Rohit Wason
Spring 2021

Math 560 Homework (#1)

Problem 1. Canada has two official languages (French and English). The distribution of responses to the question: "What is your mother tongue?" is provided.

Solution. a) What probability should replace "?" in the distribution?

Let the probability that a Canadian speaks English be represented by $P(E) = 0.59$, French by $P(F) =?$, Asian/Pacific by $P(A) = 0.07$ and Other by $P(O) = 0.11$.

Since these are exhaustive (they complete the selection of languages spoken in Canada), $P(E) + P(F) + P(A) + P(O) = 1$. Therefore we have:

$$\begin{aligned} P(F) &= 1 - (P(E) + P(A) + P(O)) \\ &= 1 - (0.59 + 0.07 + 0.11) \\ &= 1 - 0.77 \\ &= \boxed{0.23} \end{aligned}$$

b) What is the probability that the mother tongue of a randomly selected Canadian is not English?

Let's call $P(E^{'})$ the probability that the mother tongue is **not** English. Again, since the given probabilities are exclusive and exhaustive, the possibilities that a randomly selected Canadian speaks a language other than English are same as the possibilities that they speak French **or** that they speak Asian/Pacific **or** they speak "Other". In other words:

$$\begin{aligned} P(E^{'}) &= P(F) + P(A) + P(O) \\ &= 0.23 + 0.07 + 0.11 \\ &= \boxed{0.41} \end{aligned}$$

□

Problem 2. Suppose that 45% of adults in a study eat enough vegetables, 40% eat enough fruit, and 25% do both.

Solution. a) What is the probability that a randomly selected adult from this study eats enough vegetables or eats enough fruit?

Let $P(V) = 0.45$ be the probability that a randomly selected adult eats enough vegetables; $P(F) = 0.40$ that they eat enough fruit; and $P(V \cap F) = 0.25$ that they do both.

Let $P(V \cup F)$ be the probability that a person eats enough vegetables **or** enough fruit. This probability would just be $P(V) + P(F)$ had these events been mutually exclusive. But since $P(V \cap F) \neq 0$, we need to discount $P(V \cap F)$ so we don't count it again (this is also called ***de Morgan's law***):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2.1}$$

$$\begin{aligned} \therefore P(V \cup F) &= P(V) + P(F) - P(V \cap F) \\ &= 0.45 + 0.40 - 0.25 \\ &= \boxed{0.60} \end{aligned}$$

b) If 3 adults are randomly selected from this study (independently of each other), what is the probability that at least one of them eats enough vegetables?

Starting in the opposite direction, we can see that the probability that a randomly selected person does **not** eat enough vegetables,

$$P(V') = 1 - P(V) = 0.55$$

Since the events of selecting the 3 persons are independent of each other (selecting one person doesn't affect which subsequent person gets picked), the probability that **none of the 3 persons eat enough vegetables** is given by

$$P(V')^3 = 0.55^3$$

Therefore, the required probability that **at lease one** of these persons eats enough vegetables is the compliment of what we found above:

$$= (1 - 0.55^3) = 0.8336$$

$\square$