (162 - 166.67)^2 + (166 - 166.67)^2] \\
&\approx 3.739926
\end{aligned}
$$

Looking up the χ^2 *distribution critical values* table, we see that the **Critical value** at $\alpha = 0.05$ and with **Degrees of freedom**, $df = (k - 1) = 5$ comes out to be, $\chi^{2*} = 11.070$.

Conclusion: Since $\chi^2 < \chi^{2*}$ we **fail to reject** the hypothesis H_0, that this is a fair die. □