

Math 560 Homework (#9, Inference of Two-Way Tables)

Problem 1. A table of two-variables is given:

Coffee	Male	Female	Total
Always	18	15	33
Sometimes	36	36	72
Never	36	9	45
Total	90	60	150

Solution. The expected cell counts are as follows:

Coffee	Male	Female
Always	$\frac{33 \times 90}{150} = 19.8$	$\frac{33 \times 60}{150} = 13.2$
Sometimes	$\frac{72 \times 90}{150} = 43.2$	$\frac{72 \times 60}{150} = 28.8$
Never	$\frac{45 \times 90}{150} = 27$	$\frac{45 \times 60}{150} = 18$

Hypothesis: H_0 : There is no association between the row & the column variables vs. H_a : There is an association between them.

The test statistic, χ^2

$$\begin{aligned} &= \sum \frac{(\text{observed count} - \text{expected count})^2}{\text{expected count}} \\ &= \frac{(18 - 19.8)^2}{19.8} + \frac{(15 - 13.2)^2}{13.2} + \frac{(36 - 43.2)^2}{43.2} + \frac{(36 - 28.8)^2}{28.8} + \frac{(36 - 27)^2}{27} + \frac{(9 - 18)^2}{18} \\ &= 0.1636364 + 0.2454545 + 1.2 + 1.8 + 3 + 4.5 \\ &= 10.90909 \end{aligned}$$

Looking up the χ^2 *distribution critical values* table, we see that the **Critical value** at $\alpha = 0.01$ and with **Degrees of freedom**, $df = (r - 1)(c - 1) = 2$ comes out to be, $\chi^{2*} = 4.605$.

Since $\chi^2 > \chi^{2*}$ we **reject** the hypothesis H_0 , that there is no association between the coffee consumption & gender of the students. \square