Rohit Wason
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Math 560 Homework (#1)

Problem 1. Canada has two official languages (French and English). The distribution of responses to the question: "What is your mother tongue?" is provided.

Solution. a) What probability should replace "?" in the distribution?

Let the probability that a Canadian speaks English be represented by $P(E) = 0.59$, French by $P(F) =?$, Asian/Pacific by $P(A) = 0.07$ and Other by $P(O) = 0.11$.

Since these are exhaustive (they complete the selection of languages spoken in Canada), $P(E) + P(F) + P(A) + P(O) = 1$. Therefore we have:

$$\begin{aligned} P(F) &= 1 - (P(E) + P(A) + P(O)) \\ &= 1 - (0.59 + 0.07 + 0.11) \\ &= 1 - 0.77 \\ &= \boxed{0.23} \end{aligned}$$

b) What is the probability that the mother tongue of a randomly selected Canadian is not English?

Let's call $P(E^{'})$ the probability that the mother tongue is **not** English. Again, since the given probabilities are exclusive and exhaustive, the possibilities that a randomly selected Canadian speaks a language other than English are same as the possibilities that they speak French **or** that they speak Asian/Pacific **or** they speak "Other". In other words:

$$\begin{aligned} P(E^{'}) &= P(F) + P(A) + P(O) \\ &= 0.23 + 0.07 + 0.11 \\ &= \boxed{0.41} \end{aligned}$$

□

Problem 2. Suppose that 45% of adults in a study eat enough vegetables, 40% eat enough fruit, and 25% do both.

Solution. a) What is the probability that a randomly selected adult from this study eats enough vegetables or eats enough fruit?

Let $P(V) = 0.45$ be the probability that a randomly selected adult eats enough vegetables; $P(F) = 0.40$ that they eat enough fruit; and $P(V \cap F) = 0.25$ that they do both.

Let $P(V \cup F)$ be the probability that a person eats enough vegetables **or** enough fruit. This probability would just be $P(V) + P(F)$ had these events been mutually exclusive. But since $P(V \cap F) \neq 0$, we need to discount $P(V \cap F)$ so we don't count it again (this is also called ***de Morgan's law***):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2.1}$$

$$\begin{aligned} \therefore P(V \cup F) &= P(V) + P(F) - P(V \cap F) \\ &= 0.45 + 0.40 - 0.25 \\ &= \boxed{0.60} \end{aligned}$$

b) If 3 adults are randomly selected from this study (independently of each other), what is the probability that at least one of them eats enough vegetables? □