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Math 560 Homework (#5, Binomial)

Problem 1.

Solution. □

Problem 2. Population data on StatVillage (a hypothetical 128-block village in Canada) is given in the tab-delimited data file `StatVillage.txt`. The variables are listed in the first line of the data file, and information about the variables included in the file is given in the file `codesForStatVillage.txt`. Use R or other computer software to answer the following questions:

```
population = read.table(file ="StatVillage.txt",
    header = TRUE)
```

Solution. (a) The variable labeled `TOTINCH` gives the total household income. Determine the proportion of households in this population with a total household income greater than $100,000$.

```
nrow(population[population$TOTINCH>100000,])/
    nrow(population)
```

$$p = \frac{57}{512} \approx 0.11$$

(b) If 100 households are selected at random with replacement from this population, what is the probability that <u>at least 10</u> of the households in the sample will have a total household income greater than $100,000$? Compute the exact answer, rounded to at least 4 decimal places.

X, the random variable denoting the number of households among $n = 100$, whose income is greater than $100,000$ follows $\approx B(n, p) = B(100, 0.11)$ from above. According to CLT, this also $\approx N(np, \sqrt{np(1-p)}) = N(11, \sqrt{11(0.89)}) = N(11, 3.129)$

The required probability, $P(X \geq 10)$ can be re-written as

$$P(\frac{X - 11}{3.129} \geq \frac{10 - 11}{3.129})$$

For $Z = \frac{X-11}{3.129}$, this is

$$= P(Z \geq \frac{-1}{3.129} = -0.32)$$
$$= 1 - P(Z < -0.32)$$
$$= 1 - 0.3745$$
$$= \boxed{0.6255}$$

$\square$