1.6.1 - Structures

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1

Consider the structure constructed in Example 1.6.2. Find the value of each of the following:

- $0 + 0 = +(\text{Bottom}, \text{Bottom}) = \text{Oberon}$
- $0E0 = E(\text{Bottom}, \text{Bottom}) = \text{Puck}$
- $S0 \cdot SS0 = \cdot(S(\text{Bottom}), S(S(\text{Bottom}))) = \cdot(\text{Titania}, \text{Bottom}) = \text{Titania}$

Do you think $0 < 0$ in this structure?

$$0 < 0 =< (\text{Bottom}, \text{Bottom}) = \text{No}$$

3

Here is a language consisting of one constant symbol, one 3-ary function symbol, and one binary relation symbol: $\mathcal{L}$ is $\{\flat, \sharp, \natural\}$.

- Describe an $\mathcal{L}$-model that has as its universe $\mathbb{R}$, the set of real numbers.

$$\mathfrak{A} = (\mathbb{R}, \flat^{\mathfrak{A}}, \sharp^{\mathfrak{A}}, \natural^{\mathfrak{A}})$$

- Describe another $\mathcal{L}$-model that has a finite universe.

$$\mathfrak{A} = (\{1, 2, 3\}, \flat^{\mathfrak{A}}, \sharp^{\mathfrak{A}}, \natural^{\mathfrak{A}})$$

4

A short paragraph explaining the difference between a language and a structure for a language.

A language is a set of symbols, and a structure is an interpretation of those symbols. A structure is a set of objects, which have meaning. For example, the language $\mathcal{L} = \{0, +, \cdot, S\}$ has the structure $\mathfrak{A} = (\mathbb{N}, 0^{\mathfrak{A}}, +^{\mathfrak{A}}, \cdot^{\mathfrak{A}}, S^{\mathfrak{A}})$, where $0^{\mathfrak{A}}$ is the number zero, $+^{\mathfrak{A}}$ is the addition function, and $\cdot^{\mathfrak{A}}$ is the multiplication function and $S^{\mathfrak{A}}$ is the unary successor function. Without a structure, the symbols in a language have no meaning.

6

Let $\mathcal{L}$ be $\{0, f, g, R\}$, where 0 is a constant symbol, f is a unary function symbol, g is a binary function symbol, and R is a 3-ary relation symbol.

Using $\mathcal{C}$, the set of all $\mathcal{L}$-terms, we define a set

$$\mathcal{C}' = (\forall t \in \mathcal{C})(\neg t)$$

The desired $\mathcal{L}$-structure can be defined as:

$$\mathfrak{C} = \{\mathcal{C}', 0^{\mathfrak{C}}, f^{\mathfrak{C}}, g^{\mathfrak{C}}, R^{\mathfrak{C}}\}$$

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