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Math 560 Homework (#10, Regression)

Problem 1. Given:
$n = 500$
Mean height of the fathers, $\bar{x} = 67.9$ (explanatory vaiable)
Standard deviation of the height of fathers, $s_x = 2.75$
Mean height of the sons, $\bar{y} = 68.7$ (response vaiable)
Standard deviation of the height of sons, $s_y = 2.83$
The correlation between the heights of the sons and their fathers, $r = 0.5$.

Solution. (a) Give a 99% prediction interval for the height of a son whose father is 70 inches tall.
Using $b_1 = r(s_y/s_x)$ and $b_0 = \bar{y} - b_1\bar{x}$ we have that

$$b_1 = 0.5(2.83/2.75) \approx 0.5145$$

$$b_0 = 68.7 - 0.5145(67.9) \approx 33.7654$$

□

Problem 2.

Year(x)	Kilobits(y)
1971	1
1980	62.5
1987	1000
1993	16000
1998	125000
2000	250000
2002	500000
2004	976562.5

Solution. (a) We build the model,

```
lmKilo=lm(Kilobits~Year, data = dram)
lmKilo$coefficients
> (Intercept)        Year
> -40886934.88       20644.12
```

to receive the coefficients $b_0 = -40886934.88, b_1 = 20644.12$. Therefore the equation of the least-square regression line is

$$\hat{y} = -40886934.88 + 20644.12(x)$$

(b)

1. Test $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 > 0$
2. Level $\alpha = 0.02$
3. Test statistic $t = \frac{b_1}{SE_{b_1}}$

```
SSE=sum(lmKilo$residuals^2)
[1] 438637466614
```

$$\begin{aligned}
\text{SSE} &= \sum (y_i - \hat{y}_i)^2 = 438637466614 \\
s &= \sqrt{\frac{\text{SSE}}{n-2}} = 270400 \\
s_x &= 11.6795 \implies SE_x = 11.6795/\sqrt{8} = 4.129327 \\
s_y &= 347560.1 \implies SE_y = 347560.1/\sqrt{8} = 122881 \\
SE_{b_1} &= \frac{s}{s_x\sqrt{n-1}} = \frac{270400}{4.129327(7)} \approx 9355
\end{aligned}$$

Therefore the test statistic,

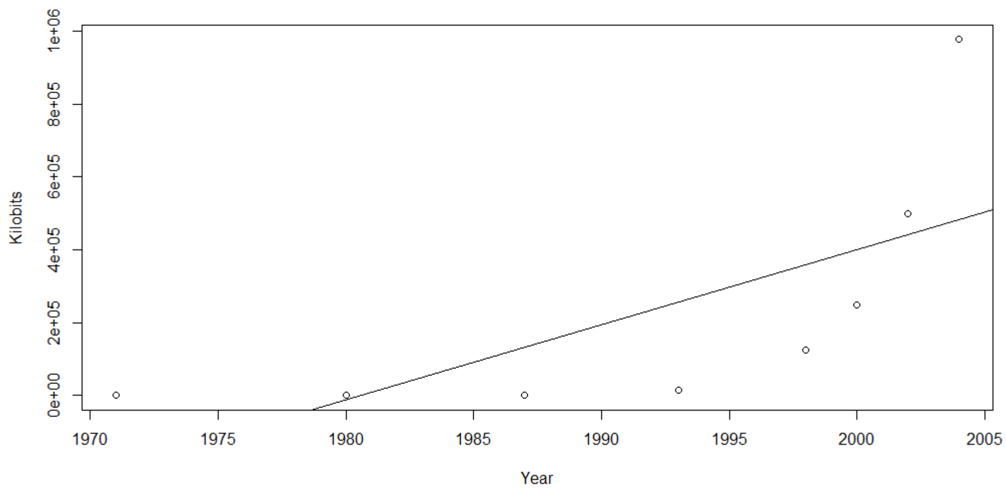
$$t = \frac{20644.12}{9355} \approx \boxed{2.2067}$$

The critical value at $\alpha = 0.02$, $\text{df} = n-p-1 = 6$ such that $P(T > t^*) = \alpha$.

$$\implies \boxed{t^* = 2.612}.$$

Since $t > t^*$, we **fail to reject** H_0.

(c) Scatterplot of Year-Kilobits, with R-line:

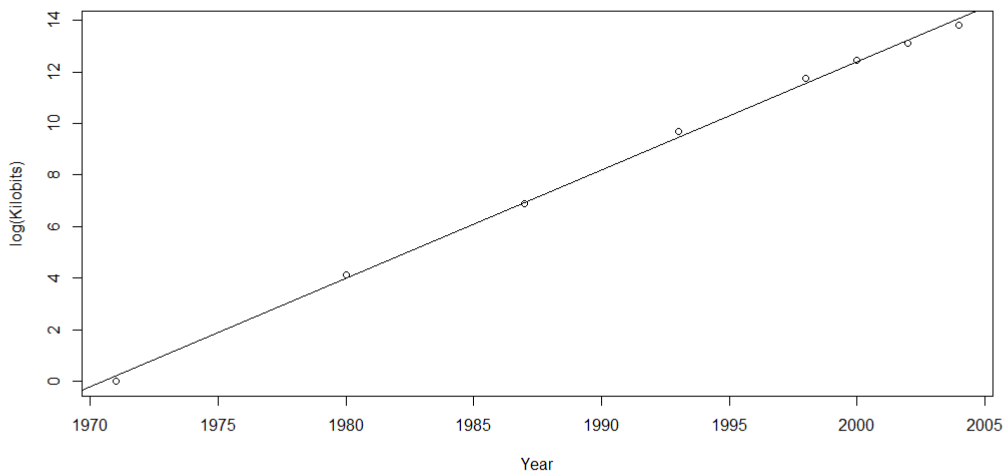


(d) To find the equation of the least-squares regression line for modeling $\ln(y)$ as a linear function of x we build a different model,

```
lmLnKilo=lm(log(Kilobits)~Year, data = dram)
lmLnKilo$coefficients
> (Intercept)        Year
> -827.6595415       0.4200238
```

to receive the coefficients $b_0 = -827.6595415, b_1 = 0.4200238$. Therefore the equation of the least-square regression line is

$$\ln(\hat{y}) = -827.6595415 + 0.4200238(x)$$



(e) For $\ln(y) = \beta_0 + \beta_1 x + \epsilon$

1. Test $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 > 0$

2. Level $\alpha = 0.02$

3. Test statistic $t = \frac{b_1}{SE_{b_1}}$

```
SSE=sum(lmLnKilo$residuals^2)
[1] 0.2438324
```

$$SSE = \sum(\ln(y_i) - \ln(\hat{y}_i))^2 = 0.2438324$$

$$s = \sqrt{\frac{SSE}{n-2}} = 0.2015905$$

$$s_x = 11.6795 \implies SE_x = 11.6795/\sqrt{8} = 4.129327$$

$$s_{\ln(y)} = 4.909217 \implies SE_{\ln(y)} = 4.909217/\sqrt{8} = 1.73567$$

$$SE_{b_1} = \frac{s}{s_x\sqrt{n-1}} = \frac{0.2015905}{4.129327(7)} \approx 0.006974173$$

Therefore the test statistic,

$$t = \frac{0.4200238}{0.006974173} \approx \boxed{60}$$

The critical value at $\alpha = 0.02, \mathrm{df} = n-p-1 = 6$ such that $P(T > t^*) = \alpha$.

$$\implies \boxed{t^* = 2.612}.$$

Since $t > t^*$, we **fail to reject** H_0.

□

Problem 3. cheese.txt

Solution. (a)

```
> lmTaste=lm(taste~acetic+h2s+lactic, data=cheese)
> lmTaste$coefficients
 (Intercept)      acetic        h2s      lactic
   -9.343375   -2.914695   3.571019   20.139395
```

The equation of the least-square regression line based on *Acetic*(x_1), *H2S*(x_2) and *Lactic*(x_3) is

$$\hat{y} = -9.343375 - 2.914695(x_1) + 3.571019(x_2) + 20.139395(x_3)$$

(b)

```
SSE=sum(lmTaste$residuals^2)
 [1] 3736.907
```

Variable	Estimate	SE
Intercept(b_0)	-9.343	22.617
Acetic(b_1)	-2.915	4.754
H2s(b_2)	3.571	1.341
Lactic(b_3)	20.139	9.454

$$\mathrm{SSE} = \sum (y_i) - \hat{y}_i)^2 = 3736.907$$

$$s = \sqrt{\frac{\mathrm{SSE}}{n-2}} = 11.55253$$

$$s_{x_1} = 11.6795 \implies \mathrm{SE}_x = 11.6795/\sqrt{8} = 4.129327$$

$$\mathrm{SE}_{b_1} = \frac{s}{s_x\sqrt{n-1}} = \frac{0.2015905}{4.129327(7)} \approx 0.006974173$$

Test for *Acetic* (x_1):

1. Test $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 \neq 0$

2. Level $\alpha = 0.05$

3. Test statistic:
$$t = \frac{b_1}{SE_{b_1}} = \frac{-2.915}{4.754} \approx -0.6132$$

4. Critical Value at $\alpha/2 = 0.025$ and Degrees of Freedom, $df = n-4 = 26$ is

```
> qt(1-0.025, 26)
[1] 2.055529
```

$$t^* = 2.055529$$

5. We **fail to reject** H_0 since $|t| < t^*$.

(c) New model for predicting Taste based only on the explanatory variables H2S and Lactic:

```
> lmTaste2=lm(taste~h2s+lactic, data=cheese)
> lmTaste2$coefficients
(Intercept)         h2s      lactic
 -21.392926    3.193556   18.791786
```

The equation of the least-square regression line based on *H2S*(x_2) and *Lactic*(x_3) is
$$\hat{y} = -21.392926 - 3.193556(x_2) + 18.791786(x_3)$$

$\square$

Project details

Introduction

According to the World Health Organization (WHO) stroke is the 2nd leading cause of death globally, responsible for approximately 11% of total deaths. This dataset is used to predict whether a patient is likely to get stroke based on the input parameters like gender, age, various diseases, and smoking status. We can use this dataset to state hypotheses of correlation between various explanatory variables (like, age, bmi, marital status, residence type, etc.) and the result that they have had a stroke.
The dataset used here can be found on Kaggle, a public competetion forum where data scientists and novices collaborate to find answers in complex datasets (kaggle.com/fedesoriano 2021). This dataset contains 5110 data points with 12 variables:

1. **id:** unique identifier
2. **gender:** "Male", "Female" or "Other"
3. **age:** age of the patient
4. **hypertension:** 0 if the patient doesn't have hypertension, 1 if the patient has hypertension
5. **heart_disease:** 0 if the patient doesn't have any heart diseases, 1 if the patient has a heart disease
6. **ever_married:** "No" or "Yes"
7. **work_type:** "children", "Govt_jov", "Never_worked", "Private" or "Self-employed"
8. **Residence_type:** "Rural" or "Urban"
9. **avg_glucose_level:** average glucose level in blood
10. **bmi:** body mass index
11. **smoking_status:** "formerly smoked", "never smoked", "smokes" or "Unknown"*
12. **stroke:** 1 if the patient had a stroke or 0 if not

Methods

According to stroke.org, a non-profit organization dedicated to public awareness about, and prevention of common causes of stroke, smoking, lack of physical activity, diabetes and obesity are leading factors that could lead to stroke (stroke.org 2021). In particular we explore the following question

Is BMI correlated to any other "quantitative" value?

We start with analysing the "class" of each variable:

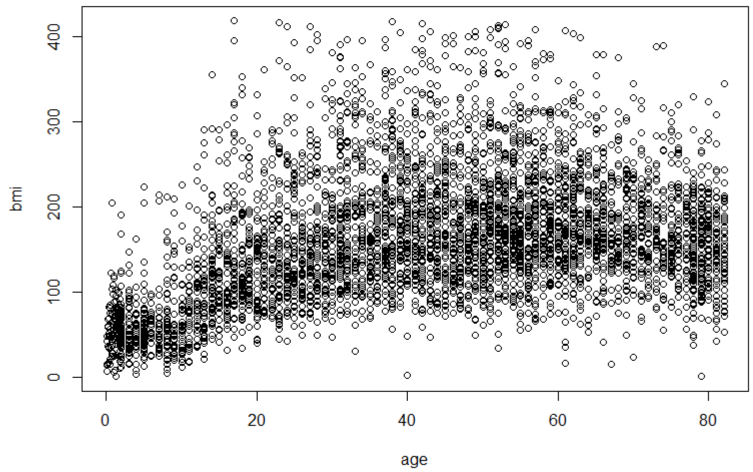
```
> sapply(stroke, class)
id                gender            age            hypertension
"integer"         "factor"          "numeric"      "integer"
heart_disease     ever_married      work_type
"integer"         "factor"          "factor"
Residence_type    avg_glucose_level bmi
"factor"          "numeric"         "factor"
smoking_status    stroke
"factor"          "integer"
```

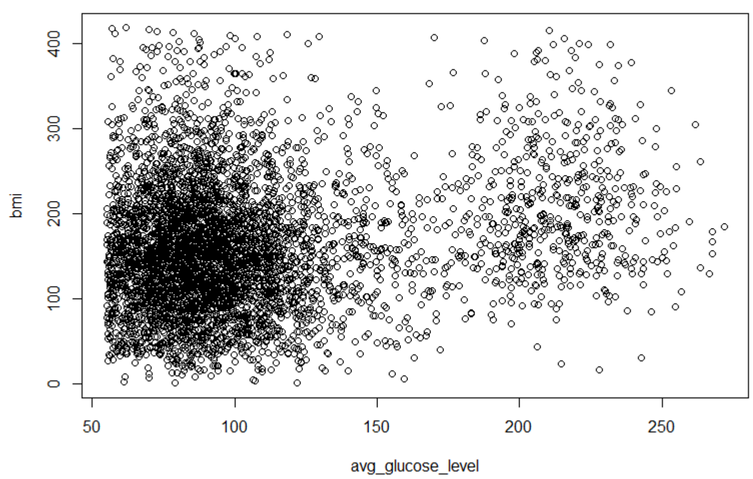
Since "bmi" is the quantity we're interested in, we notice that it is listed as "factor" and not "numeric". This is because there are values like "N/A" for some items. We filter those out:

```
> stroke2=stroke[stroke$bmi!='N/A',]
> stroke2=transform(stroke2, bmi=as.numeric(bmi))
> dim(stroke2)
[1] 4909   12
```

Now the interesting "numeric" variables are **Age** and **Average glucose level**

At this point we can plot "bmi" against each of these variables





And build a linear-regression model

```
> lmStroke=lm(bmi~age+avg_glucose_level, data=stroke2)
> lmStroke$coefficients
(Intercept)       age(x1)       avg_glucose_level (x2)
96.902336         1.080037      0.179388
```

Therefore the equation of the least-square regression line is

$$\hat{y} = 96.902336 + 1.080037(x_1) + 0.179388(x_2)$$

Variable	Estimate	SE
Intercept	96.90234	2.91577
Age	1.08004	0.04556
Avg glucose level	0.17939	0.02313

Next steps

1. Make hypotheses for "age" and "average glucose level"
2. Calculate test statistics for each of these hypotheses
3. Conclude whether the hypotheses can or cannot be rejected

Works Cited

kaggle.com/fedesoriano. “Stroke Prediction Dataset”. 2021. Web. 31/03/2021. <https://www.kaggle.com/fedesoriano/stroke-prediction-dataset?select=healthcare-dataset-stroke-data.csv>.

stroke.org. “Stroke Risk Factors You Can Control”. 2021. Web. 31/03/2021. <https://www.stroke.org/en/about-stroke/stroke-risk-factors/stroke-risk-factors-you-can-control-treat-and-improve>.