MATH 560 - Exam 1

Rohit Wason

3/3/2021

Problem 1.

Solution. *(a) Since the given values of E, are exhaustive, the probability that a randomly selected elite typist makes exactly one error in a written report*

$$\begin{aligned}
&= P(E = 1) = 1 - (P(E = 0) + P(E = 2)) \\
&= 1 - (0.63 + 0.12) \\
&= \boxed{0.25}
\end{aligned}$$

(b) The mean of the distribution of E is given by

$$\begin{aligned}
\mu_E &= \sum e_i p_i \\
&= 0(0.63) + 1(0.25) + 2(0.12) \\
&= \boxed{0.49}
\end{aligned}$$

(c) For the standard deviation

$$\begin{aligned}
\sigma_E^2 &= \sum e_i^2 p_i - \mu_E^2 \\
&= 0^2(0.63) + 1^2(0.25) + 2^2(0.12) - 0.49^2 \\
&= \boxed{0.4899}
\end{aligned}$$

Therefore $\sigma_E \approx 0.6999$

(d) To get the probability that at least one of three S.R.S has two errors we observe that the probability that a randomly selected person does not have 2 errors is $P(2') = P(0) + P(1) = 0.88$.

Since the events of selecting the 3 persons are independent of each other (selecting one person doesn't affect which subsequent person gets picked), the required probability is $P(2')^3 \approx \boxed{0.6814}$

Problem 2.

Solution. *It is given that*
$\mu_G = 134$
$\sigma_G = 78$
$\mu_E = 188$
$\sigma_E = 46$
$\rho_{GE} = -0.5$

(a) *The monthly mean for the total utility bill,*

$$\mu_{G+E} = \mu_G + \mu_E = 134 + 188 = 322$$

(b) *For the monthly standard deviation for the total utility bill,*

$$\sigma_{G+E}^2 = \sigma_G^2 + \sigma_E^2 + 2\rho\sigma_G\sigma_E = 78^2 + 46^2 + 2(-0.5)(78 \times 46) = 4612$$

Therefore $\sigma_{G+E} \approx 67.91$

Problem 3.

Solution. *A skewed continuous distribution is given with*
$\mu = 40$
$\sigma = 40$
$n = 64$

(a) *Since the distribution is Normal, the mean of the sample mean is same as population mean. Therefore*

$$\mu_{\bar{x}} = \mu = 40$$

(b) *Similarly, the standard deviation of the sample mean is given by*

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 40/\sqrt{64} = 5$$

(c) *The Central Limit Theorem states that the* ***mean time between texts****,* $\bar{x}$ *follows the Normal distribution,* $N(\mu_{\bar{x}}, \sigma_{\bar{x}})$ *and is given by*

$$N(40, 5)$$

and the probability that the mean time for 64 independent pairs of consecutive text messages is more than 42 minutes is

$$\begin{aligned}
&P(\bar{x} > 42) \\
&= P(z > \frac{42 - 40}{40}) \\
&= P(z > 0.05) = 1 - P(z \leq 0.05) \\
&= 1 - pnorm(0.05) \approx \boxed{0.48}
\end{aligned}$$

Problem 4.

Solution. *The following are given:*
$n = 25$
$\sigma = 3$
$\mu_{\bar{x}} = 5.43$
$\sigma_{\bar{x}} = 2.98$

(a) *To compute a* 90% *two-sided confidence interval, we first note that since the Normal Table evaluates probability* ***up to*** *a certain statistic, let's call it* z^**, we need to "add the tail" to the probability before we look up* z^**. In other words,*

$$z^* = qnorm(.90 + \frac{1-.90}{2}) = qnorm(.95) \approx 1.6449 \tag{1}$$

The required ***margin of error*** *is given by*

$$m = z^*\sigma/\sqrt{n} \approx 0.9869 \tag{2}$$

The 90% *two-sided confidence interval, therefore*

$$\begin{aligned} CI &= (\mu_{\bar{X}} - 0.9869, \mu_{\bar{X}} + 0.9869) \\ &= (5.43 - 0.9869, 5.43 + 0.9869) \\ &= \boxed{(4.4431, 6.4169)} \end{aligned}$$

(b) *To find the smallest sample size needed so that the margin of error,* m*, is no more than* 0.5 *we use (2) above,*

$$m = z^*\sigma/\sqrt{n} = 1.6449(3)/\sqrt{n}$$

For the desired inequality $m \leq 0.5$*, we can say*

$$\begin{aligned} \frac{1.6449(3)}{\sqrt{n}} &\leq 0.5 \\ \sqrt{n} &\geq 9.87 \\ n &\geq 97.4 \end{aligned}$$

We conclude that the sample size must be ***at least 98****.*

Problem 5.

Solution. *The following are given:*
$\mu_0 = 8$
$n = 6$
$\bar{x} = mean(c(7.7, 7.4, 7.5, 7.9, 6.8, 7.9)) \approx 7.53$
$\sigma = 0.5$
$\alpha = 0.05$

*The **null-hypothesis** is stated as* $H_0 : \mu = 8$ *and the **alternate hypothesis** is* $H_a : \mu \neq 8$.

*The **test statistic** we want to calculate is:*

$$\begin{aligned} z &= \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \\ &= \frac{7.53 - 8}{0.5/\sqrt{6}} \\ &\approx -2.3025 \end{aligned}$$

Looking up -2.3025 *in the Normal table, we get* $P(Z < z) = 0.01065292$ *however since this is a **two-sided** alternate (less than or greater than* μ_0*), we shall use* $2 \times P(Z < z) = 0.02130584$.

This value is significantly smaller than significance-level, $\alpha = .05$. *Hence* H_0 *is to be **rejected**.*