

1. Q — Find the general solution of the recurrence equation

$$g_{n+2} = 3g_{n+1} - 2g_n.$$

A — The recurrence can be written as a homogenous equation,
 $g_{n+2} - 3g_{n+1} + 2g_n = 0$. On summing:

$$\sum_{n=0}^{\infty} g_{n+2} - 3 \sum_{n=0}^{\infty} g_{n+1} + 2 \sum_{n=0}^{\infty} g_n = 0 \quad (1)$$

$$\begin{aligned} \sum_{n=0}^{\infty} g_{n+1} x^n &= g_1 + g_2 x + g_3 x^2 + g_4 x^3 + \dots \\ x \sum_{n=0}^{\infty} g_{n+1} x^n &= g_1 x + g_2 x^2 + g_3 x^3 + g_4 x^4 + \dots \\ &= f(x) - g_0 \\ \therefore \sum_{n=0}^{\infty} g_{n+1} x^n &= \frac{f(x) - g_0}{x} \end{aligned}$$

$$\boxed{r_n = \frac{g_0 - g_1}{2} (-1)^n + \frac{g_0 + g_1}{2}}$$

2. Q — Find the general solution of the recurrence equation

$$g_{n+2} = 3g_{n+1} - 2g_n.$$

A — The recurrence can be written as a homogenous equation,