Rohit Wason
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Math 560 Homework (#7, Inference of Proportions)

Problem 1. Given:
Sample ratio for the treatment group, $\hat{p_1} = \frac{82}{200745} \approx 0.0004085$
Sample ratio for the control group, $\hat{p_2} = \frac{162}{201229} \approx 0.0008050$
Pooled ratio, $\hat{p} = \frac{162+82}{200745+201229} \approx 0.0006070$

Solution. Hypothesis: $H_0 : p_1 = p_2$ vs. $H_a : p_1 \neq p_2$
Test statistic,

$$\begin{aligned} z &= \frac{\hat{p_1} - \hat{p_2}}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \\ &= \frac{0.0004085 - 0.0008050}{\sqrt{0.0006070(1-0.0006070)(\frac{1}{200745} + \frac{1}{201229})}} \\ &\approx -5.10328 \end{aligned}$$

The correspnding P-Value for the two-sided test $= 2 \times pnorm(-5.10328) \approx 0.0000003$, which is considerably smaller than $\alpha = 0.5$ so we **reject the null-hypothesis**. □

Problem 2. A red six-sided die is rolled 100 times and a six comes up 15 times. A blue six-sided die is rolled 500 times and a six comes up 100 times. Let p_1 be the probability that the red die comes up as a six, and let p_2 be the probability that the blue die comes up as a six. Compute a two-sided 95% confidence interval for $p_1 - p_2$ based on the Normal approximation.

Solution. Given:
$\hat{p_1} = \frac{15}{100} = 0.15$
$\hat{p_2} = \frac{100}{500} = 0.20$
$D = \hat{p_1} - \hat{p_2} = -0.05$

Standard error for D,

$$\begin{aligned}\sigma_{\hat{p_1}-\hat{p_2}} &= \sqrt{\frac{\hat{p_1}(1-\hat{p_1})}{n_1}+\frac{\hat{p_2}(1-\hat{p_2})}{n_2}}\\ &= \sqrt{\frac{0.15(1-0.15)}{100}+\frac{0.20(1-0.20)}{500}}\\ &= 0.03993745\end{aligned}$$

We know that for the two-sided 95% confidence interval, $z^* = 1.959964$
Therefore the margin of error

$$\begin{aligned}&= z^* \times \sigma_{\hat{p_1}-\hat{p_2}}\\ &= 0.07827596\end{aligned}$$

So a 95% confidence interval for $p_1 - p_2$ is $(-0.12827596, 0.02827596)$ □

Problem 3.

Gender	Juvenile	Adult	Total
Female	198	302	500
Male	696	1304	2000
Total	894	1606	2500

Solution. Joint/Marginal Probability Distribution:

Gender	Juvenile	Adult	Total
Female	$198/2500 = 0.0792$	$302/2500 = 0.1208$	0.2
Male	$696/2500 = 0.2784$	$1304/2500 = 0.5216$	0.8
Total	0.3576	0.6424	1

(a) Observed

$P(\text{Female}AND\text{Juvenile}) = 0.07152$
$P(\text{Female}AND\text{Juvenile}) = 0.07152$

(b) We find $\chi^2 = \sum \frac{(\text{observed}-\text{value}-\text{expected}-\text{value})^2}{\text{expected}-\text{value}}$ □